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Strongly Correlated Flat-Band Systems, Renormalization Group Flows from Holography, Large-N Collective Fields And Holography, Permeable Conformal Walls And Holography, Ads/CFT Correspondence, Chaotic Fast Scrambling At Black Holes, Higgs Bosons In A Nonminimal Supersymmetric Model, Doer" Is "Merely A Fiction Added To The Deed -- The Deed Is Everything" (Deleuze: 1968b: 45), Time As An, "Indirect Representation," (C2 35-6), The Viewer Experiences The Movement Of Time Itself, Which Images, Scenes, Plots And Characters Presuppose Or Manifest In Order To Gain Any Sort Of Movement Whatsoever, And Other Editorials Of Similar Kind And Interest : Der Kalkül Des Natürlichen Schliessen —Natural Deduction (Gentzen) And Begriffsgeschichte —History Of Concepts (Gadamer), Grundwissenschaft Der Exakten Naturwissenschaften — Foundational Science Of The Exact Natural Sciences (Moritz Alfred Geiger, Description Of Mathematics) And Gegenstandphänomenologie—Object-Phenomenology and Phenomenology Of The Object (Moritz Alfred Geiger) Models

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for the very long time behavior. They argue that the lifetime of a de Sitter phase cannot exceed the Poincaré recurrence time. This is supported by recent results of Kachru, Kallosh, Linde and Trivedi. **Naureen Goheer et al JHEP07 (2003)056 doi:10.1088/1126-6708/2003/07/056: The trouble with de Sitter space STOP. Ben Freivogel1, 2, Veronika E. Hubeny2,3, Alexander Maloney1,4, Robert C. Myers5,6, Mukund Rangamani2,3 and Stephen Shenker1** study the AdS/CFT correspondence as a probe of inflation. They assume the existence of a string landscape containing at least one stable AdS vacuum and a (nearby) metastable de Sitter state. Standard arguments imply that the bulk physics in the vicinity of the AdS minimum is described by a boundary CFT. **Ben Freivogel et al** argue that large enough bubbles of the dS phase, including those able to inflate, are described by mixed states in the CFT. Inflating degrees of freedom are traced over and do not appear explicitly in the boundary description. They nevertheless leave a distinct imprint on the mixed state. Analytic continuation allows us, in principle, to recover a large amount of nonperturbatively defined information about the inflating regime. Work also shows that no scattering process can create an inflating region, even by quantum tunneling, since a pure state can never evolve into a mixed state under unitary evolution. **Ben Freivogel et al JHEP03 (2006)007 doi:10.1088/1126-6708/2006/03/007 Inflation in AdS/CFT STOP. A** recent proposal by Ryu and Takayanagi for a holographic interpretation of entanglement entropy in conformal field theories dual to supergravity on anti–de Sitter space is generalized to include entanglement entropy of black holes living on the boundary of anti–de Sitter space. The generalized proposal is verified in boundary dimensions d=2 and d=4 for both the uv-divergent and uv-finite terms. In dimension d=4 an expansion of entanglement entropy in terms of size L of the subsystem outside the black hole is considered. A new term in the entropy of dual strongly coupled conformal-field theory, which universally grows as L2lnL and is proportional to the value of the obstruction tensor at the black hole horizon, is predicted. **Entanglement Entropy of Black Holes and Anti–de Sitter Space/Conformal-Field-Theory Correspondence Sergey N. Solodukhin Phys. Rev. Lett 97, 201601 – Published 14 November 2006 STOP. Jorge Noronha, Miklos Gyulassy, and Giorgio Torrieri** show that far zone Mach and diffusion wake "holograms" produced by supersonic strings in anti–de Sitter space/conformal field theory (AdS/CFT) correspondence do not lead to observable conical angular correlations in the strict Nc→∞ supergravity limit if Cooper-Frye hadronization is assumed. However, a special nonequilibrium "neck" zone near the jet is shown to produce an apparent sonic boom azimuthal angle distribution that is roughly independent of the heavy quark's velocity. results indicate that a measurement of the dependence of the away-side correlations on the velocity of associated identified heavy quark jets at the BNL Relativistic Heavy Ion Collider and CERN LHC will provide a direct test of the nonperturbative dynamics involved in the coupling between jets and the strongly coupled quark-gluon plasma implied by AdS/CFT correspondence **Di-Jet Conical Correlations Associated with Heavy Quark Jets in anti–de Sitter Space/Conformal Field Theory Correspondence Jorge Noronha, Miklos Gyulassy, and Giorgio Torrieri Phys. Rev. Lett. 102, 102301 – Published 13 March 2009 STOP.** Entanglement entropy for a spatial partition of a quantum system is studied in theories which admit a dual description in terms of the anti-de Sitter (AdS) gravity one dimension higher. A general proof of the holographic formula which relates the entropy to the area of a codimension 2 minimal hypersurface embedded in the bulk AdS space is given. The entanglement entropy is determined by a partition function which is defined as a path integral over Riemannian AdS geometries with non-trivial boundary conditions. The topology of the Riemannian spaces puts restrictions on the choice of the minimal hypersurface for a given boundary conditions. The entanglement entropy is also considered in Randall-Sundrum braneworld models where its asymptotic expansion is derived when the curvature radius of the brane is much larger than the AdS radius. Special attention is payed to the geometrical structure of anomalous terms in the entropy in four dimensions. Modification of the holographic formula by the higher curvature terms in the bulk is briefly discussed. **Dmitri V. Fursaev JHEP09 (2006)018 doi:10.1088/1126-6708/2006/09/018 Proof of the holographic formula for entanglement entropy STOP. Tatsuma Nishioka1, 2, Shinsei Ryu3 and Tadashi Takayanagi2** review recent progress on the holographic understanding of the entanglement entropy in the anti-de Sitter space/conformal field theory (AdS/CFT) correspondence. In general, the AdS/CFT relates physical observables in strongly coupled quantum many-body systems to certain classical quantities in gravity plus matter theories. In the case of our holographic entanglement entropy, its gravity dual turns out to be purely geometric, i.e. the area of minimal area surfaces in AdS spaces. One interesting application is to study various phase transitions by regarding the

entanglement entropy as order parameters. Holographic calculations nicely reproduce the confinement/deconfinement transition. Results can also be applied to understanding the microscopic origins of black hole entropy. **Tatsuma Nishioka et al 2009 J. Phys A: Math. Theor 42 504008 doi:10.1088/1751-8113/42/50/504008 Holographic entanglement entropy: an overview STOP. R.B. Mann** finds a class of black hole solutions to a (3+1) dimensional theory gravity coupled to abelian gauge fields with negative cosmological constant that has been proposed as the dual theory to a Lifshitz theory describing critical phenomena in (2+1) dimensions. These black holes are all asymptotic to a Lifshitz fixed point geometry and depend on a single parameter that determines both their area (and size) and their charge. Most of the solutions are obtained numerically, but an exact solution is also obtained for a particular value of this parameter. The thermodynamic behaviour of large black holes is almost the same regardless of genus, but differs considerably for small black holes. Screening behaviour is exhibited in the dual theory for any genus, but the critical length at which it sets in is genus-dependent for small black holes. **R.B. Mann JHEP06 (2009)075 doi:10.1088/1126-6708/2009/06/075 Lifshitz topological black holes STOP.** Substantial collective flow is observed in collisions between large nuclei at BNL RHIC (Relativistic Heavy Ion Collider) as evidenced by single-particle transverse momentum distributions and by azimuthal correlations among the produced particles. The data are well reproduced by perfect fluid dynamics. A calculation of the dimensionless ratio of shear viscosity η to entropy density s by Kovtun, Son, and Starinets within anti-de Sitter space/conformal field theory yields $\eta/s=\hbar/4\pi k_B$, which has been conjectured to be a lower bound for any physical system. Motivated by these results, **Laszlo P. Csernai, Joseph I. Kapusta, and Larry D. McLerran** show that the transition from hadrons to quarks and gluons has behavior similar to helium, nitrogen, and water at and near their phase transitions in the ratio η/s. They suggest that experimental measurements can pinpoint the location of this transition or rapid crossover in QCD. **Strongly Interacting Low-Viscosity Matter Created in Relativistic Nuclear Collisions Laszlo P. Csernai, Joseph I. Kapusta, and Larry D. McLerran Phys. Rev. Lett. 97, 152303 – Published 12 October 2006 STOP.** Fermi liquid theory explains the thermodynamic and transport properties of most metals. The so-called non-Fermi liquids deviate from these expectations and include exotic systems such as the strange metal phase of cuprate superconductors and heavy fermion materials near a quantum phase transition. **Thomas Faulkner1, Nabil Iqbal2, Hong Liu2,*, John McGreevy2, David Vegh3** use the anti–de-Sitter/conformal field theory correspondence to identify a class of non-Fermi liquids; their low-energy behavior is found to be governed by a nontrivial infrared fixed point, which exhibits nonanalytic scaling behavior only in the time direction. For some representatives of this class, the resistivity has linear temperature dependence, as is the case for strange metals. **Published Online August 5 2010 Science 27 August 2010: Vol. 329 no. 5995 pp. 1043-1047 DOI: 10.1126/science.1189134 Strange Metal Transport Realized by Gauge/Gravity Duality Thomas Faulkner1, Nabil Iqbal2, Hong Liu2,*, John McGreevy2, David Vegh3 STOP. Mitsutoshi Fujita, Kenji Fukushima, Tatsuhiro Misumi, and Masaki Murata** use the soft-wall anti–de Sitter/QCD model to investigate the finite-temperature effects on the spectral function in the vector channel. The dissociation of the vector meson tower onto the anti–de Sitter black hole leads to the in-medium mass shift and the width broadening in a way similar to the lattice QCD results for the heavy quarkonium at finite temperature. We also show the momentum dependence of the spectral function and find it consistent with screening in a hot wind. **Finite-temperature spectral function of the vector mesons in a holographic QCD model Mitsutoshi Fujita, Kenji Fukushima, Tatsuhiro Misumi, and Masaki Murata Phys. Rev. D 80, 035001 – Published 4 August 2009 STOP.** A model of 3+1 dimensional gravity with negative cosmological constant coupled to abelian gauge fields has been proposed as a gravity dual for Lifshitz like critical phenomena in 2+1 dimensions. The finite temperature behavior is described by black holes that are asymptotic to the Lifshitz fixed point geometry. There is a one-parameter family of charged black holes, where the magnitude of the charge is uniquely determined by the black hole area. These black holes are thermodynamically stable and become extremal in the limit of vanishing size. The theory also has a discrete spectrum of localized objects described by non-singular spacetime geometries. The finite temperature behavior of Wilson loops is reminiscent of strongly coupled gauge theories in 3+1 dimensions, including screening at large distances. **Ulf H. Danielsson and Lárus Thorlacius JHEP03 (2009)070 doi:10.1088/1126-6708/2009/03/070 Black holes in asymptotically Lifshitz spacetime STOP. Robert C. Myers and Matthias C. Wapler** study the charge transport properties of fields confined to a (2+1)-dimensional defect coupled to (3+1)-dimensional super-Yang-Mills at large-

Nc and strong coupling, using AdS/CFT techniques applied to linear response theory. The dual system is described by Nf probe D5- or D7-branes in the gravitational background of Nc black D3-branes. Surprisingly, the transport properties of both defect CFT's are essentially identical — even though the D7-brane construction breaks all supersymmetries. They find that the system possesses a conduction threshold given by the wave-number of the perturbation and that the charge transport arises from a quasiparticle spectrum which is consistent with an intuitive picture where the **defect acquires a finite** width. We also examine finite-λ modifications arising from higher derivative interactions in the probe brane action. **Robert C. Myers and Matthias C. Wapler JHEP12 (2008)115 doi:10.1088/1126-6708/2008/12/115 Transport properties of holographic defects STOP.** Many condensed matter experiments explore the finite temperature dynamics of systems near quantum critical points. Often, there are no well-defined quasiparticle excitations, and so quantum kinetic equations do not describe the transport properties completely. The theory shows that the transport coefficients are not proportional to a mean free scattering time (as is the case in the Boltzmann theory of quasiparticles), but are completely determined by the absolute temperature and by equilibrium thermodynamic observables. Recently, explicit solutions of this quantum critical dynamics have become possible via the anti-de Sitter/conformal field theory duality discovered in string theory. This shows that the quantum critical theory provides a holographic description of the quantum theory of black holes in a negatively curved anti-de Sitter space, and relates its transport coefficients to properties of the Hawking radiation from the black hole. **Subir Sachdev and Markus Müller** review how insights from this connection have led to new results for experimental systems: (i) the vicinity of the superfluid–insulator transition in the presence of an applied magnetic field, and its possible application to measurements of the Nernst effect in the cuprates, (ii) the magnetohydrodynamics of the plasma of Dirac electrons in graphene and the prediction of a hydrodynamic cyclotron resonance. **Subir Sachdev and Markus Müller 2009 J. Phys.: Condens. Matter 21 164216 doi:10.1088/0953-8984/21/16/164216 Quantum criticality and black holes STOP.** Following system is studied with sine qua non properties of asymptotic behaviour, Solutional behaviour .Renormalization Group Flows from Holography, Large-N Collective Fields And Holography, Strongly Correlated Flat-Band Systems, Permeable Conformal Walls And Holography, Ads/CFT Correspondence, Chaotic Fast Scrambling At Black Holes, Higgs Bosons In A Nonminimal Supersymmetric Model, Higgs Scalar-Pseudoscalar Mixing, Renormalization-Group-Improved Higgs Sector, Molecular Dynamics, Ads/CFT Correspondence And The Renormalization Group Flow And Molecular Dynamics-Based Triangulation Algorithm, Symmetry Breaking, Renormalization Group Flows From Holography, Renormalization Group Evolution Of Neutrino Parameters And Majorana Phases, Large-N Collective Fields And Holography, Matrix Theory And Heterotic Strings, and others.

Key words: Strongly Correlated Flat-Band Systems, Permeable Conformal Walls And Holography, Ads/CFT Correspondence, Chaotic Fast Scrambling At Black Holes, Higgs Bosons In A Nonminimal Supersymmetric Model, Higgs Scalar-Pseudoscalar Mixing, Renormalization-Group-Improved Higgs Sector, Molecular Dynamics, Ads/CFT Correspondence And The Renormalization Group Flow And Molecular Dynamics-Based Triangulation Algorithm, Symmetry Breaking

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The Basic Feelings That Makes The Pathology Possible Through Various Societal Accepted Avenues.

Donot See Think, Read And Write Again And Again: You Start Seeing Things Which Are Not There; There Might Be Necessity Of Two Or More Correction Terms.

Like Psychic DNA, The Collective Unconscious Contains "Inherited" Psychic Material That Links Us Not Only To Other Humans In The Present But Also To Our Ancestors From The Past. According To Jung's Theory, Though Each Of Us Appears To Function Independently, In Actuality We're All Tapped Into The Same Global Mind. Symbols Interact With And Condition Our Biohologram. The Collective Unconscious Is Essentially A Hologram. Symbols Arise From And Are Embedded In The Environment As Holographic Fields Of Energy. They Are Morphogenic Veils Of Primal Forces. If Your Brain Acts Like A Self-Contained Hologram, It Is Possible Your Consciousness Is Actually A Piece Of A Much Larger Hologram Of Overall Human Consciousness. Jung Noticed That Patterns Spontaneously Appear Over And Over Around The World. They Also Appear As Our Ancestral Memories Or Holographic Wisdom Field. In The Archetypal World, Everyone Is The Same, All Around The World... We Are All Gods, And Our Emotional Addictions To Pain And Suffering, Contempt, Insecurities, Doubt, Failure, Is Holographically-Recorded And Can Be Holographically Healed. All Archetypes Are A Form Of Human Expression That Is Both Holographic And Physical. Physical Formations Of Archetypal Sequences Cause Humans To Behave In Parallel Manners To Each Ancestor That Is Associated. Integration Is A Function Of Intentionality -- Conscious And Unconsciously Maintained, Or Incorporated. Integration Occurs Both Without Effort, As A Redesign Of The Central Processor Of Our Minds, And Voluntarily As A Deliberate Effort To Understand, Find Meaning, And As Rectification Of Our Behavior Towards Others And Towards Ourselves. Imagination Is Structured By The Archetypal Potentials Of The Unconscious. Archetypes Structure The Possibility To Generate

And Entertain Such Ideas. The Archetype Itself Cannot Be Known But Structures Everything We Come To Know. Their Totality Functions As A Psychic Organ. Universal Themes Appear In Distinct Cultural Garb. Over Millennia, All The Archetypes Have Emerged In Stories, Deities And Cultural Forms. One Of The Striking Points Of Religious Faith Is That They Aren't True. In Early History People Didn't Know The 'Real' Reasons Things Happened, So It's No Surprise That Their Explanations Were Wrong. But Then, Why Would We Want To Retrieve Such Superstitions? Kuhn Reminds Us That Even The Most Absurd Or Confused Explanation Of A Phenomenon Can Find Acceptance In The Absence Of A Competing Idea. Once Any Explanation Is Offered For A Phenomenon, It Has An Explanation From That Time On. Succeeding Speculations Might Be Able To Explain The Relevant Phenomena Better Than Its Predecessors. Deities, As Archetypal Role Models, Are The Opening Gambit In The Drama Of Understanding Human Subjectivity. Folk Tales Function The Same Way. Deities Or Archetypes May Have Evolved To Smooth Interpersonal Relations By Including An Understanding Of Human Types, Along With Rules For Helping The Different Types Relate With One Another. They Are Reflections Of Ancient Evolutionary Pressures, With A Dash Of Neuroanatomy. We Still Have Yjr Same Pressures Today So The Ancient Archetypes Still "Work", Regardless Of Objective Existence Of Gods And Goddesses. Burke (2003) Describes How The Underlying Pattern Or Strange Attraction Between Transference And Holography Extends To Other Processes Both Within And Outside The Field Of Psychology. Such Processes Include Projection, Projective Identification, Splitting, Memory, Biology, Creative Discovery, Theology, Synchronicity, Chaos, And Nonlocality. Identifying The Similar Patterns Of These Processes Demonstrates The Existence Of An Underlying Holographic Archetype In Which Essential Qualities Of The Whole Are Present In Each Of The Parts Of The Whole. The Autonomy Of The Overall Human Is Present In Each Conscious And Unconscious Component Part Of The Human Psyche. Cognition, Itself Is A Holographic Archetype. Many Essential Qualities Of The Whole Are Reflected Or Contained In Each Of The Parts That Make Up That Whole. It Is A Subtle Net Of Metaphor, Analogy And Simile. Holographic Archetypes Effectively Echo Their Resonant Patterns Through Literal And Symbolic Reflectaphors. The Passing Forms Of The Holographic Archetype Include The Hologram, Psychic Structure, Synchronicity, Wisdom Traditions, Memory, And The Process Of Scientific Discovery, Chaos In

Physics, Nonlocality And Virtuality In Physics. As Unconscious Structuring Principle, The Archetype Lies Beyond Normal Consciousness And Emerges Suddenly And Dynamically From The (Holographic) Psychoid Field, With A Powerful Emotional Charge That Invests It With Significance. Everything That Happens Is Conditioned By The Moment In Which It Happens. The Universal Field Imposes The Conditions. Matter Is Not Inert But Receptive To Holistic Patterning. If The Mythic World Taught Our Ancestors How To Manipulate The Empirical World, It Also Taught Them To Manipulate That Mythic Narrative Itself For Control Purposes. Socioeconomic Power Enforces Its Mythic Narrative. The Psychoid Field Imposes Holistic Function. Autonomous Inner Forces Arouse Compelling Opportunities To Enact Archetypal Behaviors. They Guide Our Perceptions And Behavior, Usually Without Our Awareness. Limbic Action Of Complexes Is A Big Part Of The Holistic Dynamics Of The Psyche. Dreams Report What Goes On Beneath The Veil Of Conscious Awareness. To The Consciousness Of The 'Thinker', Knowledge Is Thought. Period. Without Thought, The Consciousness Of The 'Thinker' Collapses Into Psychosis. With Archetypes Come The Potential For Wisdom, Relatedness, Sociality, Ambiguity, Paranoia, Projection, Identification, Denial, Inflation, Sub Personalities (Fragmentation), Defensiveness, Obsession, Hypnotic Dissociation, The Contagion Of Participation Mystique, Mythologizing, Complexes, Compensation, And Self-Delusion. Compensation May Calm Or Disturb Consciousness. There Is No Imperative For The Ego To Integrate These Alternative Perspectives, Private And Public Myths. The Unconscious Can Produce Deep Wisdom And Utter Nonsense. It Is Up To The Ego To Discriminate. The Value Of Myths Is Purely Heuristic, Not Pragmatic. Jung In The 21st Century: Synchronicity And Science By John Ryan Haule Mind Control Countermeasures

…….Yet This Is A Misleading Distinction That Signals Merely A Shift In Emphasis Rather Than Approach, Since Subjectification Is The Means Through Which Modern Power Operates In Foucault's Later Writings. In A Series Of Historical Studies, Foucault Analyzes The Formation Of The Modern Subject

From The Perspectives Of Psychiatry, Medicine, Criminology, And Sexuality, Whereby Limit-Experiences Are Transformed Into Objects Of Knowledge. His Works Are Strongly Influenced By An Anti-Enlightenment Tradition That Rejects The Equation Of Reason, Emancipation, And Progress. Foucault Argues That An Interface Between Modern Forms Of Power And Knowledge Served To Create New Forms Of Domination. With Thinkers Like Sade, Nietzsche, And Bataille, Foucault Valorizes Transgressive Forms Of Experience, Such As Madness, Violence, Or Sexuality That Break From The Prison Of Rationality. Where Modern Societies "Problematize" Forms Of Experience Such As Madness, Illness, And Sexuality, That Is, Turn Them Into Governmental Problems, Into Areas Of Life In Need Of Control And Regulation, Foucault In Turn Queries The Social Construction Of “Problems” By Uncovering Their Political Motivations And Effects And By Challenging Their Character As Natural, Necessary, Or Timeless. In What He Calls A "Diagnostic Critique" That Combines Philosophy And History (1989: 38-39, 73), Foucault Attempts To Clarify The Nature Of The Present Historical Era, To Underline Its Radical Difference From Preceding Eras, And To Show That Contemporary Forms Of Knowledge, Rationality, Social Institutions, And Subjectivity Are Contingent Socio-Historical Constructs Of Power And Domination, And Therefore Are Subject To Change And Modification. Foucault's Ultimate Task, Therefore, Is "To Produce A Shift In Thought So That Things Can Really Change" (Quoted In O'Farrell 1989: 39). The Goal Of Foucault's Historico-Philosophical Studies, As He Later Came To Define It, Is To Show How Different Domains Of Modern Knowledge And Practice Constrain Human Action And How They Can Be Transformed By Alternative Forms Of Knowledge And Practice In The Service Of Human Freedom. Foucault Is Concerned To Analyze Various Forms Of The "Limit Experience" Whereby Society Attempts To Define And Circumscribe The Boundaries Of Legitimate Thought And Action. The Political Vision Informing Foucault's Work Foresees Individuals Liberated From Coercive Social Norms, Transgressing All Limits To Experience, And Transvaluing Values, Going Beyond Good And Evil, To Promote Their Own Creative Lifestyles And Affirm Their Bodies And Pleasures, Endlessly Creating And Recreating Themselves. Foucault Denies There Can Be Any Basis For Objective Descriptive Statements Of Social Reality Or Universal Normative Statements That Are Not Socially Conditioned And Locally Bound. He Tries To Show That All Norms, Values, Beliefs, And Truth Claims Are Relative To The Discursive Framework

Within Which They Originate. Any Attempt To Write Or Speak About The Nature Of Things Is Made From Within A Rule-Governed Linguistic Framework, An "Episteme," That Predetermines What Kinds Of Statements Are True Or Meaningful. All Forms Of Consciousness, Therefore, Are Socio-Historically Determined And Relative To Specific Discursive Conditions. There Is No Absolute, Unconditioned, Transcendental Stance From Which To Grasp What Is Good, Right, Or True. Foucault Refuses To Specify What Is True Because There Are No Objective Grounds Of Knowledge; He Does Not State What Is Good Or Right Because He Believes There Is No Universal Standpoint From Which To Speak. Universal Statements Merely Disguise The Will To Power Of Specific Interests; All Knowledge Is Perspectival In Character. For Postmodern Theorists Like Foucault, The Appeal To Foundations Is Necessarily Metaphysical And Assumes The Fiction Of An Archimedean Point Outside Of Language And Social Conditioning. Habermas (1987) Rightly Finds Perplexing An Approach That Raises Truth Claims While Destroying A Basis For Belief In Truth, Which Takes Normative Positions While Suppressing The Values To Which They Are Committed. For Critique To Be Justified And Effective, It Should Preserve Standards By Which To Judge And Evaluate, But Foucault's Total Critique Turns Against Itself And Calls All Rational Standards Into Question. Richard Rorty, The Attack On Theory, And Renunciation Of Radical Politics

Deleuze's Analysis Begins By Coming To New Understandings Of The Concepts Of The Image And Movement. The Image, Above All, Is Not A Representation Of Something, That Is, A Linguistic Sign. This Definition Relies Upon The Age-Old Platonic Distinction Between Form And Matter, In Its Modern Saussurean Form Of Signifier-Signified. Rather, Deleuze Wants To Collapse These Two Orders Into One, And The Image Thus Becomes Expressive And Affective: Not An Image Of A Body, But The Body As Image (C1 58). This Collapse Comes About With Reference To Two Philosophers, Henri Bergson And Charles Sanders Pierce. Deleuze Dedicated A Book-Length Study To The Former Entitled Bergsonism (1968), And His Use Of His Notions Of Movement And Time In The Cinema Texts Is Already Presaged By This Text. Movement For Bergson, Deleuze Argues, Is Not Separable From The Object Which Moves: (I Am Not Movement: Italics Mine) They Are Literally The Same Thing. Thus, No Representative Relationship

Can Be Established Without Artificially Halting The Flow Of Movement And Thus Misconstruing The Frozen 'Element' As Self-Sufficient. (***See the befuddling similarity of holographic principle of Modern Quantum Mechanics and the Deleuzean idea; italics mine)*** There Is Only The Flow Of Movement Which Expresses Itself In Different Ways. Among Other Things, This Is One Of Deleuze's Critiques Of Phenomenology (C1 56, 60). Thus The Early Cinema Is Characterised For Deleuze By The Reign Of What He Calls The Sensory-Motor Schema. This Schema Is The Unity Of The Viewed And The Eye That Views In Dynamic Movement. This Model Of The Movement-Image Is Precisely The Nature Of Cinema, For Deleuze. It Does Not Falsify Movement By Extracting Segments And Stringing Them Together In A Representative Fashion, But Creates A Wide Range Of Expressive Images. It Is In Order To Come To Terms With The Varieties Of Movement-Images That Deleuze Turns To Pierce, Who Developed, "The Most Extraordinary Classifications Of Images And Signs . . ." (C2 30). ……. With Some Alterations, Pierce's Semiotic Classifications To Describe The Use Of Movement-Images In Cinema, And Their Centrality Before The Second World War The Movement From The First Text To Cinema 2: The Time-Image Has A Significance Closely Related To Kant's So-Called Copernican Revolution In Philosophy. Up Until Kant, Time Was Subject To The Events That Took Place Within It, Time Was A Time Of Seasons And Habitual Repetition (See (3) (C) Above); It Was Not Able To Be Considered On Its Own, But As A Measure Of Movement (C2 34-5; KCP Iv.). One Element Of Kant's Achievement For Deleuze, As We Have Seen, Is His Reversal Of The Time-Movement Relationship: He Establishes Time Itself As An Element To Which Movement Must Be Subordinated, A Pure Time. In The Cinema, Deleuze Argues, A Similar Reversal Takes Place. The Historico-Cultural Reason Behind This Reversal Is The Event Of World War Two Itself. With The Great Truths Of Western Culture Put So Deeply In Question By The Before Unimaginable Methods Employed And Their Forthcoming Results, The Sensory-Motor Apparatus Of The Movement-Image Are Made To Tremble Before The Unbearable, The Too-Much Of Life's Possibilities, The Potential Of The Present (C2 35). No Longer Could The Dogmatic Truths That Had Guided Society, And Cinema To An Extent, Allow The Apparently 'Natural' Movement From One Thing To The Next In An Habitual Fashion: 'Natural' Links Precisely Lost Their Efficacy. And With The Use Of Unnatural Or False Links, Which Do Not Follow The Sequence Or Narrative

Affect Of The Movement-Image, Time Itself, The Time-Image, Is Manifested In Cinema (Deleuze Considers Orson Welles To Be The First Auteur To Make Use Of The Time-Image (C2 137)). Rather Than Finding Time As An, "Indirect Representation," (C2 35-6), The Viewer Experiences The Movement Of Time Itself, Which Images, Scenes, Plots And Characters Presuppose Or Manifest In Order To Gain Any Sort Of Movement Whatsoever. Along With This 'External' Reason, There Is Also For Deleuze A Motivation Within Cinema Itself To Go From The Movement-Image To The Time-Image. The Movement Image Has The Tendency, Thanks To The Habitual Experience Of Movement As Normal And Centered, To Justify Itself In Relation To Truth: As Deleuze Argues With Regard To The Dogmatic Image Of Thought (See (3)(D) Above), There Is The Presupposition That Thought Naturally Moves Towards Truth. Of Course, Deleuze Suggests, Cinema, When Truly Creative, Never Relied Upon This Presupposition, And Yet, "The Movement-Image, In Its Very Essence, Is Answerable To The Effect Of Truth Which It Invokes While Movement Preserves Its Centres." (C2 142). In Questioning Its Own Presuppositions, Deleuze Argues, Cinema Moved Towards A New, Different, Way Of Understanding Movement Itself, As Subordinate To Time. (Wikipedia: Gilles Deleuze)

From His Early Writings On, Nietzsche, Like Kierkegaard, Rails Against A Life-Denying Rationalism And Idealist Philosophy Which Champions' Reason Over The Passions. Nietzsche Interprets The "Subject" As A Mere Construct, An Idealized Sublimation Of Bodily Drives, Experiences, And A Multiplicity Of Thoughts And Impulses. This "Little Changeling," On Nietzsche's View, This Subject, "Is Believed in More Firmly than Anything Else on Earth," But Is For Him A Simple Illusion Created Out Of Modern Desperation To Have A Well-Grounded Identity. Belief In The Subject Is Promoted By The Exigencies Of Grammar Which Utilize A Subject/Predicate Form, Giving Rise To The Fallacy That The "I" Is A Substance, Whereas It Is Really Only A Convention Of Grammar (Nietzsche 1968b: 37-38). For Nietzsche, "The Doer" Is "Merely A Fiction Added To The Deed -- The Deed Is Everything" (1968b: 45). "The Subject," He Concludes, Is Thus But A Shorthand Expression For A Multiplicity

Of Drives, Experiences, And Ideas. In The Spirit Of Enlightenment, Nietzsche Also Polemicizes Against Metaphysics, Arguing That It Illicitly Generalizes From Ideas In One Historical Epoch To The Entirety Of History. Against This Form Of Philosophical Universalism, Nietzsche Argues "There Are No Eternal Facts, Just As There Are No Absolute Truths. Consequently, What Is Needed From Now On Is Historical Philosophizing, And With It The Virtue Of Modesty" (Nietzsche 1986: 13). Castigating Traditional Philosophy And Values From A Critical Enlightenment Perspective, Nietzsche Anticipates Later Postmodern Critiques Of Metaphysics, Assailing The Concept Of Enduring Knowledge, The Notion Of A Transcendental World, And Presenting Metaphysical Thought As A Thoroughly Obsolete Mode Of Thinking. He Attributes The "Metaphysical Need," At The Heart Of Philosophies Such As Schopenhauer's, To Primitive Yearnings For Religious Consolation For The Sufferings Of Life And He Urges "Free Spirits" To Liberate Them And Pursue Thinking And Living Experimentally (1986: 8). Nietzsche's Attack On Foundationalism, Universalizing Thought, And Metaphysics Thus Undertakes A "Postmodern" Turn In Philosophy Through A Radical Deconstruction Of Modern Theory. But While Deconstructionist Philosophies Typically Terminate In The No, Merely Seeking To Unravel A Positive Modern Value System Into A Heap Of Disconnected Fragments, Nietzsche Starts And Finishes With A Big Yes, A Life-Affirming Value, Deconstructing Only To Reconstruct. Moving Far Away From Schopenhauerian Pessimism, Back Toward A Greek View Of Tragedy, Toward A Dionysian View Of Existence, Nietzsche Seeks "A Justification Of Life, Even At Its Most Terrible, Ambiguous, And Mendacious" (1968a: 521), A Justification Found In Art, Creativity, Independence, And The Emergence Of "Higher Types" Of Humanity. Yet Nietzsche's Perspectivism Denies The Possibility Of Affirming Any Absolute Or Universal Values: All Ideas, Values, Positions, And So On Are Posits Of Individual Constructs Of A Will To Power, Which Are To Be Judged According To The Extent To Which They Do Or Do Not Serve The Values Of Life, Creativity, And Strong Individuality. For Nietzsche There Are No Facts, Only Interpretations, And He Argues That All Interpretation Is Constituted By The Individual's Perspectives And Is Thus Inevitably Laden With Presuppositions, Biases, And Limitations. For Nietzsche, A Perspective Is Thus An Optic, A Way Of Seeing, And The More Perspectives One Has At One's Disposal, The More One Can See, And The Better One Can Understand And Grasp Specific Phenomena. To Avoid Limited And

Partial Vision One Should Learn "How To Employ A Variety Of Perspectives And Interpretations In The Service Of Knowledge" (Nietzsche 1968a: 119). THE POSTMODERN TURN IN PHILOSOPHY: THEORETICAL PROVOCATIONS AND NORMATIVE DEFICITS By Steven Best And Douglas Kelner Http://Www.Gseis.Ucla.Edu/Faculty/Kellner/Kellner.Html

And Just As During Much Of The Twentieth Century, Any Plea For Greater Social Justice Could Be Successfully Damned As Communist, Likewise Today, Any Strategy To Eradicate Suffering Is Likely To Be Condemned In Similar Reactionary Terms: Either Wire Head Hedonism Or Revamped Brave New World. This Response Is Not Just Facile And Simplistic. If It Gains Currency, The Result Is Morally Catastrophic. Of Course, The Abolitionist Issue Rarely Arises. Typically, Universal Bliss Is Still More-Or-Less Unthinkingly Dismissed As Technically Impossible. Insofar As The Prospect Is Even Contemplated - Grudgingly - It Is Usually Assumed That The New Regime Would Be Underwritten Day-By-Day With Drugs Or, More Crudely, Electrodes In The Pleasure-Centres. These Techniques Have Their Uses. Yet In The Medium-To-Long-Term, Stopgaps Won't Be Enough. All Use Of Psychoactive Drugs May Be Conceived As An Attempt To Correct Something Pathological With One's State Of Consciousness. There's Something Deeply Wrong With Our Brains. If What We Had Now Was OK, We Wouldn't Try To Change It. But It Isn't, So We Do. Mature Biological Psychiatry Will Recognise Inadequate Innate Bliss As A Pandemic Form Of Mental Ill-Health: Good For Selfish DNA In The Ancestral Environment Where The Adaptation Arose, But Bad For Its Throwaway Vehicles, Notably Us. The Whole Gamuts Of Behavioural Conditioning, Socio-Economic Reform, Talk-Therapies - And Even Euphoriant Superdrugs - Are Just Palliatives, Not Cures, For A Festering Global Illness. Its Existence Demands A Global Eradication Program, Not Idle Philosophical Manifestos And Scientific Belle's Letters. But One Does One's Best. The Ideological Obstacles To Genetically Pre-Programmed Mental Super-Health Are Actually More Daunting Than The Technical Challenges. To Be Cured, Hypo-Hedonia Must Be Recognised As A Primarily Genetic Deficiency-Disorder. Designer Mood-Brighteners And Anti-Anxiety Agents To Alleviate It Are Sometimes Branded "Lifestyle-Drugs"; But This Is To Trivialize A Serious Medical Condition Which Must Be Corrected At Source.

Happily, Our Hereditary Neuropsychiatric Disorder Is Likely To Become Extinct Within A Few Generations As The Reproductive Revolution Unfolds. Aversive Experience, And The Poisonous Metabolic Pathways That Mediate Its Textures, Will Become Physiologically Impossible Once The Genes Coding Its Neural Substrates Have Been Eliminated. We Won't Miss Its Corrupting Effect When It's Gone. In The Medium-Term, The Functional Equivalent Of Aversive Experience Can Help Animate Us Instead. Late In The Third Millennium And Beyond, Its Functional Successors May Be Expressed As Gradients Of Majestic Well-Being. On This Scenario, Our Descendants Will Enjoy A Civilisation Based On Information-Bearing Pleasure-Gradients: Whether Steep Or Shallow, We Simply Don't Know. Such A Global Species-Project Does Not Have The Desperate Moral Urgency Of Eliminating The Phenomenon Of Darwinian Pain - Both "Mental" And "Physical", Human And Non-Human Alike. Abolishing Raw Nastiness - Sometimes Vile Beyond Belief - Remains The Over-Riding Ethical Priority. One Doesn't Have To Be An Outright Negative Utilitarian To Acknowledge That Getting Rid Of Agony Takes Moral Precedence Over Maximizing Pleasure. But Both Genetic Fundamentalists And Gung-Ho Advocates Of Better Living Through Chemistry Today Agree On One Crucial Issue. There Is No Sense In Sustaining A Legacy Of Mood-Darkening Metabolic Pathways Out Of Superstitious Deference To Our Savage Past. When Bernard Marx Tells The Savage He Will Try To Secure Permission For Him And His Mother To Visit The Other Place, John Is Initially Pleased And Excited. Echoing Miranda In The Tempest, He Exclaims: "O Brave New World That Has Such People In It." Heavy Irony. Like Innocent Miranda, He Is Eager To Embrace A Way Of Life He Neither Knows Nor Understands. And Of Course He Comes Unstuck. Yet If We Swallow Such Fancy Literary Conceits, Then Ultimately The Joke Is On Us. It Is Only Funny In The Sense There Are "Jokes" About Auschwitz. For It Is Huxley Who Neither Knows Nor Understands The Glory Of What Lies Ahead. A Utopian Society In Which We Are Sublimely Happy Will Be Far Better Than We Can Presently Imagine, Not Worse. And It Is We, Trapped In The Emotional Squalor Of Late-Darwinian Antiquity, Which Neither Know Nor Understand The Lives Of The God-Like Super-Beings We Are Destined To Become. **A Defence Of Paradise-Engineering Brave New World By Aldous Huxley Brave New World (1932) Is One Of The Most Bewitching And Insidious Works Of Literature Ever Written**

Identity Is A Characteristic Of The Primitive Mentality And The Real Foundation Of Participation Mystique, Which Is Nothing But A Relic Of The Infantile Non-Differentiation Of Subject And Object, The Primordial Unconscious State. The Further We Go Back Into History, The More We See Personality Disappearing Beneath The Wrappings Of Collectivity. Any Individual's Existential Condition Is Just A Typical Variation On The Collective Themes Of The Eternal Human Condition. Our Particular Individuality Emerges From A Collective Mythic Existence. Individuals May Have An Unconscious Identity With Some Era, Person, Group, Or Object. Participation Mystique Means A Mystical Connection In Which One Cannot Clearly Distinguish Self From Other, Bound To It By A Direct Relationship Which Amounts To Partial Identity Or Transference. Unfortunately, That Is Tantamount To Giving Part Of Yourself Away. It Is The Mostly Unconscious Instinctive Human Tie To Symbolic Fantasy And Mythic Emanations Leading To Literalisms And Concretism, The Attempt To Turn The Imaginal Into Events. Mythological Motifs Project Themselves Into Situations And Objects, Including Other Persons. Everything Unconscious About Ourselves Is Projected. When The Boundary Between Consciousness And The Unconscious Is Blurred, Either Intentionally Or Unintentionally, The Ego Has A Hard Time Differentiating Between What Belongs To Itself And What Belongs To The Other. The Two Become Identified With Each Other. When A Projection Is Strongly Fixated, It May Be Next To Impossible To Separate One's Projected Material From The Person Who Is Carrying The Projection. Whether It Is A Person Or An Object, Suddenly, All Those Things Outside Become Carriers, Unconsciously, Of What Resides Within. Such Illusory States Left Unconscious Turn Toxic And Consuming. Denial Leaves Room For Serious Errors Of Fact. It Is A Warning Against Extreme Conscious Attitudes Which Inevitably Get Compensated By The Shadow. In The Process Jung Termed Enantiodromia, Projections Can Break Or Revert Into Their Opposites As Psyche Seeks To Equilibrate Or Rebalance Itself. Rightly Or Wrongly, Every Phenomenon Is Subject To Being Named, Labelled And Interpreted, Including The Sense Of Self As The Biological Basis Of Consciousness. In The Struggle To Narrate Its Experience, The Mind Tends To Label Phenomena And To Spontaneously Generate Ideation And Imaginally Fill In The Gaps In Awareness. Subcultures Are Dynamic Marketplaces For Knowledge

Construction, Authenticating And Authorizing Claims Within An Overall Belief Structure. Such Authority Structures May Be Benign, Toxic, Exploitive Or Malignant. It's Only A Small Step From Known Psychological Mechanisms That Generate Illusions To Perceiving An Entity 'Out There' With The Archetypal Characteristics Of A Personality - E.G. The Pater Noster Or Any 'Other', The Not-I, Who Interacts With Us In The Way We Interact Morally With Others. This Is Where Social Selection Really Favors Mirror Neurons That Can Produce Neurological Tricks We Interpret Subjectively, According To Our Beliefs. Social Order Is Organized Around Both Faulty And Plausible Knowledge Of Reality. Ritual Is A Creative Process Used To Construct Meaning Worlds. But They Don't Reliably Produce Intuitive Experience With Consensual Meaning. Most Often Such Experience And Interpretations Are Idiosyncratic, Cannot Be Validated, And Are Rooted Purely In Beliefs As Assumed Truths. Most Of This Retrieved Esoteric Knowledge Reflects Superstitious Ideas Arising Prior To The Seventeenth Century. Archaic Consciousness Of The Paleolithic, Mesolithic, And Neolithic Lies Buried Within Us, But To Try To Relive Them Today May Mean Being Less Than Wholly Present Now. Such Beliefs Effectively 'Teleport' One Back In Time Into Less Sophisticated Modes Of Ideation. In Archaic Identity There Is No Differentiation Between Object And Subject And No Distinction Between Lived Experience And What The Subject Believes He Or She Perceives About The World. Perceptual Competence Is Self-Assessed In Grandiose Terms Without Cognitive And Emotional Balance. Credibility Is Conferred By The Untutored Crowd Based On Otherwise Unworthy Persons, With Charisma And Emotional Appeal. Orthodoxy Of Interpretation Gives Rise To Organizations. The Recognition And Withdrawal Of Projections Usually Provokes A State Of Disenchantment Or, Conversely, Elation And Inflation Of The Ego. However, These Processes Can Also Open The Way To A Practice Of The Symbolic Life And Of Human Relations Without Too Much Alienation Or Mystification In The Therapeutic Process. This Is A ‘Knowing’ Of The Things Of Life And Their Inherent Mysteries Through The Experience Of The Mundane As Well As The Spiritual. It Is An Immersion In The Mysteries Of Nature And The Seeking Of Knowledge Through Mystical Participation. Jung In The 21st Century: Synchronicity And Science By John Ryan Haule Mind Control Countermeasures

"Mind (As Well As Metals And Elements) May Be Transmuted,
From State To State; Degree To Degree; Condition To Condition;
Pole To Pole; Vibration To Vibration. True Hermetic Transmutation
Is A Mental Art."--The Kybalion.
The Universe Is Mental"--The Kybalion.
Under, And Back Of, The Universe Of Time, Space And Change,
Is Ever To Be Found The Substantial Reality--The Fundamental
Truth."--The Kybalion.
"The Universe Is Mental--Held In The Mind Of THE
ALL."--The Kybalion
THE ALL Creates In Its Infinite Mind Countless Universes,
Which Exist For Aeons Of Time--And Yet, To THE ALL, The
Creation, Development, Decline And Death Of A Million Universes
Is As The Time Of The Twinkling Of An Eye."--The Kybalion.
"The Infinite Mind Of THE ALL Is The Womb Of Universes."--The
Kybalion.
"The Half-Wise, Recognizing The Comparative Unreality Of
The Universe, Imagine That They May Defy Its Laws--Such
Are Vain And Presumptuous Fools, And They Are Broken Against
The Rocks And Torn Asunder By The Elements By Reason Of
Their Folly. The Truly Wise, Knowing The Nature Of The Universe,

Use Law Against Laws; The Higher Against The Lower; And By
The Art Of Alchemy Transmute That Which Is Undesirable Into
That Which Is Worthy, And Thus Triumph. Mastery Consists Not
In Abnormal Dreams, Visions And Fantastic Imaginings Or
Living, But In Using The Higher Forces Against The Lower--Escaping
The Pains Of The Lower Planes By Vibrating On The Higher.
Transmutation, Not Presumptuous Denial, Is The Weapon Of The
Master."--The Kybalion.
(Notice The Insidious And Incendiary Similarity With Huxley's Brave New World And Prakruti Kshobha Models Of Earlier Papers. This Is Also One Of The Reasons Why People Donot Tell What Brahman Is; In Fact It Is Brahman-Anti Brahman As Specified)
"While All Is In THE ALL, It Is Equally True That THE
ALL Is In ALL. To Him Who Truly Understands This
Truth Hath Come Great Knowledge."--The Kybalion.
(This Is Something Like Telling Individual General Ledger Is In Cosmic General Ledger, But Cosmic General Ledger Is In Itself Probably Speaks Of The Conservative Nature Of The Zero The Final State Of Consciousness Turiya)
"As Above, So Below; As Below, So Above."--The Kybalion.
As Stars Behave So Does The Star Stuff (Human Beings); Nature Is Fooling You; Everything Is An Act Of Simulation. Italics Mine
"Nothing Rests; Everything Moves; Everything
Vibrates."--The Kybalion.

(Verily There Cannot Be Any Other Statement That Makes String Theorists Happy!)
"Everything Is Dual; Everything Has Poles; Everything
Has Its Pair Of Opposites; Like And Unlike Are The Same;
Opposites Are Identical In Nature, But Different In Degree;
Extremes Meet; All Truths Are But Half-Truths; All Paradoxes
May Be Reconciled."--The Kybalion.
(Like Anti Samuelsson Models PROPOSED Each Proposition Can Be Explained With Its Advantages And Disadvantages BY THESIS AND ANTI THESIS. This Is Just Mockery Of Models That Are Built And The Thesis And Antithesis Propounded)
"Everything Flows Out And In; Everything Has Its Tides;
All Things Rise And Fall; The Pendulum-Swing Manifests
In Everything; The Measure Of The Swing To The Right,
Is The Measure Of The Swing To The Left; Rhythm
Compensates"--The Kybalion.
(Somewhere Else It Is Mentioned That AL+GD For A Probationer CEO; High Passions Make Results Of A Company Or For That Matter Country Gratification And Deprivation Itself Which Is Never Conservative)
"Every Cause Has Its Effect; Every Effect Has Its Cause;
Everything Happens According To Law; Chance Is But A
Name For Law Not Recognized; There Are Many Planes Of
Causation, But Nothing Escapes The Law."--The Kybalion

.(Note That There Are Many Laws Of Causation At Different Planes. Probably James Grier Miller Is One Who Spoke Of A System To Be Recognised At Three Levels, Obne At Its Own, One Supra And Another Infra; I Am Writing This Out Of Memory, I Am Not Sure Whether It Is James Grier Miller Who Suggested Cross Level Hypothesis)
"Gender Is In Everything; Everything Has Its Masculine
And Feminine Principles; Gender Manifests On All
Planes."--The Kybalion.
The Manifestation Of Mental Gender May Be Noticed All Around Us In Everyday Life. The Magnetic Persons Are Those Who Are Able To Use The Masculine Principle In The Way Of Impressing Their Ideas Upon Others. The Actor Who Makes People Weep Or Cry As He Wills, Is Employing This Principle. And So Is The Successful Orator, Statesman, Preacher, Writer Or Other People Who Are Before The Public Attention. The Peculiar Influence Exerted By Some People Over Others Is Due To The Manifestation Of Mental Gender, Along The Vibrational Lines Above Indicated. In This Principle Lies The Secret Of Personal Magnetism, Personal Influence, Fascination, Etc., As Well As The Phenomena Generally Grouped Under The Name Of Hypnotism.
(It All Depends On You What You Want To Be Angel Or A Demon; It Is The Same Energy That Creates And Destroys)
"The Possession Of Knowledge, Unless Accompanied By A
Manifestation And Expression In Action, Is Like The
Hoarding Of Precious Metals-A Vain And Foolish Thing.
Knowledge, Like Wealth, Is Intended For Use. The Law Of
Use Is Universal, And He Who Violates It Suffers By
Reason Of His Conflict With Natural Forces."--The Kybalion.

To Change Your Mood Or Mental State--Change Your
Vibration."--The Kybalion.
(Believe Me: One Can Create Prakruti Siddhi And Prakruti Kshobha, If One Understands People' Feelings; To Do It Positive Way Are To Be That Of An Angel, And The Negative Way That Of A Demon)
"To Destroy An Undesirable Rate Of Mental Vibration,
Put Into Operation The Principle Of Polarity And
Concentrate Upon The Opposite Pole To That Which
You Desire To Suppress. Kill Out The Undesirable By
Changing Its Polarity."--The Kybalion.
(There Cannot Be Better Way To Mitigate If Not Obliterate Totally Terrorism; World Is Held Ransom By Vested Interests; What Is Befuddling Is If Some Groups Can Do It And Do It To The Detriment Of Many, Why Not Government Do It ,Help Poor And Downtrodden By A Better Way And Given Protection To The Threatened?)
"Mind (As Well As Metals And Elements) May Be Transmuted
From State To State; Degree To Degree, Condition To
Condition; Pole To Pole; Vibration To Vibration."--The Kybalion.
"Rhythm May Be Neutralized By An Application Of The Art
Of Polarization."--The Kybalion.
(Dreams Can Be Manipulated By Conscious Actions And Stirring Subconscious; Mind Weaves A Beautiful Fabric Of Fantasy; It May Be A Theory-Remember The Snake Swallowing Its Own Tail Dream- Or Night Mare)
"Nothing Escapes The Principle Of Cause And Effect,

But There Are Many Planes Of Causation, And One May Use
The Laws Of The Higher To Overcome The Laws Of The
Lower."--The Kybalion.
(There Cannot Be More Damning Indictment Of The System Which Can Use The Higher Authority To Muzzle The Lower, However Institutionalised It Is)
The Wise Ones Serve On The Higher, But Rule On The Lower.
They Obey The Laws Coming From Above Them, But On Their
Own Plane, And Those Below Them They Rule And Give Orders.
And, Yet, In So Doing, They Form A Part Of The Principle,
Instead Of Opposing It. The Wise Man Falls In With The Law,
And By Understanding Its Movements He Operates It Instead
Of Being Its Blind Slave. Just As Does The Skilled Swimmer
Turn This Way And That Way, Going And Coming As He Will,
Instead Of Being As The Log Which Is Carried Here And
There--So Is The Wise Man As Compared To The Ordinary
Man--And Yet Both Swimmer And Log; Wise Man And Fool,
Are Subject To Law. He Who Understands This Is Well On
The Road To Mastery."--The Kybalion.
(Gargi Asks Yajnavalkya: What Is The Best Job In This World? Reply Is Searching For Christ Or Brahman. That Is True, Albeit Materialistically It Might Not Be Remitting With Fat Bank Balance)
"THE ALL Is MIND; The Universe Is Mental."--The Kybalion

The Kybalion, By Three Initiates, [1912], At Sacred-Texts.Com

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of the characterstics of systems which satisfy the condition of cosmological constant or any other for that matter or variable or axiomatic predication. . There are lots of zeroes corresponding and concomitant to the second law of black holes. Infact as many as that of extant and existential blackholes exist. At the outset, it is to be stated that there are hadrons in every system. Supersymmetry between forces and matter, with both open and closed strings; no tachyon; group symmetry is SO (32) and its axiomatic predications, predicational anteriorities, character constitution shall be extant and existential in very many systems, and the characterstics are taken in to consideration in the classification scheme. Many systems have such fundamental instabilities like that of quantum gravity and characterstics of those systems form the citadel and fulcrum, bulwark and manor, mainstay and reinforcement, alcazar and chateau theory has a fundamental instability on which the classification schémas are valid. There are lots of systems which follow the axioms of string theory. It is the characterstics of this system which are taken in to consideration in the classification scheme. "All institutionalization involves common moral as well as other values. Collectivity obligations are, therefore, an aspect of every institutionalized role. But in certain contexts of orientation-choice, these obligations may be latent ...." (Parsons, 1951, p. 99). There are various systems that have the same bastion, support system, stylobate and sentinel as that of the Deleuzean terms and predications and phenomenological methodologies systemized. Look at this beautiful passage whose stability analysis would be eye opener. To "deconstruct" is not the same as to destroy. Deconstruction attempts to undo logical contradictions, to overturn rigid conceptual oppositions while releasing new concepts and meanings that could not be included in the old system. At the heart of Western metaphysics, for example, Derrida finds the opposition between "speech" and "writing." This binary logic functions in an illicit way to establish speech as the means of giving "presence" to the world, while writing is deemed derivative and inferior In Derrida's sense of "grammatology," however, all production of meaning is writing and subject to the infinite play of signification. By taking away the transcendental signified and advancing the concept of "differance" (language organized around difference and deferred, or mediated, understandings), Derrida, like Nietzsche, wants to leave us without transcendental illusions, metaphysical unities, and foundations that constrain thought and creativity. **THE POSTMODERN TURN IN PHILOSOPHY: THEORETICAL PROVOCATIONS AND NORMATIVE DEFICITS by Steven Best and Douglas Kelner http://www.gseis.ucla.edu/faculty/kellner/kellner.html** Each and every system has electrons, neutrons and protons and for that matter quarks. There shall be strong nuclear force and weak nuclear force. There are many systems that satisfy the criterion specified by the equation, principle or statement in question. Characterstics of the investigating systems form the bastion for the classification scheme and doxa thereof. Systemic differentiation is conducted. Despite gravity being constant, there exists gravity between two objects, and this could be taken as a system. Depending upon some parametric representationalities, functionalities, advantageousness, appropriateness, benefit, facilitation, fittingness, helpfulness, instrumentality, merit, practicality, serviceability, suitability, and usefulness, utility, these systems could be classified in to various categories. In respect of an equation, there shall be many systems that satisfy the given equation. Equations themselves could be by the utilization of the model solved term by term as has been exemplified and illustrated many time in the previous papers. There is lot of systems that could be brought in to the orbit of and gamut of the theory in question which the investigatable systems satisfy the axiomatic predications and postulation alcovishness of the systems in question. Towards the end of classificational consummation, consolidation, corporation and concatenation we take the characterstics of the systems, the predicational interiorities, ontological consonance and primordial exactitude, accolytish representations, functional topology, apocryphal aneurism and atrophied asseveration, event at contracted points, and other parameters as the bastion and stylobate of the stratification purposes. Such totalistic entities would have easy paradigm of relational content, differentiated system of expressly oriented actions with primary focus and locus of homologues receptiveness and differentially instrumental activity, variable universalism and particularism, imperative compatibilities and structural variabilities, interactional dynamical orientation, institutionalization and internalisation of pattern variables common attitudinal orientation of constituionalisation of internalized dispositions, and a qualitative gradient of structural differentiation and ascribed particularistic solidarity abstraction or interactional dynamics, internal differentiation, structural morphology, formal characterization, concept formulation, phenomenological methodologies, constituent structure, transformational minimal condition, paradigmatic feasibilities, programmatic plausibilities, comparative variability, normative aspect of expectational prediction,

projection and prognostication as consideration of the investigatory systems. Any scale can be used that is convenient to the classification scheme. It is important to note that the scheme of classification and the stratification doxa must not be adversely antagonistic, inexorably irreconcilable, antithetically antipodal, diametrically opposed, repugnantly retrogressive, inimically inverse, violatively unsimilar, diversely dissimilar, antipathetically anti thetical, conflictingly combative, obstructively pugnacious, inimically obstructive, repellently restrictive, disputatiously gainsaying or conformingly pugnacious to the axiomatic predications and postulation alcovishness of the theory in question or the equation representative or constitutive thereof. Sole intention, main objective and primary aim is twofold. One is towards the end of circumvention of the extra equation and the concomitant and corresponding variable therein. Second is avoidance of clustered congest, swarmed huddle, mustered pack, sardine squash, and swamp throng in the scheme of classification. Only thing that is sought out is the consonance in the entire diaspora and body fabric of the systems under study. This statement is true and holds unmistakably true for all the papers and I sincerely entreat readers to remember the statement and read the paper against this background. When we write A+B we mean by that B is being added to A or vice versa. It is like adding milk to water and water to milk. There may or may not be a time gap. As said earlier there may be many systems that satisfy the conditions of the equations and those systems that are investigatory or investigatable are taken in to consideration based on their characterstics in the classification doxa. When there are more than two entities, we can take logarithm and find the value of that factor to be taken with anti log to obtain prediction and projected values of the model. In case of A-B, we are removing B from A and that means B is eating up A. These factors are taken in to consideration in the application of the model to equations. Log (ab) and Log (a+b) is well defined. Model stands out as universal testament and template for application to each sentence and equation what with the quantification process done and the correlations well defined. In the eventuality of non existence of any connection at some phase, the model would have the accentuation and attrition coefficients and detritions coefficients as zero rendering the equations of concatenation simpler. Projection formula which incorporates in its diaspora the initial values provide autheticative determination, unimpeachable validation, incontrovertible establishment , apodictic evidence, reliable roll out, unfailing cinquecento quattrocento trecento, incontrovertible indication of the final finale, notwithstanding appellation, appellative, brand, cognomen, compellation, designation, flag, handle, identification, label, moniker, nomen, slot, style, surname, tab, tag, term, title designation, appellation, appellative, class, classification, denomination, description, epithet, of the classification scepter, scenario, scimitar, schottische. There is pure and impure consciousness in every one. Gratification producing and deprivation producing one's can be easily classified from individual general ledgers. Similar analogy holds for collective general ledger and cosmic general ledger or nature's general ledger. It is also to be noted that while dealing with equations towards the end of consummation of the measure, it is necessary that the two variables are to be classified in to three sections and each one would have the adventitious and decidedly stated relationship whereby the fundamental equations are drawn up and the analysis made. All the parametric representationalities, conditionalities, orientationalities remain unequivocal as stated in the variables stated in to consideration section and are different from module to module. In essence the paper is to be read as holistic one with the sole intention, primary objective and raison d' etre to build a TOE. Towards the end of circumvention of typing hundreds of superscripts and subscripts which would be a sardine squash and the concomitant operational difficulties, model is presented in piece meal of nine modules. Logarithms are to be taken in respect of those which incorporate more than one variable in bra-ket. Values of log (ab) and log (a+b) are readily available. Anti logarithm shall be taken at the end to predict, project, prognosticate the value of the variable. This is true for tensors, vectors and other variables too. Affirmational assertion, and explanation justification, statement of vindication, annotational commentary, and explicational glossary for each and every system changes and physical interpretation of results is one thing that is to be with earnest endeavour and feverish and febrile expediency. Editions never ever mean the same and identicality of parameters and this has been explicitly and unmistakably stated in the model apriori itself. Akin and analogous, cognate and concurrent, correspondent and congruent, comparable and complementary, synonymous and duple, tantamount and agnate, commensurate and correlative representation is only to highlight the importance and subterfuge the replication of work. It is my fervent solicitation to kindly bear with me for any lapses, notwithstanding the orchestrated efforts for a paper without any minor errors. Postulation predication, conclusive presumption, differential presuppositions, underscored decidedly axiomatic

statement of the statement, equation form the bye word or the watch word in the aggrandizement-amplification, caricature and crock, understatement- unembellishment, elocution-emphasis, enunciation-inflection, announcement-argument, articulation, assertion, asseveration, choice of words, commentary and communication, declaration and definition, delivery-diction, elucidation-emphasis, enunciation-execution, explanation, exposition-formulation, idiom, interpretation and intonation tone and tenor of stratification. Now, how do find the reaction of systems to these singularities. You do the same thing a boss does for you. "Problematize" the events and see how you behave. I will resort to "pressure tactics". "intimidation of deriding report", or "cut in the increment" to make you undergo trials, travails and tribulations. I am happy to see if you improve your work; but may or may not be sad if you succumb to it and hang yourself! We do the same thing with systems. systems show conducive response, felicitous reciprocation or behave erratically with inner roil, eponymous radicality without and with blitzy conviction say like a solipsist nature of bellicose and blustering particles, or for that matter coruscation, trepidiational motion in fluid flows, or seemingly perfidious incendiaries in gormandizing fellow elementary particles, abnormal ebullitions, surcharges calumniations and unwarranted(you think so but the system idoes not!) unrighteous fulminations. Amount of emotion or affect that is appropriate or expected in an given form of interaction. Particular individuals and diffuse obligations (see c and d) are associated with affectivity, whereas contacts with many individuals (universalistic) in a bureaucracy may be devoid of emotion and characterized by affective neutrality. Affective neutrality may refer to self discipline and the deferment of gratification (eg Weber's spirit of capitalism). In contrast, affectivity may be associated with expressing emotions. Adams and Sydie also refer to affective neutrality being associated with ego control (p. 15). (Parsons: Wikipedia). So the point that is made here is "like we problematize the "events" to understand the human behaviour we have to "problematize" the events of systems to understand their behaviour. This statement is made in connection to the fact that there shall be creation or destruction of particles or complete obliteration of the system (blackhole evaporation) or obfuscation of results. Some systems are like "inside traders" they will not put signature atoll! How do you find they did it! Anyway, there are possibilities of a CIA finding out as they recently did! So we can do the same thing witn systems to. This is accentuation, corroboration, fortification, .fomentatory note to explain the various coefficients we have used in the model as also the dissipations called door. In the bank example we have clarified that various systems are individually conservative, and their conservativeness extends holisticallytoo.that one law is universal does not mean there is complete adjudication of nonexistence of totality or global or holistic figure. Total always exists and "individual" systems always exists, if we donot bring Kant in to picture! For the time being let us not! Equations would become more ensorcelled and frenzied..... philosophy merges with ontology; ontology merges with univocity of being; analogy has always a theological vision; not a philosophical vision; one becomes adapted to the forms of god; self and world; the univocity of being does not mean that there is one and the same being; on the contrary, beings are multiple and different they are always produced by disjunctive synthesis; and they themselves are disintegrated and disjoint and divergent; membra disjuncta.the univocity of being signifies that that being is a voice that is said and it is said in one and the same "consciousness". In consciousness research, two rival sets of theories can be recognized: (A) Scientific material interpretations of consciousness are based on axioms that view consciousness in the context of highly advanced intentional processing of information in which subject-object relations evolve, and (B) humanistic interpretations of consciousness are based on axioms that view consciousness in the context of, say, "centered pulsations" that enable a conscious agent to act from his or her center of awareness. In this paper I will argue for the selection of axioms that favor humanistic interpretations of consciousness. The Pursuit of Autonomy Interdisciplinary Observations to Human Consciousness: Wautischer, Helmut (1993) the Pursuit of Autonomy. Interdisciplinary Observations to Human Consciousness: Social Neuroscience Bulletin, 6 (4). pp. 52-56.Everything about which consciousness is spoken about being is the same for everything for which it is said like gravity; it occurs therefore as an unique event for everything; for everything for which it happens; eventum tantum; it is the ultimate form for all of the forms; and all these forms are disjointed; it brings about resonance and ramification of its disjunction; the univocity of being merges with the positive use of the disjunctive synthesis, and this is the highest affirmation of its univocity like gravity; it is the eternal resurrection or a return itself, the affirmation of all chance in a single moment, the unique cast for all throws; a simple rejoinder for Einstein's god does not play dice; one being, one consciousness, for all forms and all times. a single instance for all that exists, a

single phantom for all the living; a single voice for very hum of voices; or a single silence for all the silences; a single vacuum for all the vacuums; consciousness should not be said without occurring; if consciousness is one unique event in which all the events communicate with each other; univocity refers both to what occurs to what it is said. This is attributable to all states of bodies and states of affairs and the expressible of every proposition. So univocity of consciousness means the identity of the noematic attribute and that which is expressed linguistically and sense fully. Univocity means that it does not allow consciousness to be subsisting in a quasi state and but expresses in all pervading reality; **CONSCIOUSNESS AND ITS UNIQUENESS June 12, 2012 at 8:40pm (See Deleuze Logic of sense, Wikipedia and Stanford encyclopedia for more details)** Atrocious contrivance and device, gratificational primogeniture, calamitous dodge and expedient, depraved, destructive, disastrous, execrable gambit and gimmick, great solace and succor, iniquitous, injurious, loathsome, low, maleficent, malevolent machination and maneuver, promethaleon of candor, frankness, honesty, honor, ingenuousness, innocence, openness, reality, sincerity, truthfulness, spiteful, stinking, ugly, unpleasant, unpropitious play and ploy, progenitor of jurisprudence and circumspection, racket and ruse, savvy, scam, stratagem, subterfuge, tactic and wile, forthrightness, honesty, truthfulness embodied and personified is the structural predisposition and dispensation of the complex. Nobody need have to take the Brahman-Anti Brahman agency to be a recalcitrant, repugnant, and refractory proposition. God gives and also takes away. That is the point made. And of course lesser gods, and God doth follow them and they follow God. That is why they are lesser gods. And wield the power on lesser mortals like us. When we mean gratification and deprivation we mean celestial Brahman Anti Brahman and Terrestial Brahman and Anti Brahman agencies producing such happiness or sadness by any means such as (blending- amalgamation, adulterant-adulteration, amalgam-amalgamation, blend- combination, composite- compound, debasement-denaturant, fusion-hybrid, intermixture-reduction, amalgam- mixture, composite- compound, fusion-mishmash, aggregate- alloy, amalgamation-blend, commixture-composite, composition-compost, conglomerate-fusion, goulash-medley, mishmash-stew, blend-coalition, commixture-compound, federation-heating, integration-junction, melting-merger, synthesis-unification, assortment- combination, adulteration-alloy, assimilation-association, batter-blend, brewed combine, composite compound, concoction confection, conglomeration cross of money, orientation(dis), penury creation, murder, mayhem plunder, pillage, apocalypse, Armageddon in various forms and permutation and combination of Manichaean artifice, agathokakological malevolence, agley chicane fourberie, fraud, furtiveness, gambit, awry and bad ,flagitious mechanizations, cacodemonic gambit, hanky-panky, intrigue, machination, deprecatory and diabolic underhandedness and wiles, energumenical skullduggery, goetic chicanery, lenocinant stratagem, malefic and maleficent strategy, malominous sell, sellout, sham subterfuge, peccable, humbug, imposture, misrepresentation peccant fraud, fraudulence, hoax, qued , scelerate pretense, ruse, sell, sellout, sham stratagems' of maladroit manouevures , sinistral ,tortious and venal sophistry, contingency enterprise, endangerment emprise, jeopardizing adventure, jejune jeremiad, avoidable inertia, latent passiveness: with each action accounted and showing balance in the nature's general ledger as also actionlessness:e). All these are recorded in Nature's general ledger. Neuron DNA encapsulates all the actions passions, actions, interactions, transactions or the lack of it at various levels of individual, collective and cosmic levels. In celestial Brahman Anti Brahman ledger there is cessation of dualities. One more point to be made is if 5=3+2, then we can say 5 gormandizes 3 or 5 gobbles up 2. Quintessentially such a statement arises out of the difference of LHS and RHS term wise. It must be remembered despite all theoretical abstractions and generalizations be it proton mass or Higgs Boson, or topology of a space, it is notwithstanding the fundamentality of nature is just a number. Just everything in Physics and mathematics is just a number. Let us assume I buy rice in Bangalore (District) worth 5k.g. I can always divide 5k.g.s of rice in categories of Bangalore (South), Bangalore (west), Bangalore (East) and Bangalore (South). On similar basis, there are billions of neutrons, protons and concomitant interactions which would satisfy the given numerical value. Example of gravity above makes things clear. Everything atleast is a function of time and someplace or some other coordinates, subordinates or superordinates. Mass of proton is 2.47 does not mean total mass of all protons is 2.47. It is this argument that is used in the classification scheme. By the very nature, self the unmonitoring but the witness consciousness agency identifies itself with something, be it a profession, an actor on the screen, when it does not have a identity. Such examples are legion, a Charlie Chaplin in "The Kid" or Raj kapoor in "Jagte Raho". These are people who just wanted to live and living without being put under surveillance every

moment is the greatest part of their life. This is "Absolute Subjectivity" of Shiva. Such a state is always craving for identification for in pure consciousness all identities are lost. It is this "identitylessness" or having very minimal identity is what pervades most in India and this is due to expansion of individual consciousness. When that happens, be it education, or wealth, the "identity' becomes so well entrenched, they become the exact opposite of "pure consciousness"," impure consciousness" (Shakti). This is a state of "Absolute or relativistic objectivity". There was this case of a lady who went to such an extent of planning, she was sure that whatever she does must work out. When it did not, she lapsed in to "identity crisis". Such thing as this might happen in well developed countries where the competition is extremal and to be achieved at any cost. Realisation that one is not the doer but the witness consciousness is just like watching a movie in which one has acted. It is in this sense "advaita", "non duality' seems to be of cardinal and preeminent importance for the analytical treat ment of "identity' and "identity lessness", both happen in space time. "Multiverses" in this sense is also like a different film of an actor released at different theaters and it is in this sense we state that that every object in space time is a multiverse. You must have totally immersed in doing a "string theory" problem or watching a "film", oblivious of surroundings. Essentially creation (space time) also is what is happening in mind and honed in to you billion times. Identification of anything with a designation or signification is one that leads to association within spacetime, which would only have pathological consequences and detrimental ramification as stated above or positive implications of "happiness" because of the identity itself. Search or 'show" has to go on! It gives a sense of consummation. a watching pot never boils; donot go to check nature; you are disturbing it; it will never show its true colours bhakti and Virakthi, gratification and deprivation, raga and dwesha is essential to live in space. It is the quantum interest thereof that produces energy to live .gratification and deprivation which form the bastion, pillar, post and stylobate and sentinel of all human actions depends on tams, rajas, and sattva of individual or insentient object. Verily tamas, rajas and satva are the characterstics of space itself, nay the behavioural pattern and attitudinal orientation. There are peaceful places like Vienna or Sringeri and there are violent places like others. Space is warped and woofed by the Akshara, the quantum information of primordial syllable Om. cosmic general ledger is the one vibration of which is brahman; quantum information possessions thereof form the sentinel of all transactions (Gargi to Yajnavalkya: my interpretation)adhocism, nonseqientiality of events, anti sequentiality of events, simultaneity of phenomenon, concurrent happenings , improbability of phenomenon happening itself are characterstics of brahman-anti brahman (Terrestial and celestial) are the cause of non predictability of happenings like in black hole, or in the areas of operation of brahman-anti brahman (Terrestial and celestial). brahman AntiBrahman actions, interactions, decisions, circumspection, adhocism, satisfies sensitive to initial conditions; topologically mixing; dense periodic orbits, the axiomatic predications and postulation alcovishness of chaos theory. Like gravity is a characteristic of space, so is tamas, rajas, sattva, with spaces knotted together by quantum information (for models see below) .accidents are preplanned actions of Terrestial and celestial Brahman AntiBrahman. All actions are perpetrated by them, be it train on fire or death. Belief systems play an important role in the determination of tamas, rajas, and satva state of human beings. There is neither cause nor effect. All actions are performed by Terrestial and celestial Brahman anti Brahman and it is off the record. psychic energy is negative equivalent to –mc^2, so that second law of thermodynamics is satisfied on a holistic basis vis-à-vis material energy ParaBrahaman is infinite individual, collective and cosmic consciousnesses are infinite (concomitant general ledgers) for they are in the evolution mode with respect to each other in that order. Later emanates from the former as transactions emanate from the conservative state of assets and liabilities. Is not zero true for infinite conservative systems inside space and time? zero is the only consummation, consolidation and conservation. if fifty out of seventy members come out of the room at whatever time you come out, talk about the grand limbo of oblivion and hibernation of lifeless world you are going to and if you are not a celebrity, then it is not coincidence, it is coordination. Nature is playing the same trick on you. That a generalised law is there and you can measure does not make things less good albeit recalcitrant, refractory and rogue particles occur time and again, by and large. Entering in to all living beings through his constant part on the new moon night, he is born therefrom next morning. Absolute ParaBrahaman (zero) projects himself again and again on the screen of individual consciousness which appears of depairing despondent world. Some where the balancing act is done on karma. cosmic general ledger reflects upon itself to keep all transactional ties conservative (Brihadarnkayaka Upanishad: 1.5 14) for models see next paper through its divine power (i

interpret man creates god; it is attribution and ascription of various Shaktis would the formation of god takes place like space and time) the self assumes (eb) the form of a deity man can contemplate and venerate, even though siva, the pure subject, can never in fact be an object of meditation (adhyeya). Until we realise our true identity with Sankara, he is worshipped and conceived to be a reality alien to ourselves. While we are in the realm of creation, he too is a creation or mode or appearing of the absolute, manifest to us in meditation, through his freedom as an eternal, omniscient being. There is no gulf between the created and the uncreated creator: nothing in reality, although an object of knowledge, ceases to be Siva: this is the reason why meditation [on this or that aspect] of reality bestows its fruit. The world of the senses and mind appears to the well awakened (suprabuddha) as a theophany, an eternal revealing of god in his creation. The doctrine of vibration declares that "there is no state in word, meaning or thought, either at the beginning, middle or end, which is not Siva." 59 to utter any word is, in reality, to intone a sacred formula.every act is a part of Siva's eternal cosmic liturgy, every movement of the body a ritual gesture (mudras), and every thought, god's thought. By what path are you not attainable? What words do not speak of you? in which meditation are you not an object of contemplation? What indeed are you not, o lord? 61 spandashakti, which accounts for the appearing of all things, is also the means by which deity in its many varied forms appears to man. Ksemaraja concludes: the ultimate object of worship of any theistic school differs not from the Spanda principle. The diversity of meditation is due solely to the absolute freedom of Spanda. 62 Sankara is not only the supreme object of devotion; as the static polarity of the absolute, he is the inner reality which holds together its Siva and Sakti outer manifestations. Phenomena are patterns of cognitions projected onto the surface of self-luminous siva-consciousness. There they become apparent, directly revealed to consciousness according to their manifest form. Siva is accordingly symbolized as the ground or surface of awareness, smooth and even like a screen (samabhittitalopama). 64 inscribed on this screen (kutfya) are the countless manifest forms which appear within it rendering it as diverse and beautiful as a fossil ammonite (falagrama). 65 Siva is the sacred ground upon which the cosmic mandalas are drawn, the absolute surface of inscription which bears the mark (cihna) of the universe. Abhinava writes: the variety of this world can only be manifest if the highest lord, who is essentially the pure light of consciousness, exists; just as a surface is necessary for a picture. if external objects were perceived in isolation then, because 'blue' and 'yellow', etc., are self-confined and the perceptions [we have of them] refer to their objects alone and so are insentient, mute and dumb in relation to one another . . . how would it be possible to be aware that an object is variegated? but just as depths and elevations can be represented by lines on a smooth wall, and we perceive [a female figure and think], similarly it is possible to be aware of differences in the variegated (contents of experience) only if all the diverse perceptions are connected together on the one wall of the universal light of consciousness. 66 Siva is the perfect artist who, without need of canvas or brush, paints the world pictures. the instant he imagines it, it appears spontaneously, perfect in every respect. The colours he uses are the varying shades and gradations of his own Spanda energy and the medium his own consciousness. Intractable disaffect, Mutinous obdurate, Rebellious recalcitrant, Seditious refractory, Insubordinate insurgent, antagonistically antipodean, discrepant incompatible, Contumacious coadjutant, Conflicting converse, Unyielding obdurate, the antagonistic and anachronistic dispositional ties of Anti Brahman are in fact the sine qua non of Brahman only (See Vishwaroopa of Mahavishnu, which contains the polarised positives on RHS and negatives on LHS). The universe is coloured with the dye of its own nature (svabhava) by the power of Siva's consciousness (a//). Rajanaka rama says: homage to him who paints the picture of the three worlds, thereby displaying in full evidence his amazing genius {pratibha)\ to Sambhu who is beautiful with the hundreds of appearances laid out by the brush of his own unique, subtle and pure energy. 68 analogously, at the microcosmic level, all the cognitions and emotions, etc., which make up the individual personality form the outward flow of essentially introverted consciousness. They are specific pulsations (vise$aspanda) or aspects of the universal pulsation (samanyaspanda) of pure t consciousness. at the lower level, within the domain of maya, they represent the play in the fettered soul of the three primary qualities (gunas) or feeling-tones' which permeate to varying degrees his daily experience. These are: 1) sattva — the quality of goodness and luminosity which accompanies blissful experience both aesthetic and spiritual. 2) Rajas — the passion or agitation which oscillates between the extremes of 'light' and 'darkness' and characterises inherently painful experiences. 3) Tamas — the torpor and delusion which accompany states of inertia and ignorance. 69 the liberated soul recognises that these three are the natural and uncreated powers of pure consciousness. for him they are

manifest respectively as: 1) Sankara 's power of knowledge (jnana) — the light of consciousness (prakasa); 2) the power of action (kriya) — the reflective awareness of consciousness (vimarsa)\ 3) the power of maya — which does not mean here the world of diversity, but the initial subtle distinction which appears between subject and object in pure consciousness. 70(doctrine of vibration: mark S.G. Dyczkowski) .to "deconstruct" is not the same as to destroy. Deconstruction attempts to undo logical contradictions, to overturn rigid conceptual oppositions while releasing new concepts and meanings that could not be included in the old system. at the heart of western metaphysics, for example, derrida finds the opposition between "speech" and "writing." this binary logic functions in an illicit way to establish speech as the means of giving "presence" to the world, while writing is deemed derivative and inferior in derrida's sense of "grammatology," however, all production of meaning is writing and subject to the infinite play of signification. By taking away the transcendental signified and advancing the concept of "differance" (language organized around difference and deferred, or mediated, understandings), derrida, like Nietzsche, wants to leave us without transcendental illusions, metaphysical unities, and foundations that constrain thought and creativity. The postmodern turn in philosophy: theoretical provocations and normative deficits by Steven best and Douglas Kelner http://www.gseis.ucla.edu/faculty/kellner/kellner.html. you cannot see beyond consciousness for that is the limit of your storage of information or the basis for understanding further information. Turiya is empty bucket; it is from which prajna the measure of arises; turiya measures the number of quantum information stores in other three states of waking, dreams, and dreamless deep sleep (Manduka Upanishad; my interpretation for models next paper).let there be path where angels tread: it is a force never born never dies the brahman an anti brahman verily there are parallel universes in (eb) in the ambit of space and time; the society of sinners and the sinned with the grammar of evil; parallel universes contain(e) multiverses in them; all physics, philosophy, break down(e) in these universes; astronomical gratification ,deprivation, glorification, mortification, projections, axes, dimensions, rotations, foldings, concurrences, motions, shocks, are the axiomatic predications of multiverses and parallel universes; death is when self (the witness consciousness; register of individual general ledger) leaves the body; ostensibly parallel seem to have (e) an objective; parallel universes donot (e) have objectives; their paradigm of relational content, differentiated system of expressly oriented actions with primary focus and locus of homologues receptiveness and differentially instrumental activity, variable universalism and particularism, imperative compatibilities and structural variabilities, interactional dynamical orientation, institutionalization and internalisation of pattern variables common attitudinal orientation of constituionalisation of internalized dispositions, and a qualitative gradient of structural differentiation and ascribed particularistic solidarity abstraction or interactional dynamics, internal differentiation, structural morphology, formal characterization, concept formulation, phenomenological methodologies, constituent structure, transformational minimal condition, paradigmatic feasibilities, programmatic plausibilities, comparative variability, normative aspect of expectational prediction, affection- appreciation, awareness- discernment, emotion- feeling, gut reaction, heart insight, intuition, judgment, keenness, perceptiveness, rationale, sensation, sense, sensitiveness, sensitivity, sentiment, susceptibility, taste, vibes, admiration, aesthetic sense, affection, appraisal, assessment, attraction, awareness, cognizance, commendation, comprehension, enjoyment, esteem, estimation, grasp, high regard, knowledge, liking, love, perceptional realization, recognitional regard, relish, respect, responsiveness, sensibility, sensitiveness, sensitivity, sympathy, understanding, valuation, acknowledgment, airing, baring, , confessional defenselessness, denudation-denunciation, disclosure, display, divulgence-exhibition, hazard, introduction, jeopardy, laying open, liability, manifestation, nakedness, openness, peril, presentation, publicity, revelation, risk, showing, susceptibility , susceptiveness,susceptivity, unfolding, unmasking, unveiling, vulnerability, vulnerableness, action, affection, appreciation and ardor, behavior, capacity, compassion, concern, cultivation and culture, delicacy, discernment and discrimination, emotion, empathy, faculty, fervor, fondness, heat, imagination, impression and intelligence, intensity and intuition, judgment, keenness- palpability, passion, pathos, pity, reaction, refinement, sensibility, sensitivity, sentiment, sentimentality , sharpness, spirit, sympathy, tangibility, taste, tenderness, understanding, warmth, fond memories, hearts and flowers, homesickness, longing, pining, reminiscence, ,schmaltz, sentimentality wistfulness, yearning satiation is only ostensible; their job is to get the job done; their business is to mind other's business ;all laws of physics and finance breakdown; it is antilaw,piscatorial Piratish and bubonic bucaneerishness; there is free transactions between in multiverses; when reflected on the screen of consciousness they appear as

extant existential universe. Transactions are astronomical and gratification deprivation, glorification and mortification incalculable; they have no barriers of region, religion, language which they create for others. theirs objectives are: corruption criminality, debasement debauchery, degradation evil immoral lewdness licentiousness, perversion sinfulness, vice viciousness, vileness wickedness, improbity corruption, atrocity decadence, degeneration degradation, depravity evil, immorality impurity, infamy iniquity, looseness lubricity, perversion profligacy, sinfulness turpitude, vice viciousness, vulgarity wickedness, disgrace bad reputation, abasement abuse, baseness black eye, blemish blur, brand comedown,contempt contumely, corruption culpability, debasement debasing, defamation degradation, derision disbarment, discredit disesteem, disfavor dishonor, disrepute disrespect, humbling humiliation ignominy ill repute, infamy ingloriousness, meanness obloquy, odium opprobrium, pollution ,put-down reproach, scandal scorn, slander slight, slur spot, stain stigma, taint tarnish, turpitude venality, enormity Horribleness , abomination atrociousness, atrocity crime and the positive aspectionalities thereof .let there not be any incursions where evil mercenaries in the garb of mendicants thrive with adversely antagonistic, inexorably irreconcilable, antithetically antipodal, diametrically opposed, repugnantly retrogressive, inimically inverse, violatively unsimilar, diversely dissimilar, antipathetically anti thetical, conflictingly combative, obstructively pugnacious, inimically obstructive, repellently restrictive, disputatious gainsaying or conformingly pugnacious Manichaean artifice, agathokakological malevolence, agley chicane fourberie, fraud, furtiveness, gambit, awry and bad ,flagitious mechanizations, cacodemonic gambit, hanky-panky, intrigue, machination, deprecatory and diabolic underhandedness and wiles, energumenical skullduggery, execrable and goetic chicanery, lenocinant stratagem, malefic and maleficent strategy, malominous sell, sellout, sham subterfuge, peccable, humbug, imposture, misrepresentation peccant fraud, fraudulence, hoax, qued , scelerate pretense, ruse, sell, sellout, sham stratagems' of maladroit manouevures , sinistral ,tortious and venal sophistry, contingency enterprise, endangerment emprise, jeopardizing adventure, jejune jeremiad, avoidable inertia, latent passiveness . statement for Mephistophelean mercenaries philosophy merges with ontology; ontology merges with univocity of being; analogy has always a theological vision; not a philosophical vision; one becomes adapted to the forms of god; self and world; the univocity of being does not mean that there is one and the same being; on the contrary, beings are multiple and different they are always produced by disjunctive synthesis; and they themselves are disintegrated and disjoint and divergent; membra disjuncta.the univocity of being signifies that that being is a voice that is said and it is said in one and the same "consciousness". everything about which consciousness is spoken about being is the same for everything for which it is said like gravity; it occurs therefore as an unique event for everything; for everything for which it happens; eventum tantum; it is the ultimate form for all of the forms; and all these forms are disjointed; it brings about resonance and ramification of its disjunction; the univocity of being merges with the positive use of the disjunctive synthesis, and this is the highest affirmation of its univocity like gravity; it is the eternal resurrection or a return itself, the affirmation of all chance in a single moment, the unique cast for all throws; a simple rejoinder for Einstein's god does not play dice; one being, one consciousness, for all forms and all times. a single instance for all that exists, a single phantom for all the living; a single voice for very hum of voices; or a single silence for all the silences; a single vacuum for all the vacuums; consciousness should not be said without occurring; if consciousness is one unique event in which all the events communicate with each other; univocity refers both to what occurs to what it is said. This is attributable to all states of bodies and states of affairs and the expressible of every proposition. So univocity of consciousness means the identity of the noematic attribute and that which is expressed linguistically and sense fully. Univocity means that it does not allow consciousness to be subsisting in a quasi state and but expresses in all pervading reality. Negative energy stabilising wormholes are considered candidates for darkmatter. An analysis of negative energy might be an useful ingredient of the thoughts of travel at speeds more than the velocity of light. Peirce explains that an "imperfect continuum" possesses "topical singularities, or places of lower dimensionality" which break continuity, while "the parts of a perfect continuum have the same dimensionality as the whole." (CP 4.642) Circular continua without singularities are time and space. (CP 6.210-212) By using math as his lens, Peirce envisioned singularities or lower dimensional holes in the imperfect cosmologic continuum, while maintaining that a probabilistic law of chance exists that entropy decreases **(Wikipedia).** On the side of philosophy**,** quite explicitly, Deleuze provides a critique of Husserl's "sleight-of-hand" change from a static categorical phenomenology to one supposedly more

dynamically constitutive of reality. This a clear cut **chagrined circumvention**, **contraventional curbing**, **defeated disgruntlement**, **dissatisfactional downer**, **dragged failure**, **fizzled foil**, **grievance hindrance**, **impedimental irritation**, letdown, nonfulfillment, nonsuccess, obstruction, old one-two, **resent mental setback**, unfulfillment of the earlier theories makes Sartre contend that human existence is a conundrum whereby each of us exists, for as long as we live, within an overall condition of nothingness (nothing-ness)—that ultimately allows for free consciousness. But simultaneously, within our being (in the physical world), we are constrained to make continuous, conscious choices. It is this dichotomy that causes anguish, because choice (subjectivity) represents a limit on freedom within an otherwise unbridled range of thoughts. Subsequently, humans seek to flee our anguish through action-oriented constructs such as escapes, visualizations, or visions (such as dreams) designed to lead us toward some meaningful end, such as necessity, destiny, determinism (God), etc. Thus, in living our lives, we often become unconscious actors—Bourgeois, Feminist, Worker, Party Member, Frenchman, Canadian or American—each doing as we must to fulfill our chosen characters' destinies. However, Sartre contends our conscious choices (leading to often unconscious actions) run counter to our intellectual freedom. Yet we are bound to the conditioned and physical world—in which some form of action is always required. This leads to failed dreams of completion, as Sartre described them, because inevitably we are unable to bridge the void between the purity and spontaneity of thought and all-too constraining action; between the being and the nothingness that inherently coincide in our self. Deleuze insists on the absolute non-resemblance between what conditions (metastable, pre-individual plane of singularities) and what is conditioned (actualized or the individuated 'thing'). Only then do "the conditions of the true genesis become apparent," Deleuze writes (Logic of Sense 105). Without addressing this issue, Deleuze's borrowings from Simondon, Lautman and structuralism remain untethered. Because of Bowden's decision to completely sidestep Deleuze's overt negotiations with phenomenology, his otherwise sophisticated commentary suggests a pedagogical narrowness. If, the omission of phenomenology strengthens Bowden's focus on the ontological priority of the event over substantial ontologies via the genealogy of figures and movements Deleuze outlines, it also incompletely articulates the problem of the sense-event as Deleuze formulates it. We have lost any idea of whom Deleuze takes to be his primary adversary, against whom he stakes out his position. This worry is most clearly manifested in Bowden's description of Deleuze's ontological prioritizing of events as offering a "transcendental ontology." For Bowden Deleuze's transcendental ontology defines events that "are ontologically prior to substances, 'all the way down'" (82). But Bowden provides a second definition: it is that the priority of events over worldly individuals means that the world of events "is not something external to the conditions of knowledge" (69). **SEAN BOWDEN the Priority of Events: Deleuze's Logic of Sense Sean Bowden, the Priority of Events: Deleuze's Logic of Sense, Edinburgh University Press, 2011, 296pp., $40.00 (hbk), ISBN 9780748643646. Reviewed by David Scott, Coppin State University Weltschmerz, agony, apprehension, blues, depression, dread, mid-life crisis, misgiving, nervousness, uneasiness, angst were expressed with such effulgent words…… aggrandizement-amplification, caricature and crock, understatement- unembellishment, elocution-emphasis, enunciation-inflection, announcement, argument, articulational assertion, asseverational choice of words, commentary and communication, declaration and definition, delivery, diction, elucidation, emphasis, enunciation-execution, explanation, exposition-formulation, idiom, interpretation and intonation about the human predicament is brought out in Being and Nothingness. Being and Nothingness offers a critique of Sigmund Freud's theory of the unconscious, based on the claim that consciousness is essentially self-conscious. Sartre also argues that Freud's theory of repression is internally flawed. According to Sartre, in his clinical work, Freud encountered patients who seemed to embody a particular kind of paradox—they appeared to both know and not know the same thing. In response, Freud postulated the existence of the unconscious, which contains the "truth" of the traumas underlying the patients' behavior. This "truth" is actively repressed, which is made evident by the patients' resistance to its revelation during analysis. Yet what does the resisting if the patients are unaware of what they are repressing? Sartre finds the answer in what Freud calls the "censor". "The only level on which we can locate the refusal of the subject," Sartre writes, "is that of the censor.** The Holographic Principle, as conceived in current physics, applies to fields, and perhaps, even to more elementary entities, called strings and branes. These more elementary entities remain hypothetical at the time of this writing. There are many layers between the level of fields and that of neurons. We

know that neurons are surrounded by a surface phospholipid membrane, which supports an electrochemical process that is fundamentally necessary to human experience. As the information supported by the membrane surface according to the Holographic Principle is proportional to its area, we should expect to find that the neural correlates of consciousness most clearly associated with those parts of the neuron that have the have a high proportion of neuronal surface area, and, in particular, receptive surface area. Dendrites account for an average of about 90% of the neuron is receptive surface area (Wong, 2002), and so would be expected to be the most important neural structures in information processes. We find this to be true **The Holographic Principle Theory of Mind MARK GERMINE Institute for Psycho science (Wikipedia).** Discordant-discrepant, dissonant-hard-line, implacable, incompatible, incongruous- inconsistent, inexorable-inflexible, inharmonious-intransigent, opposed, reluctant, unappeasable, uncompromising, unfriendly some of these expatiations and enumerations might sound, a more generalised interpretations is given by a Vedantin. Look at this arresting passage: The teacher continued to be silent. When addressed a second and third time he said: "I am teaching, but you do not follow. The Self is silence." The undetermined and unthinkable character of the Brahman is a consequence of the absolute's eternal and immutable nature. To concede the existence of a real universe is, from the Vedantin's point of view, to posit the existence of a reality apart from the Brahman. Nor can we simply identify a real universe with the absolute unless we are prepared to compromise its unchanging, absolute status. The criterion of authenticity is immutability. Reality never changes; only that which is less than real can appear to do so. Reality is constant in the midst of change. What this means essentially is that there is change although nothing changes. This impossible situation is reflected in the ultimate impossibility of change itself. That which does not exist prior to its changing and at the end, after it has changed, must be equally non-existent between these two moments. Although the world of change appears to be real, it cannot be so. Change, according to the Vedantin, presupposes a loss of identity. Reality cannot suffer transformation; if it were to do so, it would become something else and the real would be deprived of its reality. The immortal can never become mortal, nor can the mortal become immortal. The ultimate nature of anything cannot change. Change of any sort is merely apparent (vivarta); the world of change and becoming is a false super-imposition (adhyaropa, adhyasa) on the absolute. In cosmic terms, the mistake (bhranti) consists of the supposition that the real Brahman is the unreal universe and the unreal universe is the real Brahman. In microcosmic terms, it is the mistake of falsely conceiving the body, mind or even one's personality to be the Self. In the same way as the image of a snake is falsely superimposed on a rope, similarly the universe is falsely projected onto the real substratum, the Brahman. Ignorance is not merely a personal lack of knowledge, but a cosmic principle. As such it is called "Maya," the indefinable factor (anirvacaniya) that brings this mistake in identity about. The reality status of this cosmic illusion is also indefinable: on the one hand it is not Brahman, the sole reality; on the other hand it is not absolutely non-existent like a hare's horn or the son of a barren woman. Brahman is the source of world appearances only in the sense of being their unconditioned ground or essential nature. The universe is false not because it has no nature of its own but because it does have one. Just as the illusion of a snake disappears when one sees that it is nothing but a rope, similarly cancellation (badha) of the empirically real occurs when the absolute reality of the Brahman is realised. Thus, according to Vedanta, appearance implies the real, while the real need not imply appearance. To appear is essentially to appear in place of the real, but to be real is not necessarily to appear. All things -exist because the absolute exists. It is their Being. Thus the very existence of phenomena implies their non-existence as independent realities. When they are known to be as they are, in the fullest sense of their existence, their phenomenal nature disappears leaving the ground of Being naked and accessible. This approach was validated by a critique of experience. The Vedanta established that space, time and the other primary categories of our daily experience can have no absolute existence. It was therefore necessary to make a distinction between relative truths — that accepted by the precritical common man — and an absolute truth discovered at a higher level of consciousness. The Saiva absolutist 17 rejects any theory that maintains that the universe is less than real. From his point of view a doctrine of two truths, one absolute and the other relative, endangers the very foundation of monism. The Kashmiri Saiva approach is integral: everything is given a place in the economy of the whole. It is equally wrong to say that reality is either one or diverse. Those who do so fail to grasp the true nature of things which is neither as well as both. "We do not" says Abhinavagupta, "base our contention that [reality] is one because of the contradictions inherent in saying that it is dual. It is your approach {paksa) that accepts this [method] While, if

[duality and oneness] were in fact [to contradict each other], they would clearly be two [distinct realities]." The Vedantin, who maintains that non-duality is the true nature of the absolute by rejecting duality as only provisionally real, is ultimately landed in a dualism between the real and illusory by the foolishness of his own excessive sophistry (vacafadurvidya). Oneness is better understood as the coextensive unity (ekarasa) of both duality and unity. They are equally expressions of the absolute. Gopinath Kaviraj says: According to Sarikara, Brahman is truth and Maya is inexplicable (anirvacaniya). Hence the [Advaitin's] endeavour to demonstrate the superiority of Advaita philosophy is turned against his own system. It tarnishes the picture of its philosophical perfection and profundity. He cannot accept Maya to be a reality, therefore his non-dualism is exclusive. The whole system is based on renunciation and elimination and thus is not all-embracing.... By accepting Maya to be Brahman (brahmamayi), eternal (nitya) and real (satyarupa), Brahman and Maya [in the Tantra] become one and coextensive. **The Doctrine of Vibration: An Analysis of the Doctrines and Practices of Kashmir Shaivism Mark S. G. Dyczkowski. Concept of being from Shaivite point of view:** Solutional behaviour and stability analysis of such systems is of paramount importance, notwithstanding the assumptions made which are not farfetched, in that it throws light on the fact whether they could be candidates for yet unsolved problems, providing Authetícative determination, unimpeachable validation, incontrovertible establishment, apodictic evidence, reliable roll out, unfailing cinquecento **quattrocento trecento,** incontrovertible indication for the wide amplitudinal ramificatory usage of the thesis propounded. The world is now the other person's world, a foreign world that no longer comes from the self, but from the other. The other person is a "threat to the order and arrangement of your whole world...Your world is suddenly haunted by the Other's values, over which you have no control"(Satre: being and Nothingness) Sartre states that many relationships are created by people's attraction not to another person, but rather how that person makes them feel about themselves by how they look at them. This is a state of emotional alienation whereby a person avoids experiencing **their subjectivity by identifying themselves with "the look" of the other**. The consequence is **conflict**. In order to maintain the person's own being, the person must control the other, but must also control the **freedom of the other "as freedom**". These relationships are a profound manifestation of "bad faith" as the for-itself is replaced with the other's freedom. The purpose of either participant is not to exist, but to maintain the other participant's looking at them. This system is often mistakenly called "love", but it is, in fact, nothing more than emotional alienation and denial of freedom. *In essence we are always simulating and selling ourselves at the cost of our freedom. Both as a script writer and participant consciousness, human beings experienceâ complete subjective feelings and sometimes absolute subjective feelings, which if remains unchecked might lead to pathological ramifications*. (italics mine) Perspicuous forbearance, Sophisticated seasoning, Partcipational observation, Background actuality, Existential worldliness, Existential strife, Predicational anteriorities, Character consonance, Ontological consonance, Primordial exactitude, Phenomenological correlates, Accolytish representation, Atrophied asseveration. Anamensial alienisms, Anchorite aperitif, Arcadian Atticism all delineated in the conditionalities and functionalities and orinetatilities of the system which incorporate the rules and regulations, axiomatic predications and postulation alcovishness of the foregoing state form the bastion, pillar, post, stylobate and sentinel of the classification scheme and doxa. appropriate and diagnostic, differentiating-discriminating, distinctive-distinguishing, emblematic- especial, essentially exclusive, fixed, idiosyncratic-inborn, inbred, indicative and individual, individualistic-inherent, individualizing and ingrained, inherent, innate, local, marked, native, normal, original, particularly- peculiar, personal, private, proper, regular, representative-singular, special, specific, symbolic, symptomatic- unique these form the classification and stratification measure that is suitable notwithstanding the constancy in many a case, as in gravity, which depends on the mass and distance for two given objects**.** Like Kant and Bergson, Deleuze considers traditional notions of space and time as unifying forms imposed by the subject. He therefore concludes that pure difference is non-spatio-temporal; it is an idea, what Deleuze calls "the virtual". (The coinage refers to Proust's definition of what is constant in both the past and the present: "real without being actual, ideal without being abstract.") Proust, Le Temps Retrouvé, ch. III: see the fourth line from the bottom of this page, or, in English translation, the thirteenth paragraph here: "I began to discover the cause by comparing those varying happy impressions which had the common quality of being felt simultaneously at the actual moment and at a distance in time, because of which common quality the noise of the spoon upon the plate, the unevenness of the paving-stones, the taste of the madeleine, imposed the past upon the present and made me hesitate as to which time I was existing

in. Of a truth, the being within me which sensed this impression, sensed what it had in common in former days and now, sensed its extra-temporal character, a being which only appeared when through the medium of the identity of present and past, it found itself in the only setting in which it could exist and enjoy the essence of things, that is, outside Time. [...] **Nothing but a moment of the past Much more perhaps; something which being common to the past and the present, is more essential than both. [...] a marvellous expedient of nature had caused a sensation to flash to me—sound of a spoon and of a hammer, uneven paving-stones**—simultaneously in the past which permitted my imagination to grasp it and in the present in which the shock to my senses caused by the noise had effected a contact between the dreams of the imagination and that of which they are habitually deprived, namely, the idea of existence—and thanks to that stratagem had permitted that being within me to secure, to isolate and to render static for the duration of a lightning flash that which it can never wholly grasp, a fraction of Time in its pure essence. When, with such a shudder of happiness, I heard the sound common, at once, to the spoon touching the plate, to the hammer striking the wheel, to the unevenness of the paving-stones in the courtyard of the Guermantes' mansion and the Baptistry of St. Mark's, it was because that being within me can only be nourished on the essence of things and finds in them alone its subsistence and its delight. It languishes in the observation by the senses of the present sterilised by the intelligence awaiting a future constructed by the will out of fragments of the past and the present from which it removes still more reality, keeping that only which serves the narrow human aim of utilitarian purposes. But let a sound, a scent already heard and breathed in the past be heard and breathed anew, simultaneously in the present and in the past, real without being actual, ideal without being abstract, then instantly the permanent and characteristic essence hidden in things is freed and our true being which has for long seemed dead but was not so in other ways awakes and revives, thanks to this celestial nourishment"While Deleuze's virtual ideas superficially resemble Plato's forms and Kant's ideas of pure reason, they are not originals or models, nor do they transcend possible experience; instead they are the conditions of actual experience, the internal difference in itself. "The concept they [the conditions] form is identical to its object."[Desert Islands, p. 36.] A Deleuzean idea or concept of difference is therefore not a wraith-like abstraction of an experienced thing, it is a real system of differential relations that creates actual spaces, times, and sensations. [See "The Method of Dramatization" in Desert Islands, and "Actual and Virtual" in Dialogues. In fact it not as if the style and substance of science and philosophers have been entirely different. look at the conformality and consonance in the Gaussian Theory and Deleuze's statements on identity and the loss of it in the following passage…….That is, identity is a continuous process that finds its limit in exhaustion. As Deleuze will write, "…the logical relation of causality is inseparable from a physical process of signalling, without which it would not be translated into action. By 'signal' we mean a system with orders of disparate size, endowed with elements of dissymmetry; by 'sign' we mean what happens within such a system, what flashes across the intervals when a communication takes place between disparates. The sign is indeed an effect, but an effect with two aspects: in one of these it expresses, qua sign, the productive dissymmetry; in the other it tends to cancel it" (dr, 20). An actualized entity just is such a signal-sign system, remaining in perpetual communication with the virtual multiplicity it actualizes, constituting its own elements and identity. Such is Deleuze's account of what Badiou refers to as the operation of the "count-for-one"… an operation that is immanent to being itself and energetic in character. yet how are we to discern the local nature of ontological situations, the manner in which there is no global, overarching global situation in which all local situations are embedded as if parts of a whole? in part the above passage already responds to this question. if series must be brought into relation to communicate in order for causality to occur, and if the resonance of series is always a matter of chance, of a throw of the dice, like Lacan's objet a where no series enjoys primacy of model over copy, then all we have are local situations without a total situation. Unlike Heidegger's analysis of Dasein where Dasein is always being-in-the-world, Deleuze's simulacra or actualizations are always divergent, enjoying only local relations. Heidegger argues that in order to understand the being of Dasein we must understand the manner in which Dasein is a being-in-the-world. By contrast, Deleuze's concept of multiplicity provides us with the principle of a new structuralism, of a new local ontology, that allows us to understand the immanent organization of a multiplicity without referring it to an embedding global space. as DeLanda so beautifully puts it in relation to the mathematician gauss, "…when gauss began to tap into these differential resources, a curved two-dimension surface was studied using the old Cartesian

method: the surface was embedded in three-dimensional space complete with its own fixed set of axes; then, using those axes, coordinates would be assigned to every point of the surface; finally, the geometric links between points determining the form of the surface would be expressed as algebraic relations between the numbers. but gauss realized that the calculus, focusing as it does on infinitesimal points on the surface itself (that is, operating entirely with local information), allowed the study of the surface without any reference to a global embedding space. basically, gauss developed a method to implant the coordinate axes on the surface itself (that is, a method to implant the coordinate axes on the surface itself (that is, a method of 'coordinatizing' the surface) and, once points had been so translated into numbers, to use differential (not algebraic) equations to characterize their relations. as the mathematician and historian Morris Kline observes, by getting rid of the global embedding space and dealing with the surface through its own local properties, 'gauss advanced the totally new concept that a surface is a space in itself'" (intensive science and virtual philosophy, 11-12).according to the old Cartesian method, we can only outline the properties of a space by relativizing it to a global space in terms of which it is then mapped. by contrast, gauss is able to explore a space in terms of its intrinsic metric and organization as a local space, without referencing it to a whole of which it is conceptualized as a part. **The Image of Thought**

"Image of thought "permeates both popular and philosophical discourse. According to this image, thinking naturally gravitates towards truth. Thought is divided easily into categories of **truth and error**. The model for thought comes from the educational institution, in which a master sets a problem and the pupil produces a solution which is either true or false. This image of the subject supposes that there are different faculties, each of which ideally grasps the particular domain of reality to which it is most suited. In philosophy, this conception results in discourses predicated on the argument that "**Everybody knows..."** the truth of some basic idea. Descartes, for example, appeals to the idea that everyone can at least think and therefore exists. Deleuze points out that philosophy of this type attempts to eliminate all objective presuppositions while maintaining subjective ones. Deleuze maintains, with Artaud, that real thinking is one of the most difficult challenges there is. Thinking requires a confrontation with stupidity, the state of being formlessly human without engaging any real problems. One discovers that the real path to truth is through the production of sense: the creation of a texture for thought that relates it to its object. Sense is the membrane that relates thought to its other. Accordingly, learning is not the memorization of facts but the coordination of thought with a reality. "**As a result, 'learning' always takes place in and through the unconscious, thereby establishing the bond of a profound complicity between nature and mind".** Deleuze's alternate image of thought is based on difference, which creates a dynamism that traverses individual faculties and conceptions. This thought is fundamentally energetic and signifying: if it produces propositions, these are wholly secondary to its development. Philosophy merges with ontology; ontology merges with univocity of being; analogy has always a theological vision; not a philosophical vision; one becomes adapted to the forms of god; self and world; the univocity of being does not mean that there is one and the same being; on the contrary, beings are multiple and different they are always produced by disjunctive synthesis; and they themselves are disintegrated and disjoint and divergent; membra disjuncta.the univocity of being signifies that that being is a voice that is said and it is said in one and the same "consciousness". Everything about which consciousness is spoken about being is the same for everything for which it is said like gravity; it occurs therefore as an unique event for everything; for everything for which it happens; even tum tan tum;it is the ultimate form for all of the forms; and all these forms are disjointed; it brings about resonance and ramification of its disjunction; the vorticity of being merges with the positive use of the disjunctive synthesis, and this is the highest affirmation of its univocity like gravity; it is the eternal resurrection or a return itself, the affirmation of all chance in a single moment, the unique cast for all throws; a simple rejoin der for Einstein's god does not play dice; one being, one consciousness, for all forms and all times. a single instance for all that exists, a single phantom for all the living; a single voice for every hum of voices or a single silence for all the silences; a single vacuum for all the vacuousness consciousness should not be said without conjuring if consciousness is one unique event in which all the events communicate with each other; univocity refers both to what occurs to what it is said. The attributable to all states of bodies and states of affairs and the expressible of every proposition So univocity of consciousness means the identity of thé noematic attribute and that which is expressed linguistically and sense-fully. Univovity.We mean that it does not allow consciousness to be subsisting in aquasi state and but

expresses in all pervading reality. (Reference: LOS: Deleuze) Deleuze sometimes spoke of getting up behind an author and "creating a monster", as if his reading methodology somehow consisted of a distortion of the philosopher's or author's thought. Certainly there is a case to be made for this with regard to the process of deterritorialization, where something is wrested from a territory and deterritorialized upon a new territory, like the animal paw that is deterritorialized from the earth and reterritorialize on the branch. however, it seems to me that the more interesting aspect of Deleuze's approach to other philosophers and art is not so much his "monstrous creations" (i seldom find them particularly monstrous), but rather their Gaussian or Riemannian style, where he explores them in terms of their own internal organization and metric, without reducing them to something alien such as history, society, biology (reductivism), or the signifier. What we find in Deleuze's approach to phenomena is a Gaussian technique. for instance, take Deleuze's books on cinema, his book on Francis bacon, or his study of sacher-masoch. In the first instance, Deleuze's carefully separates cinema from narrative and the signifier, studying it in terms of its specific organization pertaining to the production of images. in the case of bacon, Deleuze doesn't look for an underlying narrative or "meaning", but instead studies the manner in which bacon composes and organizes his images and lines, both in terms of their production and actuality, so as to liberate a logic of sensation. finally, in approaching sacher-masoch, Deleuze allows Sacher-Masoch's novels to speak for themselves in terms of their desire and relation to pain, stalwartly refusing to reduce masochism to the complement of sadism. in each case, we have a local exploration of a "space" of multiplicities that is extremely precise. Sean Bowden the priority of events: **Deleuze's logic of sense sean Bowden, the priority of events: Deleuze's logic of sense, Edinburgh university press, 2011, 296pp., $40.00 (hbk), isbn 9780748643646 Reviewed by David Scott, Coppin state university and Gaussian spaces and multiplicities a note on Gaussian spaces and multiplicities posted by larval subjects under uncategorized.** What you "see" is not always what you "get." Many people mistakenly take their own visions literally without "seeing through" the various possibilities that are outside their belief system, knowledge or skill base. Deeper reality is not remote in the physical sense but in a psychological sense. The archetypes of the collective unconscious are arrayed behind the scenes of current worldwide conditions, of crisis and confusion. They mirror our own states back at us, whether we perceive them as such or interpret them plausibly or not. The noise of ordinary consciousness and beliefs drowns out the signal. Unconsciousness is the background of our ordinary awareness. Our organism is very much at the center of such effects. The organismic source is our human bodies and the focus of human consciousness. The fantasy principle dethrones reality, but can be dissociative or compensatory. The human mind is a meme-Scape. Pre-conceived concepts vie with structures, concepts with images. Like scientists who ignore assumed truths, we leapfrog over our beliefs and personality deficits, claiming idiosyncratic imagination is literal reality. It couldn't be further from the truth and symbolism is utterly lost. The metaphor that might heal us enslaves us. Perhaps images like the holographic universe have an implicate order. Can we have a sense of the cosmos in the world without projecting myriad fantasies on it that we embrace literally? Has the world become so horrible it is unreasonable to be realistic? We may need to look at our drives and wishes, rather than the fantasy content. Psyche constructs reality. Our experience of so-called reality is always mediated by our image of it. Even if all the contents of the psyche are real, that doesn't mean they are realistic. That psyche is real is still a radical proposition, but psychic politics certainly color the self-image and ideas of everyone. We observe and participate with images It is not a question of nature or nurture (genes alone or experience alone). Rather, everything is both. We inherit the structures that make our experience what it is. But the structure itself is "empty," and each human culture "fills" it with its own specific adaptations. It is difficult to define an archetype and set boundaries that distinguish it from others. In a hologram each part contains all the information but in lower resolution. Archetypes have this holographic quality. There are patterns within patterns within patterns. Some overlap with others, and some are nested inside others. Archetypal realities, passed on through DNA, are expressed in distinctive neuronal tracts in the brain. They include customs and laws regarding property, incest, marriage, kinship, and social status or roles; myths and legends; beliefs about the supernatural; gambling, adultery, homicide, schizophrenia, and the therapies to deal with them. A mythic and visionary language of immediate experience encompasses themes of deepest, highest, and ultimate concern. Most fantasy-based individuals are at a complete loss to coherently explain their own conventional behavior much less anomalous events and their deep meaning, much less the cultural unconscious or mythological unconscious matrix. But they try, and become utterly entrenched in their belief that

they are right about the nature of the world and reality. We have pseudo-memories about our personal lives. Why not more so for our collective life? The subject matter often revolves around catastrophe, creation and the mythopoeic forces of mankind. Ignorant of such dynamics, interpretive mistakes and displaced psychic contents proliferate into errors of fact. Propaganda, media distortions, memes, and disinformation compound the social problem of misapprehension further. Shameless self-promotion by personalities of such ideas leads to cults. They make up myths about the myths of by-gone eras. Roiling unconscious images can be fatally confusing. Thought illusions culminate in projections and projections of mythology. Jung suggested symbols live only as long as they are pregnant with meaning. Philosophy arose from criticism of myth, from discussing and challenging it. In science, we criticize, reject and eliminate theories. At the edge of the abyss of the unknown, new signs and symbols emerge. Credible theories and paradigms must include biology, physics, and neurophysiology. One of the reasons people "see God", or a guru, or anomalies may be because our brains are constructed to see reality through the eyes of others. There are heaps of mirror neurons which are there to make us feel the 'other'. Mirror neurons do for psychology what DNA did for biology. They provide a unifying framework and help explain a host of mental abilities. As in the psychochemical processes of empathy or falling in love, a complex feedback loop sustains a state of mind. But when we empathically transpose ourselves into someone else's position, we expose ourselves to that reality -- cognitively and emotionally. The unconscious complicates empathy, both ways. Mirror neurons might well play a role in bonding, language and self-awareness. Naively, we take too much as self-evident. But 'seeing' does not always 'believe', though many make this error or leap in logic and formulate their choices and future accordingly. Yet, there is only one way to learn what consciousness is. Experience. But we have no satisfactory explanatory edifice for consciousness. Would such a theory release in each of us our own inner knowledge of the creativity of our own consciousness, and its infinite possibilities? The problem is trying to define a verb, a dynamic, as if it were a noun. But we do recognize the effect of consciousness. It functions to mediate states of consciousness, high and low psychobiological arousal. Consciousness is the subconscious lifted up by the physical body. When the body fails, the consciousness collapses back into the subconscious. All our thoughts come from the subconscious which can see our intentions but not our world. This relates somehow to intention being imaginary and not of the physical frictionized world (King). Gerald Edelman postulates that the flows of information in the brain are mediated through're-entrant' feedback loops. As evolution provides new cognitive functions, new re-entrant loops are established. Even language itself is an archetype -- a chaotic field of dynamic associations. A subtle net of tropes, grammar, symbols, and meaning, the program language begins in limbic resonance. Some phenomena generate their own language patterns, nomenclature, and internal coherence of meaning and representation. n a holographic universe, even time and space could no longer be viewed as fundamentals. Because concepts such as location break down in a universe in which nothing is truly separate from anything else, time and three-dimensional space would also have to be viewed as projections of this deeper order. At its deeper level reality is a sort of super hologram in which the past, present, and future all exist simultaneously. This suggests that given the proper tools it might even be possible to someday reach into the super holographic level of reality and pluck out scenes from the long-forgotten past. Or not. A fantasy of such penetration or phenomenon inside the head is not the same as that penetration. **Jung in the 21st Century: Synchronicity and science By John Ryan Haule Mind Control Countermeasures.** Adversely antagonistic, inexorably irreconcilable, antithetically antipodal, diametrically opposed, repugnantly retrograde, inimically inverse, violatively unsimilar, diversely dissimilar some statements in Vedanta, it is not superfluous or redundant. It is evolutionary. Autheticative determination, unimpeachable validation, incontrovertible establishment, apodictic evidence, reliable roll out, unfailing cinquecento **quattrocento trecento,** incontrovertible indication of the evolutionary process towards understanding is ever present in all, its thematic and discursive form. it is the relentless holding on to a theory this is despicable albeit the theory is not. Look at this paragraph. It is the 'I am the body/mind' belief that gives rise to the 'I am not (e) the world' belief. These two beliefs are (=) co-created."Consciousness projects (e&eb) the appearance of the mind, body and world by taking the shape of thinking, sensing and perceiving." "Attention is Consciousness with an object. When the object vanishes, attention simply remains what it always is, Consciousness.""There is (=) no purpose to (e) meditation. The purpose is already accomplished." "Everything that is experienced is experienced by, through, in and as Consciousness." "The seen cannot be separated from seeing and seeing cannot be separated from (e)

Consciousness." "The Reality of any experience is not hidden in the appearance; it is expressed by the appearance." "Once we see that everything is Consciousness... Maya still dances, but it is a dance of love not seduction." There are some truly excellent sections such as a long one taking, as a starting point, the following quotation from the painter Cézanne: "Everything vanishes, falls apart, doesn't it? Nature is always the same but nothing in her that appears to us lasts. Our art must render the thrill of her performance, along with her elements, the appearance of all her changes. It must give us a taste of her Eternity." Rupert begins: "That statement must be one of the clearest and most profound expressions of the nature and purpose of art in our era." And he goes on for some 14 pages to elaborate on this claim, examining the nature of the 'elements' Existence, appearance and Consciousness and their relationship. "This Reality is the support or ground of the appearance. The appearance may be an illusion, but the illusion itself is real. There is an illusion. It has Reality." He refers to the rope-snake metaphor and says that: "We do not see anything new. We see in a new way." "So that we know that nature is real, that there is something present, that there is a reality to it, even if everything that appears to us is insubstantial and fleeting." **The Transparency of Things' by Rupert Spira Book Review by Dennis Waite The following is a review of Rupert's book: 'The Transparency of Things: Contemplating the Nature of Experience'. There is an essay from the book here and the essay on Cézanne may be read at the awakened eye website For models see Mitigation Of Beam-Induced Backgrounds, Multi-Particle Azimuthal Correlations, Search For Extra Dimensions, Multi-Higgs-Boson Cascade, Top Quark Pair (Tt) Production Charge Asymmetry, $\Delta Y \equiv Yt - Yt^{-}$, Reflects The Asymmetry In Tt^{-} Production, Ultra-Relativistic Electrons And Other Letters Of Interest: A Synecdochal Syncretism: Et Lux In Tenebris Lucet: Light Shines In The Darkness Models.** What then is the relation of Higgs boson and consciousness? Functionalist approaches generally assume that conscious experience appears as a novel property at a critical level of computational complexity. On the surface this would seem to deal with issues 1 and 3, however a conscious threshold has neither been identified nor predicted, and there are no apparent differences in electrophysiological activities between non-conscious and conscious activity. Regarding the nature of experience (why we are not unfeeling "zombies") functionalism offers no testable predictions. Problem 2) of 'binding' in vision and self is often attributed by functionalists to temporal correlation (e.g. coherent 40 Hz), but it is unclear why temporal correlation per se should bind experience without an explanation of experience. As functionalism is based on deterministic computation, it is also unable to account for Penrose's proposed non-computability (4), or free will (5). Something may be missing. To address these issues, various proposals have been suggested in which macroscopic quantum phenomena are connected to the brain's known neural activity. For the problem of unitary binding, Marshall (1989) suggested that coherent quantum states known as Bose-Einstein condensation occurred among neural proteins (c.f. Penrose, 1987; Bohm and Hiley, 1993; Jibu and Yasue, 1995). Pre-conscious to conscious transitions was identified by Stapp (1992) with collapse of a quantum wave function in pre-synaptic axon terminals (c.f. Beck and Eccles, 1992). In another proposal, protein assemblies called microtubules within the brain's neurons are viewed as self-organizing quantum computers ("orchestrated objective reduction - Orch OR" e.g**. Penrose and Hameroff, 1995; Hameroff and Penrose 1996a; 1996b; c.f. Hameroff 1997; 1998a; 1998b; 1998c; 1998d). Stability analysis of such theories would certainly provide a gateway for further bringing in the "reality" in space time.** String theory suggests that the graviton particle would not be detected from a particle collision because it would have "jumped" into another dimension. So, if the graviton exists and the theory is correct, there should be a "gap" in the energy profile of all the particles ejected from the collision. To understand this, think of breaking the pack of balls in a game of pool. The total energy of all the balls after the collision will depend on the force with which the cue ball was hit. So, after the collision, the energy of all the balls is measured and added-up to make sure it is the same as the energy of the cue ball. BUT - imagine that there is some energy missing, because one of the pool balls has "gone missing" - into a higher dimension. (POSTED BY THEREALJEFFHAL**) Talbot says** "If the universe is a hologram, in some sense it suggests that there may be two very drastically different levels of reality: the concrete reality that we see when we look at [things]... and at some deep level there's a level of reality where everything dissolves into an ocean of energy that is holographically interconnected.". A theory of everything must encompass all the variations however contradictory it may be. Nor need any theory be sent in to grand limbo of oblivion and hibernation or a differentiated groundless less ness

because there are some irregularities which are like audit discrepancies need to be rectified. Although the language of religion and science seem different, both are talking about the central order of nature relative to DNA and information, as is Peirce, Petrus, Bacon, and pharaonic Egypt. This path-ordering, active information suggests that the central pattern or law of our holographic cosmos may be modeled on the biophysical battleground of gene regulation in bacteriophage Lambda, one of nature's most efficient and highly evolved mechanisms identified by experimental praxis. Egyptian texts support that the Lambda lifestyles of lysogeny and lysis can be understood as two paths to two living systems, photosynthesis and its reversal chemiluminescence, the cosmic key for evolution of mind and continuity. The texts support that the Lambda genome is the world-heart of two ways. The viral lifestyle of lysogeny controlled by cI protein results in matter, classical spacetime, our world of projected shadow, photosynthesis, sexual genesis or vertical gene transfer, mind in the human body, and the imperfect continuum. In contrast, the viral lifestyle of lysis controlled by cro protein and lactose metabolism results in transformation to energy, the quantum, chemiluminescence, asexual genesis or HGT, mind in the cosmos, and the perfect continuum. Some scientists believe our cosmos is entangled with another (Chown 2007). On the holographic quantum level, perhaps this is just the competitive entanglement of cI and cro proteins cycling in perpetual motion—an inherent law of nature elucidating the assembly of things. In light of modern experiments, physicist John Wheeler (1988) speculated that the act of conscious observation functions with quantum mechanics, catalyzing a probability or outcome. So, if an observing consciousness has knowledge of the classical path and its magnetic field orientation, an act of measurement or choice might entangle space to reify the holographic quantum paths to a perfect continuum. Peirce guesses that to be drawn into a new system, a particle must have the right mass, the right velocity in the right direction, the right attraction, and it must present itself at the right point (EP 1. 270). Similarly, a finite probability exists for any particle that approaches the black hole event horizon to bounce back, dependent on the incoming particle's energy, its charge, and its projection of the orbital momentum on the axis of rotation of the black hole (Kuchiev 2003; 2004; 2004a), the same conditions in Egyptian texts (King 2004; 2006).Only an approximate conclusion remains: chance may be knowledge of the cosmos' holographic mode of operation and magnetic fields; habit breaking may result in the transformation of energy by a violation of the second kind; and evolution of mind or energy might be possible through HGT via a cosmic viral code of Lambda-genesis. "Order is simply thought embodied in arrangement;" (CP 6.490). Accordingly, the original meaning of the signs may relate to a viral biophysics of crystallized mind for human evolution via a natural chemical pathway to a perfect continuum. Again, this semiotics of evolvability and order recurs in time with the chaotic dream of reason that projects a pervasive falsification of our perceptual world. As Wallace Stevens envisions in "Connoisseur of Chaos", perhaps "the pensive man may see." **Peircean Process Metaphysics Origin of Science Quantum and Classical Laws Ancient Egyptian Evolutionary Biophysics** Adversely antagonistic, inexorably irreconcilable, antithetically antipodal, diametrically opposed, repugnantly retrograde, inimically inverse, violatively unsimilar, diversely dissimilar are the views of epistemologists in the analysis of raison d'etre of knowledge. Epistemology is the study of knowledge and those things closely related to it: justification, what it takes for you to be justified, the relation between knowledge and justification, whether you can have any justified beliefs at all, and if so, how you come to know (or justifiably believe) things, how you can use what you know (or justifiably believe) to come to know (or justifiably believe) other things, and whether and why it's valuable to know instead of merely having justified and true beliefs. The literature on epistemology is vast. Here's a very brief summary of some epistemological discussions. Concerning knowledge, many epistemologists think knowledge is justified true belief, where the justification you have is linked to the truth of the matter in the right kind of way, though what this way is a matter of debate; some epistemologists think knowledge can't be analyzed this way. Concerning justification's relation to knowledge, some epistemologists think we don't need to be justified to know, and some think we do need to be. Concerning what it takes to be justified, some epistemologists think that what it takes for you to be justified are only factors internal to the believer (Internalists). Others think it takes an external factor, like reliable or well-functioning cognitive faculties (Externalists). Skeptics argue that we can't have any justified beliefs at all, and many epistemologists reply to the skeptic's arguments. Concerning how we use what we know (justifiably believe) to come to know (justifiably believe) other things, some epistemologists (Foundationalists) argue that there are bedrock propositions that we know (justifiably believe), and we build our knowledge (justified beliefs) on those. Others (Coherentists) argue that

there aren't bedrock propositions; rather, a set of beliefs is justified as a whole, and several beliefs can be mutually supporting. Concerning the value of knowledge, some argue that knowledge is intrinsically valuable. Others have argued that knowledge is valuable only because of the role it plays in practical reasoning, and others argue that knowledge isn't more valuable than justified and true belief, but there are other epistemic states such as understanding, that do have value above their proper subparts. **Edited y Matthew McGrath (University of Missouri, Columbia). It is not as if there is an anathema or misnomer in the philosophy or physics. Infact most of the pnysicists have used philosophical ideas towards the end of consummation of their their and their concomitant justification thereof.** The eternal return produces becoming-active. It is sufficient to relate the will to nothingness to the eternal return in order to realize that reactive forces do not return. However far they go, however deep the becoming-reactive of forces, reactive forces will not return. The small, petty, reactive man will not return. Affirmation alone returns, this that can be affirmed alone returns, joy alone returns. Everything that can be denied, everything that is negation, is expelled due to the very movement of the eternal return. We were entitled to dread that the combinations of nihilism and reactivity would eternally return too. The eternal return must be compared to a wheel; yet, the movement of the wheel is endowed with centrifugal powers that drive away the entire negative. Because Being imposes itself on becoming, it expels from itself everything that contradicts affirmation, all forms of nihilism and reactivity: bad conscience, ressentiment..., we shall witness them only once. [...] The eternal return is the Repetition, but the Repetition that selects, the Repetition that saves. Here is the marvelous secret of a selective and liberating repetition. There is no need to remind the reader that neither the image of a centrifugal movement nor the concept of a negativity-rejecting repetition appears anywhere in Nietzsche's writings, and indeed Deleuze does not refer to any text in support of this interpretation. Further, one could highlight that Nietzsche never formulates the opposition between active and reactive forces, which constitutes the broader framework of Deleuze's interpretation. For some years, Marco Brusotti has called attention to the fact that Deleuze introduced a dualism that does not exist in Nietzsche's writings. To be sure, the German philosopher describes a certain number of "reactive" phenomena (for example, in the second essay of the Genealogy of Morality, § 11, he talks about "reactive affects" [reaktive Affekte], "reactive feelings" [reaktive Gefühlen], reactive men [reaktive Menschen]); but these are nonetheless the result of complex ensembles of configurations of centers of forces that remain in themselves active. Neither the word nor the concept of "reactive forces" ever appears in Nietzsche's philosophy. **After the discovery of the two principles of thermodynamics began a debate about the dissipation of energy and the thermal death of the universe which framed the modern renewal of the debate between the linear and circular conceptions of time**. Scientists such as Thomson, Helmholtz, Clausius, and Boltzmann and-by way of Kant, Hegel and Schopenhauer-philosophers such as Dühring, Hartmann, Engels, Wundt and Nietzsche have tried to address this problem by using the force of scientific argumentation and of philosophical discussion. Whoever believed in an origin and a final end to the motion of the universe (be it in the physical form of the gradual loss of heat, or in the metaphysical form of a final state of the "world process"), relied on the second principle of thermodynamics or on the demonstration of the thesis of Kant's first cosmological antinomy. On the contrary, those who refused to admit a final state to the universe used Schopenhauer's argument of infinity a parte ante-according to which if a final state were possible, it should already have established itself in the infinity of time past-to propose henceforth a number of alternative solutions. Scientists would propose the hypothesis that energy could have re-concentrated after a cosmic conflagration, thus reversing the tendency towards dissipation. Those belonging to the monistic and materialistic tradition relied on the first principle of thermodynamics and on the infinity of matter, space and time, and regarded the universe as an eternal succession of new forms. A certain critical agnosticism was widespread among scientists and philosophers, oftentimes through a reaffirmation of the validity of Kant's antonymic conflict, this movement avoided to take a stand on specifically speculative issues. Other German philosophers, like Otto Caspari, or Johann Carl Friedrich Zöllner, had reintroduced an organicist and pan-psychical conception of the universe, investing atoms with the ability to escape any state of balance. Indeed, it is probably one of Otto Caspari's works, The Correlation of Things (Der Zusammenhang der Dinge. Gesammelte philosophische Aufsätze (Bleslau: Trewendt, 1881)), which awakened Nietzsche's interest for all things cosmological, in that summer of 1881, in Sils-Maria. addressing Schopenhauer and Eduard von Hartmann's mystical pessimism according to which the world is the creation of a stupid and blind essence (which, after having created the world by mistake, comes to the realization that it had made a mistake and

strives to return it to nothingness) Caspari stresses that it is nothing short of mystical to imagine that the world may have been borne out of a an originary and undifferentiated state. Where would it have drawn the first impulse? But, continues Caspari, even if the world had received this first impulse from some deus ex machina, there is no doubt that, in the temporal infinity of past time thus far, it would have either attained the end of the process (but this is impossible because the world would then have ended), or it would be necessarily bound to repeat indefinitely this original mistake, and the entire process that accompanies it. But then, what is the process of the world? We must now take one more step back and understand further the process of the world according to von Hartmann. Hartmann objects that the regressive movement postulated by Schopenhauer is possible only in thought: it remains nothing more than an "ideal postulate" with no real object and which "does not teach us anything about the real process of the world that unfolds in a movement contrary to this backwards movement of thought" (Hartmann, Philosophie des Unbewussten, third edition (1871): 772). Hartmann affirms that if unlike Schopenhauer one admits the reality of time and of the world process, one must also admit that the process must be limited in the past and therefore that there must be an absolute beginning. In Hartmann's mind, failure to do so would result in positing the contradictory **concept of an accomplished infinity**: "The infinity that from the point of view of regressive thinking, remains an ideal postulate, which no reality may correspond to, must, for the world, whose process is, on the contrary, a progressive movement, open up to a determinate result; and here the contradiction comes to light" (Hartmann (1871): 772). What really "comes to light" in this passage is the fact that Hartmann does not provide a demonstration but a petitio principii. Indeed, the concept of the world process analytically contains the concept of a beginning of the world. In all rigor, it is therefore impossible to demonstrate these concepts with reference to each other. Secondly, Hartmann's view that one is bound to accept the reality of the world process even if one rejects the ideality of Schopenhauer's time is mistaken. Hartmann believes that if time is real there must be a world process with both an **absolute beginning and an absolute end. Without any justification, Hartmann jumps from Schopenhauer's negated time to oriented time.** Hartmann's view is that the world process leads into a final state absolutely identical to the initial state. However, it follows from this that even as the cosmic adventures of the unconscious come to a close, we are still haunted by the specter of a new will and of another beginning of the world process. This exposes a serious internal flaw of Hartmann's system insofar as it jeopardizes the possibility of a final liberation from existence and suffering. This is why in the last pages of his work, "On the Last Principles," he painstakingly calculates the degree of probability of a reawakening of the volitional faculty of the unconscious. Insofar as the will is entirely free, unconditioned and a-temporal, the possibility of a new volition is left to pure mathematical chance and is therefore ½. Hartmann further stresses that if the will were embedded in time, the probability of the repetition would amount to 1 and the process of the world would be bound to begin again, in an eternal return which would completely preclude the possibility of a final liberation. Fortunately, this is not the case since-according to Hartmann's remarkable logic-the world-process develops through time, but the original will is outside of time. In fact, one may even affirm, along the lines of Hartmann's peculiar theory of probability, that every new beginning gradually reduces the probability of the next beginning: let n be the number of times that the will is realized, the probability of any new realization is ½ n. "But it is clear that the probability ½ n diminishes as n increases, in a way that suffices to reassure us in practice" (Hartmann (1869): 663). In his famous work entitled **On the Conservation of Force (1847), Hermann von Helmholtz** had divided the totality of the energy in the universe between potential energy and kinetic energy and affirmed the reciprocal convertibility of the two. In 1852, William Thomson pointed out that there exists a sub-ensemble within kinetic energy, heat, which, once it has been generated, is no longer entirely convertible into potential energy-or into any other form of kinetic energy. Considering that the (partial) reconversion of heat into labor is possible only in situations that present a disparity in temperature, and that heat tends to pass from warmer to cooler bodies by spreading on an even temperature level through space, Thomson concluded that the universe tends towards a final state where any energetic transformations, every movement and every form of life will cease. Caspari's atoms (which bring to mind those in Leibniz's Monadology) resemble some sort of biological monads, endowed with internal states. For Caspari, every atom obeys the ethical imperative to participate in the conservation of the general organism and its movement does not only follow the simple physical kind of interaction but also an a priori law ensuring. If a balance of forces had been attained at any moment, this moment would still be going on: therefore, it never happened. The present state contradicts this proposition.

Supposing that a certain state rigorously identical with the present state had, one day, existed, this supposition is not refuted by the present state. As one of the infinite possibilities, it is necessary that the present state had been given anyway, since until now an infinite period of time has already unfolded. **If equilibrium were possible, it must have occurred**; and if the present state has already taken place, then so too the one that preceded it as well as the one preceding that one. Therefore it has already taken place a second time, a third time and so on. And likewise it shall take place again a second time, a third time. Innumerable times forwards and backwards. This amounts to saying that all becoming occurs within a repetition of an innumerable number of absolutely identical states. [.] The immovability of forces, their equilibrium is a conceivable case, but it has not occurred. As a result the number of possibilities is greater than the number of realities. -The fact that nothing identical recurs may be explained not thanks to chance, but only thanks to an intention infiltrated within the essence of force. Indeed, supposing an enormous amount of cases, the random occurrence of the same combination is more probable than the same combination never recurring. The quantum of force in the universe is determinate and not "infinite": let us beware [hüten wir uns] from such conceptual extravaganza! Therefore the number of situations, modifications, combinations and developments of this force is doubtless enormous and practically "immeasurable," but in any case this number is determinate and not infinite. On the other hand, the time in which the universe exerts its force is infinite. That is to say, that force is eternally identical and eternally active: -until the present instant an infinity has already taken place, that is to say that all possible developments must have already taken place. Consequently, the present development must be a repetition and therefore both this that was born from it and this that shall be born from it and so on both forwards and backwards. Everything has taken place an innumerable number of times because the overall situation of all forces always recurs (FP 11 [202] of 1881). Nietzsche regards the mechanistic vision as more plausible and less anthropomorphic than organicism. However, faced with the two major **cosmological models of his time, the mechanistic model and the organic model, Nietzsche wishes to return its polymorphous, proteiform, unstructured** and chaotic character to nature of which the perfectly non-theological and non-teleological theory of eternal return is the strongest seal. This is the first of the "new battles" which come to whoever is aware of the consequences of the death of God: take any antropomorphism away from nature. In the preparatory papers, the third book of the Gay Science is entitled "Gedanke eines Gottlosen / Thoughts of a Godless One." Aphorism 109 of this book, which immediately follows the famous aphorism against the shadows of God, **Boltzmann's theory belongs to the third phase of the debate on thermodynamics and cosmology. The publication of Thomson's brief paper "On a Universal Tendency in Nature to the Dissipation of Mechanical Energy" of 1852 signals the beginning of the scientific controversies on the problem of the dissipation of energy and opened the first phase of the debate, which was announced in the Reflexions on the Motor Powers of Fire by Sadi Carnot and whose conclusion is represented by Clausius's recapitulative article on the concept of entropy in 1865.[44] The second phase started in 1867 when, at the forty-first congress of German scientists and doctors, Clausius gave a lecture on "The Second Principle of the Mechanistic Theory of Heat," where he applied the results of his research on thermodynamics to the universe**. It is true that in his famous lecture of 1854, Helmholtz had already presented the cosmic consequences of the second principle, but Clausius' contribution had a strong impact on German culture. This is because in this lecture he robustly rejected the possibility to consider the universe as an eternal and self-renewing circle, an ewiger Kreislauf in which force and matter are in constant transformation, as was heretofore affirmed by the materialism of the scientists and philosophers, and he did so in the name of the second principle of thermodynamics. In this way, the debate on the principles of thermodynamics gained great importance in European Culture starting in 1867. In the two first phases, it is Thomson's mechanism that predicts the thermal death of the universe. In the third phase, on the contrary, the meaning of the term mechanism changes radically.[46] In accordance with the apocalyptic climate of this period dominated by the "rebirth of idealism," the "overcoming of scientific materialism" and the "bankruptcy of science", the mechanistic paradigm which had accompanied the birth of modern science became challenged on **account of the second principle of thermodynamics**. According to the theorem of the quasi-periodicity of the motions of mechanical systems demonstrated by Poincaré as part of the problem of the three bodies (1890), a mechanical system must evolve according to a quasi-periodical movement and consequently it must always return-sooner or later-to the initial state. **An easily established theorem informs** us that a limited world obeying solely the laws of

mechanics shall always pass through a state closely similar to its initial state. On the contrary, according to established experimental laws (supposing we grant them absolute value and wish to push their consequences to the end), the universe is directed towards a final state, which once it is attained, it shall not be able to escape. In this final state, which shall be like a sort of death, all material bodies shall be at rest at the same temperature. **Poincaré's theorem seems** therefore incompatible with the second principle of thermodynamics, which predicts a unidirectional movement of all natural phenomena until the whole universe is brought to a total standstill. Wilhelm Ostwald and the entire energeticist school of thought contended that the principles of thermodynamics were fundamentally new, and could not be re-incorporated to traditional physics and that they should serve as a basis for a new science that regards the qualitative diversity of energy and its tendency to degradation as its axioms. Against energeticism and in an effort to bring entropic phenomena back into the theoretical framework of mechanism, Ludwig Boltzmann introduced the concept of probability in physics, not as an instrument of calculation, but as an explicative principle. In Boltzmann's statistical thermodynamics, the increase of entropy assumed by Clausius is re-interpreted as an increase in molecular chaos. As a result, it becomes possible to explain mechanistically the evolution of closed systems endowed with increasing entropic value, without it committing us to granting absolute value to the second principle of thermodynamics. **Moreover, one no longer needs to fear the thermal death of the universe insofar as the state of equilibrium will in principle** never be complete, but rather will be attained only statistically, leaving open the possibility of fluctuations towards less probable states. Boltzmann's critics remarked that this hypothesis involved two paradoxes called the objection of reversibility (Umkehreinwand), and that of repetition (Wiederkehreinwand). I shall only address here the second one since it coincides with the theory of the eternal return. Based on Poincaré's theorem quoted above, Ernst Zermelo objected to Boltzmann that his model of the universe suggested that after a finite (if admittedly very long) time the system would return to its initial position. In his first response to Zermelo, Boltzmann avoids committing himself directly to cosmological questions and he only observes that, in the case of concrete thermodynamic systems, the time of recurrence may be extremely long. For example, in normal conditions of pressure and temperature, one-centimeter cube of gas requires 1010 years to reach a molecular configuration identical to the original one! However, following a response by Zermelo, Boltzmann wrote a new article where he outlines a cosmological picture that he will re-use later in his conclusion to his famous Lectures on Gas Theory. In this cosmological picture, Boltzmann considers the universe as a closed system with constant entropy, within which some fluctuations occur, creating islands of negative entropy. Our solar system originates in one of these fluctuations. As Clausius correctly pointed out, the entropy of our solar system increases constantly as the solar system gets closer to the state of chaos and of the thermal death of the rest of the cosmos. However, in other zones of the universe, some new fluctuations and new islands appear, so that thermal death is never generalized. Here we are given a grand cosmic image, in which the solar system and the sparkle of life that was lit on planet earth are only a fluctuation of order from within a dominant entropic tendency. Life, and the order on which it is based are exceptions, transitory forms taking place in the realm of the shapeless, they are islands of the cosmos that will soon be re-absorbed into chaos. According to Poincaré's theorem, our island will have to be reborn, to develop and die innumerable times in a strictly identical fashion. This happened an infinite number of times during the past eternity and it shall take place again an infinite number of times in the eternity to come. **Boltzmann accepts the "paradox" of recurrence-that is the eternal return of the same-as a legitimate consequence of the probabilistic conception of thermodynamics**. It may be rejected for ethical reasons, it may be stored away as an abstract speculation or dismissed along with other cosmic fantasies, but it cannot be rejected on the basis of any rigorously scientific viewpoint. **The Eternal Return: Genesis and Interpretation BY PAUL D'IORIO(excerpts). Nietzsche found the template of a materialistic cosmology based upon the first principle of thermodynamics. During the same year, he could have found a model of an organistic solution to the problem of thermal death of the universe as well as a discussion on the conformation of space in Zöllner's book On the Nature of Comets** (Johann Carl Friedrich Zöllner, Über die Natur der Kometen. Beiträge zur Geschichte und Theorie der Erkenntnis (Leipzig, Engelmann, 1872), 299 f. and 313 f.); Assumed Association In Space Time Of Particles Or Men Leads To Detrimental Ramification Be In Terms Of Progress Of “Subject Matter” Or In The Pathological Disequilibrium Of Gratification Deprivation Complex Which Again Has Mental Turbulence In Case Collective Consciousness Approve It (Raaga) Or Pernicious Implications In Case The Theory Is Not

Accepted; After All What Is A Theory: Set Of Correlations And Causations Which Are “Tested” Against A Collective Consciousness Background, Which Yields “Results’ Which Are Themselves In “Question” Due To The Very Perception And The Simulation Prospective. The Definition Of Quantum Theorists' Terms, Such As Wavefunctions And Matrix Mechanics, Progressed Through Many Stages. For Instance, Erwin Schrödinger Originally Viewed The Electron's Wavefunction As Its Charge Density Smeared Across The Field, Whereas Max Born Reinterpreted It As The Electron's Probability Density Distributed Across The Field. Although The Copenhagen Interpretation Was Originally Most Popular, Quantum Decoherence Has Gained Popularity. Thus The Many-Worlds Interpretation Has Been Gaining Acceptance Moreover, The Strictly Formalist Position, Shunning Interpretation, Has Been Challenged By Proposals For Falsifiable Experiments That Might One Day Distinguish Among Interpretations, As By Measuring An AI Consciousness Or Via Quantum Computing. More Or Less, All Interpretations Of Quantum Mechanics Share Two Qualities: They Interpret Formalism—A Set Of Equations And Principles To Generate Predictions Via Input Of Initial Conditions They Interpret A Phenomenology—A Set Of Observations, Including Those Obtained By Empirical Research And Those Obtained Informally, Such As Humans' Experience Of An Unequivocal World Two Qualities Vary Among Interpretations: Ontology—Claims About What Things, Such As Categories And Entities, Exist In The World Epistemology—Claims About The Possibility, Scope, And Means Toward Relevant Knowledge Of The World. Now How Does A Person In A Crowd Behave? They He Would Due To Herd Mentality; Ok; That Is One Theory; Simulation By Terrestial Brahman Anti Brahman Will Make Them Behave In A Highly Organised And Coordinated Manner; With Association Of Things And Individuals Already Established Resultant Orientation Of The Actions Hall Be Only Oriented Towards The Figment Of Imagination And Product Of Puerile Prognostication Formally Established By Terrestial Brahman Anti Brahman Before Such A Coordinated And Organised Execution Of Exercise Is Conducted; Analogically, If A Simulation Is Going On In A Bunch Of Particles Or Electrons, And If Some Agency Say Celestial Brahman Anti Brahman Agency Is Simulating It, There Shall Be Coordination And Organisation So That You See “Reality” While The Truth Shall Remain Hidden. Result Is The Suppression Of Information, Delineation Of Disinformation, And The Concomitant Development Of Mushrooms Of Concocted And Fabricated Theories To That “Reality”; In The Case Of Terrestial Brahman Anti Brahman Agency Intention May Be To Bulldoze And Bowdlerize You, Baffle And Befuddle You In To Submission; In The Case Of Celestial Brahman Anti Brahman Agency Only Reason Could Be To Shield The Truth Being Disseminated; But Why?;May Be You Should Never Become God Which You Are Trying To; May Be God Is Really Playing Dice; What Does That Result In To ; You Become A Scezophreniac Associating Again And Again What You Had Been Until Now Of Particles And Other Concomitants Again And Again Leading To Flawed Inferences. Confusing The Epistemic With The Ontic, Like If One Were To Presume That A General Law Actually "Governs” Outcomes—And That The Statement Of A Regularity Has The Role Of A Causal Mechanism—Is A Category Mistake. In A Broad Sense, Scientific Theory Can Be Viewed As (=) Offering Scientific Realism—Approximately True Description Or Explanation Of The Natural World—Or Might Be Perceived With Antirealism. A Realist Stance Seeks The Epistemic And The Ontic, Whereas An Antirealist Stance Seeks Epistemic But Not The Ontic. In The 20th Century's First Half, Antirealism Was Mainly Logical Positivism, Which Sought To Exclude Unobservable Aspects Of Reality From Scientific Theory. Since The 1950s, Antirealism Is More Modest, Usually Instrumentalism, Permitting Talk Of Unobservable Aspects, But Ultimately Discarding The Very Question Of Realism And Posing Scientific Theory As A Tool To Help Humans Make Predictions, Not To Attain Metaphysical Understanding Of The World. The Instrumentalist View Is Carried By The Famous Quote Of David Mermin, "Shut Up And Calculate", Often Misattributed To Richard Feynman. Quantum Mechanical Wave Function (Absolutely Squared) Describes The Completed Interference Pattern, It Must Describe An Ensemble Interpretations Of Quantum Mechanics From Wikipedia. Philosophy Of Science, The Distinction Of Knowledge Versus Reality Is Termed Epistemic Versus Ontic. A General Law Is A Regularity Of Outcomes (Epistemic), Whereas A Causal Mechanism May Regulate The Outcomes (Ontic). A Phenomenon Can Receive Interpretation Either Ontic Or Epistemic. For Instance, Indeterminism May Be Attributed To Limitations Of Human Observation And Perception (Epistemic), Or May Be Explained As A Real Existing Maybe Encoded In The Universe (Ontic). Confusing The Epistemic With The Ontic, Like If One Were To Presume That A General Law Actually "Governs” Outcomes—And That The Statement Of A Regularity Has The Role Of A Causal Mechanism—Is A Category

Mistake. Interpretations Of Quantum Mechanics From Wikipedia. Reality Is Constant In The Midst Of Change. Note The Conservation Of Individual, Collective And Cosmic General Ledgers Are What Are Being Spoken Of. Not The Changing Transactions, But The Immutable Refractory Zero. (Italics Mine) What This Means Essentially Is That There Is Change Although Nothing Changes. Like There Are Changing Transactions And Still The Assets And Liabilities Will Be Equal. This Impossible Situation Is Reflected (E&Eb) In The Ultimate Impossibility Of Change Itself. That Which Does Not Exist Prior To Its Changing And At The End, After It Has Changed, Must Be Equally Non-Existent Between These Two Moments. Although The World Of Change Appears To Be Real, It Cannot Be So. (Real)Change, According To The Vedantin, Presupposes A Loss Of Identity. Reality Cannot Suffer Transformation If It Were To Do So, It Would Become Something Else And The Real Would Be Deprived Of Its Reality. The Immortal Can Never Become Mortal, Nor Can The Mortal Become Immortal The Doctrine Of Vibration: An Analysis Of The Doctrines And Practices Of Kashmir Shaivism Mark S. G. Dyczkowski. Concept Of Being From Shaivite Point Of View: In His Famous Work Entitled On The Conservation Of Force (1847), Hermann Von Helmholtz Had Divided The Totality Of The Energy In The Universe Between Potential Energy And Kinetic Energy And Affirmed The Reciprocal Convertibility Of The Two. In 1852, William Thomson Pointed Out That There Exists A Sub-Ensemble Within Kinetic Energy, Heat, Which, Once It Has Been Generated, Is No Longer Entirely Convertible Into Potential Energy-Or Into Any Other Form Of Kinetic Energy. Considering That The (Partial) Reconversion Of Heat Into Labor Is Possible Only In Situations That Present A Disparity In Temperature, And That Heat Tends To Pass From Warmer To Cooler Bodies By Spreading On An Even Temperature Level Through Space, Thomson Concluded That The Universe Tends Towards A Final State Where Any Energetic Transformations, Every Movement And Every Form Of Life Will Cease: The Eternal Return: Genesis And Interpretation BY PAUL D'IORIO: The Ultimate Nature Of Anything Cannot Change. Change Of Any Sort Is Merely Apparent (Vivarta). The World Of Change And Becoming Is A False Super- Imposition (Adhyaropa, Adhyasa) On The Absolute. It Appears So On The Screen Of Individual, Collective Consciousness, Albeit One Cannot Be So Sure Of Cosmic, For There Are Many Reasons That Cosmic Transactions Also Had To Be Subjected To Change By ParaBrahaman. In Cosmic Terms, The Mistake (Bhranti) Consists Of The Supposition That The Real Brahman Is The Unreal Universe And The Unreal Universe Is The Real Brahman. In Microcosmic Terms, It Is The Mistake Of Falsely Conceiving The Body, Mind Or Even One's Personality To Be The Self. Note We Have Taken Self As Witness Consciousness. Notwithstanding There Is Nothing Wrong In The Conception Of Self As A Dynamical Presupposition, In That Individual Consciousness And Collective Consciousness And Even Cosmic Consciousness Evolutes In To ParaBrahaman. In The Same Way As The Image Of A Snake Is Falsely Superimposed On A Rope, Similarly The Universe Is Falsely Projected Onto The Real Substratum, The Brahman. Ignorance Is Not Merely A Personal Lack Of Knowledge, But A Cosmic Principle. As Such It Is Called "Maya," The Indefinable Factor (Anirvacaniya) That Brings This Mistake In Identity About. The Reality Status Of This Cosmic Illusion Is Also Indefinable: On The One Hand It Is Not Brahman, The Sole Reality; On The Other Hand It Is Not Absolutely Non-Existent Like A Hare's Horn Or The Son Of A Barren Woman. Note Here The Vedantin Avoids Talking About Zero Which We Have Avidly And Assiduously Stressed About. Nothing Is The Factor That Exists. Also See the Remarks about Satre's Being And Nothingness. Brahman Is The Source Of World Appearances Only In The Sense Of Being Their Unconditioned Ground Or Essential Nature. The Universe Is False Not Because It Has No Nature Of Its Own But Because It Does Have One. The Doctrine Of Vibration: An Analysis Of The Doctrines And Practices Of Kashmir Shaivism Mark S. G. Dyczkowski. Concept Of Being From Shaivite Point Of View: It Is The 'I Am The Body/Mind' Belief That Gives Rise To The 'I Am Not The World' Belief. These Two Beliefs Are (=) Co-Created."Consciousness Projects The Appearance Of The Mind, Body And World By Taking (Eb) The Shape Of Thinking, Sensing And Perceiving.""Attention Is Consciousness With An Object. When The Object Vanishes, Attention Simply Remains What It Always Is Consciousness.""There Is No Purpose To Meditation. The Purpose Is Already Accomplished.""Everything That Is Experienced Is Experienced By, Through, In And As Consciousness.""The Seen Cannot Be Separated From Seeing And Seeing Cannot Be Separated From Consciousness." "**The Reality Of Any Experience Is Not Hidden In The Appearance; It Is Expressed By The Appearance The Transparency Of Things'** By Rupert Spira Book Review By Dennis Waite The Following Is A Review Of Rupert's Book: 'The Transparency Of Things: Contemplating The Nature Of Experience'. There Is An

Essay From The Book Here And The Essay On Cézanne May Be Read At The Awakened Eye Website Nature Rejects The Naiveté That Seeks Absolute Truth. We Are Beginning To Realize, Individually And Culturally, That "Realities" Are All Human Constructions. The Task Becomes One Of "Catching Ourselves In The Act" Of Creating Our Own "Reality" From The Flow Of Events. Human Truth Is Always An Engagement Of Mind With Experience. The Challenge Of The Therapist In These Times Of Chaotic Change Is To Validate The Concept That We Don't Have To Fear The Collapse Of What We Think We Are. Strange Attractors: Transference, Holography, And An **Archetype Burke, J. (2003). Strange Attractors: Transference, Holography, And An Archetype (Doctoral Dissertation, Pacifica Graduate Institute, 2003).** When we get to the topic and understanding of the Transcendental Logic, the brilliant and baffling Transcendental Deduction (TD) Recall the two movements just discussed, the one from experience to its conditions and the one from the forms of valid inference to the concepts that we must use in all judging (the Categories). This duality led Kant to his famous question of right (quid juris) (A84=B116): with what right do we apply the Categories, which are not acquired from experience, to the contents of experience? (A85=B117). Kant's problem here is not as arcane as it might seem. It reflects an important question: How is it that the world as we experience it conforms to our logic? In briefest form, Kant thought that the trick to showing how it is possible for the Categories to apply to experience is to show that it is necessary that they apply (A97).TD has two sides, though Kant never treats them separately John Marlado, accounts, attributes, and ascribes the philosophy of Nishida KitarÅ is lucid, insightful, and deeply informative despite the highly questionable argument that gives the book its structure. In comparison with other, technically more accurate accounts of Nishida's philosophy, it is broader in scope than most but more single-minded in its approach. The analysis is for the most part limited to four books that have been translated into English, out of approximately 16 volumes of philosophical essays in Japanese, but it aims to capture the contours and most important details of Nishida's entire philosophy. To appreciate the achievement of this account, one would need only to read, in English translation or in the Japanese original, any of the volumes with their streams of repetitive, puzzling expressions and circuitous, often dead- end argumentation. Wilkinson's elucidation of details in the passages he samples makes it easier to understand why Nishida is considered Japan's greatest academic philosopher. A critical look at the frame of the book and some of its own claims reveals where, to my mind, it has gone astray. The argument that frames this book is repeated throughout: Nishida's philosophy is an attempt to articulate, in a characteristically Western philosophical manner, the Zen experience he had as a young man. Nishida first made the attempt by using the conceptual frameworks of Western philosophers, and after they failed him he developed his own, largely by recasting Buddhist insights. In the end, a philosophical articulation of the Zen worldview in Western terms or even in Eastern terms recast by conceptual, Western philosophical methods approved impossible. What Nishida attempted is incommensurable with Western philosophy. While the author first offers these statements as hypotheses (p. 2), one premise of his argument that there is such a thing as "the Zen world-view" or "the Zen experience ‘‘remains unquestioned; and another that Nishida's pivotal experience was the Zen experienceâis asserted as "beyond question" (p. 151; see also pp. 28, 48, 10, 114).Wilkinson's presentation of Zen in the first chapter is drawn largely from D. T. Suzuki, with help from Suzuki's portage Abe Masao; some comments on time, birth, and death in; and a few other sources. The result is a caricature that imagines Zen as an invariable, unitary experience. In fact, Dagan’s teachings on practice as the manifestation of enlightenment do not sit well with D. T. Suzuki's emphasis on a satori experience, and Suzuki does not enlighten us about what Zen in its various practices has historically been. The author would have benefited from acquaintance with other presentations by Zen teachers, such as that of ShunryÅ Suzuki'sZen Mind Beginner's Mind, with historical accounts of Zen traditions, and with contrasts between DÅgen's view of time and Sam Â· sÄraand that of other Buddhist philosophers. A little research in contemporary Zen scholarship would have shown how implausible is the talk of "nirvana or awareness of mu" (p. 24), "the basic Zen position [that] never varies" (p. 3), "the Zen vision" (pp. 5, 7), "Zen epistemology" , "Zen experience" (p. 59), "the conception of time involved in Zen" (p. 15), and "the fundamental Zen assertion that there is no absolute time" (p. 16). Even if DÅgen's notion that time and being are nondual (p. 16) epitomized most of Zen, Nishida's early notion of "a transcendent, unchanging reality apart from time" (p. 51) would contrast sharply with it. Equally questionable is Wilkinson's contention that Nishida's "pure experience" is equivalent to Zen satori. The notion of pure experience Nishida developed in his first major work may be pivotal for his entire philosophy, but he never claims or implies that any part of his philosophy

presupposes an exceptional experience like satori that grounds his convictions and renders them unverifiable to those who have not had the experience, as Wilkinson intimates (pp. 55, 159, and 161). Nishida presents his notions of pure experience and intellectual intuition, the grasp of its unity, and later... Perennialism and Constructivism is studied by Randolph T Dible : experiencer or the observer and the transcendental subjectivity itself Pure objectivity is essentially a deconstruction, apophatic and negativa The encountering of the experiencer or observer—transcendental subjectivity itself—at the foundation of the world leads inevitably to the recognition of pure objectivity as ultimate reality (which can be taken as its ultimate deconstruction, analogous to the apophatic or via negativa), from which objects derive their value, weight, significance, meaning or objectivity. In this way, pure objectivity can be seen as the supra-self-evident Axiological Axiom, so to speak, even Unconditional Love, in romantic terms. This axiology (value theory) has a structure inverse to the relationship between transcendental subjectivity as the radical unity of pure self-reference and on the other hand, the world of forms, as mere traces (representations, indications) of the unique, original "first distinction" Spencer-Brown speaks of at the foundation of his calculus. That is, all forms (i.e., distinctions, differences) would reduce to being the first distinction, also known as the marked state, which can be called penultimate reality (pure self-reference or transcendental subjectivity: the Spirit which animates us), except that forms are complimentary to their content, which is their objectivity or value, which would reduce to the unmarked state or ultimate reality. It is the incongruity of form (thoughts; Whitehead's "negative prehensions") and value (feelings; Whitehead's "positive prehensions," or my notion of objectivity, meaning and qualia; in short, the non-formal aspects of experience) that holds forms open and keeps them from absolute reduction. This accounts for the brute, concrete persistence of the "functional illusion"-- to use a term from Dzogchen Buddhism-- of the world. Thus this system has an axiology of metaphysical objectivity grounded on the ideal of pure objectivity as the source of all value, meaning and significance, itself the very fecundity of profundity, which is the motive of drawing the distinction in the first place **(Wikipedia, Kant's writings and Logic Of Sense)** Nano rings were created by accident while intending to make quantum dots. They have interesting optical properties associated with excitons and the Aharonov–Bohm effect. Application of these rings used as light capacitors or a buffer includes photonic computing and communications technology. Analysis and measurement of geometric phases in mesoscopic rings is ongoing. Several experiments, including some reported in 2012, show Aharonov-Bohm oscillations in charge density wave (CDW) current versus magnetic flux, of dominant period h/2e through CDW rings up to 85 **μm** in circumference above 77 K. This behavior is similar to that of the superconducting quantum interference devices (seeSQUID). **Aharonov–Bohm nano rings (Wikipedia) Stability analysis of nano rings would help progressively accentuate the research about the materials division that uses nano rings.** From the profound contemplation and wisdom of the Buddha and Mahabodhisattvas, we know there is no such reality. Instead, egolessness (non-self) is the only path to understand the reality of the deluded life. All existences are subject to the law of causes and conditions. These include the smallest particles, the relationship between the particles, the planets, and the relationship between them, up to and including the whole universe! From the smallest particles to the biggest matter, there exists no absolute independent identity. Egolessness (non-self) implies the void characteristics of all existence. Egolessness (non-self) signifies the non-existence of permanent identity for self and existence (dharma). Sunyata stresses the voidness characteristic of self and existence (dharma). Sunyata and egolessness possess similar attributes. As we have discussed before, we can observe the profound significance of sunyata from the perspective of inter-dependent relationships. Considering dharma-nature and the condition of nirvana, all existences are immaterial and of a void-nature. Then we see each existence as independent of each other. But then we cannot find any material that does exist independent of everything else. So egolessness also implies void-nature! From the observation of all existences, we can infer the theory of nirvana and the complete cessation of all phenomena. From the viewpoint of phenomena, all existences are so different from each other, that they may contradict each other. They are so chaotic. In reality, their existence is illusionary and arises from conditional causation. They seem to exist on one hand, and yet do not exist on the other. They seem to be united, but yet they are so different to one another. They seem to exist and yet they do cease! Ultimately everything will return to harmony and complete calmness. This is the nature of all existence. It is the final resting place for all. If we can understand this reality and remove our illusions, we can find this state of harmony and complete calmness. **Teachings in Chinese Buddhism (6) sunyata (emptiness) in the Mahayana context (Wikipedia) Many models**

have been given concatenating Buddhist doctrines and Vaccum energy. These models essentially throw light on the essence of self(witness consciousness) and the cosmic general ledger (cosmic consciousness) while the imbalance in the gratification deprivation complex always remain unbalanced when it comes to individual general ledger , advocating the evolutionary theory of individual consciousness to cosmic consciousness. It is to be noted that it is not the question of knowing all the transactions but the wisdom of what lies behind the debit- Credit, Debit-Debit, or Credit-Credit principle that is most important for assimilation. Look at this simple thesis to prove the point stated in the foregoing. Thesis: 'The World Has A Beginning In Time, And In Space It Is Also Enclosed In Boundaries.' Proof: 'For If One Assumes That The World Has No Beginning In Time, Then Up To Every Given Moment In Time An Eternity Is Elapsed, And Hence An Infinite Series Of States Of Things In The World, Each Following Another, Has Passed Away. But Now The Infinity Of A Series Consists Precisely In The Fact That It Can Never Be Completed Through A Successive Synthesis. Therefore An Infinitely Elapsed World-Series Is Impossible, So A Beginning Of The World Is A Necessary Condition Of Its Existence, Which Was The First Point To Be Proved (Kant, Critique Of Pure Reason, Tr. By R. Guyer And A. W. Wood (Cambridge: Cambridge University Press, 1998): B 454, P. 470). As Quoted In The Eternal Return: Genesis And Interpretation BY PAUL D'IORIO"The Cyclical Hypothesis, So Heavily Criticized By Nietzsche (VP II 325 And 334), Arises In This Way." In Fact, Nietzsche Was Not Criticizing The Cyclical Hypothesis But Only The Particular Form Of That Hypothesis Presented In Vogt's Work. All Of Nietzsche's Texts Without Exception Speak Of The Eternal Return As The Repetition Of The Same Events Within A Cycle Which Repeats Itself Eternally. If Deleuze's Interpretation Holds That The Eternal Return Is Not A Circle, Then What Is It? A Wheel Moving Centrifugally, Operating A "Creative Selection," "Nietzsche's Secret Is That The Eternal Return Is Selective" Says Deleuze: The Eternal Return Produces Becoming-Active. It Is Sufficient To Relate The Will To Nothingness To The Eternal Return In Order To Realize That Reactive Forces Do Not Return. However Far They Go, However Deep The Becoming-Reactive Of Forces, Reactive Forces Will Not Return. The Small, Petty, Reactive Man Will Not Return. Affirmation Alone Returns, This That Can Be Affirmed Alone Returns, Joy Alone Returns. Everything That Can Be Denied, Everything That Is Negation, Is Expelled Due To The Very Movement Of The Eternal Return. We Were Entitled To Dread That The Combinations Of Nihilism And Reactivity Would Eternally Return Too. The Eternal Return Must Be Compared To A Wheel; Yet, The Movement Of The Wheel Is Endowed With Centrifugal Powers That Drive Away The Entire Negative. Because Being Imposes Itself On Becoming, It Expels From Itself Everything That Contradicts Affirmation, All Forms Of Nihilism And Reactivity: Bad Conscience, Ressentiment..., We Shall Witness Them Only Once. [...] The Eternal Return Is The Repetition, But The Repetition That Selects, The Repetition That Saves. Here Is The Marvelous Secret Of A Selective And Liberating Repetition. There Is No Need To Remind The Reader That Neither The Image Of A Centrifugal Movement Nor The Concept Of A Negativity-Rejecting Repetition Appears Anywhere In Nietzsche's Writings, And Indeed Deleuze Does Not Refer To Any Text In Support Of This Interpretation. Further, One Could Highlight That Nietzsche Never Formulates The Opposition Between Active And Reactive Forces, Which Constitutes The Broader Framework Of Deleuze's Interpretation. For Some Years, Marco Brusotti Has Called Attention To The Fact That Deleuze Introduced A Dualism That Does Not Exist In Nietzsche's Writings. To Be Sure, The German Philosopher Describes A Certain Number Of "Reactive" Phenomena (For Example, In The Second Essay Of The Genealogy Of Morality, § 11, He Talks About "Reactive Affects" [Reaktive Affekte], "Reactive Feelings" [Reaktive Gefühlen], Reactive Men [Reaktive Menschen]); But These Are Nonetheless The Result Of Complex Ensembles Of Configurations Of Centers Of Forces That Remain In Themselves Active. Neither The Word Nor The Concept Of "Reactive Forces" Ever Appears In Nietzsche's Philosophy. The Eternal Return: Genesis And Interpretation BY PAUL D'IORIO Some Parts Are Deleted Due To Spatial Constraints. In the midst of thoughts about the eternal return we find at least two other thematic axes On the one hand, the view of the **world as a constant flux of forces without any goal, law, or rules of becoming. A chaos sive natura de-divinized and de-anthropomorphized which constitutes the "ontological substratum**" of the whole of Nietzsche's reflexions On the other hand, an ensemble of fragments of an anthropologico-sociological character, designing a path of liberation leading to the creation of superior individuals by way of a profound transformation of their instinctual structure. This transformation must be achieved by a practice of solitude and internal struggle towards the liberation from the ancient representations of the world and from the incorporated herd

values. For an analysis of these thematic perspectives, see Paolo D'Iorio, La linea e il circolo. Cosmologia e filosofia dell'eterno ritorno in Nietzsche (Genova: Pantograf, 1995): 233-322. With most of the parameters quantified, with assumptions there must be picturesque view of the self and non self, space and time, science and philosophy with the concatenational consummations. After The Chapter "On Redemption," Where Zarathustra Dares Not Expose His Doctrine, The Eternal Return Begins To Be Enunciated In Part Three Of The Work. In The First Place, It Is The Dwarf Who Formulates It In The Chapter "On The Vision And The Riddle." Facing The "Gate Of The Instant" Which Symbolizes The Two Infinities That Stretch Towards The Past And The Future, The Dwarf Whispers: "All Truth Is Crooked, Time Itself Is A Circle" The Dwarf Represents The Spirit Of Gravity, And He Embodies The Herd Morality, "The Belittling Virtue" Which Is The Title Of Another Chapter From Part III. The Dwarf Can Endure The Eternal Return Without Great Difficulties Because He Has No Aspirations; Unlike Zarathustra He Does Not Wish To Climb The Mountains That Symbolize Elevation And Solitude. In Two Unpublished Notes, From The Summer And The Fall Of 1883, Nietzsche Writes: The Doctrine Is At First Favored By The Rabble, Before It Gets To The Superior Men. The Doctrine Of Recurrence Will First Smile To The Rabble, Which Is Cold And Without Any Strong Internal Need. It Is The Most Ordinary Of Life Instincts, Which Gives Its Agreement First. Hence, The Content Of The Doctrine Is The Same, But Whereas The Dwarf Can Endure It (Because He Interprets It According To The Pessimistic Tradition For Which "Nothing Is New Under The Sun"), Zarathustra, Who Is The "Advocate Of Life" Regards The Eternal Return As The Strongest Objection To Existence, And As The Rest Of The Dream Suggests, He Does Not Yet Succeed In Accepting It After The Vision At The Gate Of The Instant, The Chapter Is Brought To An End By The Enigma Of The Shepherd. The Eternal Return: Genesis And Interpretation BY PAUL D'IORIO .How does the memory affect the GD complex or for that matter the gratification deprivation of an individual or a collective identity? Non-Markovian local-in-time master equations give a relatively simple way to describe the dynamics of open quantum systems with memory effects. Despite their simple form, there are still many misunderstandings related to the physical applicability and interpretation of these equations. Here, we clarify these issues both in the cases of quantum and classical master equations. Further introduction of the concept of a classical non-Markov chain signified through negative jump rates in the chain configuration. **E-M Laine et al 2012 J. Phys. B: At. Mol. Opt. Phys. 45 154004 doi:10.1088/0953-4075/45/15/154004 Local-in-time master equations with memory effects: applicability and interpretation.** Supposing That There Were Indeed An "Energy Of Contraction" Constant In All Centers Of Force Of The Universe, It Remains To Be Explained Where Any Difference Would Ever Originate. It Would Be Necessary For The Whole To Dissolve Into An Infinite Number Of Perfectly Identical Existential Rings And Spheres, And We Would Therefore Behold Innumerable And Perfectly Identical Worlds COEXISTING [Nietzsche Underlines This Word Twice] Alongside Each Other. Is It Necessary For Me To Admit This? Is It Necessary To Posit An Eternal Coexistence On Top Of The Eternal Succession Of Identical Worlds? The Eternal Return: Genesis And Interpretation BY PAUL D'IORIO. Science is usually portrayed as dehumanizing. Brave New World epitomizes this fear. "The more we understand the world, the more it seems completely pointless" (Steven Weinberg). Certainly science can seem chilling when conceived in the abstract as a metaphysical world-picture. We may seem to find ourselves living in a universe with all the human meaning stripped out: Participants in a soulless dance of molecules, or harmonics of pointlessly waggling superstrings and their braneworld cousins. Nature seems loveless and indifferent to our lives. What rights have we to be happy? **(Brave New World: Aldous Huxley).** Religion can afford to be dogmatic but science cannot. Look at this passage which counters both scientists and philosophers alike: However, Hartmann Objects That The Regressive Movement Postulated By Schopenhauer Is Possible Only In Thought: It Remains Nothing More Than An "Ideal Postulate" With No Real Object And Which "Does Not Teach Us Anything About The Real Process Of The World That Unfolds In A Movement Contrary To This Backwards Movement Of Thought" (Hartmann, Philosophie Des Unbewussten, Third Edition (1871): 772). Hartmann Affirms That If Unlike Schopenhauer One Admits The Reality Of Time And Of The World Process, One Must Also Admit That The Process Must Be Limited In The Past And Therefore That There Must Be An Absolute Beginning. In Hartmann's Mind, Failure To Do So Would Result In Positing The Contradictory Concept Of An Accomplished Infinity: "The Infinity That From The Point Of View Of Regressive Thinking, Remains An Ideal Postulate, Which No Reality May Correspond To, Must, For The World, Whose Process Is, On The Contrary, A Progressive Movement, Open Up To A Determinate Result; And Here The

Contradiction Comes To Light" (Hartmann (1871): 772). What Really "Comes To Light" In This Passage Is The Fact That Hartmann Does Not Provide A Demonstration But A Petitio Principii. Indeed, The Concept Of The World Process Analytically Contains The Concept Of A Beginning Of The World. In All Rigors, It Is Therefore Impossible To Demonstrate These Concepts With Reference To Each Other. Secondly, Hartmann's View That One Is Bound To Accept The Reality Of The World Process Even If One Rejects The Ideality Of Schopenhauer's Time Is Mistaken. Hartmann Believes That If Time Is Real There Must Be A World Process With Both An Absolute Beginning And An Absolute End. Without Any Justification, Hartmann Jumps From Schopenhauer's Negated Time To Oriented Time. The Eternal Return: Genesis And Interpretation BY PAUL D'IORIO. In the last years several theoretical papers discussed if time can be an emergent property deriving from) quantum correlations. Here, to provide an insight into how this phenomenon can occur, authors present an experiment that illustrates Page and Wootters' mechanism of "static" time, and Gambini et al. subsequent refinements. A static, entangled state between a clock system and the rest of the universe is perceived as evolving by internal observers that test the correlations between the two subsystems. They implement this mechanism using an entangled state of the polarization of two photons, one of which is used as a clock to gauge the evolution of the second: an "internal" observer that becomes correlated with the clock photon sees the other system evolve, while an "external" observer that only observes global properties of the two photons can prove it is static.Phys. Rev. A 89, 052122 (2014)DOI:10.1103/PhysRevA.89.052122 arXiv: 1310.4691 [quant-ph] **Time from quantum entanglement: an experimental illustration Ekaterina Moreva.** Time is an entanglement .When the new ideas of quantum mechanics spread through science like wildfire in the first half of the 20th century, one of the first things physicists did was to apply them to gravity and general relativity. The results were not pretty. It immediately became clear that these two foundations of modern physics were entirely incompatible. When physicists attempted to meld the approaches, the resulting equations were bedeviled with infinities making it impossible to make sense of the results. Then in the mid-1960s, there was a breakthrough. The physicists John Wheeler and Bryce DeWitt successfully combined the previously incompatible ideas in a key result that has since become known as the Wheeler-DeWitt equation. This is important because it avoids the troublesome infinites—a huge advance. But it didn't take physicists long to realise that while the Wheeler-DeWitt equation solved one significant problem, it introduced another. The new problem was that time played no role in this equation. In effect, it says that nothing ever happens in the universe, a prediction that is clearly at odds with the observational evidence. This conundrum, which physicists call 'the problem of time', has proved to be thorn in flesh of modern physicists, who have tried to ignore it but with little success. Then in 1983, the theorists Don Page and William Wootters came up with a novel solution based on the quantum phenomenon of entanglement. This is the exotic property in which two quantum particles share the same existence, even though they are physically separated. Entanglement is a deep and powerful link and Page and Wootters showed how it can be used (e) to measure time. Their idea was that the way pair of entangled particles evolves is a kind of clock that can be used to measure change. But the results depend on how the observation is made. One way to do this is to compare the change in the entangled particles with an external clock that is entirely independent of the universe. This is equivalent to god-like observer outside the universe measuring the evolution of the particles using an external clock. In this case, Page and Wootters showed that the particles would appear entirely unchanging—that time would not exist in this scenario. But there is another way to do it that gives a different result. This is for an observer inside the universe to compare the evolution of the particles with the rest of the universe. In this case, the internal observer would see a change and this difference in the evolution of entangled particles compared with everything else is an important a measure of time. This is an elegant and powerful idea. It suggests that time is an emergent phenomenon that comes about because of the nature of entanglement. And it exists only for observers inside the universe. **Time is an entanglement** (Wikipedia). Danish religious philosopher Soren Kierkegaard carried out a systematic critique of the pretensions of reason and an abstract rationalism which he believed that the modern age was nurturing. Condemning reflection as a "danger" that ensnares people in logical delays and machinations, Kierkegaard compared it to a prison. Reflection is for him a form of captivity, a bondage which "can only be broken by [passionate] religious inwardness" (1978: 81). **Reflection seduces individuals into thinking its possibilities are "much more magnificent than a paltry decision"** (1978: 82). It leads them to act **"on principle,"** to dwell on the deliberation of the context of their actions and the calculation of their worth or outcome. Kierkegaard argues that this drives away feeling, inspiration, and

spontaneity, all of which are crucial for true inner being and a vital relation to God. For Kierkegaard, as Nietzsche would later agree, genuine inner being (and culture) is characterized by the tautness and tension of the soul which characterizes passionate existence. But the "coiled springs of life relationships ... lose their resilience" in reflection (1978: 78) and "everything becomes meaningless externality, devoid of [internal] character" (1978: 62). Kierkegaard thus contributes to the development of an irrationalist tradition that has echoes in some later postmodern thought. Kierkegaard might well have agreed with his contemporary Fyodor Dostoyevsky, who wrote: "An intelligent [reflective] man cannot seriously become anything ... excessive consciousness is a disease" (1974: 3, 5). In an age overtaken by rules and regulations, genuine action -- which Kierkegaard assumes to be subjective and spontaneous -- is frustrated at every turn. Complaining that we are too "sober and serious" (1978: 71) even at banquets, Kierkegaard bemoans the fact that even suicides are premeditated (1978: 68)! "That a person stands or falls on his actions is becoming obsolete; instead, everybody sits around and does a brilliant job of bungling through with the aid of some reflection and also by declaring that they all know very well what has to be done" (1978: 73). Thus, it is passion, not reflection, that guarantees "a decent modesty between man and man [and] prevents crude aggressiveness" (1978: 62). "Take away the passion and the propriety also disappears" (1978: 64). The ambiguity in the word "passion" may cause some confusion here. To say that the age and its individuals are "passionless" is not to say there are no emotions whatsoever, but rather that there is no true spiritual inwardness and depth, no intensively motivated action and commitment. It suggests that passion exists only in a simulated, pseudo-form, "the rebirth of passion" through "talkativeness" (1978: 64). "Chattering" for Kierkegaard gets in the way of "essential speaking" and merely "reflects" inconsequential events (1978: 89-99). Hence, in “the present age,” emotions -- which in fact are all too pronounced -- have been transformed into negative forces. Anticipating Nietzsche's genealogy of the "slave revolt" in morality, Kierkegaard claims that the "enthusiasm" of the prior age of Revolution, a "positively unifying principle,” has become a vicious "envy," a "negatively unifying principle" (1978: 81), a leveling force in its own right insofar as those lacking in talent and resources want to tear down those who have them. **THE POSTMODERN TURN IN PHILOSOPHY: THEORETICAL PROVOCATIONS AND NORMATIVE DEFICITS By Steven Best and Douglas Kelner http://www.gseis.ucla.edu/faculty/kellner/kellner.html.** In November 2011, the United States government argued before the US Supreme Court that it wants to continue utilizing GPS tracking of individuals without first seeking a warrant. In response, Justice Stephen Breyer questioned what this means for a democratic society by referencing Nineteen Eighty-Four. Justice Breyer asked, "If you win this case, then there is nothing to prevent the police or the government from monitoring 24 hours a day the public movement of every citizen of the United States. So if you win, you suddenly produce what sounds like 1984... "[65]. What is to be said in unmistakable terms is that when it comes to national security and innocent citizens there is question of compromise even it be an infringement and encroachment of so called privacy which has been made a word to subterfuge the activities that are far said to be fair. Quantum theory has provoked intense discussions about its interpretation since its pioneer days. One of the few scientists who have been continuously engaged in this development from both physical and philosophical perspectives is Carl Friedrich von Weizsäcker. The questions he posed were and are inspiring for many, including the authors of this contribution. Weizsäcker developed Bohr’s view of quantum theory as a theory of knowledge. **Harald Atmanspacher, Hans Primas** show that such an epistemic perspective can be consistently complemented by Einstein’s ontically oriented position. **Time, Quantum and Information 2003, pp 301-321 Epistemic and Ontic Quantum Realities Harald Atmanspacher, Hans Primas Robert W. Spekkens** present a toy theory that is based on a simple principle: the number of questions about the physical state of a system that are answered must always be equal to the number that are unanswered in a state of maximal knowledge. Many quantum phenomena are found to have analogues within this toy theory. These include the noncommutativity of measurements, interference, the multiplicity of convex decompositions of a mixed state, the impossibility of discriminating nonorthogonal states, the impossibility of a universal state inverter, the distinction between bipartite and tripartite entanglement, the monogamy of pure entanglement, no cloning, no broadcasting, remote steering, teleportation, entanglement swapping, dense coding, mutually unbiased bases, and many others. The diversity and quality of these analogies is taken as evidence for the view that quantum states are states of **incomplete knowledge** rather than states of reality. A consideration of the phenomena that the toy theory **fails to reproduce, notably, violations of Bell inequalities** and the existence of a Kochen-Specker theorem,

provides clues for how to proceed with this research program.DOI: http://dx.doi.org/10.1103/PhysRevA.75.032110 **Evidence for the epistemic view of quantum states A toy theory Phys Rev A 75, 032110 – Published 19 March 2007 Robert W. Spekkens In such specialized and autonomous areas, it is to the eminent and erudite authors to interpret the contradistinctions and contradictions' in the dove tailing explaining. This I feel would be yeoman service in the creation of Nature's general ledger. M. S. Leifer and O. J. E. Maroney** examine the relationship between quantum contextuality (in both the standard Kochen-Specker sense and in the generalized sense proposed by Spekkens) and models of quantum theory in which the quantum state is maximally epistemic. We find that preparation noncontextual models must be maximally epistemic, and these in turn must be Kochen-Specker noncontextual. This implies that the Kochen-Specker theorem is sufficient to establish both the impossibility of maximally epistemic models and the impossibility of preparation noncontextual models. The implication from preparation noncontextual to maximally epistemic then also yields a proof of Bell's theorem from an Einstein-Podolsky-Rosen-like argument. DOI: http://dx.doi.org/10.1103/PhysRevLett.110.120401 Received 25 August 2012Published 20 March 2013 © 2013 American Physical Society **Maximally Epistemic Interpretations of the Quantum State and Contextuality Phys. Rev. Lett 110, 120401 – Published 20 March 2013 M. S. Leifer and O. J. E. Maroney.** Epistemological, onitic and ontological aspects particularly of quantum mechanics have invited profound thinkers to contribute their mite to the growing literature on the subject, which includes the contributions from the subterranean realm and ceratoid dualism of philosophy like that Chaos theory of Deleuze, Derrida , Kierkegaard and others. **Andrei Khrennikov** shows that the Dirac-von Neumann formalism for quantum mechanics can be obtained as an approximation of classical statistical field theory. This approximation is based on the Taylor expansion (up to terms of the second order) of classical physical variables – maps f: $\Omega \rightarrow R$, where Ω is the infinite-dimensional Hilbert space. The space of classical statistical states consists of Gaussian measures ρ on Ω having zero mean value and dispersion $\sigma 2(\rho) \approx h$. This viewpoint to the conventional quantum formalism gives the possibility to create generalized quantum formalisms based on expansions of classical physical variables in the Taylor series up to terms of nth order and considering statistical states ρ having dispersion $\sigma 2(\rho) = h\ n$ (for $n = 2$ we obtain the conventional quantum formalism). **December 2005, Volume 18, Issue 7, pp 637-650 Date: 28 Nov 2005 Generalizations of Quantum Mechanics Induced by Classical Statistical Field Theory Andrei Khrennikov** It is widely accepted that consciousness or, in other words, mental activity is in some way correlated to the behavior of the brain or, in other words, material brain activity. Since quantum theory is the most fundamental theory of matter that is currently available, it is a legitimate question to ask whether quantum theory can help us to understand consciousness. Several approaches answering this question affirmatively, proposed in recent decades, will be surveyed. It will be pointed out that they make different epistemological assumptions, refer to different neurophysiological levels of description, and adopt quantum theory in different ways. For each of the approaches discussed, these imply both problematic and promising features which will be indicated. **Discrete Dynamics in Nature and Society Volume 2004 (2004), Issue 1, Pages 51-73 http://dx.doi.org/10.1155/S102602260440106X Quantum theory and consciousness: an overview with selected examples Harald Atmanspacher Copyright © 2004 Hindawi Publishing Corporation. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.** Another thought provoking and investigatory article is by Peter G. Lewis et al. Perhaps the quantum state represents information about reality, and not reality directly. Wave function collapse is then possibly no more mysterious than a Bayesian update of a probability distribution given new data. **Peter G. Lewis et al** consider models for quantum systems with measurement outcomes determined by an underlying physical state of the system but where several quantum states are consistent with a single underlying state—i.e., probability distributions for distinct quantum states overlap. Significantly, they demonstrate by example that additional assumptions are always necessary to rule out such a model. DOI: http://dx.doi.org/10.1103/PhysRevLett.109.150404 **Distinct Quantum States Can Be Compatible with a Single State of Reality Phys. Rev. Lett. 109, 150404 – Published 9 October 2012 Peter G. Lewis et al** Communication complexity of a quantum channel is the minimal amount of classical communication required for classically simulating a process of state preparation, transmission through the channel and subsequent measurement. It establishes and perpetuates a limit on the power of quantum communication in terms of classical resources. **Alberto**

Montina shows that classical simulations employing a finite amount of communication can be derived from a special class of hidden variable theories where quantum states represent statistical knowledge about the classical state and not an element of reality. This special class has attracted strong interest very recently. The communication cost of each derived simulation is given by the mutual information between the quantum state and the classical state of the parent hidden variable theory. Finally, find that the communication complexity for single qubits is smaller than 1.28 bits. The previous known upper bound was 1.85 bits. DOI: http://dx.doi.org/10.1103/PhysRevLett.109.110501 **Epistemic View of Quantum States and Communication Complexity of Quantum Channels Phys. Rev. Lett 109, 110501 – Published 12 September 2012 Alberto Montina for models see elsewhere .** Stability analysis of what? Quantum state representing reality or our knowledge of reality? Does the quantum state represent reality or our knowledge of reality? In making this distinction precise, **Nicholas Harrigan, Robert W. Spekkens** are led to a novel classification of hidden variable models of quantum theory. They show that representatives of each class can be found among existing constructions for two-dimensional Hilbert spaces. Approach also provides a fruitful new perspective on arguments for the nonlocality and incompleteness of quantum theory. Specifically, **Nicholas Harrigan, Robert W. Spekkens** show that for models wherein the quantum state has the status of something real, the failure of locality can be established through an argument considerably more straightforward than Bell's theorem. The historical significance of this result becomes evident when one recognizes that the same reasoning is present in Einstein's preferred argument for incompleteness, which dates back to 1935. This fact suggests that Einstein was seeking not just any completion of quantum theory, but one wherein quantum states are solely representative of our knowledge. Hypothesis is supported by an analysis of Einstein's attempts to clarify his views on quantum theory and the circumstance of his otherwise puzzling abandonment of an even simpler argument for incompleteness from 1927. **February 2010, Volume 40, Issue 2, pp 125-157 Date: 09 Jan 2010 Einstein, Incompleteness, and the Epistemic View of Quantum States Nicholas Harrigan, Robert W. Spekkens** statistical model of the probabilistic description of physical reality has been studied by many authors. **Andrei Khrennikov posits** a contextual statistical model of the probabilistic description of physical reality. Here contexts (complexes of physical conditions) are considered as basic elements of reality. There is discussed the relation with QM. He proposes a realistic analogue of Bohr's principle of complementarity. In the opposite of the Bohr's principle, **Andrei Khrennikov** principle has no direct relation with mutual exclusivity for observables. To distinguish his principle from the Bohr's principle and to give better characterization, he changes the terminology and speaks about supplementarity, instead of complementarity. Supplementarity is based on the interference of probabilities. It has quantitative expression trough a coefficient which can be easily calculated from experimental statistical data. There is need not appeal to the Hilbert space formalism and noncommutativity of operators representing observables. Moreover, in our model there exists a pair of supplementary observables which cannot be represented in the complex Hilbert space. There are discussed applications of the principle of supplementarity outside quantum physics. **October 2005, Volume 35, Issue 10, pp 1655-1693 The Principle of Supplementarity: A Contextual Probabilistic Viewpoint to Complementarity, the Interference of Probabilities and Incompatibility of Variables in Quantum Mechanics Andrei Khrennikov Foundations of Physics** Description of the vacuum in Yang-Mills theory and arrive at a physical interpretation of the pseudoparticle solution and the attendant violation of symmetries has been done by **R. Jackiw and C. Rebbi.** It is here vaccuum stability forms the bastion, pillar, post and stylobate of the mechanisms of yang mills Theory and action of a particle. The existence of topologically inequivalent classical gauge fields gives rise to a family of quantum mechanical vacua, parametrized by a CP-nonconserving angle. The requirement of **vacuum stability** against gauge transformations renders the vacua chirally noninvariant. DOI: http://dx.doi.org/10.1103/PhysRevLett.37.172 **Vacuum Periodicity in a Yang-Mills Quantum Theory Phys. Rev. Lett 37, 172 – Published 19 July 1976 R. Jackiw and C. Rebbi.** Three-dimensional Yang-Mills and gravity theories augmented by gauge-invariant mass terms are analyzed. These topologically nontrivial additions profoundly alter the particle content of the models and lead to quantization of a dimensionless mass-coupling-constant ratio. The vector field excitations become massive, with spin 1 (rather than massless with spin 0), and the mass provides an infrared cutoff. The gravitation acquires mass, mediates finite-range interactions, and has spin 2 (rather than being absent altogether); although its mass term is of third derivative order, there are no ghosts or acausalities. DOI:

http://dx.doi.org/10.1103/PhysRevLett.48.975 © 1982 The American Physical Society **Three-Dimensional Massive Gauge Theories Phys. Rev. Lett 48, 975 – Published 12 April 1982 S. Deser et al** All scattering amplitudes in the maximally supersymmetric N=4 super-Yang–Mills theory possess a new, dual superconformal symmetry which extends the previously discovered dual conformal symmetry of MHV amplitudes. To reveal this property we formulate the scattering amplitudes as functions on the appropriate dual superspace. Rewritten in this form, all tree-level MHV and next-to-MHV amplitudes exhibit manifest dual superconformal symmetry. We propose a new, compact and Lorentz covariant formula for the tree-level NMHV amplitudes for arbitrary numbers and types of external particles. The dual superconformal symmetry is broken at loop level by infrared divergences. However, we provide evidence that the dual conformal anomaly of the MHV and NMHV superamplitudes is the same and, therefore, their ratio is dual conformally invariant. We show this explicitly for the six-particle amplitudes at one loop. Authors conjecture that these properties hold for all, MHV and non-MHV, superamplitudes in N=4 SYM both at weak and at strong coupling. **Nuclear Physics B Volume 828, Issues 1–2, 21 March 2010, Pages 317–374 Dual superconformal symmetry of scattering amplitudes in N=4 super-Yang–Mills theory J.M. Drummonda DOI: 10.1016/j.nuclphysb.2009.11.022.** By comparison with numerical results in the maximal Abelian projection of lattice Yang–Mills theory, it is argued that the nonperturbative dynamics of Yang Mills theory can be described by a set of fields that take their values in the coset space SU(2)/U(1). The Yang–Mills connection is parameterized in a special way to separate the dependence on the coset field. The coset field is then regarded as a collective variable, and a method to obtain its effective action is developed. It is argued that the physical excitations of the effective action may be knot solitons. A procedure to calculate the mass scale of knot solitons is discussed for lattice gauge theories in the maximal Abelian projection. The approach is extended to the SU (N) Yang–Mills theory. A relation between the large N limit and the monopole dominance is pointed out. **Physics Letters B Volume 458, Issues 2–3, 8 July 1999, Pages 322–330 An effective action for monopoles and knot solitons in Yang–Mills theory Sergei V. Shabanov DOI: 10.1016/S0370-2693(99)00612-7.** It is here the quintessential formulation of consciousness occurs. Like a man standing on the threshold of infinity trying to ponder what lies beyond the veil, which separates the seen from unseen a true seeker tries to lift the individual general ledger to the cosmic general ledger. Like in Darwinian evolution, mind also has to evolve from individual consciousness to cosmic consciousness. In Kaishmiri Shaivism one withdraws from the finite to the infinite, but one also goes on an outward journey from the infinite to the finite, because both the finite and the infinite have an intimate connection. The finite is not seen as unreal, but as a symbol of the infinite. There is no real distinction between them. Those two movements constitute Spanda, a key concept in Kaishmiri Shaivism. Spanda is the pulsation of the Absolute in different phases of being. There are no opposites like subject and object, unity or duality, absolute or relative. They are just different phases of the universal vibration of the Absolute. The goal is to realize or be at once infinite and finite. One does not turn away from appearances (like in Advaita Vedanta), but one realizes that the Absolute manifests all things. Spanda, the eternal pulsation of the Absolute, oscillates between a passion to create and dispassion from the created. Through it the Absolute transform itself into all things and then returns back into the emptiness of its undifferentiated nature. In Kaishmiri Shaivism the Absolute is seen as pure consciousness (=being). The Absolute is an eternal all-pervasive principle, the highest reality, the nature of all entities eternally and blissfully at rest within its own nature. The Absolute is the nature of the Self (and thus of us all). The Absolute is divine, it is Shiva, the Lord of the Universe. It is full of conscious activity through which it generates the universe, and reabsorbs it into itself at the end of each cycle of creation. Thus we speak of monism, as everything resides within this one absolute consciousness. It sustains all things, it embraces all things, and it pervades all things. All things are appearances within the absolute consciousness, but nevertheless real (in contrast to Advaita Vedanta where appearances are seen as unreal or illusionary). All things appear external (out there, outside ourselves), but they do not have a being on their own. They do not exist as separate entities on their own. Everything is contained within consciousness. What we see as objects are manifestations of consciousness. The event which constitutes the universe is always internal events happening within consciousness because their essential nature is consciousness itself. If a physical object were totally material, and independent or external to consciousness, it could never be experienced. The universe and consciousness are two aspects of a whole. The universe is an attribute of consciousness which bears consciousness as its substance. Doctrine of Vibration: S.G.Dyczkowski What you "see" is not always what you "get." Many people

mistakenly take their own visions literally without "seeing through" the various possibilities that are outside their belief system, knowledge or skill base. Deeper reality is not remote in the physical sense but in a psychological sense. The archetypes of the collective unconscious are arrayed behind the scenes of current worldwide conditions, of crisis and confusion. They mirror our own states back at us, whether we perceive them as such or interpret them plausibly or not. The noise of ordinary consciousness and beliefs drowns out the signal. Unconsciousness is the background of our ordinary awareness. Our organism is very much at the center of such effects. The organismic source is our human bodies and the focus of human consciousness. The fantasy principle dethrones reality, but can be dissociative or compensatory. The human mind is a meme-Scape. Pre-conceived concepts vie with structures, concepts with images. Like scientists who ignore assumed truths, we leapfrog over our beliefs and personality deficits, claiming idiosyncratic imagination is literal reality. It couldn't be further from the truth and symbolism is utterly lost. The metaphor that might heal us enslaves us. Perhaps images like the holographic universe have an implicate order. Can we have a sense of the cosmos in the world without projecting myriad fantasies on it that we embrace literally? Has the world become so horrible it is unreasonable to be realistic? We may need to look at our drives and wishes, rather than the fantasy content. Psyche constructs reality. Our experience of so-called reality is always mediated by our image of it. Even if all the contents of the psyche are real, that doesn't mean they are realistic. That psyche is real is still a radical proposition, but psychic politics certainly color the self-image and ideas of everyone. We observe and participate with images It is not a question of nature or nurture (genes alone or experience alone). Rather, everything is both. We inherit the structures that make our experience what it is. But the structure itself is “empty,” and each human culture “fills” it with its own specific adaptations. It is difficult to define an archetype and set boundaries that distinguish it from others. In a hologram each part contains all the information but in lower resolution. Archetypes have this holographic quality. There are patterns within patterns within patterns. Some overlap with others, and some are nested inside others. Archetypal realities, passed on through DNA, are expressed in distinctive neuronal tracts in the brain. They include customs and laws regarding property, incest, marriage, kinship, and social status or roles; myths and legends; beliefs about the supernatural; gambling, adultery, homicide, schizophrenia, and the therapies to deal with them. A mythic and visionary language of immediate experience encompasses themes of deepest, highest, and ultimate concern. Most fantasy-based individuals are at a complete loss to coherently explain their own conventional behavior much less anomalous events and their deep meaning, much less the cultural unconscious or mythological unconscious matrix. But they try, and become utterly entrenched in their belief that they are right about the nature of the world and reality. We have pseudo-memories about our personal lives. Why not more so for our collective life? The subject matter often revolves around catastrophe, creation and the mythopoeic forces of mankind. Ignorant of such dynamics, interpretive mistakes and displaced psychic contents proliferate into errors of fact. Propaganda, media distortions, memes, and disinformation compound the social problem of misapprehension further. Shameless self-promotion by personalities of such ideas leads to cults. They make up myths about the myths of by-gone eras. Roiling unconscious images can be fatally confusing. Thought illusions culminate in projections and projections of mythology. Jung suggested symbols live only as long as they are pregnant with meaning. Philosophy arose from criticism of myth, from discussing and challenging it. In science, we criticize, reject and eliminate theories. At the edge of the abyss of the unknown, new signs and symbols emerge. Credible theories and paradigms must include biology, physics, and neurophysiology. One of the reasons people "see God", or a guru, or anomalies may be because our brains are constructed to see reality through the eyes of others. There are heaps of mirror neurons which are there to make us feel the 'other'. Mirror neurons do for psychology what DNA did for biology. They provide a unifying framework and help explain a host of mental abilities. As in the psychochemical processes of empathy or falling in love, a complex feedback loop sustains a state of mind. But when we empathically transpose ourselves into someone else's position, we expose ourselves to that reality -- cognitively and emotionally. The unconscious complicates empathy, both ways. Mirror neurons might well play a role in bonding, language and self-awareness. Naively, we take too much as self-evident. But 'seeing' does not always ‘believe', though many make this error or leap in logic and formulate their choices and future accordingly. Yet, there is only one way to learn what consciousness is. Experience. But we have no satisfactory explanatory edifice for consciousness. Would such a theory release in each of us our own inner knowledge of the creativity of our own consciousness, and its infinite possibilities? The problem is trying to define a

verb, a dynamic, as if it were a noun. But we do recognize the effect of consciousness. It functions to mediate states of consciousness, high and low psychobiological arousal. Consciousness is the subconscious lifted up by the physical body. When the body fails, the consciousness collapses back into the subconscious. All our thoughts come from the subconscious which can see our intentions but not our world. This relates somehow to intention being imaginary and not of the physical frictionized world (King). Gerald Edelman postulates that the flows of information in the brain are mediated through 're-entrant' feedback loops. As evolution provides new cognitive functions, new re-entrant loops are established. Even language itself is an archetype -- a chaotic field of dynamic associations. A subtle net of tropes, grammar, symbols, and meaning, the program language begins in limbic resonance. Some phenomena generate their own language patterns, nomenclature, and internal coherence of meaning and representation. In a holographic universe, even time and space could no longer be viewed as fundamentals. Because concepts such as location break down in a universe in which nothing is truly separate from anything else, time and three-dimensional space would also have to be viewed as projections of this deeper order. At its deeper level reality is a sort of super hologram in which the past, present, and future all exist simultaneously. This suggests that given the proper tools it might even be possible to someday reach into the super holographic level of reality and pluck out scenes from the long-forgotten past. Or not. A fantasy of such penetration or phenomenon inside the head is not the same as that penetration. **Jung in the 21st Century: Synchronicity and science By John Ryan Haule Mind Control Countermeasures.** The lower domain of experience is what we do not directly observe, the quantum realm, while the higher level is what we ordinarily observe, the classical realm. Based purely on experiment, the formulation of quantum theory initially placed the great divide between the quantum and classical domains between the measuring apparatus and the particle (Herbert, 1985). However, in the early 1930s, John von Neumann, in his rigorous mathematical treatise on quantum mechanics, found no support for such a division, compelling him to conclude that the wave for was collapsed by consciousness. Von Neumann's treatise has been called the "quantum bible" and "the most influential book on quantum theory ever written" (Herbert, 1985). The von Neumann formulation leads us to conclude that the lower realms of quantum reality form a virtually limitless array of potentials that are inherently incapable of realization without the observation of a knowing entity, which we identify with consciousness (Kafatos and Nadeau, 1990). We thus have a correspondence between quantum theory, the hierarchical structure implied by the Holographic Principle, and the higher orders of experience, leading to the full expression of the Conscious Universe. David Bohm's theory of the implicate and explicate orders involves a holographic Principle that is fully consistent with the Holographic Principle discussed here (Bohm 1980; 1986; Bohm and Hiley, 1993). According to Bohm, there is an implicate order that represents the universal, holographic subtext of reality, and which unfolds in every moment to produce the explicate order that we all observe. Thomas Germinario (2004) has equated the implicate order with the unconscious process, and the explicate order with conscious process. He emphasizes the nature of dreams within the implicate order, and the importance of the dream work for maintaining a healthy mind through integration of the implicate order and our daily lives. Allan Combs and Mark Holland (1990) connected the implicate order or holomovement (Bohm, 1980) with Carl Jung is theory of synchronicity (Jung and Pauli, 1955), with the implicate order providing a holographic medium through which apparently disconnected individuals become connected. The principle of synchronicity, the instantaneous connection of people and events beyond the senses, has been equated with the quantum-physical principle of non-locality (Combs and Holland, 1990; Germine, 1991), and has been proposed to be the fundamental mechanism of conscious process (Germine, 1991).Applying the Holographic Principle theory of mind, subjects may be connected synchronously in the manner of two individuals conversing through cellular phones, where the radio signals are transmitted through a distant satellite. The difference here is that the radio signals travel at the speed of light, and thus the there is some miniscule time lapse between the sending and receiving ends. The Holographic Principle implies a much more distant and instantaneous communication pathway, with the analogue of the satellite being the holographic boundary of the Universe, as well as a much richer communication, potentially involving a transmission of thoughts and feelings. Our ego-consciousness seems to mask the universal relatedness implied by the Holographic Principle, and it is perhaps only through transcendence of the ego-consciousness that the higher orders of experience can become conscious. In the phenomenon of synchronicity, there seems to be a meaningful connection between individuals that breaks through the barrier of ego-consciousness. Such a connection is reported

by many individuals in the course of dreams, when the ego-consciousness has been suspended, at times informing the dreamer of something that has happened in the life of a meaningfully-connected individual. **The Holographic Principle Theory of Mind MARK GERMINE Institute for Psycho science** Impassioned-intense Connectionist neuroscience, in an effort to attain relevance to clinical practice, has recently given us a theory of the relationship of the unconscious to consciousness which is dualistic (Viamontes and Beitman, 2007), an ad hoc theory of conclusions based on assumptions, the most erroneous of which are that representations of information are information, and that such representations elaborate themselves through a process of representations of representations in neuronal circuits (Bechtel and Abrahamsen,1991). As theory, the connectionist models are lacking heuristic value and **are inimical to the progress** of basic and clinical neuroscience. This is not to denigrate or minimize the importance of neuronal connections, but these connections are rather like the wires in a radio that plays a symphony. The radio, in and of itself, is incapable of playing the symphony. The radio doesn't do not write the symphony, nor does it broadcast the Passionate-raging symphony over the airways. Yet, if we loosen one wire, the symphony is not manifested through the radio. The brain is not subject to the same vulnerability, in that it has parallel and redundant networks of processing, and, it is in this sense that the connectivity of the brain becomes important. Connectionism, to the extent that it accepts consciousness and the unconscious at all, attributes them to separate groups of "circuits" in the brain, with consciousness-processing circuits having a limited but detailed ability to analyze information, and unconscious-processing processing more information in less detail (Viamontes and Beitman, 2007). There is an element of truth in this view in that information that has importance and meaning resonates within the brain at higher levels of experience and at higher levels of holographic recursion. This resonance involves what we know, what we feel, and what we are capable and willing to consciously realize. The Universe, as William Blake noted, can be seen in a grain of sand. Indeed, the grain of sand requires the Universe to exist. The human organism requires the Self to exist. The Self is not a simple amalgam of sensory experiences, as suggested in the connectionist model (Blinder, 2007). This amalgam view of the Self can be damaging when applied to psychotherapy, as the connectionists suggest (Blinder, 2007). The Self is ours forever and ever, from eternity to eternity. As Milton suggested, the mind is its own place. Our deepest purpose is for our minds to resonate with the supra-conscious levels of experience, much as it is purpose of the radio to play the symphony. If the symphony is not to our liking, we can turn the radio off. So, perhaps, the transpersonal consciousness can turn our own individual and collective radios off, or perhaps turn the volume down, if we are not in harmony with the Universal Mind. Physics has given us many enigmas. Quantum theory itself is an enigma. But some of the enigmas seem to involve the limitation of our own abilities of discernment. The thought experiment of "Schrödinger's Cat," as it is often interpreted (e.g. Gribbin, 1984), assumes that the cat is not conscious, and is incapable of observation in the context of collapse of the wave function. This is due, in large part, to the false dichotomy of information and experience. Our observation of other animals indicates, to those that are attuned to the animal mind, who can "feel" the animal mind, that, at the very least, all mammals are conscious. It would seem that the great neuropsychiatrist, Stanley Cobb (1948), may have been correct in his attribution of levels of consciousness to a wide variety of organisms: "...lower animals with no cerebrum appear to be conscious...even plants such as the sunflower that turns towards strong light may have a vague awareness or warmth and comfort. There are many degrees of consciousness and it is my contention that it is integrated at many levels like other important functions of the central nervous system."With respect to the theory of mind, recent mainstream thinking remains classical and mechanistic. Such views of mind are advocated by our most in prominent neuroscientists (e.g. Changeux, 1985; Gazzaniga, 1985) as neuroscience research attempts to explain all mental function on the basis of pure mechanism and the localized function of specific areas of the brain. Francis Crick, the distinguished Nobel Laureate, had a very influential second career as a neuroscientist. He concluded that we are basically soulless creatures, that all mental processes could be reduced to identifiable, neural correlates in the brain, and that, in fact, we are nothing but a collection of neurons (Crick, 1995). Crick even went so far as to say that what we call the soul is a group of neurons located in the prefrontal lobes of the brain. Another Nobel Laureate, Gerald Edelman (1987), has written a critically acclaimed book on "neural Darwinism," which purports that the brain is "circuitry" is constructed through a developmental process of "survival of the fittest" neurons and neuronal connections. At the same time the total purposelessness and lack of direction in evolution is expounded, again to critical acclaim, by Richard Dawkins (1986). The idea that

there essentially no self or soul imbued with agency and self determination was expounded by Gilbert Ryle (1949/2002) with his parody of the "ghost in the machine," and this work continues to be influential among many scholars of the mind. Philosophers such as Patricia Churchland (1986), who coined the term "neurophilosophy," take an "eliminative materialist" view of belief, free will, and consciousness. Other philosophers, such as Daniel Dennett (1991) would make experience "epiphenomenal," a by-product of brain processes with no effect whatsoever on the function of mind. However, unlike neuroscience, which is still in its infancy, the philosophy of mind has had a long history, including such notable figures as Plato, Aristotle, Descartes, Kant, Locke, Hume, Kierkegaard, and James, to name just a few. Perhaps, as noted by Ervin Laszlo (1974): "in today's world, most of the traditional functions of cognitive synthesis have atrophied and are ignored and neglected." Hopefully, what we have presented here will be a step in the right direction for what Laszlo calls a "conceptual synthesis" to help fill the need for meaningful engagement is such a world. The fundamental elements of such a synthesis have been described by Laszlo as follows: Conceptual synthesis performs at least five basic functions in the guidance of affairs. They are the mystical, the cosmological, the sociological, the pedagogical or psychological, and the editorial functions. The mystical function inspires in man a sense of mystery and profound meaning related to the universe and of himself in it. The cosmological function forms images of the universe in accord with local knowledge and experience, enabling men to describe and identify the structure of the universe and the forces of nature. The sociological function validates supports and enforces social order, representing it in accord with the nature of the universe, or as the natural or right form of social organization. The pedagogical or psychological function guides individuals through stages of life, teaching ways of understanding themselves and others and presenting desirable responses to life is challenges and trials. Finally, the editorial function of conceptual is to define some aspects of reality as important and credible and hence to be attended to, and other aspects unworthy of serious attention. **The Holographic Principle Theory of Mind MARK GERMINE Institute for Psycho science** Kant sums up this inversion and its spirit early in the Critique of Pure Reason: Up to now it has been assumed that all our cognition must conform to the objects; but all attempts to find something about them a priori through concepts that would extend our cognition have, on this presupposition, come to nothing. Hence let us once try whether we do not get farther with the problems of metaphysics by assuming that the objects must conform to our cognition, which would agree better with the requested possibility of an a priori cognition of them, which is to establish something about objects before they are given to us. This would be just like the first thoughts of Copernicus, who, when he did not make good progress in the explanation of the celestial motions if he assumed that the entire celestial host revolves around the observer, tried to see if he might not have greater success if he made the observer revolve and left the stars at rest. (B xvi) Onticology, like all variations of object-oriented ontology, is realist in its orientation. In defending a realist ontology onticology holds that the vast majorities of objects, actants, beings, or entities are independent of humans and are what they are regardless of whether any humans regard them or register them. In short, onticology rejects any anthropomorphic, idealist, or anti-realist thesis to the effect that to be is to be the correlate of mind, spirit, the body, the human, and language or otherwise. While it is certainly the case that knowledge is necessarily dependent on the object to which it relates, the converse does not hold true. Objects are not dependent on being known, regarded, perceived, or spoken about. As such, and to put it in Aristotlean terms, knowledge is an accident of objects, not objects an accident of knowledge. As Althusser so nicely puts it, "[n]o doubt there is a relation between thought-about-the-real and thisreal, but it is a relation of knowledge, a relation of adequacy or inadequacy of knowledge, not a real relation, meaning by this a relation inscribed in that real of which the thought is the (adequate or inadequate) knowledge" (Reading Capital, 96). Althusser goes on to remark that "[t]he distinction between a relation of knowledge and a relation of the real is a fundamental one: if we did not respect it we should fall irreversibly into either speculative or empiricist idealism" (ibid.). Onticology categorically endorses Althusser's verdict. It is a fundamental necessity to distinguish between those relations that belong to the object and those that belong to knowledge. Contemporary philosophy continuously confuses these two very different sorts of relations. Naturally the question arises of how it is possible to surmount our relation to the object so as to determine whether objects themselves possess the properties we encounter in relating to objects. In other words, given that we can only ever relate to the object in relating to the object how is it possible to surmount this relation to get at the being of the object itself? Much more will have to be said about this later– and the answers will be

surprising with respect to standard prejudices about realism –however, for the moment it can be said that onticology takes its epistemological inspiration from the transcendental realism of Roy Bhaskar. Among other things, Bhaskar sought to provide a transcendental grounding for the sciences. Insofar as onticology defends the thesis that the field of being is much more vast than the field of objects investigated by the natural sciences, it parts way with the thesis that the domain of being is exhausted by the domain of natural objects. However, the general form of Bhaskar's argument holds for our realist purposes. A transcendental argument seeks to elucidate the conditions under which certain acknowledged practices and forms of cognition are possible. Kant, for example, asked what must be the case for mathematical judgments to be possible. How is it both that we are able to extend our knowledge, as if by magic, through mathematical judgments and, more significantly, that these judgments are able to provide genuine knowledge of the world despite the fact that these forms of reasoning are not based on experience? Part of Kant's argument consisted in claiming that mind imposes the forms of space and time on the data of experience. In other words, space and time are not attributes of being itself but rather of the mind that regards being. Insofar, Kant argues, as mathematics is ultimately a rumination on the nature of space and time taken in their most abstract form and insofar as the mind imposes space and time on the manifold of sensation, it thus follows that a priori judgments about the nature of spatio-temporal relations are possible that anticipate the structure of actual-space times without directly experiencing these space-times. Why? Because any manifold of sensation must necessarily be structured by these forms imposed by intuition. **Onticology– A Manifesto for Object-Oriented Ontology Part I Posted by larval subjects under Object-Oriented Philosophy My Question Is This: You Say, "Granted There Are Deeper And Deeper And More Primitive Forms Of "Knowledge" Or Abstraction Into Mechanism, But I Wouldn't Say That The Atomic Level Brings A New Epistemic World Into View." But Doesn't Any Kind Of Knowledge, However Primitive, Imply Bringing Some Kind Of Epistemic World Into View Or Proto-View?** Joel: David, Yes Exactly. My Question Is, Just How Far Down The Complexity Gradient Can The Meaning Of 'Knowledge,' Or 'Semiotics, Or Specifically Representation (Maya) Be Stretched Without Breaking? I Can Comfortably Stretch It To The Genetic Level, As The Proto-Epistemic, Where Genetic Codes 'Represent' Phenotypic Structures Or Modes, And The Whole Of Evolution Learns From Its Mistakes And Successes As That Knowledge And History Is Encoded Into And As The Recapitulation Of The Embryogenesis Of The Organism. But An Atom, In My Model, While Certainly Reacting To Its Environment From Its Own Attractor, Essence, And 'Interiority' (Prehension) Isn't Making Representations Of It To Do So. But The Key To The Power And 'Essence' Of Representation (And Simultaneously Its Limitation) Is The Capacity Of Abstraction And Choice. Evolution Does This At A Rudimentary Level (And Anthropomorphizing A Bit) With Its Sending Into The World Its 'Random' Variations, Organismic Ideas, In A Sense. Its 'Mind' Is Literally On The Outside As Our Living World And Biosphere. A 'Choice' Is Made In The Biospheric Interiority, And Knowledge For The Evolutionary/Embryogenetic Trajectory Is Gained By The "Differential Reproductive Success" Of These Individuals Lives Through Time, As The Increasing Intelligence, Representation, And Capacity For Choice Is Directly Injected Into The Flow Through Sexual Selection. But Where Is The Encoded/Abstracted (Enfolded) Representation Of Options And A Choice Among Them Made At The Atomic Level? I Am Working With The Most Cutting-Edge And Coherent Models For The Atom (Including My Own) That I Can Find Among The Heterodoxy, And I See No Evidence Of Representation And Choice At This Level, Or Rather No Real Way To Further Stretch The Meaning Of The Terms To This Level. Sure There Is Uncertainty From Infinite Difference And Immanent Causation (See Bohm On Infinite Causation), But Without Representation There Really Is No Choice (Although Crudely We Could Call A 'Bifurcation' A 'Choice' By The System As A Whole). So I Am Simply Saying That Representation At The Atomic Level Is Entirely Enfolded And These Are Really Just Ontic Level Phenomena. The Atomic Level, In My View, Is Pre- Not Yet Proto-Epistemic, Which I Place At The Genetic Level Where We Find The First Codes And Primitive Representation. This Just Means That There Is A Real Distinction Between Ontic And Epistemic (And A Gradient Between Them) And We Can't Collapse The Two Into Just The Ontic-Epistemic. And Yet, With The Distinction Intact, And The Gradient To Explore, They Remain Nondual, Univocal, ONE Or Ontic.What I Would Say Exists At All Levels (My Own Self-Aware Myth About The Given) Is What I Call The "Symbiogenesis Of Subject And Object". This Is The Spinozan Essence Of Dynamic Stability And Growth, And Leibnizian Prehension. Also Called The "Nucleation Of Observability" In Spinbitz. It Is From This Nucleus Of The Observer (Fuller) In A "Point-Free

Geometry" (Whitehead) Of Pointless "Points Of View" As Conditions Of Boundary That Wilber's "All Is Perspective" Finds Its Ontic Grounding. And This Symbiogenesis Is Also A Rudimentary Or Deep-Level Native Or Primitive Intelligence, Evolution At The Involution Of The Subject-Object Interface. It Is An Exploration Of Creative Learning And Direct Immanent Awareness, Unmediated (Enfolded) By Representational Forms (Unfolded). It Just Is, In Waves (Cosmic Vertebrae) Before It Abstracts, Represents And Incarnates The Flesh Of What Could Be. Matter Is Enfolded Seed Of The Flower Of Its Abstraction. Maya Is This Depth And Recursion Of Brahma Into And Through The Boundary Conditions Of Abstract Relation. And Brahma Is The Span Of The Emergent Rigor Of The Cosmic Ergodic Spine That Opens The Recursions Into The Field Of Infinite Difference. In Immanent-Transcendent Waves, Literally And Empirically A Limit-Cycle In Ergodicity, It Enfolds And Unfolds From Anatom To Anatomy, From Infinite Intelligence Or Omnirelational Awareness Of Self, Through The Representational Depths Of Self As Self-Consciousness. Omni-Evolution, Trans-Dynamics, And The Anatom Posted On February 21, 2013A Discussion With Tom Huston, Part II The Following Is Part 2 Of An Excerpt From A Discussion Initiated By Tom Huston On His Facebook Page, Edited And Embellished For Clarity. Find Part One, Here. In This Part We Dig Into The Nature Of The "Anatom" And The "Ergodic Spine" Of An Electro-Fractal Cosmos, And Into The Trans-Dynamic Integration Between Being And Becoming, And The X-Interface−The Crossroads Of The Ontic-Epistemic (Brahma And Maya) And The Subject-Object Polarities. Univocal_Dynamics_V1 Tom Huston Is A Founding Member Of Integral Institute And A Former Editor Of Enlighten Next Magazine. His Webpage Is Tomhuston.Com. David Marshall Is A Writer And Editor Living In Chicago. No meaning, value, or normativity to be found in nature, that there is nothing **natural beings ought to be,** but that, rather, these judgments arise from us (See last post of larval subjects) which has led some to raise valuable questions about the coherence of these claims. The problem arises when the following three propositions are taken together: 1. there is nothing outside of nature.2. Beings have no intrinsic meaning, purpose, or value (in and of themselves, there's nothing they ought to be). 3. Value judgments about what beings and being ought to be arise from us and beings like us (bonobo apes, dolphins, institutions, birds of paradise, etc). The problem arises between proposition 1 and 3. How can it both be true that there is nothing outside of being and those normative judgments belong to us and other beings capable of making normative judgments, not nature? The problem arises from restricting these judgments to humans and beings capable of making these judgments. In making such a claim it seems as if we're saying that there's something outside of nature, something that is beyond nature, thereby violating the first thesis and potentially reintroducing the nature/culture distinction. By contrast, if we say that normative judgments are the special domain of those living beings with the proper degree of sentience to make such judgments, then we seem to reintroduce the nature/culture distinction and fall back into the sorts of problems that I outlined in my last post (and that are so nicely critiqued by thinkers such as Latour). Is there a way out of this? I don't know. Great_Chain_of_Being_2_ (lighter) ASIDE: It seems to me that this is really what the debate between realism and anti-realism, realism and socio-linguistic constructivism surrounding the new materialisms and speculative realists has really been all about. It's very easy to treat this as an abstract, academic debate: "Are you a realist or are you an anti-realist?", as if it were just a matter of what happens to be true. But it seems to me that this debate has, in reality, always been about politics. As I outlined in my last post, we have perpetually seen how appeals to the real and natural have been used in the name of oppressive power, inscribing both the exploitation of nature and the oppression of various people in the very fabric of being itself. Theistic theology and realist ontology have perpetually been used in the name of what Deleuze called "State Philosophies" or philosophies that ontologize contingent orders of power and privilege (e.g., "the great chain of being" used to justify patriarchy, monarchy, serfdom, poverty, etc, and appeals to nature used to justify poverty and racial inequality (The Bell Curve), patriarchy (evolutionary psychology), heteronormativity, capitalism, etc). Because arrangements of power and inequality are always contingent in the sense that there's no marked difference in the capacities of peoples, power always looks for a transcendent supplement that would provide justification through ontological necessity. Antirealism– from the Greek atomists to present –became the radical and emancipatory gesture because it revealed the lie behind all of these forms of social organization or their inherit contingency or arbitrariness. Realism, by contrast, has all too often functioned as an apologetics for arbitrary power and social organizations. Here it's worth recalling what Foucault said about science: "…Even before we know to what extent something like Marxism or psychoanalysis is

analogous to a scientific practice in its day-to-day operations, in its rules of construction, in the concepts it uses, we should be asking the question, asking ourselves about the aspiration to power that is inherent in the claim to being a science. The question or questions that have to be asked are: "What types of knowledge are you trying to disqualify when you say that you are a science? What speaking subject, what discursive subject, what subject of experience and knowledge are you trying to minorize when you say 'I speak this discourse, I am speaking a scientific discourse, and I am a scientist.' What theoretico-political vanguard are you trying to put on the throne in order to detach it from the massive, circulating, and discontinuous forms that knowledge can take?" (Society Must Be Defended, 10). All of these questions hold equally for claims to something being real. What is one trying to minorize when claiming something is real? What becomes privileged? What is excluded? It is these questions that have been at the heart of the realism debates, for as Spencer-Brown taught us, every distinction has a marked and unmarked space, draws attention to something to be included and pushes something into the unconscious or the domain of the invisible, hidden, or veiled. This is above all the case with evocations of the real. However, as I've tried to show antirealism leads to its own problems. First, so long as we exclude real beings from our ontological inventory, we are unable to fully understand how power functions ("such and such a set of cultural formations have been thoroughly debunked, yet people still live as if they believed them"). Not only do we not fully understand the sources of the problems due to too much focus on the discursive and semiotic, but we deny ourselves valuable sites of political intervention at the level of infrastructure. Second, we are prevented from addressing things such as cultural racism, such as that found in Heidegger's privileging of the West and the Greeks and Germans in particular. We do a good job addressing biological and theological racism, heteronormativity, and sexism by showing how it is a cultural construction, but when faced with racism such as Heidegger's where he argues that there's something "unique" about the Greek event, it's language, and about the German language, or Badiou/Zizek's racism with respect to the "Pauline Event", we really have no response. Here someone like Jared Diamond or Fernand Braud el is needed to explain global-geographical inequalities. Third, the tools of the cultural turn really do not provide us with the means of thinking the ecological as a site of the political. For this reason, I've tried to formulate a third way– which might be called "constructivist realism" or "constructivist naturalism" –that retains the insights of the cultural turn, while also allowing a robust place for the material. I don't claim to be original in this. I think that many such as Manuel DeLanda, Deleuze and Guattari, Stacy Alaimo, Karen Barad, Jane Bennett, etc., are up to something similar. It's a vast project that requires the work of a multitude of voices, especially given the way in which the culturalism pervades humanities. **The Relational and the Non-Relational: Notes towards an Immanent and Pluralist Theory of Meaning Posted by larval subjects under uncategorized k-bigpic I have deleted or reformulated some sentences for which I beg pardon attributable to spatial constraints** What is correlation, what is causation? What is coordination? What is coincidence? These thoughts have bothered me since long. I have been able to excavate some reliable material from Wikipedia which I would like to share. "All views are partial and contingent – that's the lesson of pluralism, and you say as much yourself when you say that we must be attentive to ways our own knowledge might contain superstitions. But it's not discourse that determines whether one view works and another doesn't – it's a confrontation with **non-discursive (non-human)** agencies." I wonder if part of the issue here is that we understand pluralism differently. For me this doesn't sound like pluralism at all. Rather, it just sounds like our **epistemological condition**. We [hopefully] come to understand those portions of existence we question or investigate. The reason we come to investigate them is largely contingent. Finally, the accounts of these features of existence we give can be mistaken. All of this is perfectly consistent with a monism. I'm not sure why what he outlines above is the lesson of pluralism. It seems to me that every realist knows this. It seems to me that ontological pluralism is something quite different. Ontological pluralism is the thesis that there are many different worlds inhabited by many different entities. Thus, for example, you would have one world; say that of Lucretius, that's only inhabited by atoms and their combinations. You would have another world; say the world of the Mongolian shaman, that's inhabited by spirits that do all sorts of things. The ontological pluralist is saying that these spirits are, that they exist, that they're real. This is something quite different than merely saying that there are partial and contingent points of view on the world. It's this saying that these entities are rather than that some person or groups of people believe that they are that is the nub of the issue. Now, I think part of the issue here is that there's an ambiguity in the term ontology. Ontology can be one of two things. On the one hand, ontology is a group

or persons set of beliefs as to what is. Here it's trivially true that there are pluralities of ontologies and the realist readily recognizes this. This is the whole reason there are debates over ontology. Mongolian shamans have their ontology, Europeans theirs, Christian fundamentalists theirs, materialists theirs, etc. When striving to understand and communicate with others it's vital to understand these ontologies because, as rhetoricians like Burke point out, our **beliefs about what is** are among the things that motivate our action. Despite having never seen bacteria I was my hands because I believe there are bacteria and viruses on door handles and whatnot and don't want to get sick. My belief about a particular thing existing is what motivates my action. And who knows, perhaps this belief is as superstitious as the belief that the crops failed because God was displeased with my community. On the other hand, ontology is a theory **about what is.** It is making a claim that something exists. This is where the rubber hits the road. The ontological pluralist seems committed to the thesis that every group set of beliefs about what exists is sufficient for granting the existence of those entities. This is what I find objectionable in Latour. Obviously I'm not bothered by Latour's suggestion that we should take into account the role that nonhumans like speed bumps, rivers, microbes, etc., play in the form that social assemblages take. But this is not all that Latour claims. This is precisely where the philosopher might balk. Unlike the ethnographer, the philosopher is not interested in what people believe exists, but rather philosophers– at least of the realist variant –are trying to figure out what is. In other words, the realist philosopher begins with the premise that not all of these beliefs about what is are true. So for the philosopher, recognizing that Mongolian shaman's believe in the existence of shamans would only be the first step. The next step would consist in determining whether there's good reason for thinking such entities really do exist, i.e., whether there's good reason for believing these entities have mind and culture independent reality. Lest readers think that I'm just picking on the supernatural here, we can ask similar questions about strings, subatomic particles, galaxies, etc., etc., etc. I guess the question really comes down to what exactly we mean by ontological pluralism. When we talk about ontological pluralism are we defending the thesis that people have different theories of being? If so, then ontological pluralism is trivially true. If this is what is meant, then I certainly share Jeremy's view that it's valuable to understand the different world's people believe in. Certainly when I was practicing as an analyst I didn't get in ontological debates with my patients and it was necessary to understand their theory of being or their ontology to properly attend to them. Or, when we talk about ontological pluralism are we defending the thesis that all these ontological theories are true and refer to really existing entities? That's quite a different claim and is not one I would defend or endorse. Now someone might object that "in both The Democracy of Objects and Onto-Cartography you defend the thesis that there are multiple worlds." This is true. Because I hold that not everything is related I'm led to the conclusion that there are diverse worlds. However, I also hold that however many worlds there might be, these worlds are nonetheless composed solely of material entities. Within the framework I propose I wouldn't suggest that there's one world where there are spirits and another world where there are souls and yet another where there are only material entities. My view is that there aren't spirits or souls in any of these worlds. One of the things I keep hearing in these discussions is that somehow the realist adopts a view from nowhere. I honestly don't understand this criticism. Investigation always occurs somewhere and requires all sorts of mediations involving technologies, experiments, etc. It's that labor of gathering evidence, conducting experiments, using technologies to observe the world, etc., that gradually gives us a body of data that allows us to say there's good reason to believe that such and such a thing exists and has these powers. Another charge seems to be that the realist refuses to recognize that their claims about the world are fallible. I find this charge particularly strange because it's precisely because the realist recognizes the difference between our theories of the world or what we say about the world and the world itself that fallibility is built into the core of his position. Realism doesn't mean one holds they have special access to the world, that they know all truths, or that they have the truth in hand, only that there are truths to be known and that we can be mistaken about things. Circling **Squares Philip** has a post responding to my quandaries about how to mesh realism and pluralism. He writes: Ontologically and metaphysically the idea of realist pluralism is no longer an issue. There are (appropriately) numerous variants but the basic idea that reality is itself pluralistic is well established. The question is political-discursive. It's what Stengers and Latour are getting at with their concepts of diplomacy and cosmopolitics. They grant, first, that all entities exist and, second, that to say that someone's cherished idol (or whatever disputed entity they hold dear) is non-existent is a **'declaration of war'** – 'this means war,' as Stengers often says. They thus shunt onto-political discourse off of the terrain of

knowledge/belief in the sense of existence/non-existence. Their basic claim seems to be that 'respect for otherness,' i.e. political pluralism, can only come from granting the entities that others hold dear an ontology, even if you don't 'believe' in them. You are thus permitted to say 'I do not follow that god, he has no hold over me' but you are not permitted to say 'your god is an inane, infantile, non-existent fantasy, grow up.' And it's not just a question of politeness (although there's that too). The point is to grant others' idols and deities an existence – one needn't agree over what that existence entails, over what capacities that entity has or what obligations it impresses upon you as someone in its partial presence but to deny it existence entirely is to 'declare war' – to deny the possibility of civil discourse, of pluralistic co-existence. I believe that it was Richard Rorty who once quipped something like claims to reconcile realism and idealism always seem to end with a triumph of idealism. We don't, in fact, get realism through such approaches, but rather just get a pervasive anti-realism. I think this is also the problem with the "non-controversial pluralism" advocated by Stengers and Latour that Phillip defends here. Such pluralism is not realism but is, in fact, a thoroughgoing social constructivism. I think this is the central problem with Latour's argument in Irreductions (these days I regret having ever defended it). In rejecting both Enlightenment critique and what he calls "reduction" he wants to say something like "The Pentacostal really is filled with the Holy Spirit", that for the 19th working scientist heat really is a fluid and phlogiston really is what allows things to burn, and that for the Greek lightning really is an expression of Zeus's anger. Latour tells us that we aren't to reduce or explain away the entities posited by another group's "ontology" but are to develop explanations from within that ontology. It might not sound particularly sexy– and it certainly doesn't tell us what is worth thinking –but I can't help but believe that philosophy is the critical and reflective investigation of basic concepts that guide our investigation of the world about us, how we ought to live our lives, and what form of governance might be best. Compare two figures. A scientist might ask, what causes depression? We can very well imagine a philosopher turning around and asking the scientist, what is causality? The scientist presupposes a concept of causality in her investigations. She uses this concept in her inquiry. Now she might have a sophisticated concept of causality or she might never have thought much about causality at all, using it in the sort of colloquial and unreflective way that Plato decried when, for example, people like Euthyphro talked about piety. A whole cascade of questions arise when we raise a question like "what is causality?" We can ask whether or not causality exists at all. We can ask how we distinguish between correlation and two events that merely accompany one another from genuine causation. This, for example, was Hume's question. But perhaps most importantly we can ask whether there is only one form of causality or many forms of causality. Is there only one-to-one causation; one cause and one effect? Is there many-to-one causation; or many events conspiring to produce an effect? Is there one-to-many causation; or one event producing a variety of different effects? We can even ask whether causality necessarily moves from past to present or whether there aren't forms of causality that move from future to past! **January 23, 2014 Pluralism and Realism:** multiple-worlds Over at Struggles Forever, Jeremy Trombley has an interesting post up on "the ontological turn" in anthropology or ethnography. I've been meaning to have a discussion with him about this as I think it's an issue many of us are struggling with. For example, the core project of The Democracy of Objects– a proj ect which I think many have missed –is to somehow reconcile some version of social constructivism with a realist ontology capable of making room for ecology (which requires realist and materialist positions as there's a fact of the matter where global warming is concerned) as well as the role played by objective agencies in social assemblages such as technologies, infrastructure, features of geography, local climates, the growth cycles of plants and animals, waste, etc. Maybe we can try to organize some cross-blog event to discuss these issues. I certainly think they're close to the heart of Jeremy, Michael of Archive Fire, Arran James, and a host of others. As an aside, I'm beginning to realize how the different sites of the political I've been outlining– semiopolitics, thermopolitics, oikopolitics (political economy), geopolitics, eropolitics (the politics of sex and desire), biopolitics, and chronopolitics (and I'm sure there are other political sites!) –are drawing me away from traditional Marxism. Assuming that classical Marxism holds that economics or the conditions and relations of production are determinative of all other sites of the political, the various sites of the political that I've been outlining would lead to the conclusion that there is not one determinative base of the political. This would not require committing Marx to flames, but rather of recognizing the phenomenon of overdetermination, or of a variety of different entangled sites of the political. But I digress. First, I find myself wondering what the ontological turn means in ethnography. Is it 1) the investigation of the different ontologies

held/proposed by different cultures? E.g., the Aztecs believed that reality was structured in this way, while the Greeks in that way, and the ancient Chinese this way, etc? Or 2) is it an investigation of how real entities– independent of cultural beliefs –influence cultural formations? Or is it a combination of both? A position that I would favor. January 20, 2014 Thoughts on the Social and Political Implications of Correlationism .Lion Mirror Truth be told, as my thought has evolved the issue of correlationism had fallen off the radar for me. Somehow the debate had come to seem too "philosophical" to me, too "scholastic", too remote from what interests me: understanding why social assemblages are organized as they are, how power functions in social assemblages, and what we might do to address that power and change things. Somehow the question of whether or not we can get out of the correlation between thinking and being just came to seem remote from these sorts of issues. Somehow it seemed too epistemological. ASIDE: Numerous discussions over the years have led me to believe that the debate over correlationism is poorly understood (or maybe I just don't understand it). On countless occasions I've heard people say "of course we must relate to things in order to know them." Well yeah, of course! I don't think this is what the critic of correlationism is getting at. It seems to me that correlationism is something more robust than the theses that we must relate to something to know it. Correlationism instead seems to require the theses that thought and being are indiscernible. Put more concretely, the correlationist is someone who argues that we either a) can never tell whether being is merely a construction of our thought (weak correlationism), or b) who argues that thought actually constructs being (strong correlationism). In other words, correlationism is another name for idealism. One can hold that we must relate to something in order to know it without being a correlationist. As an aside I should also add that I am a correlationist about some things. For example, I think money is something constructed by society and is therefore a strong correlationist when it comes to money. At any rate, for a long time I'd become rather indifferent to debates about correlationism and philosophies of access. I had learned the lessons of speculative realism– which I could have also learned, I think, from Deleuze and Guattari, the new materialist feminists, actor-network theorists such as Latour –and had moved on. However, occasionally you come across a tone of phrase that pitches something in a different light. In The Cut of the Real, Katerina Kolozova writes, …the political problem of contemporary philosophy identified by the 'new realists' is, in fact, the product of a more fundamental epistemic problem. In his book After Finitude, Quentin Meillassoux calls this problem 'correlationism' and identifies it as an essentially post-Kantian legacy, which continues to dominate and limit philosophy. As a matter of fact, correlationism lies at the heart of postmodern theory and consists in the premise that thought can only 'think itself,' that the real is inaccessible to knowledge and human subjectivity, and that there is nothing but discursive constructs that fully determine thinking and that are meth0dologically accounted for all the way down. (1 – 2) Thought thinking only itself: Thought only encountering itself. In the jargon of postmodern and poststructuralist lingo, this would be the thesis of infinite semiosis, where signs ("thoughts") only ever relate to other signs. Within this framework, discursivity comes to be the hegemonic framework defining all of being. At the level of politics and social theory more generally, if the correlationist thesis is true the consequences are clear: all social phenomena are discursive and all solutions to social and political problems will be discursive. The sole sphere of the political will be the discursive and all questions of politics will be questions of speech-acts and interpretation. The problem here is not that many theorists recognize that the discursive and semiotic plays an important role in the social and the political. It does and I've repeated this tirelessly. The problem is with what happens when thought or the semiotic becomes a hegemon, an "all", foreclosing our ability to recognize other forms of power. What I've wanted to say is that not all power functions discursively. In my last post and elsewhere I spoke of some other forms of politics: Thermopolitics: The politics surrounding **energy in the form of calories and fuels such as gasoline** and coal, and how our life and our very bodies are structured by energy dependencies and by being trapped in particular distributive networks that render these forms of energy available. I'm being quite literal when I speak of energy, talking about the effects, for example, of the absence of food in certain educational environments on cognition, for example; and am generally hostile to metaphorical extensions of the concept of energy which I see as erasing the dimension of real materiality. Geopolitics: The role that features of natural and built geography such as mountain ranges, rivers, oceans, soil conditions, roads, housing design, etc., play in the form that social relations take and how they impact individual bodies. Chronopolitics: The way in which the structurations of time organize what is possible for us. For example, the structuration of the working day, how much we can say and comprehend at

any given time, the impact of things like the invention of the clock, etc. **Oikopolitics: This would be the domain of political economy described so well by Marxists**. So five different types of politics: Semiopolitics (or what currently dominates critical theory), thermopolitics, geopolitics, chronopolitics, and oikopolitics. No doubt there are other sites of the political or political struggle that we could speak of, but this is a good start. Also, it should be obvious that these aren't exclusive domains, but are entangled in all sorts of important ways. For example, something might take place at the level of semiopolitics (speech, law, rhetoric, norms, and communication) that has all sorts of impact at the level of thermopolitics. Congress might decide to cut programs that fund school meal programs. This, in turn, will have a thermodynamic impact on those students that go without the calories they need developmentally and cognitively to function in a particular way. There is an entanglement here of semiopolitical and thermopolitical domains. The young student here has been constrained both at the level of **semiotic phenomena and thermodynamic structures**. The point is that if true, semiotic intervention (speech-acts, protests, interpretations, deconstructions, etc) will not be an appropriate response to all political problems because social formations are not entirely structured by the semiotic. The child in that school does not suffer from a lack of the right signs, but from a lack of calories needed to run the engine of his thought and body. Certainly semiotic interventions might be needed to render that energy available, but it is the energy itself that is at issue and the absence of that energy that forms the spider web entangling him in his position. A correlationist perspective tends to erase this as even being a site of the political. **Larval Subjects January 28, 2014 Ontological Anarché Posted by larval subjects under Uncategorized January 27, 2014 Roden on Pluralism Over at enemy industry Roden has an excellent post up on the pluralism discussion. Check it out here. January 25, 2014 Different Senses of Pluralism and Ontology Posted by larval subjects under Uncategorized Responding to one of my comments over at his blog, Jeremy Trombley:** I have deleted restricted, reformulated some sentences .Kindly bear with me. Intention is to make the discursive enucleation consistent with the subject in question**.** Recourse to epistemology of the quantum mechanics generic and fervor of string theory is a sine qua non for the further comprehension of the theory and ontology .How would the world appear to us if its ontology was that of classical mechanics but every agent faced a restriction on how much they could come to know about the classical state? We show that in most respects it would appear to us as quantum. The statistical theory of classical mechanics, which specifies how probability distributions over **phase space evolve** under Hamiltonian evolution and under measurements, is typically called Liouville mechanics, so the theory we explore here is Liouville mechanics with an epistemic restriction. The particular epistemic restriction we posit as our foundational postulate specifies two constraints. The first constraint is a classical analog of Heisenberg's uncertainty principle; the second-order moments of position and momentum defined by the phase-space distribution that characterizes an agent's knowledge are required to satisfy the same constraints as are satisfied by the moments of position and momentum observables for a quantum state. The second constraint is that the distribution should have maximal entropy for the given moments. Starting from this postulate, authors derive the allowed preparations, measurements, and transformations and demonstrate that they are isomorphic to those allowed in Gaussian quantum mechanics and generate the same experimental statistics. They argue that this reconstruction of Gaussian quantum mechanics constitutes additional evidence in favor of a research program wherein quantum states are interpreted as states of incomplete knowledge and that the phenomena that do not arise in Gaussian quantum mechanics provide the best clues for how one might reconstruct the full quantum theory. I: http://dx.doi.org/10.1103/PhysRevA.86.012103 **Reconstruction of Gaussian quantum mechanics from Liouville mechanics with an epistemic restriction Phys. Rev. A 86, 012103 – Published 10 July 2012 Stephen D. Bartlett, Terry Rudolph, and Robert W. Spekkens.** Deleuze in fact characterizes modes of existence, with their powers and capacities. The answer is this: Deleuze approaches modes of existence, ethically speaking, not in terms of their will, or their conscious decision making power (as in Kant), nor in terms of their interests (as in Marx, for example), but rather in terms of their drives. For Deleuze, conscious will and preconscious interest are both subsequent to our unconscious drives, and it is at the level of the drives that we have to aim our ethical analysis. Here, I would like to focus on two sets of texts on the drives taken, not from Nietzsche and Spinoza, but rather from Nietzsche and Leibniz (Leibniz being one of the first philosophers in the history of philosophy to have developed a theory of the unconscious). The first set of texts comes from Nietzsche's great early book entitled Daybreak, published in July 1881. Nietzsche first approaches the question of the drives by giving us an everyday scenario:

"Suppose we were in the market place one day," he writes, "and we noticed someone laughing at us as we went by: this event will signify this or that to us according to whether this or that drive happens at that moment to be at its height in us—and it will be a quite different event according to the kind of person we are. One person will absorb it like a drop of rain, another will shake it from him like an insect, another will try to pick a quarrel, another will examine his clothing to see if there is anything about it that might give rise to laughter, another will be led to reflect on the nature of laughter as such, another will be glad to have involuntarily augmented the amount of cheerfulness and sunshine in the world—and in each case, a drive has gratified itself, whether it be the drive to annoyance, or to combativeness or to reflection or to benevolence. This drive seized the event as its prey. Why precisely this one? Because, thirsty and hungry, it was lying in wait" (D 119). This is the source of Nietzsche's doctrine of perspectivism ("there are no facts, only interpretations"), but what is often overlooked is that, for Nietzsche, it is our drives that interpret the world, that are perspectival—and not our egos, not our conscious opinions. It is not so much that I have a different perspective on the world than you; it is rather that each of us has multiple perspectives on the world because of the multiplicity of our drives—drives that are often contradictory among themselves. "Within ourselves," Nietzsche writes, "we can be egoistic or altruistic, hard-hearted, magnanimous, just, lenient, insincere, can cause pain or give pleasure" (Parkes, pp. 291-292). We all contain such "a vast confusion of contradictory drives" (WP 259) that we are, as Nietzsche liked to say, multiplicities, and not unities. Moreover, these drives are in a constant struggle or combat with each other: my drive to smoke and get my nicotine rush is in combat with (but also coexistent with) my drive to quit. This is where Nietzsche first developed his concept of the will to power—at the level of the drives. "Every drive is a kind of lust to rule," he writes, "each one has its perspective that it would like to compel all the other drives to accept as a norm" (WP 481). DELEUZE AND THE QUESTION OF DESIRE: TOWARD AN IMMANENT THEORY OF ETHICS Daniel W. Smith. **In on a Possible Physical Metatheory of Consciousness Miroljub Dugic et al** show that the modern quantum mechanics, and particularly the theory of decoherence, allows formulating a sort of a physical metatheory of consciousness. Particularly, the analysis of the necessary conditions for the occurrence of decoherence, along with the hypothesis that consciousness bears (more-or-less) well definable physical origin, leads to a wider physical picture naturally involving consciousness. This can be considered as a sort of a psycho-physical parallelism, but on very wide scales bearing some cosmological relevance. Open Systems & Information Dynamics Volume 9, Number 2, 153 (2002) arXiv: quant-ph/0212128. On a broader perspective and background, the theoretical computation of the universe is justified.IN physics and cosmology, digital physics is a collection of theoretical perspectives based on the premise that the universe is, at heart, describable by information, and is therefore computable. Therefore, according to this theory, the universe can be conceived of as either the output of a deterministic or probabilistic computer program, a vast, digital computation device, or mathematically isomorphic to such a device. Digital physics is grounded in one or more of the following hypotheses; listed in order of decreasing strength. The universe or reality: is essentially informational (although not every informational ontology needs to be digital); is essentially computable (the pancomputationalist position); can be described digitally; is in essence digital; is itself a computer (pancomputationalism); is the output of a simulated reality exercise. Following Jaynes and Weizsäcker, the physicist John Archibald Wheeler wrote the following: [...] it is not unreasonable to imagine that information sits at the core of physics, just as it sits at the core of a computer. (John Archibald Wheeler 1998: 340) It from bit. Otherwise put, every 'it'—every particle, every field of force, even the space-time continuum itself—derives its function, its meaning, its very existence entirely—even if in some contexts indirectly—from the apparatus-elicited answers to yes-or-no questions, binary choices, bits. 'It from bit' symbolizes the idea that every item of the physical world has at bottom—a very deep bottom, in most instances—an immaterial source and explanation; that which we call reality arises in the last analysis from the posing of yes–no questions and the registering of equipment-evoked responses; in short, that all things physical are information-theoretic in origin and that this is a participatory universe. (John Archibald Wheeler 1990: 5) David Chalmers of the Australian National University summarised Wheeler's views as follows: Wheeler (1990) has suggested that information is fundamental to the physics of the universe. According to this 'it from bit' doctrine, the laws of physics can be cast in terms of information, postulating different states that give rise to different effects without actually saying what those states are. It is only their position in an information space that counts. If so, then information is a natural candidate to also play a role in a fundamental theory of consciousness. We are led to a conception of the

world on which information is truly fundamental, and on which it has two basic aspects, corresponding to the physical and the phenomenal features of the world. (Wikipedia). **Participatory consciousness forms the bastion, pillar, post, stylobate and fulcrum of the redressal measures and corrective steps taken for circumvention of many problems, identified and unidentified, spread widest commonalty. Anne Baring's sententious and pithy article will be good to have a timely outlook.** I would like to open my contribution to this meeting with the words of Ken Wilber that I quoted in the first Millennium Symposium: 'The secret impulse of life is towards greater consciousness. Maybe the evolutionary sequence is from matter to body, to mind, to soul, to spirit - each transcending and including, each with a greater depth and greater consciousness and wider embrace' Ken Wilber, and with these from Rupert Sheldrake's contribution to the same meeting: "It is possible to make quite a good case that the sun could be conscious and if the sun, then why not the stars? Why not the galaxies? Why should there not be galactic minds? And if they communicate with each other then the distances are far too great for light to be the medium of transmission. We are into inter-galactic telepathy." After many decades of study, I know of only one diagram in all religious traditions that can adequately explain these statements; that can give us both a blue-print of the many levels of consciousness and a route-map to help us reach beyond our present level of understanding. I would like to introduce a diagram of the four worlds of Kabbalah to illustrate the theme that spirit has brought these four worlds into being and that we, living in the fourth and manifest world know nothing of the existence of the other three invisible ones. It is the clearest diagram I know of that can explain to us the multi-levelled structure of consciousness and our place in the great chain of being. We are indeed, as Wilbur suggests, evolving from matter to body, to mind, to soul, to spirit but where have we come from? It is possible that, as participants in the emanation of spirit into manifest life, we have also come from spirit to soul, to mind, to body, to matter. Everything, from the immense galactic energies to the minute forms of matter participates in one life, is generated from one creative spirit. There is nothing outside this life, this spirit. This diagram suggests that truth is not an article of faith but the experience of participating in the undiscovered reality of what has brought us into being. We cannot know truth until we have entered into communion with that reality. Continuing in similar vein **Michel Bitbol dilates upon the dialectic deliberation and polemical argumentation between** hermeneutists and eliminativists. When he formulated the program of Neurophenomenology, Francisco Varela suggested a balanced methodological dissolution of the "hard problem" of consciousness. He shows that his dissolution is a paradigm which imposes itself onto seemingly opposite views, including materialist approaches. I also point out that Varela's revolutionary epistemological ideas are gaining wider acceptance as a side effect of a recent controversy between hermeneutists and eliminativists. Finally, he emphasizes a structural parallel between the science of consciousness and the distinctive features of quantum mechanics. This parallel, together with the former convergences, point towards the common origin of the main puzzles of both quantum mechanics and the philosophy of mind: neglect of the constitutive blindspot of objective knowledge. Phenomenology and the Cognitive Sciences Phenomenology and the Cognitive Sciences **Phenomenology and the Cognitive Sciences 2002, Volume 1, Issue 2, pp 181-224 Science as if situation mattered Michel Bitbol.** Caroline Williams investigated affective density of the political and its effect on our understanding of political subjectivity. Taking up Spinoza's challenge to think about affect beyond corporeal embodiment, he argues that there is a modality of affectivity that cannot simply be inscribed within the borders of subjectivity. theorising affect as an impersonal force anchored in a relational ontology that gives due recognition to the circulation of affects, as well as to their ambivalent structure in creating sites of identification, and he utilises this ontology to reflect on the dynamic of the political and the shape of political subjectivity. Williams argues that Spinoza's philosophy (through ideas of conatus and imagination) offers the conceptual resources to reconfigure the composition of affective subjectivity as a transindividual social bond and as an unconscious dynamic of ethico-political existence. **Subjectivity (2010) 3, 245–262. doi:10.1057/sub.2010.15 Affective processes without a subject: Rethinking the relation between subjectivity and affect with Spinoza Caroline Williams** Phenomenological Time bears ample evidence and infallible observatory and apodictic evidence albeit with a sense of chagrined circumvention and defeated disgruntlment circularity of both psychological and phenomenological interpretations of art which are sometimes insightful, but most often work on the logic of cyclical time and unavoidable drives and representations played out as a 'natural' order: where 'men can't avoid being boys' a circularity which valorizes a helplessness/truthfulness conundrum. One begins with a classical phenomenological

fallacy with Duchamp's Fountain, does the urinal keep returning or do we keep returning to the urinal? Put in this way, it is a male question, something that phenomenology conveniently conceals when positing the "I", or indeed, the "we". But we must be careful not to simply assume that the phenomenological first person is white, heterosexual, able-bodied and male, providing a foundation upon which knowledge should be based as a universal condition. Deleuze's second critique of time is linked to Kant. This model of time extends not only to phenomenology but to the phenomenological interpretation of art and is seen as a straight line. In the Critique of Pure Reason, Kant cuts up circular time and reassembles it as a series of sensory experiences which memory processes and which constitute subjectivity. Whereas the circular model of time (and interpretation) inscribes fatalism into the subject and the world, the phenomenological method is in danger of solipsism, where the subject constitutes the world. Much of embodied philosophy and enactivist theories of cognition go down this line with a plethora of art historical analyses by the likes of Rosalind Krauss, Michael Fried and others taking the lead from Merleau-Ponty. These approaches successfully challenge the **homunculus model of a Cartesian mind-body** dualism where there is a supervisor in the mind who processes experience and even consciousness and to whom all representations are directed. But phenomenology is mainly concerned with perceptual consciousness, the processing of colors, shapes, forms, motion in art and the world. The senses and sensorimotor processes are the fundamental pathways by which the interior is connected to the exterior in a chiasm of self-constitution in the world. The problem with this is that we explore the art through our sense perceptions, and what does art reveal to us?—our sense perceptions! Such solipsistic approaches have real problems relating complex and abstract conceptual production to bodily and sensorimotor contingencies, and they nearly always posit the subject as the site and object of discovery in the world. To be fair, phenomenology has moved on to rather more sophisticated models of intersubjectivity, taking into account the specificities of different kinds of bodies and ways of being beyond the white, male heterosexual as the common denominator, yet many naïve approaches to art perpetuate the notion that perceptual, rather than conceptual processes and evolutionary neural patterns are primary, fundamental ways to understand the art experience In fact, these aspects are only part of art production and reception, aspects which are often filtered out or made known to us in order for us to parody such trigger responses, as in much of Duchamp's work, which holds in contempt such retinal approaches to art. If we take the **subject of abstract expressionism** in the 1950s or later minimalist art, art history, teaching and interpretation still insists on putting the act of perceiving as fundamental to the constitution not only of the phenomenological first person but also of art itself. The male act of usually standing up, which consists of directing a stream with the hand(s) into a container or on a flat surface, sand or snow in order to leave patterns and traces (the usual child's prank) underlies the logic of macho 1950s Abstract Expressionist culture, especially that of Jackson Pollock, so effectively parodied by Carolee Schneeman's Vagina Scroll, 1975. Here, painting or writing is a kind of a discharge which invests the **object and space with** the phenomenological "I", disciplined by existential struggle and reflected back by the traces, signatures and gestures left behind in the work of art, rediscoverable as an element in the making of other subjectivities.It is precisely this kind of solipsism which is blind to the art event as something well in excess of re-establishing habits of subjective self-identity through the production and interpretation of art. The attempt to reinforce a phenomenological identity upon the event by personalizing sense experience is unable to see art as something which remodels the phenomenological 'I' and neural plasticity itself. We learn new technologies and redraw our neural pathways through such new and spontaneous experiences which draw us into intersubjective worlds. Duchamp's Fountain and Deleuze's Repetition and Différence Grégory Minis sale (*I have made some changes*) The relation of different groups to deconstruction is settled by its relation to groups. An American enthusiast--one who has had a role in the development of our affection for Derrida--notices that the motivational chains are phobically driven by concerns with social scale. Richard Rorty: "He wants to figure out how to break with the temptation to identify himself with something big. . . ."(10) And: "So I take Derrida's importance to lie in his having had the courage to give up the attempt to unite the private and the public, to stop trying to bring together a quest for private autonomy and an attempt at public resonance and utility. He privatizes the sublime, having learned from the fate of his predecessors that the public can never be more than beautiful."(11) Notice is made here of what is indeed the telling feature. Levinas is praised, for example, for his alarm when "the social will [is] sought in an ideal of fusion . . . the subject losing himself in a collective representation, in a common ideal. . . . It is the collectivity which says 'us,' and which,

turned toward the intelligible sun, toward the truth, experience, the other at his side and not face to face with him. The tellingly titled D'un ton apocalyptique contains passages that are particularly excited: These people situate themselves outside the ordinary, but they have in common this: they describe themselves as having an immediate and intuitive relation with mystery. And they want to attract, to seduce, and lead others to the mystery, through mystery. This agogic function of the leader of men, of the duce, of the Führer, of the leader, places him above the crowd that he manipulates with the aid of a small numbers of adepts joined together in a sect with a secret language, a clique or a small party with its ritualized practices. The mystifiers pretend to have exclusive access to the privilege of a secret mystery. . . . The revelation or the unveiling of the secret is something that they jealously reserve for themselves. Jealousy is here a major characteristic. There is not narcissism and non-narcissism; there are narcissisms that are more or less comprehensive, generous, open, extended. What is called non-narcissism is in general but the economy of a much more welcoming, hospitable narcissism, one that is much more open to the experience of the other as other. I believe that without a movement of narcissistic reappropriation, the relation to the other would be absolutely destroyed; it would be destroyed in advance. The relation to the other--even if it remains asymmetrical, open, without reapporpriation--must trace a movement of reappropriation in the image of oneself for love to be possible, for example. Love is narcissistic. Beyond that, there are little narcissisms, there are big narcissisms. . . .(23)

Justice of the Pieces Deconstruction as a Social Psychology Douglas Collins Sabine Wilke cogently argues for the rise of the aesthetic dimension in philosophical writing by showing how the metaphorical and poetic style of Heidegger, the principles of phrase configurations with Adorno, or the topographic writing of derrida replace through their very form something what traditionally would have been subject to discursive exposure The concern shifts from telling to showing, showing first and foremost that something remains untold. An exemplary book in this aesthetic drive is Avital Ronell's Telephone Book where the playful layout and explicit break with traditional design interrupt the sequential logic of the paragraphs thereby performing the very argument at stake: open up new circuits of meaning and signification. The engagement with form and publishing formats is only consequential since media theory as exposed by Walter J. Ong, Eric A. Havelock, Parry Miman or Marshall McLuhan besides many others made it clear that different communication technologies condition the mode of thought of a given culture The fractals of Benoit Mandelbrot or the butterfly shaped Lorenz attractor are the popular icons of this turn in the sciences. For communication studies and philosophy, Vilém Flusser was one of the pioneer thinkers reflecting the rise of what he called “techno-images.” The photograph is the paradigmatic case which condenses the whole crises of Western thought and culture on its grainy surface. It subverts the linear logic of writing.6 Flusser's central diagnosis detects a shift from the grand narratives associated with the literate mind and Modernity towards creating and envisioning the world through image based practices. Writing himself in a succinct, visually inspired style, Flusser contents that the web of meaning, spun throughout the centuries by our textual practices, has become transparent. We discover the void between the letters through which the world escapes: “We are alienated from the world of text, precisely because we see through them as our own product.”7 As a consequence we are confronted with a world of dissociated facts and information bits. Flusser hails this fragmented world as the advent of the zero dimensional point-universe that will provide the new raw material for our image based practices. Techno-images are nothing else than a conscious, inherently artistic drawing-together of point-elements as exemplified first and foremost by the silver-grains of the photograph but also by more recent pixel based techno-images. Similar to blueprints, designs or statistical curves, the weather map for example visualizes and contracts a tremendous amount of meteorological data from widely distributed stations into a single image of high- and low pressure
areas, temperatures, wind speeds, etc. Techno-images “compute” in this sense the universe of unimaginable, abstract information-bits into a concrete surface – the image. Echoing Paul Feyerabend, these image practices have to be understood as creative practices where the truth of scientific models has been replaced by its aesthetic quality

Aesthetics, Writing, Networked Computers Jörg Muller Dissertation Submitted to the Division of Media and Communications of The European Graduate School in Candidacy for the Degree of Doctor of Philosophy

Lothar Schäfer (1) and Sisir Roy (2) in Quantum Reality, the Importance of Consciousness in the Universe, the Discovery of a Non-Empirical Realm of Physical Reality, and the Convergence with Ancient Traditions of Indian and Western Philosophy investigate metaphysical aspects that include 1) the discovery of a non-empirical part of physical reality in a realm of potentiality; 2) the emanation of the empirical world out of a realm of non-material

forms; 3) the discovery that the nature of physical reality is that of an indivisible Wholeness – the One; and 4) panpsychism: the possibility that the One is aware of its processes like a Cosmic Consciousness. The convergence of powerful traditions of seemingly disparate cultures is particularly important to point out in the present process of globalization, when a unifying view is needed to avoid controversy and conflict. To quote them in extenso : This is the nonlocality of the quantum world. Menas Kafatos and Robert Nadeau (1990) have drawn a remarkable conclusion from this phenomenon: If reality is nonlocal, the nature of the universe is that of an undivided wholeness. Because our consciousness has emerged from this wholeness and is part of it, it is possible to conclude that an element of consciousness is active in the universe: a Cosmic Consciousness. Pre-reflective self-consciousness is pre-reflective in the sense that (1) it is an awareness we have before we do any reflecting on our experience; (2) it is an implicit and first-order awareness rather than an explicit or higher-order form of self-consciousness. Indeed, an explicit reflective self-consciousness is possible only because there is a pre-reflective self-awareness that is an on-going and more primary self-consciousness. Although phenomenologists do not always agree on important questions about method, focus, or even whether there is an ego or self, they are in close to unanimous agreement about the idea that the experiential dimension always involves such an implicit pre-reflective self-awareness. In line with Edmund Husserl (1959, 189, 412), who maintains that consciousness always involves a self-appearance (Für-sich-selbst-erscheinens), and in agreement with Michel Henry (1963, 1965), who notes that experience is always self-manifesting, and with Maurice Merleau-Ponty who states that consciousness is always given to itself and that the word 'consciousness' has no meaning independently of this self-givenness (Merleau-Ponty 1945, 488), Jean-Paul Sartre writes that pre-reflective self-consciousness is not simply a quality added to the experience, an accessory; rather, it constitutes the very mode of being of the experience: This self-consciousness we ought to consider not as a new consciousness, but as the only mode of existence which is possible for a consciousness of something (Sartre 1943, 20 [1956, liv]). The notion of pre-reflective self-awareness is related to the idea that experiences have a subjective 'feel' to them, a certain (phenomenal) quality of 'what it is like' or what it 'feels' like to have them. As it is usually expressed outside of phenomenological texts, to undergo a conscious experience necessarily means that there is something it is like for the subject to have that experience (Nagel 1974; Searle 1992). This is obviously true of bodily sensations like pain. But it is also the case for perceptual experiences, experiences of desiring, feeling, and thinking. There is something it is like to taste chocolate, and this is different from what it is like to remember what it is like to taste chocolate, or to smell vanilla, to run, to stand still, to feel envious, nervous, depressed or happy, or to entertain an abstract belief. Yet, at the same time, as I live through these differences, there is something experiential that is, in some sense, the same, namely, their distinct first-personal character. All the experiences are characterized by a quality of mineness or for-me-ness, the fact that it is I who am having these experiences. All the experiences are given (at least tacitly) as my experiences, as experiences I am undergoing or living through. All of this suggests that first-person experience presents me with an immediate and non-observational access to myself, and that consequently (phenomenal) consciousness consequently entails a (minimal) form of self-consciousness. To put it differently, unless a mental process is pre-reflectively self-conscious there will be nothing it is like to undergo the process, and it therefore cannot be a phenomenally conscious process. The mineness in question is not a quality like being scarlet, sour or soft. It doesn't refer to a specific experiential content, to a specific what; nor does it refer to the diachronic or synchronic sum of such content, or to some other relation that might obtain between the contents in question. Rather, it refers to the distinct givenness or the how it feels of experience. It refers to the first-personal presence or character of experience. It refers to the fact that the experiences I am living through are given differently (but not necessarily better) to me than to anybody else. It could consequently be claimed that anybody who denies the for-me-ness of experience simply fails to recognize an essential constitutive aspect of experience. Such a denial would be tantamount to a denial of the first-person perspective. It would entail the view that my own mind is either not given to me at all — I would be mind- or self-blind — or is presented to me in exactly the same way as the minds of others. There are also lines of argumentation in contemporary analytical philosophy of mind that is close to and consistent with the phenomenological conception of pre-reflective self-awareness. Alvin Goldman provides an example: [Consider] the case of thinking about x or attending to x. In the process of thinking about x there is already an implicit awareness that one is thinking about x. There is no need for reflection here, for taking a step back from thinking about x in order to examine it…When we

are thinking about x, the mind is focused on x, not on our thinking of x. Nevertheless, the process of thinking about x carries with it a non-reflective self-awareness (Goldman 1970, 96). A similar view has been defended by Owen Flanagan, who not only argues that consciousness involves self-consciousness in the weak sense that there is something it is like for the subject to have the experience, but also speaks of the low-level self-consciousness involved in experiencing my experiences as mine (Flanagan 1992, 194). As Flanagan quite correctly points out, this primary type of self-consciousness should not be confused with the much stronger notion of self-consciousness that is in play when we are thinking about our own narrative self. The latter form of reflective self-consciousness presupposes both conceptual knowledge and narrative competence. It requires maturation and socialization, and the ability to access and issue reports about the states, traits, dispositions that make one the person one is. To claim that every kind of self-consciousness is conceptual is overly cognitive. Bermúdez (1998), to mention one further philosopher in the analytic tradition, argues that there are a variety of **nonconceptual forms of self-consciousness** that are "logically and ontogenetically more primitive than the higher forms of self-consciousness that are usually the focus of philosophical debate" (1998, 274; also see Poellner 2003). This growing consensus across philosophical studies supports the phenomenological view of pre-reflective self-consciousness. (References: Stanford Encyclopedia, Post Modersnist Theories, Internet Encyclopedia of Philosophy) Korzybski pointed out that there are two ways to slice easily through life; to believe everything or to doubt everything. Both ways save us from thinking. So, from time to time you have to give yourself a Virtual Reality Check. The hallmark of postmodern philosophy has been disbelief or skepticism of all "metanarratives," or translations of reality. Postmodernism has even turned its profound skepticism on such important humanist concepts as "objective truth" and reason.Yet, for a deconstructionist postmodern society, individually we are still riddled with superstition and gullibility, and open to manipulation through our belief systems as any politician, philosopher, clergy, or salesperson will attest. Further, most people are painfully naive when it comes to even the simplest scientific understanding. Most of us don't have a clue about the fundamental nature of physical reality or our own psychological nature, and our ability to be fooled by our senses and mind, or methods of social and technological persuasion. We may think we know, but the penetration is deeper than we dare imagine. Nature rejects the naiveté that seeks absolute truth. We are beginning to realize, individually and culturally, that "realities" are all human constructions. The task becomes one of "catching ourselves in the act" of creating our own "reality" from the flow of events. Human truth is always an engagement of mind with experience.. We don't need to fear the collapse of our personalistic belief system (the "box" we live in), nor our belief in absolute truth. **A strong desire to engage in the "quest for uncertainty" complements our anxiety that perhaps there is no absolute, objective ground to reality**. The warrant of Truth is ever elusive when we deconstruct the foundational justifications of our convenient notions about the way the world works. It is easy to confuse what is actually the creation of beliefs with the "discovery of Truth," a common goal of science and theology. Science offers no ontological Scientific Picture of the World but substitutes a number of applied theories as meta-theory. It's close, powerful and useful: it 'works' and allows use to do work. But the still mysterious "true" metatheory may reveal the basis of consciousness. But who knows when we will lift that veil? Permeating the living reality of our culture are certain contagious notions, fads and trends that have the ability to influence the way we think about the nature of Reality and ourselves. Some of them are toxic and can consume you; others are just whims of pop culture infectious and benign as the common cold. **Jung** described the concept of psychic contagion by certain archetypal forces inherent in the human psyche, which manifest in our spiritual lives and belief systems. He spoke of both conscious and unconscious contamination. Notions like this range from simple superstitions to scientific concepts, to urban myths. Notions sweep through our culture and insinuate themselves within its fabric, as fads, whether they are "real" or not, they can be influential. They mood alter us, make us feel we belong. An analysis of these notions is useful in distinguishing a common human phenomenon from any potential "alien influence" which may or may not be exerted on us from an unknown source. **Strange Attractors: Transference, Holography, and an Archetype Burke, J. (2003). Strange Attractors: Transference, Holography, and an Archetype (Doctoral dissertation, Pacifica Graduate Institute, 2003).** In consideration to the Nobel Proze for Chemistry this year(2014), a stability nalysis of the proble in to single molexues would be a contemporaneous question that has to be taken cognisace of and given importance to . Determination of molecular structure by geometry optimization became routine only after efficient methods for calculating the first derivatives of the energy

with respect to all atomic coordinates became available. Evaluation of the related second derivatives allows the prediction of vibrational frequencies if harmonic motion is estimated. More importantly, it allows for the characterization of stationary points. The frequencies are related to the eigenvalues of the Hessian matrix, which contains second derivatives. If the eigenvalues are all positive, then the frequencies are all real and the stationary point is a local minimum. If one eigenvalue is negative (i.e., an imaginary frequency), then the stationary point is a transition structure. If more than one eigenvalue is negative, then the stationary point is a more complex one, and is usually of little interest. When one of these is found, it is necessary to move the search away from it if the experimenter is looking solely for local minima and transition structures. The total energy is determined by approximate solutions of the time-dependent Schrödinger equation, usually with no relativistic terms included, and by making use of the Born–Oppenheimer approximation, which allows for the separation of electronic and nuclear motions, thereby simplifying the Schrödinger equation. This leads to the evaluation of the total energy as a sum of the electronic energy at fixed nuclei positions and the repulsion energy of the nuclei. Notable exceptions are certain approaches called direct quantum chemistry, which treat electrons and nuclei on a common footing. Density functional methods and semi-empirical methods are variants on the major theme. **Orlando Alvarez** discusses Polyakov's quantization of the string in the presence of a boundary allowing for an arbitrary topology for the world sheet. In addition to the dynamical conformal factor discovered by Polyakov, there are a finite number of new degrees of freedom if the surface is more complicated than a sphere or a disc. The quantization of the Liouville theory in an arbitrary topology is also discussed. A one-loop calculation shows that the model is renormalizable if one performs a mass renormalization and an additive field renormalization. The renormalization group equations have a perturbative infrared unstable fixed point in all topologies. Copyright © 1983 Published by Elsevier B.V. **Nuclear Physics B Volume 216, Issue 1, 25 April 1983, Pages 125–184 Theory of strings with boundaries: Fluctuations, topology and quantum geometry Orlando Alvarez DOI: 10.1016/0550-3213(83)90490-X** Quantum geometry (the modern loop quantum gravity involving graphs and spin-networks instead of the loops) provides microscopic degrees of freedom that account for black-hole entropy. However, the procedure for state counting used in the literature contains an error and the number of the relevant horizon states is underestimated. In our paper a correct method of counting is presented by **Marcin Domagala and Jerzy Lewandowski.** Results lead to a revision of the literature of the subject. It turns out that the contribution of spins greater than 1/2 to the entropy is not negligible. Hence, the value of the Barbero–Immirzi parameter involved in the spectra of all the geometric and physical operators in this theory is different than previously derived. Also, the conjectured relation between quantum geometry and the black-hole quasi-normal modes should be understood again. **Marcin Domagala and Jerzy Lewandowski 2004 Class. Quantum Grav 21 5233 doi:10.1088/0264-9381/21/22/014 Black-hole entropy from quantum geometry** Quantum geometry predicts that a universe evolves through an inflationary phase at small volume before exiting gracefully into a standard Friedmann phase. This does not require the introduction of additional matter fields with ad hoc potentials; rather, it occurs because of a quantum gravity modification of the kinetic part of ordinary matter Hamiltonians. Authors draw the cognizance of application of the same mechanism that can explain why the present day cosmological acceleration is so tiny. DOI: http://dx.doi.org/10.1103/PhysRevLett.89.261301 **Inflation from Quantum Geometry Phys. Rev. Lett 89, 261301 – Published 12 December 2002 Martin Bojowald** The loop quantum cosmology of the closed isotropic model is studied with special emphasis on a comparison with traditional results obtained in the Wheeler-DeWitt approach by **Martin Bojowald** .Investigation includes the relation of the dynamical initial conditions to boundary conditions such as the no-boundary or the tunneling proposal and a discussion of inflation from quantum cosmology. **DOI: http://dx.doi.org/10.1103/PhysRevD.67.124023 Loop quantum cosmology, boundary proposals, and inflation Phys. Rev. D 67, 124023 – Published 23 June 2003 Martin Bojowald and Kevin Vandersloot** In loop quantum cosmology, the universe avoids a big bang singularity and undergoes an early and short super-inflation phase. During super-inflation, non-perturbative quantum corrections to the dynamics drive an inflaton field up its potential hill, thus setting the initial conditions for standard inflation. **Shinji Tsujikawa et al** show that this effect can raise the inflaton high enough to achieve sufficient e-foldings in the standard inflation era. They also analyse the cosmological perturbations generated when slow-roll is **violated after super-inflation** and show that loop quantum effects can in principle leave an **indirect signature** on the largest scales in the CMB, with some loss of power and

running of the spectral index. **Shinji Tsujikawa et al 2004 Class. Quantum Grav 21 5767 doi:10.1088/0264-9381/21/24/006 Loop quantum gravity effects on inflation and the CMB Quantum geometry from quantum information** is the study of **Florian Girelli and Etera R Livine .**Loop quantum gravity defines the quantum states of space geometry as spin networks and describes their evolution in time. We reformulate spin networks in terms of harmonic oscillators and show how the holographic degrees of freedom of the theory are described as matrix models. This allows us to make a link with non-commutative geometry and to look at the issue of the semi-classical limit of loop quantum gravity from a new perspective. This work is thought of as part of a bigger project of describing quantum geometry in quantum information terms. **Florian Girelli and Etera R Livine 2005 Class. Quantum Grav 22 3295 doi:10.1088/0264-9381/22/16/011 Reconstructing quantum geometry from quantum information: spin networks as harmonic oscillators.** Quantum geometric mechanics is the topic of study by **Dorje C. Brody Lane P. Hughston** establishing a clear relation between the quantum mechanics and quantum geometry and also the entangled states. The manifold of pure quantum states can be regarded as a complex projective space endowed with the unitary-invariant Fubini–Study metric. According to the principles of geometric quantum mechanics, the physical characteristics of a given quantum system can be represented by geometrical features that are preferentially identified in this complex manifold. Here we construct a number of examples of such features as they arise in the state spaces for spin, spin 1, spin and spin 2 systems, and for pairs of spin systems. A study is then undertaken on the geometry of entangled states. A locally invariant measure is assigned to the degree of entanglement of a given state for a general multi-particle system, and the properties of this measure are analysed for the entangled states of a pair of spin particles. With the specification of a quantum Hamiltonian, the resulting Schrödinger trajectories induce an isometry of the Fubini–Study manifold, and hence also an isometry of each of the energy surfaces generated by level values of the expectation of the Hamiltonian. For a generic quantum evolution, the corresponding Killing trajectory is quasiergodic on a toroidal subspace of the energy surface through the initial state. When a dynamical trajectory is lifted orthogonally to Hilbert space, it induces a geometric phase shift on the wave function. The uncertainty of an observable in a given state is the length of the gradient vector of the level surface of the expectation of the observable in that state; a fact that allows us to calculate higher orders corrections to the Heisenberg relations. A general mixed state is determined by a probability density function on the state space, for which the associated first moment is the density matrix. The advantage of a general state is in its applicability in various attempts to go beyond the standard quantum theory, some of which admit a natural phase-space characterization. **Journal of Geometry and Physics Volume 38, Issue 1, April 2001, Pages 19–53 Geometric quantum mechanics Dorje C. Brody Lane P. Hughston DOI: 10.1016/S0393-0440(00)00052-8** Expansion (unmesa) of [consciousness] or the creative intuition {pratibha) is (=) [experienced] in the interval which divides two [moments] of differentiated perception (vikalpa) It is here that they arise and disappear. The Sastras and Agamas proclaim with reasoned argument that it is (=) free of thought-constructs {Nirvikalpa) and precedes (e) Path to Liberation all mental representations of any object. None can deny that a gap exists between perceptions insofar as two moments of thought are (=) invariably divided. This [gap] is (=) the undifferentiated unity of all the countless manifestations. Similarly, in the outer more objective sphere, where change consists (e) of the alterations in the configurations of manifest appearances (Abhasa\ the transition from one to (e&eb) another corresponds to (e&eb) a phase of pure luminosity that marks (eb) the beginning of one form and the end of another. The world of manifestation and differentiated perceptions (Yikalpa) thus extends from one Centre to (eb) the next. **What is meant again is the realisation or the evolution of individual consciousness or general ledger to be the cosmic general ledger is the sole criterion for the Satchidananda state where all dualities cease** Although it is never in fact divorced from (e) the subject who resides there, the ignorant fail (e) to grasp this fact and so, cut off from (e) the Centre, the world of objectivity becomes (=) for them the sphere of Maya. Bhagavatotpala quotes the Light of Consciousness amvitprakdsa): This ever pure experience (suddhanubhava) is variegated by (e) each form [revealed within it] Even so it remains unstained (nirmala) when moving to another. Just as a cloth which is naturally white, once dyed, cannot (e) change colour without [first] becoming white again, similarly the pure power of awareness, (citi) once coloured by (e) form, is pure [again] at the Centre where that form is (=) abandoned and from whence it proceeds to (e&eb) another. In his Essence of Vibration (Spandasarpdoha), K§Emaraja explains that the rise and fall of every individual perception in the field of awareness is (=) a specific pulsation of consciousness. This is exactly

the individual general ledger transaction lifted to the transaction in cosmic general ledger. During The Initial Instant Of Perception, T Consciousness Is Manifestly Apparent And The Yogi, Participating In Its Plenitude, Observes The Outer World Without Being Attached To Any Particular Divine Body And Sacred Circle Of The Senses Singling It Out From Any Other, Like A Man Who Observes A City From A High Mountain Peak. He Sees The Outer World Reflected Within His Consciousness Free Of Thought-Constructs And So 'Stamps* The Outer On The Inner While Absorbing The Object And Means Of Knowledge In The Pure Subject Which Grasps Them As The Expansion Of His Own Nature. K§Emaraja Says: By Penetrating Into Bhairavimudra, The Yogi Observes The Vast Totality Of Beings Rising From, And Dissolving Into, The Sky [Of Conscious- Ness], Like A Series Of Reflections Appearing And Disappearing Inside A Mirror. Through The Practice Of Bhairavimudra The Yogi Realises That He Is The Substratum Consciousness (Adhisfhatr) Which Both Underlies And Is The Essence Of All Things. He Discovers That Phenomena Have No Independent Existence Apart From Him And So Are, In This Sense, Void. At The Same Time, He Realises That Because All Things Are Consciousness, They Are Far From Unreal. He Views The Outer World Yet Sees It Not. Beyond Both Voidness And Non-Voidness He Penetrates Into The Supreme Abode (Pararn Padam) Of Siva's Consciousness. [The Powers Of The Senses] Endowed With The Attributes Of The Great Union [Between Subject And Object] Whose Form Is The Wakening Of Man's Spiritual Potential (Kuntfalinf), Fill [With Consciousness] The Outer Clatter Of Diversity (Bhedatfambara) Born Of Its Intense Power And Are Then Established In The Unobscured Abode Of The Void Of Consciousness To Shine [There] Eternally. Thus Residing Beyond Being And Non-Being, The Sole Protector Of The Unity Which Is Tranquil And Expanding [Consciousness], Whose Glory Is All-Embracing And Form Unobscured, Is Called Bhairavimudra. Through The Practice Of Bhairavimudra, The Yogi Unites The Universal Vibration Of T Consciousness With The Individual Pulsation Of Objectivised 'This' Consciousness. The Two Aspects Of Consciousness Are Now In A State Of Equilibrium Like The Two Pans Of An Evenly Weighed Balance And The Yogi Experiences The Pure Knowledge (Suddhavidya) That: 'I Alone Am All Things'. Thus Becoming The Master Of The Wheel Of Energies He Is Free, Like Siva, To Create And Destroy. When [The Yogi] Is Well Established, Without Wavering, Solely In The Integral Egoity Of His Authentic Nature, The Spanda Principle, And Is Absorbed In Contemplation (Samavisfa), He Becomes One With It The Doctrine Of Vibration (Tanmaya). Then . . . Dissolving And Creating The Universe By Means Of His Introverted And Extroverted Absorption, He Destroys And Creates All Things Out Of Sarikara, His Innate Nature. [Thus] He Assumes The State Of The Universal Experiencer And Having Absorbed All That Is To Be Experienced From [The Grossest Level] — Earth — To (The Subtlest) — Siva — He Reaches The State Of The Supreme Subject By Progressively Recognising [His Identity With Him]. Thus, Introverted And Extroverted Absorption Both Lead To The Recognition Of The Pulsation (Spanda) Of One's Own Consciousness. This Is Where Individual General Ledger Becomes Cosmic General Ledger. At The Level Of Consciousness Corresponding To Siva's Basic State (Saryibhavdvastha), The Alternation From Inner To Outer Is Instantaneously Resolved Into The Vibration Of His Nature. When The Yogi Finally Comes To Be Constantly Aware Of This Reality, His Enlightenment Is Full And Perfect. Freed Of All Means (Anupaya) And Delighting In The Power Of His Bliss (Anandasakti), He Knows And Does Whatever He Pleases. The Yogi Seeking Self-Realisation Must Acquire Mastery Over This Movement. Ksemaraja Stresses That The Doctrine Of Vibration Teaches That Liberation Can Only Be Achieved By First Withdrawing All Sense Activity In Introverted Contemplation (Nimilanasamadhi) To Then Experience The 'Great Expansion' (Mahdvikdsd) Of Consciousness While Recognising This To Be A Spontaneous Process Within It. This Is Done Through The Practice Of Kramamudra. A Passage From The Now Lost Kramasiitra Explains: Although The Adept's Attention [May Be] Outwardly Directed, He Enjoys Contemplative Absorption Through The Introverted Aspect Of Kramamudra. Initially He Turns Inward From The Outside World And [Then] From Within [Himself] He Exits Into The Outer World Under The Influence Of His Absorption. There Is No Necessity Of Any Glorification Or Mortification Here. It Is Just Like Joining A Job And Retiring And Realising All That Happened Is Just By His Own Effort And Nothing Else. Thus The Sequence (Krama) In This Attitude (Mudra) [Ranges Through] Both Inner And Outer. The Yogi Must Pervade The Surface Level Of Awareness (Yyutthana) With The Same Bliss He Experiences Plunging Into The Depths Of Con-Templative Absorption (Samadhi). Note Here That Samadhi Is A Death Like State And Still Being Matter And Information Stored You Experience Consciousness. Submerging Himself And Emerging Repeatedly From Samadhi,

He Eventually Recognises That The Unity Of Consciousness Pervades Both States: The Best Of Yogi's, Who Has Achieved A State Of Complete Absorption Even When Risen From Meditation, [Inwardly] Vibrating Like A Drunkard In Blissful Inebriation From The After-Effects Of The Nectar Of Con- Templation, Sees All Things Dissolving In The Sky Of Consciousness Like A Cloud In The Autumn Sky. He Plunges Repeatedly Within Himself Divine Body And Sacred Circle Of The Senses And Becomes Aware Of His Identity With Consciousness By The Practice Of Introverted Contemplation. Thus Even When He Is Said To Have Risen From Absorption, He Is One With [His] Experience Of It. Ksemaraja Goes On To Explain That This Practice Is Called 'Mudra' Because It Both Fills The Adept With Bliss (Mud) And Is Itself The Bliss Of Consciousness. Moreover, It Dissolves Away (Dra) All Bondage And 'Stamps' The Universe Of Experience With The Seal (Mudra) Of The Fourth State (Turiya) Of Enlightened Consciousness Beyond, And Including, The Three States Of Waking, Dreaming And Deep Sleep. It Is Called 'Krama' Because It Is The Root Source Of All Emanation And All Other Conscious Processes Which Succeed One Another In Ordered Sequence (Krama) And Is, At The Same Time, Their Successive (Krama) Appearance As Well. By The Practice Of Kramamudra The Opposites Fuse And Siva And Sakti Unite. They Yogi Comes To Experience The Simultaneous Pervasion Of All The Lower, Grosser Categories Of Existence By The Higher And The Presence Of The Lower In The Higher. Commencing His Practice In A Low Form Of Bhairavimudra, The Yogi Conjoins The Outer With The Inner; Then, In Kramamudra, He Fills Both The Outer With The Inner And The Inner With The Outer. When He Achieves Perfection In This Two-Fold Movement, He Attains To The Highest Form Of Bhairavimudra In Which The Two Merge Completely In The Experience Of The Absolute (Anuttara), Free Of All Differentiation And Polarities. If He Fails To Maintain Awareness Of This State, He Again Falls Into Kramamudra Until He Has Finally Completely Merged All The Highest States In The Lower And The Lower In The Higher. He Then No Longer Needs To Resort To Any Means (Anupaya) To Achieve Liberation. All He Says Or Does Anything He Perceives Or Thinks, Instantly Occasions In Him The Highest Level Of Consciousness. Thus The Fruit Of Bhairavimudra Is The Wonder (Camatkara) Or Amazement (Vismaya) That Overcomes The Yogi When He Reaches The Plane Of Union (Yogabhumika) Y Xsl Where All Opposites Merge In The Radiance Of The Great Light Of Consciousness. The Stanzas On Vibration Teach: How Can One Who, As If Astonished, Beholds His Own Nature As That Which Sustains [The Existence Of Everything] Be Subject To This Painful Round Of Transmigration (Kusrti)? The Yogi, Recognising His True Nature To Be The Supreme Subject, Is Astonished To Suddenly Discover That The Individual He Thought He Was, Caught Up In The Trammels Of Thought And Living In A World Enmeshed In The Web Of Time And Space, Does Not Really Exist At All. He Experiences A 'Turning About' (Paravrtti) In The Deepest Seat Of The Doctrine Of Vibration Consciousness As He Penetrates His True Nature. The Sudden Eruption Of This Intuition (Pratibha) Arouses In Him A Cry Of Amazement As He Transcends All Thought-Constructs And, Perfectly Absorbed In His Own Nature, Is Liberated. The Path To Liberation Essentially, Spanda Doctrine Is Concerned With Two Matters. The First Is To Impart To Those Who Are Fit To Receive The Teachings A Deeper Understanding Of The Ultimate Goal Of Life (Upeya). When We Have Understood What Truly Benefits Us And Is Worth Attaining And What, On The Contrary, Is Of No Real Value But Stands In The Way Of This Attainment, We Can Begin To Make Progress Towards Our Goal. This Is Spanda Doctrine's Second Concern, Namely, To Show The Way In Which We Can Develop Spiritually Through Siva's Grace And The Right Application Of The Means To Realisation That It Teaches. When Both These Aspects Of The Teaching Have Been Correctly Understood And Applied, The Spanda Yogi Achieves A Clear And Permanent Realisation Of His Goal And Is Liberated, Thus Fulfilling The Ultimate Aim Of The Teaching. The Doctrine Of Vibration Is Not Meant For The Spiritually Dull. It Is Not For The Worldly Whose Consciousness, Clouded By Ignorance, Is As If Dreaming, Even During The Waking State Of Daily Life, The Dream Of Its Own Thought-Constructs. The Teachings Are Meant For Those Who Are Awake (Prabuddha), Those Who, Full Of Faith And Reverence, Are Always Alert And Intent On Discerning The True Nature Of Ultimate Reality. This Reality Is Understood In Three Basic Ways. The First Is Purely Transcendental. The Stanzas Choose This Aspect As The One Which Formally Defines It Most Specifically. Ultimate Reality Transcends All The Opposites, Including Subject And Object. This Does Not Mean, However, That It Is An Unconscious Void, A Mere Absence Of All Existence. In Fact, This Negative Characterization Of Reality (Which Includes Also A Denial Of All That Is Unconscious) Implies A Positive Immanence In Which The Opposites Are United In The Oneness Of Pure Consciousness That Is

Equally Siva And Spanda, His Universal Activity. These Two Seemingly Contrasting Aspects Are Reconciled In The Third, Namely, Reality Understood As The Essential Nature Of All Things. Although Universal And Everywhere The Same, It Is Understood To Be The Essential And Specific Nature Of Each Existent As Its 'Own Nature' (Svabhava). In The Case Of The Individual Soul It Is Even More Specific, More Personal As His Own 'Own Nature' (Svasvabhava). Note Here That Svabhava Is Not Svasbhava. In Fact We Made A Statement That Svabhava Is The Very Characterstics Of Space Time Like, Crime Zones, Silent Zones, School Zones We Classify Space In To. Belonging To None Other Than Oneself It Is The Pure Subjectivity That Perceives Experiences, Enjoys, Reflects, Thinks And Senses As Well As Being The Conscious Agent Who Creates Every Possible Form Of Experience In All The States Of Consciousness. The Liberating Knowledge Of Reality Thus Corresponds To Our Regaining Possession Of Ourselves (Svatmagraha). We Must Lay Hold Of Ourselves And Abide In Our Authentic Nature. Reality Coincides With Our Own Most Fundamental State Of Being (Svasthiti), Free Of All Contrasts And Contradictions. Once We Have Overcome The Negative Forces That Arise From Our Ignorance And Prevent Us From Abiding In Ourselves, We Are Liberated. To Do This, We Must Penetrate Through The Pulsing Fluctuations Of Objectively Experienced States And Perceptions At The Surface Level Of Consciousness And Gain Insight Into The Timeless Rhythm Of Our Own Nature Manifest In The Universal Arising And Falling Away Of All Things. We Are Not Freed Of The Trammels Of Perpetual Change By Setting It Aside; On The Contrary, We Must Gain Insight Into The Recurrent Cycles Of Creation, Persistence And Destruction, Or Else Be Bound By Our Ignorance. This Spiritual Ignorance Consists Essentially Of Our Contracted State Of Consciousness And So Can Only Be Effectively Countered By Expanding It To Reveal Our Own Authentic Nature As This Expanded State Itself, Which Is The Universal Vibration (Samanyaspanda) Of Consciousness. Vibration Of Strings Which Gives Shape To Objects Is Itself A Information, A Sine Qua Non Of Individual Consciousness. The Spanda- Yogi Treads The Path Of Consciousness Expansion. The Movement From The Contracted To The Expanded State Marks The Transition From Ignorance To Understanding, From The Dispersion And Incompleteness Of A Form Of Consciousness Entirely Centred On An Objectively Perceived And Discursively Represented Reality To A Direct, Intuitive Awareness Of The Unity And Integral Wholeness Of Our Own Absolute Spanda Nature. Along The Way To This Supreme Realisation Consciousness Develops, As Veil After Veil Is Lifted, Until It Becomes Full And Perfect In The Absolute Which Encompasses Within Itself All Possible Formats Of Experience. As Abhinava Says: [This Realisation] Is The Supreme Limit Of Plenitude And As Such There Can Be No Higher Attainment. Any [Other] Attainment [We Can] Conceive Issues From A State That Falls Short Of [This] Perfection. Once* [This] Uncreated Fullness Has Been Attained, Pray Tell, What Other Fruit Can There Be [Beyond It]? The Fettered Soul's Contracted State Of Consciousness Binds Him Because He Is Deprived Thereby Of The Subtle, Intuitive Insight Into The Underlying Unity Of Existence And His Attention Is Focused Instead On Its Gross, Outer Diversity Easily Apparent To Everybody, However Restricted His Consciousness May Be. However, Although The Fettered Soul In This State Is Ignorant Of This Unity, This Does Not Mean That His Knowledge Of Diversity Is False. Ignorance Entails A Form Of Knowledge Which, Although Quite Correct, Is Binding. We Are Not Absolutely Ignorant Of Reality For If We Were We Would Be Totally Unconscious. Spiritual Ignorance Is Always Linked With Some Degree Of Consciousness. Those Subject To The Round Of Birth And Death Are Not Inert Clods Of Earth. This Also Leads To Conclusion That Shakti Responsible Is Impure Because Of Maya Imbibed In It. Thus, Although Ignorance Obscures Consciousness, It Is Wrong To Think Of It, As Dualist Saivites Do, In Terms Of A Defiling Impurity That Shrouds It Like A Cloth Covering Ajar. Spiritual Ignorance Can Be Nothing But Consciousness Itself, Albeit In A Limited State. Siva, Who Is Universal Consciousness, Is The Innate Nature Of Both Its Contracted And Expanded States, Both Of Which Are Forms Of Knowledge, Namely: 1) Supreme Knowledge (Parajnana) Defined As The Revelation Of One's Own Innate Nature As The One Reality Which Is The Being Of All Things. 2) Inferior Knowledge (Aparajndna) Which Jayaratha Explains Results From The Mental Activity (Yyapard) Of The Individual Subject Whose Consciousness Is Contracted. It Consists Of The Mental Representations (Vikalpa) He Forms Of Himself And His Object, Of The Type I Know This'. The Lower Knowledge Obscures The Higher And Binds The Soul By Breaking Up His Direct, Pervasive Awareness Of His Own Pure Consciousness Nature, Free Of Mental Representation. The Stanzas On Vibration Teach: Operating In The Field Of The Subtle Elements, The Arising Of Mental Representation Marks The Disappearance Of The Flavour Of The Supreme Nectar Of Immortality; Due To

This [Man] Forfeits His Freedom. As We Have Already Seen, Three Factors Are Necessary For Perception And Thought To Be Possible, Namely, The Perceiving Subject, The Means Of Knowledge And The Object Perceived. Rajanaka Rama, In His Commentary On The Stanza Cited Above, Explains At Length That These Three Factors Correspond To Three Major Divisions In The Lower Thirty-One Categories Of Existence, Namely: 1) The Object. This Consists Essentially Of The Five Primary Sensations Which Are The Subtle Elements {Tanmatras) Of Smell, Taste, Sight, Touch And Sound Along With The Five Gross Elements — Earth, Water, Fire, Air And Ether — Of Which These Sensations Are The Perceivable Qualities. 2) The Means Of Knowledge. This Consists Of The Senses And The Inner Mental Organ. 3) The Subject. At This Level, The Subject Is The Individual Soul (Purusa) Whose Consciousness Is Contracted By The Five Obscuring Coverings (Kancukas) Of Limited Knowledge And Action, Attachment, Natural Law And Time Along With Maya, Their Source. All These Categories Belong To The Impure Creation (Asuddhasrsti), Which Is The Sphere Of Maya Where The Lower Order Of Knowledge Operates And Subject And Object Are Divided. Above Them Are Five More Categories Which Belong To The Pure Creation (Suddhasrsfi) Where Subject And Object Are Still United. The Highest Of Those Categories Are Siva And Sakti. Combined They Represent The State Of Pure T Consciousness And Its Sentient Subjectivity (Upalabdhrta), Respectively. The Next Category Is Called Sadasiva. Here Faint Traces Of Objectivity Appear In The Pervasive, Undivided Consciousness Of Siva And Sakti. Consciousness, Now Full Of The Power Of Knowledge (Jnanasakti), Views The All In A State Of Withdrawal (Nimesa), Shining Within, And At One With Its Own Nature. T Consciousness Predominates Over 'This' Consciousness Which It Encompasses In The Awareness That: 'I Am This [Universe]' (Aham-Idam). Next Comes The Category 'Isvara' Corresponding To The Awareness: 'This (Universe) Is Me' (Idam-Aham). 'This' Consciousness Takes The Upperhand Over T Consciousness And Unfolds Externally Full Of The Creative Power Of Action (Kriya&Akti). The All Now Becomes More Clearly Manifest As An Independent Reality. It Is Still Experienced As One With Consciousness But Is No Longer Fully Merged Within It. Finally, When Both Subjective And Objective Aspects Share An Equal Status In The Two-Fold Awareness That: 'I Am This (Universe) And This (Universe) Is Me (Ahamidam-Idamaham)\ Pure Knowledge (Fuddhavidya), The Last Of These Categories, Emerges. The Pure Categories Are The Experience Of The Impure Categories When They Are Recognised To Be One With Consciousness. They Are Experienced Within The Domain Of The Pure Universal Subject The Enlightened Yogi Realises Himself To Be. Mental Representations (Yikalpa) Emerge From This Pure Awareness And Subside Into It In Consonance With The Rhythm Of The Emanation And Withdrawal Of The Lower Categories. Impelled By The Universal Will, This Movement Is Spontaneous And Free. Free Of All Hopes And Fears The Enlightened Yogi Sees All Things As Part Of This Eternal Cosmic Game, Played In Harmony With The Blissful Rhythm Of His Own Sportive Nature At One With All Things. The Stanzas On Vibration Teach: Everything Arises [Out Of) The Individual Soul And He Is All Things. Being Aware Of Them, He Perceives His Identity [With Them]. Therefore There Is No State In The Thoughts Of Words Or [Their] Meanings That Is Not Siva. It Is The Enjoyer Alone Who Always And Everywhere Abides As The Object Of Enjoyment. Or, Constantly Attentive, And Perceiving The Entire Universe As Play, He Who Has This Awareness (Sawvitti) Is Undoubtedly Liberated In This Very Life. According To The Doctrine Of Vibration, Only Liberation In This Life (Jivanmukti) Is Authentic Liberation. Liberation After Death (Yideha- Mukti) In Some Form Of Disembodied State Free Of All Perceptions And Notions Of The World Of Diversity Is Not The Ultimate Goal. Ksemaraja Stresses That Liberation Is Only Possible By Realising One's Own Identity With The Whole Universe, However Difficult This May Be. Similarly, He Maintains That The Suspension Of All Mental And Sensory Activity, Which Takes Place In The Introverted Absorption Of Contemplation With The Eyes Closed (Nimilanasamadhi) That Leads To Identification With Transcendent Consciousness Is Complemented And Fulfilled By The Cosmic Vision Had Through The Expansion Of Consciousness That Takes Place In Contemplative Absorption With The Eyes Open (Unmilanasamadhi). Consequently, Ksemaraja Explains That The First Of The Three Sections, Into Which He Divides The Stanzas, Deals With The Former Mode Of Contemplation And The Second Section With The Latter. Significantly, The Last Stanza Of The Second Section Ends With The Declaration That 'This Is The Initiation That Bestows Siva's True Nature'. In Other Words, This Realisation, Attained Through The Expanding Consciousness Of Contemplation With The Eyes Open, Initiates The Yogi Into The Liberated State, Which Is Identification With Siva Whose Body Is The Universe. In Order To Attain This Expanded State Of Liberated Consciousness, The Yogi Must

Find A Spiritual Guide Because The Master (Guru) Is The Means To Realisation. The Master Is For His Disciple Siva Himself For It Is He Who Through His Initiation, Teaching And Grace, Reveals The Secret Power Of Spiritual Discipline. Instructing In The Purport Of Scripture He Does More Than Simply Explain Its Meaning: He Transmits The Realisation It Can Bestow. The Master Is At One With Siva's Divine Power Through Which He Enlightens His Disciple. It Is This Power That Matters And Makes The Master A True Spiritual Guide, Just As It Was This Same Power That Led The Disciple To Him In His Quest For The Path That Leads To The Tranquility That Can Only Be Found 'In The Abode Beyond Mind'. The Master Is The Ferry That Transports The Disciple Over The Ocean Of Thought — If, That Is, The Disciple Is Ready. The Disciple Must Be 'Awake' (Prabuddha) 2% Attending Carefully To The Pulse Of Consciousness. This Alert State Of Wakefulness Is At Once The Keen Sensitivity Of Insight As Well As The Receptivity Of One Who Has No Other Goal To Pursue Except Enlightenment. The Highest, Most Perfect Relationship The Disciple Can Have With His Master Is Such As It Is With Siva Himself: One Of Identity. The Exchange That Takes Place Between Them Is An Internal Dialogue Within Universal Consciousness, Their Common Identity (Svabhava). Limiting Itself To A Point Source (Anu) And Obscured By The Thought-Constructs Born Of Doubt And Ignorance, Consciousness Assumes The Guise Of The Disciple Who Seeks To Attain The Expanded Fullness Of His Master's Consciousness. The Master, On The Other Hand, Embodies The Aspect Of Consciousness Which Responds To The Inquiring Consciousness Of His Disciple. Free Of The Notions Of 'Self And 'Other', When The Disciple Is Liberated By His Grace, It Is The Master Who In Reality Liberates Himself. Although Ksemaraja Assures Us That The Master Can By Himself Enlighten His Disciple By The Initiation He Imparts To Him And The Other Means (Yukti) N He Adopts, Even So, He Is Not The Only Guide On The Path. Apart From The Master There Is Scripture And, Above All, One's Own Personal Experience, Because, As Abhinava Says: The Knowledge [Acquired] By Gradually [Coming To Understand The Meaning Of] The Scriptures And Following The Master [Who Knows Them] Leads, [When] Confirmed For Oneself, To The Realisation Of One's Own Identity With Bhairava. It Is Important To Know The Scriptures. God Reveals Himself Through Them; They Are One Of The Forms In Which He Is Directly Apparent In This World. They Teach Man What Is Worth Attaining And What Should Be Avoided And So Like A Boat Convey Him Across The Ocean Of Profane Existence (Samsara) To The Other Shore Where God's True Nature Is Revealed To Him. However, The Study Of The Scriptures Is Of Value Only If Accompanied By The Spiritual Knowledge That Results From Personal Experience. Mahesvarananda Writes: Being Well Versed In The Nature Of Deity Is One Thing, But Being Well Versed In The Sacred Scripture Is Another, Just As The Peace Of That Abode Is One Thing And What Worldly People Experience Is Another. Vasugupta, Who Found The Sivasutra, Knew The Means To Realisation (Yukti) As Well As The Scriptures And Had Fully Experienced The One Ultimate Reality. Therefore, Ksemaraja Declares Him To Be Amongst The Best Of Teachers. The Stanzas On Vibration (That Ksemaraja Attributes To Vasugupta) Accordingly Transmit The Secrets Of The Sivasutra In Accord With Scripture, Sound Reasoning And Personal Experience. The Latter Is Particularly Important For The Spanda Yogi; He Is Not Interested In Wasting His Time In Useless Discussion About The Experience Of Consciousness Expansion And Its Fruits, For That Can Only Be Known For Oneself. The Yogi Can Achieve This Experience Either Through Faith In The Master Or Personal Insight (Svapratyayatah) Acquired By Unswerving Devotion To God. Ksemaraja Accordingly Quotes A Passage From The Bhagavadgita Where Krsna Says: Those I Deem To Be The Best Yogis Who Fix Their Thoughts On Me And Serve Me, Ever Integrated [In Themselves], Filled With The Highest Faith. But While The Yogi's Development Depends On Faith And Personal Experience Of The Higher States Of Consciousness, He Can, And Must, Strengthen His Conviction In The Light Of Reason .When Reason (Upapatti) And Direct Insight (Upalabdhi) Work Together, They Serve As A Means To Liberation. Reason Alone Cannot Help Us, But When It Is Based On An Intuitive Insight Of Fundamental Principles Along With A Direct Experience Of Reality, Error Is Eradicated And The Yogi Is Freed. In This Way The Awakened Yogi Realizes His Inherent Spiritual Power (Svabala) With Which He Exerts Himself To Distinguish Between The **Motions Of Individualized Consciousness And The Universal Vibration (Samanyaspanda)** Of The Collective Consciousness That Is Their Ultimate Ground And Firm Foundation. Note The Information Of Individual, Collective And Cosmic General Ledger Is Alone Responsible For Creation And Also Destruction. Thus, Although The Doctrine Taught In The Stanzas On Vibration Accords With Scripture, It Is Supported By Reason And Above All By Personal Experience. Thus, For Example, The Seventeenth Stanza

Describes The Difference In The Manner In Which The Well Awakened And The Unawakened Experience Their Own Nature (Atmopalambha While The Eighteenth Describes The Experience Of The Well Awakened In The Three States Of Waking, Dreaming And Deep Sleep. Indeed, Rajanaka Rama, One Of The Commentators, Explains That The First Sixteen Stanzas Establish On The Basis Of Personal Experience (Svanubhava) That One's Own True Nature Is Independent Of The Body. Similarly, The Remaining Stanzas Also Discuss The Direct Experience Of One's Own Nature, But This Time As The Unity Of All Things. This Direct Experience, In Its Diverse Aspects, Is Both The Means By Which The Yogi Develops His Consciousness As Well As His Ultimate Goal. **Doctrine Of Vibration: Mark S.G.Dyczkowski Models See Next Paper** From the point of view of the object, the expansion (unmesa) of this pulse is represented(eb) by the initial desire to perceive (didrksa) a particular object, while the contracted (nimesha) phase is (=) the withdrawal of attention from the object previously perceived. From the point of view of the perceiving subjectivity, the phases are (=) reversed, so that the initial desire to perceive marks (eb) the contraction (nimesha) of subjective consciousness while the falling away of the previous perception is (=) its expansion (unmesa). At the higher level, where these two phases are experienced within (eb) consciousness, they represent (eb) the state of the categories of Isvara ('this universe is me') and Sadasiva ('I am this universe'). Utpaladeva says: Expansion (unmesa\ which is in (eb) the external manifestation [of objectivity], is (=) Kvaratattva while contraction (Niemba), which is in the internal manifestation [of subjectivity], is Sadasiva. ***There is no glorification or mortification here. It just like joining the job and retiring the involution and evolution phases***.(italics mine)The Doctrine of Vibration At this level all the powers of consciousness fuse (e&eb) and both phases are (=) manifest as part of one reality. This unity is in fact apparent (eb) to everybody at each moment. However, within the domain of Maya, which is the sphere of differentiated perceptions (Yikalpa), it is clearly manifest (eb) only at the juncture (Madhya) between two cognitions. In this Centre resides (eb) the void (kha) of consciousness (free of (e) thought-constructs) which divested of (e) diversity, digests into (eb) itself all the psycho-physical processes that give (eb) life to the multiplicity of perceptions. **Here it is important to note that the when the cosmic general ledger zero reflects upon itself and it is projected on the individual consciousness you see a film an illusory animations wherein creation itself becomes subjective wherein the individual consciousness is both the perceiver the perceived being the image on the screen on the screen of individual consciousness**. The yogi moves from the particular vibrations of consciousness at its periphery to (e&eb) the universal throb of the Heart in the Centre. As Abhinava explains: The self-reflective awareness in (eb) the Heart of pure consciousness, present (eb) at the beginning and end of each perception, within (eb) which the entire universe is (=) dissolved away without residue, is (=) called in the scriptures, the universal vibration of consciousness (samanyaspanda) and is (=) the outpouring (uccalana) [of awareness] within one's own nature. All the categories of existence (tattvas) are united in (eb) the Heart of the Centre where the life-giving elixir of Siva's consciousness floods (e&eb) one's own inner nature. To reside in the Centre is to abide by (e) the law of totality (gramadharma) in a state which transcends (e) the workings of the mind (unmana).Consciousness (jnana) with Light as its support, residing in (eb) the Centre between being and non-being is (=) known as the act of abiding in (eb) one's own abode as (=) the perceiving subjectivity (dratfrtva) free of (e) all obscuration. That which has been purified by (e) pure awareness **(suddhavijnana)** is called the transcendent (viviktavastu), said to be (=) the mode of being (v/7//) of the law of totality (gramadharma) through (e&eb) which everything is (=) easily attainable. The power in the Centre (madhyasakti) is (=) the eternal Present Beyond time it is the source of both past and future. To be established there is to abide without a break in Rama, the supreme enjoyer, in every action of one's life. Rama is Siva, the supreme cause Who pervades the fourteen aspects which embrace the entire universe of experience, namely, moving, standing, dreaming, waking, the opening and closing of the eyes, running, jumping, exertion, knowledge [born] of the power of the senses, the [three] aspects of the mind, living beings, names and all kinds of actions. This came as a natural development in Spanda doctrine not only for **this reason but also because the universal ego is experienced as the inner dynamics of absolute consciousness. To conclude our summarial exposition of the Divine Means, which is centred on the direct experience of this pure ego (and hence on Spanda in this form), we turn now to a brief description of its inner, cyclic activity. We shall do this** by examining Abhinava's esoteric exegesis of the symbolic significance of the word 'A HAM', which in Sanskrit means T, and symbolises by its form the ego's dynamic nature. “Dasein Does Not Fill Up A Track Or Stretch ‘Of Life’ — One Which Is Somehow Present-At-Hand — With The Phases Of Its

Momentary Actualities. It Stretches Itself Along In Such A Way That Its Own Being Is Constituted In Advance As A Stretching-Along. The 'Between' Which Relates To Birth And Death Already Lies In The Being Of Dasein … It Is By No Means The Case That Dasein 'Is' Actual In A Point Of Time, And That, Apart From This, It Is 'Surrounded' By The Non-Actuality Of Its Birth And Death. Understood Existentially, Birth Is Not … Something Past In The Sense Of Something No Longer Present-At-Hand; And Death Is Just As Far From Having The Kind Of Being Of Something … Not Yet Present-At-Hand But Coming Along … Factical Dasein Exists As Born; And, As Born, It Is Already Dying, In The Sense Of Being-Towards-Death. As Long As Dasein Factically Exists, Both The 'Ends' And Their 'Between' *Are*, And They Are In The Only Way Possible On The Basis Of Dasein's Being As *Care* … As Care, Dasein Is The 'Between'." **— Martin Heidegger (Thomas Piel - Traducteur), Being And Time** Path to Liberation On without knowledge that everything is manifest within consciousness is illusory or unreal in that sense alone. 78 Things are more real or more tangibly experienced according to their own essential nature (svabhava) to the degree in which we recognise that they are appearances (abhasa) within absolute consciousness As Jayaratha says: Just as images manifest in a mirror, for example, are essentially mere appearances, so too are [phenomena] manifest within conscious- ness. Thus, beause they are external, [phenomena] have no being (sattva) of their own. The Lord says this [not with the intention of saying anything about the nature of things] but in order to raise the level of consciousness of those people who are attached to outer things; thus everything in this sense is essentially a mere appearance. [Knowing this], in order to quell the delusion of duality, one should not be attached to anything external. 79 The ultimate experience is the realisation that everything is contained within consciousness. We can discover this in two ways. Either we merge the external world into the inner subject, or we look upon the outer as a gross form of the inner. In these two ways we come to recognise that all things reside within our own consciousness just as consciousness resides within them. This all-embracing inwardness is only possible if there is an essential identity between the universe and consciousness. The events which constitute the universe are always internal events happening within consciousness because their essential nature is consciousness itself. 80 We can only account for the fact that things appear if there is an essential identity between consciousness and the object perceived. If a physical object were really totally material, that is, part of a reality independent of, and external to, consciousness, it could never be experienced. Abhinava says: The existence or non-existence of phenomena within the domain of the empirical (iha) cannot be established unless they rest within consciousness. In fact, phenomena which rest within consciousness are apparent (prakasamana). And the fact of their appearing is itself their oneness {abheda) with consciousness because consciousness is nothing but the fact of appearing {prakasa). If one were to say that they were separate from the light of [that consciousness] and that they appeared [it would be tantamount to saying that] 'blue' is separate from its own nature. However, [insofar as it appears and is known as such] one says: 'this is blue 1 . Thus, in this sense, [phenomena] rest in conscious- ness; they are not separate from consciousness. The Doctrine of Vibration The universe and consciousness are two aspects of the whole, just as quality and substance constitute two aspects of a single entity. The universe is an attribute (dharma) of consciousness which bears (dharmiri) it as its substance. It is said that 'substance' is that resting in which this entire group of categories manifests and is made effective. Now, if you don't get angry [we insist that] this entire class of worlds, entities, elements and categories (tattva) rests in consciousness and [resting in it] is as it is. Thus consciousness contains everything in the sense that it is the ground or basis (adhard) of all things, their very being (satta) and substance from which they are made. But, unlike the Brahman of the Advaita Vedanta, it is not the real basis (adhisthand) of an unreal projection or illusion. Consciousness and its contents are essentially identical and equally real. They are two forms of the same reality. Consciousness is both the substratum and what it supports: The perceiving awareness and its object. In this respect, the Kashmiri Saiva is frankly and without reserve an idealist. Although he does not deny the reality of the object, his position is at odds with most commonly accepted forms of realism. The realist maintains that the content perceived is independent of the act of perception. The content is only accidentally an object of perception and undergoes no change in the process of being perceived. His contention, however, is essentially unverifiable; to verify it, we would have to know an object without perceiving it. This, from the Kashmiri Saiva point of view, is not possible. Objects of which we have no knowledge may indeed exist, but they are knowable as objects only if they are related to subjects who perceive them. In this sense, if there were no subjects, there could be no objects. 86 The subject, however, as opposed to the object is, in terms of the phenomenology of

perception, apparent to himself. He is self-luminous (svaprakdsa). Thus, conscious- ness (the essence of subjectivity) is one s own awareness by virtue of which all things exist. The realist maintains that consciousness clearly differs from its object insofar as their properties are contrary to each other. The Saivite idealist. However, says that the object is a form of awareness (vijnanakara)™ The objective status of the object is cognition itself. Perception manifests its object and renders it immediately apparent (sphuta) to those who perceive it. It does not appear at any other time. If 'blue' were to exist apart from the cognition of k blue\ two things would appear: 'blue' and its cognition, which is not the case. It is the perception of the object which constitutes its manifest nature. An entity becomes an Integral Monism of Kashmiri Saivism object of knowledge not by virtue of the entity itself but by our knowledge of it. If objects had the property of making other objects appear, it would be possible for one object to make another appear in its own likeness. 'Blue' is perceived to be 'blue' because it is manifest as such to the perceiver. As Abhinava points out: The [nature of an] object of knowledge could not be established through a means of knowledge totally unrelated to it — a crow does not become white because a swan [sitting next to it] is white. Perception, on the other hand, is immediately apparent to conscious- ness. It is self-luminous in the sense that it is directly known without need of being known by any ulterior acts of perception and makes its object known at the same time. Adopting the Buddhist Yogacara doctrine that things necessarily perceived together are the same (sahopa- lambhaniyamavada), the Saivite affirms that because the perceived is never found apart from perception, they are in fact identical. Reality (satya) is the point where the intelligible and the sensible meet in the common unity of being; it cannot be said to exist in itself outside, and apart from, knowledge or vision. Bhagavatotpala in his commentary on the Stanzas on Vibration quotes: Once the object is reduced to its authentic nature, one knows [the true nature of] consciousness. What then [remains of] objectivity? What [indeed could be] higher than consciousness? Consciousness is essentially active. Full of the vibration of its own energy engaged in the act of perception, it manifests itself externally as its own object. When the act of perception is over, consciousness reabsorbs the object and turns in on itself to resume its undifferentiated inner nature. Knowledge (jnana) manifests internally and externally as each individual entity.... Once knowledge has assumed that form it falls back [into itself]. The Yogacara Buddhist similarly maintains that consciousness creates its own forms. But, according to him, because the perceived and perception are identical; there is no perceived object at all. The so-called outer world is merely a flux of cognitions, it is not real. He is firmly committed to a doctrine of illusion. The reality of consciousness from The Doctrine of Vibration his point of view is established by proving the unreality of the universe. "All this consists of the act of consciousness alone", says Vasubandhu, "because unreal entities appear, just as a man with defective vision sees unreal hair or a moon, etc."He points to dreams as examples of purely subjective constructs which appear to be objective realities. The apparent reality dreams possess is not derived from any concrete, objective world, but merely from the idea of objectivity. While the Yogacara does not say that an idea has, for example, spatial attributes, it does have a form manifesting them. While he agrees with the Saiva idealist that appearances have no independent existence apart from their appearing to consciousness, he maintains that for this reason they are unreal. The creativity of consciousness consists in its diversification in many modes having apparent externality; it is not a creation of objects. While the Kashmiri Saivite agrees that the world is pure conscious- ness alone, he maintains that it is such because it is a real creation of consciousness. The effect is essentially identical with the cause and shares in its reality. Matter and the entire universe are absolutely real, as 'congealed' (sty ana) or 'contracted' (samkucita) forms of consciousness. "This God of consciousness", writes Ksemaraja, "generates the universe and its form is a condensation of His own essence (rasa)" m By boiling sugarcane juice it condenses to form treacle, brown sugar and candy which retains its sweetness. Similarly, consciousness abides unchanged even though it assumes the concrete material form of the five gross elements. The same reality thus abides equally in gross and subtle forms. Consequently no object is totally insentient. Even stones bear a trace (vasana) of consciousness, although it is not clearly apparent because it is not associated with the vital breath (prana) and other components of a psycho-physical organism. Somananda goes so far as to affirm that physical objects, far from being insentient, can only exist insofar as they are aware of themselves as existing. The jar performs its function because it knows itself to be its agent. Indeed, all things are pervaded by consciousness and at one with it and hence share in its omniscience. Thus, Siva, Who perceives Himself in the form of physical objects, is the one ultimate reality. "The jar knows because it is of my nature", writes Somananda, "and I know it because I am of the jar's nature. I know

because I am of Sadasiva's nature and He knows because He is of my nature; Yajnadatta [knows] because he is of Siva's nature and Siva [knows] because He is of Yajnadatta's nature". Integral Monism of Kashmiri Saivism Everything in this sense is directly perceived by absolute conscious- ness, and this direct perception (pratyaksa) unifies the knowable into a single, undivided whole. This is the central concept behind a doctrine originally expounded by Narasimha called 'the non-dualism of direct perception' (pratyaksadvaita). This states that consciousness is essentially perceptive and that its perception of all things operates throughout the universe. Insofar as phenomena are clearly evident (sphufa) to us, everything is directly perceived by absolute consciousness, with which our individual consciousness is identical. This direct perception unfolds everywhere; the one true reality, it is alone and without companion or rival (nihsapatna). Even though it remains one, it can, by its very nature, perceive distinctions (bheda) between one entity and another, without this engendering any division within it. We distinguish between two entities in empirical terms on the basis of their mutual exclusion (anyonyabhava). The relative distinction {bheda) between them is essentially the perceived difference between their respective characteristics. Despite this difference they are united within the purview of a single cognition insofar as they are equally both manifest appearances. This cognition is the undivided essence (rasa) or 'own nature' (svabhava) of both. Encompassed by the 'fire of consciousness', there is no essential difference between them. Just as when an emerald and ruby reflect each other's light, the ruby is reddish-green and the emerald greenish-red, similarly everything is connected with everything else as part of the single variegated (vicitra) cognition of absolute consciousness. Mahesvarananda writes: The Supreme Lord's unique state of emotivity (asadharanabhava) is the outpouring of pure Being (mahasatta). It is manifest as the brilliance (sphuratta) of the universe which, if we ponder deeply, [is realized to be] the single flavour (ekarasa) of the essence of Beauty which is the vibration of the bliss of one's own nature. In this way all things are in reality one although divided from the one another sharing as they do the 'single flavour' (ekarasa) of the pure vibration of consciousness. Kashmiri Saiva Realism Kashmiri Saivism as a whole has been variously called a form of 'realistic idealism', 'monistic idealism', ‘idealistic monism' and 'concrete monism'. It is easy to understand why Kashmiri Saivism is The Doctrine of Vibration said to be 'idealistic' and 'monistic', but in what sense is it also 'realistic*? The answer to this question is of no small importance in trying to under- stand the central idea behind its metaphysics and the fundamental importance of the concept of Spanda, in this seemingly impossible marriage between monistic idealism and pluralistic realism. The Kashmiri Saiva approach understands the world to be a symbol of the absolute, that is, as the manner in which it presents itself to us. Again we can contrast this view with that of the Advaita Vedanta. The Advaita Vedanta understands the world to be an expression of the absolute insofar as it exists by virtue of the absolute's Being. Being is understood to be the real unity which underlies empirically manifest separateness and as such is never empirically manifest. It is only transcendentally actual as 'being-in-itself. The Kashmiri Saiva position represents, in a sense, a reversal of this point of view. The nature of the absolute, and also that of Being, is onceived as an eternal becoming (satatodita), a dynamic flux or Spanda, 'the agency of the act of being'. It is dentified with the concrete actuality of the fact of appearing, not passive unmanifest Being. Appearance (abhasa) alone is real.ppearing (prakasamanatva) is equivalent to the fact of being (astitva). Ksemaraja writes in his commentary on the Stanzas on Vibration: Indeed, all things are manifest because they are nothing but manifestation. The point being that nothing is manifest apart from manifestation The absolutely unmanifest, from this point of view, can have as little existence as the space in a lattice window of a sky-palace. Nay, even less, because even that space can appear as an imagined mage manifest within consciousness. Everything is real according to the manner in which it appears. Even an illusion is in this sense real, insofar as it appears and is known in the manner in which it appears. The empirical and the real are identical categories of thought. As Abhinava says: Thus this is the supreme doctrine (upanisatf), namely that, when- ever and in whatever form [an entity] appears, that then is its particular nature. Perhaps at this stage a brief comparison with Heidegger's ideas might prove to be enlightening and not altogether out of place. According to Heidegger's phenomenology of Being, reality is intelligible in a two-fold Integral Monism of Kashmiri Saivism manner as 'phenomenon' and 'logos'. Heidegger defines what he means by 'phenomenon' as: "that-which-shows-itself. The manifest . . . phenomena are then the collection of that which lies open in broad daylight or can be brought to the light of day — what the Greeks at times implicitly identified as 'ta onta' (the things-which-are)". 127 In his later writings Heidegger drops the term 'phenomenon' in preference for the verbal form 'phainesthai' in order to emphasize even more the actuality or

presentational property of Being. Explaining this new form of the term he writes: "Being disclosed itself to the ancient Greeks as 'physis'. The etymological roots 'phy-' and 'pha-' designate the same thing: 'phyein', the rising-up or upsurge which resides within itself as 'phainesthai', lighting-up, self-showing, coming-out, appearing-forth." Heidegger contrasted his notion of phenomenon with semblance (Schein) and with appearing (Erscheinung). In the case of semblance a thing can show itself as that which it is not, as when fool's gold shows itself to be gold. The ancients always allied semblance with non-being. Heidegger points out, however, that semblances are grounded in showings, and so does Abhinava. Both Heidegger and Abhinava consequently maintain that all semblances have a real basis and are to be treated as instances of phenomena along with the so-called real showing or manifestation of non-deceptive objects. So Heidegger states that: 'how- ever much seeming, just that much being'. Thus self-showing or appearing defines Being as phenomenon, but this definition of Being is as yet incomplete. Being is not only self-showing but 'logos' which Heidegger explains means 'discourse' (Rede) in the sense of 'apophansis': 'letting-be-seen'. Phenomenology, which according to Heidegger is the only correct study of Being, means 'letting-be-seen-that-which- shows-itself. This is true of Saiva Paramadvaita as well. The reality of the world demands recognition; we are forced to accept the direct presentation of the fact of our daily experience. As Abhinava says: "if practical life, which is useful to all persons at all times, places and conditions were not real, then there would be nothing left which could be said to be real." A thousand proofs could not make 'blue' other than the colour blue. The reality of whatever appears in consciousness cannot be denied. Objects appear; they do not cease to do so by a mere emphatic denial. The manifestation of an entity in its own specific form is a fact at one level of consciousness; it is real. The appearing of the same entity in the same form but recognised to be a direct representation of the absolute is also a fact, but at another level of consciousness. It is no more or less real than the first. 'As is the state of consciousness, so is the experience,' says Abhinava. Although the nature of the absolute is discovered at a higher level of consciousness, The Doctrine of Vibration nonetheless it presents itself to us directly in the specific form in which we perceive things; otherwise there would be no way in which we could penetrate from the level of appearing to that of its source and basis. Abhinava writes: Real is the entity iyastu) that appears in the moment of direct perception (sak$atkara), that is to say, within our experience of it. Once its own specific form has been clearly determined one should, with effort, induce it to penetrate into its pure conscious nature. All things are known to be just as they present themselves. The concrete actuality of being known (pramiti), irrespective of content, is itself the vibrant (spanda) actuality of the absolute. Liberating knowledge is gained not by going beyond appearances but by attending closely to them. "The secret," Mahesvarananda says, "is that liberation while alive (jivanmukti) is the profound contemplation of Maya's nature." No ontological distinction can be drawn between the absolute and its manifestations because both are an appearing (dbhasa), the latter of diversity and the former of 'the true light of consciousness which is beyond Maya and is the category Siva'. Those who have attained the category of Pure Knowledge above Maya and have thus gone beyond the category of Maya, see the entire universe as the light of consciousness . . . Just as the markings [on a feather] are nothing apart from the feather, the feather [is nothing apart from] them, similarly, when the light of consciousness is manifest, the whole group of phenomena is manifest as the light of consciousness itself. Within the sphere of Maya, every entity's 'own nature' (svabhdva) corresponds to its specific manifest form. Accordingly it is defined as that which distinguishes it from all else and from which it never deviates. Above the sphere of Maya, that is, above the level of objectivity, is the domain of the subject. At this level, everything is realised to be part of the fullness of the experience r and hence no longer bound by the conditions which impinge on the object. Here the part is discovered to be the whole, that is, consciousness in toto. In this sphere beyond relative distinctions, the yogi realises that (all) the categories of existence are present in every single category. m The yogi experiences every individual particular as the sum total of everything else. He recognises that all things have one nature and that every particular is all things. This is the 'essence' (sard) or co-extensive unity (samarasya) of all things. Integral Monism of Kashmiri SaivismWe have established that reality is manifest according to how [and the degree in which] the freedom of consciousness reveals it and that [this freedom] is the womb of all forms. Just as 'sweetness' is present in its entirety in every atom of the sugarcane, so each and every atom [of the universe] bears within itself the emanation of all things. This is the level of consciousness in which the absolute reflects on itself realising to its eternal delight and astonishment (camatkara) its own integral nature. The reality of the world of diversity is not denied, but experienced in a new mode of awareness free of time and space in

the eternal omnipresence of the Here and Now. [Phenomenal forms of awareness] such as 'this [exists]', born of the colouring [imparted to the absolute] by the limitations engendered by the diversifying power of time {kalakalana) also emanate within the Supreme Principle. There [at that level], Fullness {purriata) is the one nature [of all things] and so everything is omnipresent; otherwise, associated with division (khancjlana), the Fullness [of the absolute] would not be full. The content of absolute consciousness consists of diverse appearings (abhdsa) which, because they are manifest through it in this way, do not compromise the wholeness of consciousness. Everything we perceive is a momentary collocation of a number of such manifestations which combine together like 4 a row of altar lamps' (dipavalT) to form the single radiant picture of the universe. The individual objects which constitute the universe are specific collocations of such 'atomic' appearings. Together they form a single unified particular which appears according to its own defining features (svalaksand). A jar, for example, consists of a number of appearances such as 'round', 4 fat\ 'earthen', 'red', etc., which together discharge a single function (arthakriya), in this case, that of carrying the appearance 'water'. They unite with each other much as the scattered rays of a lamp come together when focused, or as the various currents of the sea together give rise to waves. Atomic appearings can combine in any number of ways, provided that they are not contrary to one another as established by the dictates of natural law (niyati). An appearance of 'form', for example, cannot combine with that of 'air'. Insofar as they share a common basis (samanyadhikaranya), a given cluster of appearances appears as a single whole. This common basis is the most prominent member of the group; the appearance 'jar' is such in the example quoted above. Any one appearance in a cluster may assume a more important or subordinate role. The result is a specific The Doctrine of Vibration awareness of an object of the form: 'here this is such.' While individual appearances do not lose their separate identity {svarupabheda) when they rest on a common basis, even so the particular object which appears according to its own characteristics (svalak$ana) is an individual reality in its own right. It is a different kind of appearance characterised by its association with the appearance of the specific location and time in which it is made manifest. The form of our experience is thus 'I now see this here'. But when we perceive each particular constituent appearance separately, each assumes a separate fixed function. Abhinava cites the following colourful example to illustrate how the various combinations of appearances account for the variety of experience: Thus even though the appearance of the beloved may manifest externally, it is as if far away in the absence of another appearance, namely, that of 'embracing'. So when the [appearing of the beloved] is associated with another appearance [namely that of 'far away'] the power (arthakriya) it formerly had of giving pleasure appears as its contrary. The form our experience assumes depends, not only on the nature of the object perceived, but also on personal factors entirely peculiar to ourselves. This theory explains this in two ways. In one sense, the object remains the same, but one or other of its constituent appearances comes to the fore according to the inclinations of the perceiver. From another point of view, we can say that the perceived object is different for each perceiver according to the difference in the prominent appearance manifest to him. Abhinava, citing as an example a golden jar, illustrates how the same object appears differently to different perceivers according to the use they wish to make of it and to their state of mind: When a person who is depressed and feels that there is nothing [of value for him in the world] sees the jar, he merely perceives the appearance 'exists' [in the form of the awareness that] 'it is\ He is not conscious of any other [of its constituent appearances] at all. An individual who desires to fetch water [perceives] the appearance 'jar'. The man who simply wants something that can be taken somewhere and then brought back [perceives] the appearance 'thing'. The man who desires money [perceives] the appearance 'gold'. The man who desires a pleasing object [perceives] the appearance 'brightness' while he who wants something solid sees the appearance 'hardness'. These 'atomic events' or appearances emerge from the pure subject's consciousness and combine together to form a total event at each moment. Integral Monism of Kashmiri Saivism Daily life (vyavahara) goes on by virtue of this ever renewed flux of appearances. They are connected together and work towards a single unified experience because they appear within the field of consciousness of the universal subject. The aggregate of appearances arises in the [supreme] subject as do [sprouts in] a rice field. Even though each sprout germinates from its own seed, they are perceived as a collective whole. Appearances rest in this way within the universal subject. 'External- ity' is itself another appearance; it arises from a distinction between appearances and the individual subject. So, although all manifestation always occurs within the subject, it appears to be external due to the power of Maya which separates the individual subject from his object. This split must occur for daily life to be possible. Only externally manifest appearances can perform their

functions; when they are merged within the subject and at one with him, they cannot do so. Daily life proceeds on the basis of the operation and withdrawal of the conditions necessary for fruitful action to be possible. Appearance in this sense represents the actualisation of a potential hidden in consciousness made possible by virtue of its dynamic, Spanda nature which is both the flow from inner to outer and back as well as the power that impels it. The emergence from, and submergence into, pure consciousness of each individual appearance is a particular pulsation (visesaspanda) of differentiated awareness. Together these individual pulsations constitute the universal pulse (samanyaspandd) of cosmic creation and destruction. Thus, every single thing in this way forms a part of the radiant vibration {sphuratta, sphurana) of the light of absolute consciousness. Light and Awareness: The Two Aspects of Consciousness Absolute consciousness understood as the unchanging ontological ground of all appearing is termed 'Prakasa'. As the creative awareness of its own Being, the absolute is called ' Vimarsa '. Prakasa and Vimarsa — the Divine Light of consciousness and the reflective awareness this Light has of its own nature — together constitute the all-embracing fullness (purnata) of consciousness. The Recognition (pratyabhijfia) school of Kashmiri Saivism develops this concept of the absolute which finds its fullest expression in Utpaladeva's Stanzas on the Recognition of God. Even though neither of these two key terms appear in the Stanzas on Vibration or the Aphorisms of Siva, they recur frequently in their commentaries. Thus, although the original formulation of the Doctrine of Vibration differs from the theology of Recognition in this respect, it was extended in the course of its development to accommodate this concept of the absolute as well. This was possible, and quite justified, insofar as the absolute understood in Pratyabhijfia terms does not, as we shall see, differ essentially from that of the Spanda school. We can, as Kashmiri Saivites themselves have done, explain one in terms of the other. The Doctrine of Vibration Prakasa: The Light of Consciousness Prakasa is the pure luminosity' (bhdna) or 'self-showing' that constitutes the essence and ultimate identity (atman) of phenomena. That things appear at all is due to the light of consciousness, and their appearing (avabhasana) is itself this Light which bestows on all things their evident, manifest nature. Established in the light of consciousness everything appears there according to its own specific nature (svabhava). Anything that supposedly does not rest in this Light is as unreal as a sky-flower. 3 Thus, according to Rajanaka Rama, unlike the light of the sun, or any other light, this Light not only makes all things apparent, it is also their ultimate source. 4 Full of its divine vibration the Light makes all things manifest and withdraws them into itself. This supra- temporal activity characterises it most specifically; devoid of it, it would be no better than an inert physical phenomenon. At the same time, this light is the conjunction (slesa) or oneness (aikdtmya) of its countless manifest forms, 6 and the collective whole (sampina'ana) of all the categories of existence. The universe is nothing but the shining of the Light within itself. It is the radiant vibration (sphuratta) of this Light, the state (avast hand) in which consciousness becomes manifest.Although the Light shines as all things at all times and hence also makes their diversity manifest, 9 penetrating each object individually as well as collectively, it is not totally 'merged' (magna) or identified with the object so as to suffer any division within itself. Our experience of any object is of the form: T see this': it is not itself an object, but the manifest form the object assumes as a luminous principle of experience. The Light is ever revealed and can never be obscured; objectivity can never cast a shadow on the light of consciousness. The Stanzas on Vibration declare: That in which all this creation is established and from whence it arises is nowhere obstructed because it is unconditioned by [its very] nature. This Light is the highest reality (paramartha). It is the 'Ancient Light' (puranaprakasa) that makes all things new and fresh every moment. It is 'always new and secret, ancient and known to all'. It is the form of the Present (vartamanarupa), the Eternal Now. Time and space are relations between the contents of consciousness; they cannot impinge on the integrity of the absolute itself. 16 Neither space nor time can divide it, for they are one with the Light that illumines them Light and Awareness: Two Aspects of Consciousness and makes them known as elements of experience. But this Light is the shining of the absolute; it is not an impersonal principle. It is the living Light of God, indeed it is God Himself, the Master Who instructs the entire universe. 18 Siva is this 'auspicious lamp', Who illumines all things. He is the Light of consciousness that reveals the presence of both the real and the unreal, of light' and 'darkness'. 20 Abhinavagupta writes: Thus Bhairava, the Light, is self-evident (svatahsiddha); without beginning, He is the first and last of all things, the Eternal Present. And so what else can be said of Him? The unfolding of the categories of existence (tattva) and creation, which are the expansion of His own Self, He illumines, luminous with His own Light, in identity with Himself, and because He illumines Himself, so too He reflects on His own nature, without His wonder

(camatkara) being in any way diminished. Since Zwicky's early observations, similar data from other galaxy clusters has yielded the same result – we consistently see that clusters of galaxies appear to By developing an awareness of the Centre, the yogi experiences the bliss of consciousness. 117 Through this gap he plunges into introverted absorption (nimilanasamadhi) and then emerges again to pervade the field of awareness between Centres and so experience the Cosmic Bliss (jagadananda) of the universal vibration of consciousness. 1 i8 He then recognises that this state pervades every aspect of experience. In this way the yogi's consciousness is no longer afflicted by the power which obscures it, hemming the Centre in on both sides with thought-constructs that seemingly deprive it of its fullness. As he realises directly his pure conscious nature as the universal ego free of all mental representations, it expands out to embrace all things within itself. Thus the realisation the Divine Means leads to, and is directly based upon, is that this pure ego is in all things just as all things are within it. In the Spanda tradition, as recorded in the Stanzas on Vibration, no such ego is recognised. ' I9 Man's authentic nature is, however, understood in personal terms as every individual's own 'own nature* (svasvabhava) which is Siva, the universal vibration of pure subjectivity (upalabdhfta). It is not surprising, therefore, that later commentators found these two conceptions to be essentially the same and accordingly identified one's own inner nature with the pure ego. This came as a natural development in Spanda doctrine not only for **this reason but also because the universal ego is experienced as the inner dynamics of absolute consciousness. To conclude our summarial exposition of the Divine Means, which is centred on the direct experience of this pure ego (and hence on Spanda in this form), we turn now to a brief description of its inner, cyclic activity. We shall do this** by examining Abhinava's esoteric exegesis of the symbolic significance of the word 'A HAM', which in Sanskrit means T, and symbolises by its form the ego's dynamic nature. The objective world of perceptions is, as we have seen, essentially a chain of thought-constructs (prapanca) closely linked to one another and woven into the fabric of diversity (vicitrata). This thought (vikalpa) is a form of speech (vac) uttered internally by the mind (citta), which is itself an outpouring of consciousness. Consciousness also, in its turn, resounds with the silent, supreme form of speech {para vac) which is the reflective awareness through which it expresses itself to itself. Consequently, the fifty letters of the Sanskrit alphabet, which are the smallest phonemic units into which speech can be analysed, are symbolic of the principal elements of the activity of consciousness. Letters come together to generate words and words go on to form sentences. In the same way the fifty phases in the cycle of consciousness represent, in the realms of denoted meaning (vacya)the sum total of its universal activity (kriya) corresponding to the principal forces (kala) which come together to form the metaphysical categories of 186 The Doctrine of Vibration experience, which in their turn appear in the grossest, most explicitly 'articulate' form as the one hundred and eighteen world-systems (bhuvana). 'A', the first letter of both AHAM and the Sanskrit alphabet, is the point of departure or initial emergence of all the other letters and hence denotes Anuttara — the absolute. 'Ha', is the final letter of the alphabet and represents the point of completion when ail the letters have emerged. It represents the state in which all the elements of experience, in the domains of both inner consciousness and outer unconsciousness, are fully displayed. It is also the generative, emission (visarga) which, like the breath, casts the inner into the outer, and draws what is outside inward. The two letters 'A' and 'Ha' thus represent Siva, the transcendental source and Sakti, His cosmic outpouring that flows back into Him. The combined 'A-Ha' contains within itself all the letters of the alphabet — every phase of consciousness, both transcendental and universal. (For a graphic representation of this analysis, see figure 1.) M, the final letter of AHAM, is written as a dot placed above the letter which precedes it. It comes at the end of the vowel series and before the consonants and so is called 'anus vara '(lit. 'that which follows the vowels') and also 'bindu ' (lit. 'dot,' 'drop,' 'point' or 'zero'). While the consonant 'M' symbolises the individual soul (purusa), 'bindu 1 rep resents the subtle vibration of T, which is the life force (jivakala) and essence of the soul's subjectivity manifest at the transcendental, supra-mental level (unmana). 12 ° It is the zero-point in the centre between the series of negative numbers, in this case the vowels which represent the processes happening internally within Siva, and the series of positive numbers — the consonants which symbolise the processes happening externally within Sakti. Bindu, as a point without area, symbolises the non-finite nature of the pure awareness (pramitibhava) of AHAM. It is the pivot around which the cycle of energies from 'A' to 'Ha' rotates, the Void in the centre from which all the powers emanate and into which they collapse. As such, it is the supreme power of action which holds subject, object and means of knowledge together in a potential state in the one Light that shines as all three 121 containing them in its repose 122

(visrdnti). Bindu is the 'knower' (jndtr), who is essentially consciousness that, though omniscient does not manifest its intelligence, like a man who knows the scriptures but having no occasion to explain them to others silently bears this knowledge within himself. As such, it symbolises the union of Siva and Sakti (sivasakti- mithunapina'a) 123 in a state of heightened potency in which they have not yet divided to generate the world of diversity. It stands, in other words, at the threshold of differentiation in the stream of emanation still contained within Siva. Expansion Commences Bindu — The Individual Soul Withdrawal Commences Figure 1 188 The Doctrine of Vibration Then, to the degree in which that which is to be accomplished by the power of action residing within it [as a potential] penetrates into the absolute, it appears initially as bindu, which is the light of pure consciousness. 124 When outer objectivity is reabsorbed into its transcendent source, bindu is the point into which all the manifest powers of consciousness are gathered and fused together. The universal potency of all the letters is thus contained in bindu which, as the reflective awareness of supreme T consciousness, 125 gives them all life. Thus bindu also marks the beginning of Siva's internal movement back to the undifferentiated absolute and so stands at the threshold of both emission and absorption without being involved in either. The three aspects of AH AM together constitute a movement from the undifferentiated source of transcendental consciousness — 'A' — through the expansion or emission of its power — 4 Ha' — to the subject — 'M 1 — which contains and makes manifest the entire universe of experience. The reverse of this movement, which of withdrawal (samhara), is represented by M-Ha-A. AHAM and M-Ha-A alternate in the rotation (ghurnana) of the reflective awareness of T consciousness as immanent Sakti emerges from transcendental Siva to then merge back into Him. As Abhinava says: The universe rests within Sakti and She on the plane of the absolute (anutiara) and this again within Sakti ... for the universe shines within consciousness and [consciousness shines] there [within the universe by the power of] consciousness. These three poles, forming a couple and merging, make up the one supreme nature of Bhairava Whose essence is AHAM- 126 At the microcosmic level, fc A' represents the initial moment when the subject begins to rise out of himself to view the object. The movement from 'A to 'Ha' marks the emergence of sensation within the field of awareness, which is represented by the fifty letters of the alphabet symbolic of the fifty aspects of the flux of consciousness leading to objectified perception. 'Nl' is tne subject who, resting content within himself when he has perceived his object, merges through the inner flow of awareness into k A\ the absolute. Then from the absolute (A) its emission (Ha) flows back into the pure subject (M) set to perceive his object. Thus all the cycles of creation and destruction are contained within AHAM through which they are experienced simultaneously as the spontaneous play of the absolute. The yogi who recognises this recurrent pulse of awareness to be the movement of his own consciousness merges his limited ego with the Path to Liberation universal ego. **This is natures general ledger** Thus he realises that its power to create, sustain and destroy all things is his own inner strength (svabaia) that he exerts effortlessly in the same state of mystical absorption (turlya) in universal consciousness that the absolute itself enjoys. In this way he shares in the three-fold awareness Siva Himself has of His own nature which Abhinava describes as follows: 4 I make the universe manifest within myself in the Sky of Consciousness. I, who am the universe, am its creator! ' — this awareness is the way in which one becomes Bhairava. 'AH of manifest creation (sadadhvari) is reflected within me, I cause it to persist 1 — this awareness is the way in which one becomes the universe. The universe dissolves within me. I who am the flame of the [one] great and eternal fire of consciousness' — seeing thus one achieves peace. 127 The experience of the liberated thus coincides with the realisation of their own divine nature which, through its power, rules and guides the cosmic order. Thus this attainment (siddhi), which is liberation itself, is in the DoctrLie of Vibration technically called 'Mastery over the Wheel of Energies' (cakresvaratvasiddhi) because the liberated soul, identified with Siva, now governs, as does Siva, the cycle of the powers that bring about the creation and destruction of all things, Here we are talking of individual consciousness becoming cosmic general ledger and the liberationist realises all destruction and creation is by him. This clearly means that he is in his own world of illusion. Creation is subjective quintessentially The Empowered Means (Saktopaya) All the practices taught in the Stanzas on Vibration are internal. Whenever ritual is mentioned, it is invariably interpreted in terms of the dynamics of the inner processes the yogi experiences and implements in the course of his yogic practice. The Doctrine of Vibration, Ksemaraja affirms, 129 is concerned entirely with these inner disciplines centred, as it is, in one way or another, on consciousness or, at least, on the inner activity of the mind. Thus the Empowered Means which, like the other categories we have discussed, is entirely internal, includes an important part of Spanda practice. Spanda

practice belonging to the Divine Means centres on one's own inherent nature (svasvabhava) as Siva, the universal perceiver and agent, that belonging to the Empowered Means on His power. Instead of arriving directly at the all-embracing emptiness of subjective conscious- ness, the yogi practising the Empowered Means realises his true nature through the fullness of its energy. Practising the Divine Means, the yogi plunges, as it were, straight into the fire of consciousness; practising the 190 The Doctrine of Vibration Empowered Means he merges with its rays. Either way the yogi is centred equally on ultimate reality. The power of consciousness is no less absolute than its possessor. To make this point Abhinava quotes the Matanga- tantra: This reality consists of the rays of [Siva's] power and is variously said to be the abode of the Lord's manifestation . . . That same [power] illumined [by Siva] is itself also luminous, unshaken and unmoving. That very [power] is the supreme state, subtle, omnipresent, the nectar of immortality, free of obscuration, peaceful, yearning for pure Being alone {vastumatra) and devoid of beginning and end. Perfectly pure, it is said to be the body [of ultimate reality]. 110 The yogi concentrates on the powers operating in all of life's activities as particular pulsations (visesaspanda) in the universal rhythm (samanyaspanda) of the power of consciousness. In this way he rises progressively from the particular to the universal until he reaches pure Being (satta), the greatest of all universals (mahasamanyd) and the highest form of Siva's power. Thus the creative power of Maya, manifest through countless lesser powers, no longer causes the yogi to stray from Siva's consciousness but becomes the means through which it can be realised 13 1 in the illuminating brilliance (sphuratta) which is Siva's pure Being. Thus by discovering the true nature of Sakti, the yogi realises himself to be Siva, its possessor Who consists of all its countless powers. Thus practise belonging to this Means leads to the same pure consciousness free of thought-constructs realised through the Divine Means. Although the ultimate realisation is instantaneous, the yogi rises to it gradually by freeing his consciousness of the limitations imposed upon it by thought. Abhinava explains: The same occurs in the Empowered Means [as does in the Divine]. At the discursive level of consciousness {yaikalpikibhumi) [where the Empowered Means functions] knowledge and action, although evident, are, for the reasons explained previously, contracted. A blazing energy [is revealed within] the one who dedicates himself to removing the burden of this contraction. [This energy eventually] brings about the inner manifestation {antarabhasa) of pure consciousness he seeks. 132 Consciousness is individualized and its power of knowledge and action contracted by the thought-constructs born of ignorance. The arising of these mental representations, as the Stanzas on Vibration say, deprives the soul of its freedom and immortal life. 133 The practise of the Path to Liberation 191 Empowered Means is meant to free the fettered soul of this constriction on his consciousness. It operates within the mental sphere (cetasy™ and is designed to purify thought (vikalpasamskara) in order to. reveal the pure consciousness which is its ground and ultimate source. Thus, the Empowered Means is concerned with the second instant of perception, during which the subject forms mental representations of his object. Thought functions on the basis of an awareness of relative distinctions between specific particulars, distinguishing them from one another and thus seemingly fragmenting the essential unity of reality. 135 The vibrant vitality of consciousness, universally manifest, is clouded like a mirror by a child's breath 136 and the soul is deprived of the liberating intuition of the one reality free of thought-constructs (nirvikalpa). Abhinava writes: The [fettered soul] is like a dancing girl who although wishing to leave the dancehall is collared by the doorkeeper of thought and thrown back onto the stage of Maya. 137 All thought is centred on objectivity and hence dislodges awareness from the plenitude of pure subjective consciousness. Thus, to regain the original state of rest (visranti) consciousness enjoys, the yogi must rid himself of thought. As thought-forms decrease, pure, thought-free awareness is strengthened 138 until the yogi is fully established in a state in which the relative distinctions (bheda) conceived between entities dissolve away. Everything appears to him as pure Being (sattdmdtra) l}9 and the entire universe shines before him pervaded by Siva's radiance. 140 His intuitive faculty (mati) thus purified, the yogi gains both the perfections (siddhi) of yogic practice and liberation (mukti). His consciousness is now like a well-polished mirror which reflects everything he desires and grants it to him. 141 Abhinava writes: Just as a man who has been ill for a long time forgets his past pain completely when he regains his health, absorbed as he is in the ease of his present condition, so too those who are grounded in pure awareness free of thought-constructs are no longer conscious of their previous [fettered] state. Consciousness, the sole truly existent reality, free of thought- constructs is made fully and evidently manifest by eliminating thesedifferentiated perceptions. The wise man should therefore exert himself to attend closely to this [state of awareness]. I42 The thought-constructs generated within consciousness do not in reality

affect it at all. They can neither break up nor add anything to the Light which shines as all things. 143 They are in fact nothing but 192 The Doctrine of Vibration consciousness itself 144 which perceives, through its power of reflective awareness (yimarsa), the multitude of objects in diverse ways, and so assumes this form. 145 Although thought-constructs are mental representations of objects once seen or present, they are products of the power of consciousness and not of the objects they represent. 146 Thought is both analytic and synthetic; 147 it serves the useful purpose of separating individual elements of experience from others and linking together those that appear to be distinct from one another so that they can be better understood. 148 It does not consist merely of false mental constructs projected onto reality that need to be wholly rejected. Thought obscures consciousness and distracts it only when it appears in the form of doubt, vacillating between alternatives. 149 Once this conflicting duality (dvaitddhivasa) 150 is eliminated, thought is purified and rests in itself as the 'thought-less thought' of pure consciousness. 151 By gradually eliminating the multitude of conflicting notions that agitate him, the yogi ultimately achieves the certainty (niscaya) corresponding to a direct awareness of his own divine nature. 152 Abhinava explains: Thought is in reality none other than pure consciousness. Even so, it serves as a means to liberation for the individual soul (anu) only when it takes the form of certainty (niscaya). 153 The yogi must eliminate every doubt and misguided notion that leads him to believe himself to be other than Siva. By developing the thought: 4 1 am Siva', it ultimately affirms itself directly as a pure awareness beyond thought without any intervening mental representations. Abhinava says: Just as the man who thinks intensely that he is a sinner becomes such, just so one who thinks himself to be Siva, and none other than He, becomes Siva. This certainty (ddrtfhya), which penetrates and affirms itself in our thoughts, coincides with an awareness free of thought- constructs engendered by a series of differentiated mental representa- tions, the object of which is our identity with Siva. 154 As thought is gradually purified, it becomes progressively clearer until its object becomes maximally apparent (sphufatama). 155 The stream of perceptive consciousness (pramana) progressively reveals each aspect of its object which, thus affirming itself with increasing clarity, reveals its ultimate nature. The yogi reflects repeatedly upon it as the object of his realisation and loving devotion, for all that is perceptible and need be known (j fie yd) is Siva alone. As Abhinava says: Path to Liberation 193 What should we say of those who before they are satisfied have to see their beloved again and again, caress her and think about her for a long time? 156 The yogi practising the Empowered Means is initiated into the Great Sacrificial Rite (mahayaga), eternally enacted at the interface between the inner and outer aspects of consciousness, by a direct infusion of awareness from his master who is the embodiment and outer symbol of the yogi's enlightened identity. ,57 The rite begins with ritual bathing (snana) which is in this case the immersion of the body of thought in the white ashes of the cosmic fuel of duality, burnt in the fire of consciousness. 158 The yogi then goes on to worship (puja) by uniting all that is pleasing to the senses in the oneness of consciousness. 159 The ritual formula (mantra) he recites is the eternal resonance of the awareness which is the pulsation of the Heart of his own consciousness. 160 Repetition (Japa) of the formula is every activity, perception, breath or thought which arises within him while plunged in the universal awareness of his true nature. 161 The mental image he visualises meditating (dhyana) on the Deity in the course of the rite, is whatever the yogi spontaneously imagines and contemplates as the outpouring of the universal creativity of consciousness. 162 Ritual gesture (mudra) is whatever bodily posture the yogi may assume when, fully absorbed in consciousness, he moves, staggering about (ghumita) as it were, drunk with the wine of self-realisation. 163 Oblation is performed by offering with devotion and awareness all the sensations which flow in through the channels of the senses to the fire of his subjectivity, which is thus inflamed (uddipita) and makes all things one with itself. 164 The outer ritual which commences in the sphere of the Individual Means thus leads naturally to the inner rite of the Empowered Means. When the yogi's practise (abhyasa) reaches fruition, the rite merges with the spontaneous activity of consciousness. This is fullness (purnata), the completion and reunification of the forces within consciousness which, through the power of ignorance, were formerly dispersed and divided. "Just as a horse driven here and there", writes Abhinava, "over plains, hills and dales follows the will of its rider, so also consciousness, driven by various expedients (bharigi), quiescent or terrific, abandoning duality, becomes Bhairava. Just as by looking repeatedly at one's own face in a mirror one comes to know that it is the same [as the image reflected], so also, [one sees] in the mirror of mental representations of meditation {dhyana), ritual (puja) and worship (area) one's own Self as Bhairava and so quickly identifies with Him. This identification is the realisation that takes place in the absolute (anuttara)." 165 194

The Doctrine of Vibration By ridding himself of the relative distinctions engendered by thought, the yogi practising the Empowered Means, illumined by the power of self- awareness of Pure Knowledge {suddhavidya), transcends the distinction between right and wrong, purity and impurity. He is led to the conviction that the pure consciousness, which is his true nature, is unaffected by whatever action he may do, whether conventionally accepted as good or bad. Abhinava quotes the Malinivijaya as saying: All here is enjoined and all prohibited. This alone, O Lord of the gods, is here prescribed as obligatory, namely that the mind be firmly applied to the true reality. It matters little how this is achieved. He whose mind is firmly established in [this] reality, even if he eats poison, is as little affected by it as are lotus petals by water. 166 Impurity is a state of seeming separation from consciousness. 167 The yogi who has freed himself of all false notions comes to realise that the true nature of consciousness can never be sullied or limited by any object appearing within it. 168 This is the realisation the ancient sages achieved through a direct intuition of reality free of intruding thought-constructs {avikalpabhava), but kept secret in order not to confuse the worldly. 169 Similarly, in reality nobody is ever bound. It is ignorance to believe bondage exists and to contrast it with a conceived state of liberation. 170 If the Self is one with Siva, how can it be either bound or released? 171 Nothing essentially distinguishes those who are bound from those who are free. 172 The difference between their states is merely conceptual. 173 Pure consciousness abides free of all such distinctions. Thus Bhagavatotpala, <n his commentary on the Stanzas, repeatedly stresses that thought- constructs obscure consciousness and misguide the individual soul. 174 Those who are bound are convinced that they are dull witted, conditioned by Karma, sullied by their sin and helplessly impelled to action by some power beyond their control. He who manages to counter this conviction with its opposite achieves freedom. 175 He who considers himself to be free is free indeed, while he who thinks himself bound remains so. Thus at the highest level of realisation, as Abhinava says: Nothing new is achieved nor is that which in reality is unmanifest, revealed- [only] the idea is eradicated that the luminous being shines not. 176 Nothing is impure, all is perfect, including Maya and the diversity it engenders. To say that illusion exists and that ignorance must be Path to Liberation 195 eradicated implies that it has a separate existence apart from conscious- ness. If this is so, it has as little reality as the shadow of a shadow, but if not,then it must be consciousness itself. Thus, as Kallata says, bondage, the binder and the bound are in fact one. 177 It is Siva Himself Who freely obscures His own nature. Siva binds Himself by Himself. 178 Concealing and revealing Himself, Siva plays His timeless game. At the Divine (sambhava) level of pure Siva-consciousness, the Spanda yogi directly lays hold of the power inherent in his own conscious nature (svabala) which gives life to the psycho-physical organism and impels the senses and mind to action. 179 In this way every thought- construct, and with it the ego, is instantly annulled in the immediacy of the pure subjectivity that remains unaltered throughout every perception and state of consciousness. The same takes place at the Empowered level b> attending to the recurrent activity — Spanda — of the subject, that is, the flux of awareness through the cyclic movement of the powers of consciousness. 180 By attending (avadhana) to this movement the thought- constructs that emerge and subside in the course of perception are seen to be part of this universal process, and, in this way purified, are no longer binding. Thus, Ksemaraja says that the Spanda teachings are concerned most directly with the Empowered Means. m The yogi who is always alert to discern the pulse of Spanda quickly realises his own authentic state of being (nijarfi bhavam). 1 * 2 He is then truly awake, not only literally, but also in the deeper sense that he is awake to his authentic nature, its power and activity. When attention (avadhana) slackens, this movement takes place unconsciously and so the thought-constructs and perceptions generated through it appear to take on an autonomous existence of their own, just as happens when we dream. 183 The spontaneity of the movement that travels between subject and object and holds them together in the pure awareness of the universal subject's identity with his cosmic object devolves into the creative activity of waking and dreaming. Man, in other words, becomes a victim of his states of consciousness and the contents that they, by their very nature, generate within themselves. 184 The Spanda teachings are not only concerned with the structure of thought and its functions, but also with the powers and properties of its vehicle, namely, speech. Speech issues out of consciousness, develops into thought to then become articulated sound. A focal point of Spanda doctrine is thus the role speech plays in the formation of thought- constructs and their purification. Although this takes place at all levels of practise below the Divine (sambhava), the Spanda teachings, meant as they are for advanced yogis, ignore the outer forms of spiritual discipline to concentrate on practise in the Empowered (sakta) psychic sphere (cetas) and what lies beyond it where speech is the pure inner awareness

(yimarsa) **Although Abhinava does not differentiate between individual consciousness and cosmic general ledger, one tends to comprehend that right interpretation is the transfor mentioned the individual consciousness to cosmic consciousness responsible for the creation and destruction which the sadhaka sees as his own power. Every one is rama and every one is Shiva. only thing that is essential is striving and concerted efforts and sustained struggle to reach.** The Doctrine of Vibration of the light of consciousness. The Doctrine of Vibration identifies this, the highest level of speech (para vac), with the universal pulse of consciousness that resounds spontaneously within it as the inner flow of its own undifferentiated awareness. 185 Beyond the realms of language, it is the transcendental consciousness in which all language is rooted and pervades all that language denotes as its essential being. Utpaladeva writes: The Supreme Voice is consciousness. It is self-awareness spontaneously arisen, the highest freedom and sovereignty of the Supreme Lord. That pulsing radiance (sphuratta) is pure Being, unqualified by time and space. As the essence [of all things] it is said to be the Heart of the Supreme Lord. 186 When the intention arises within consciousness to discern its own brilliance manifest in the world of denotations and denoted meanings, speech turns from the supreme transcendental level to that of immanence and assumes the form of a pure intuitive awareness (pratibha) which perceives and comprehends its universal manifestation. This is the voice of intuition (pasyanti), which grasps the meaning inherent inwardly in all words and externally in all that they denote. Analogous to the non- discursive, instinctual knowledge animals possess, it is a pure generic perception not yet formed into language in which the act of denotation, its object and that which denotes it are indistinguishable. Illumined by the voice of intuition birds migrate in their due seasons, the cock crows at dawn and young mammals suck at the breast. 187 Infants similarly reflect and respond instinctively to their environment by virtue of this intuitive sense 188 and through it come to grasp the link between words and the objects they denote. As they learn to speak, they begin to form concepts and so the next two levels of speech develop. One is the outer corporeal speech (yaikhari) and the other the subtler, inner discourse (antah- saryijalpa) of thought that forms at the intermediate level (madhyama) where the ratiocinating mind stands between the higher levels of intuition and its outer verbal expression. In this way the development of speech in infancy reflects its progressive actualisation in every spoken word. A hymn to the Goddess quoted by Bhagavatotpala describes this process well: Therefore, O Supreme Goddess, the highest form of speech should be worshipped as the [universal] cause that establishes the existence of all things by insight {niscaya) into their nature (artha) brought about by their manifestation through the superimposition [of verbal designations]. O Mother, insight into the true nature [of things] is nothing but the Path to Liberation 197 act of intent of that [same speech], apart from which [speech itself and all that it expresses] could not attain to its own nature. Again, in that state [speech] is said to be the light of one's own nature. Free of division and succession it is attainable [only] by the yogi. Then from the state of intent, O Siva, speech [assumes] the nature of thought as the radiant pulse (sphuraria) of desire to speak of that which is in the domain of words. Then consisting of words, it bears a clearly expressed meaning, for if [speech] were not such, meaning could not be understood. 189 personal experience clearly proves that thought is invariably associated with speech. 190 Thought is a function of language. Through it we communicate to ourselves a mental image of the world about us and can construct complex ideas about ourselves. Language is the fabric from which our world of ideas is woven. Mental representation which orders the influx of sensation and presents us with a meaningful, picture of the world, memory, the elaboration of ideas and the shifting tides of emotion are all intimately connected with language and through it to the consciousness which underlies them. To think of language as nothing more than a system of denotation based on a commonly accepted convention (sanketd) fails to fully account forks inherent power to convey meaning (vacakasakti). In order to learn the convention we must be born with an innate ability to grasp meaning, and this ability is not itself learned nor found anywhere within the domain of convention. Lacking this ability we would be caught in an infinitely expanding system of denotation in which each element pointed to some other within it, without ever coming to rest anywhere. Unless we can couple the word 'jar' with the object it denotes, explaining that the word 'pot' is a synonym of the word 'jar' would leave us none the wiser. 191 The connection between word and meaning is only explicable if we postulate that it is an inherent property of the power of awareness to link one with the other. Language must be grounded in the pure cognitive awareness (prama) of consciousness which stands beyond, and yet illumines, the sphere of experience we define and understand through the medium of language. As Abhinava says: Someone may hear another person speak, but if his awareness (prama)

is obscured, he is unable to rise, unconscious as he is, to the level of the experiencing subject [who understands] what has been said. He only grasps the outer successive (sound) of what the other person says and thus can only repeat it as would a parrot. An understanding of its meaning presupposes that he has caught hold of his own power of awareness (prama) by attaining the autonomy [of the conscious, universal subject]. **Note that we have defined sachitananada as one which understand Truth and Untruth, Bliss and Unbliss, and Knowledge and ignorance where all these are equal. Truth=Untruth, Bliss=Unbliss and Ignorance=Knowledge** The Doctrine of Vibration Outer, articulate speech consists of a series of ordered phonemic elements produced and combined by the vocal organs to form meaningful words. In order for this to be possible, these elements must also be grounded in consciousness (prama). The articulated phonemes are merely outer, gross manifestations of the phonemic energies (yarnagrama) held in a potential state within consciousness. This 'mass of sounds* (sabdarasi) is the light of consciousness (prakdsd) which makes the universe manifest and contains all things within itself. In other words, it is the totality of consciousness expressed as the collective awareness symbolised by all the letters corresponding to the introverted subjectivity of Siva Himself. The power through which this potential actualises itself into speech and the world of denotation is technically called * Mdtrkd*. It is the reflective awareness (vimarsa) and radiance (sphuratta) of the supreme subject — the 'mass of sounds' (Jabdarasi) — and the undivided wonder Siva experiences when He contemplates the universe He gathers up into Himself in the form of countless words (vdcaka) and their meanings (vdcya). 193 Mdtfkdsakti is manifest in the second movement of consciousness after the primal vibration of the pure luminosity of the 'mass of sounds', as the state of pure potency which arises when its unsullied subjectivity begins to turn away from itself and is associated with faint traces of objectivity (dmrsya- cchdyd). 194 Mdtrkd contains within itself the various aspects of objectivity that, although not yet manifest, are ready to issue forth. Thus this power, at one with Siva, is called 'Mdtrkd' because she is the mother (mdtrkd) of the universe that she contains within herself as does a pregnant woman her child. 195 The circle of the powers of Mdtrkd (mdtrkdcakra) consists of the phonemic energies contained in AH AM, the universal ego. 196 When grasped in its entirety at its source, these energies elevate the consciousness of the enlightened, but when split up and dispersed give rise to the obscuring forces (kald) which lead the ignorant away from realisation. The fettered soul is ignorant of the pure egoity that is the source of speech and so it generates, through its powers, the many thought-constructs that deprive him of the awareness of unity and obscure Siva's universal activity. 197 The Stanzas on Vibration declare: He who is deprived of his power by the forces of obscuration {kala) and a victim of the powers arising from the mass of sounds (sabdardsi) is called the fettered soul. 198 The powers [of speech] are always ready to obscure his true nature as no mental representation can arise that is not penetrated by speech. 199 The rays of phonemic energies emanate from the light of Siva, the Path to Liberation 199 'mass of sounds' (sabdarasi) in eight groups. They constitute the powers of the inner mental organ and the five senses, figuratively arranged in a circle around the sacred shrine {pi(ha) of Matrka&akti who manifests externally as the body. 200 The eight classes and the names of the deities presiding over them are as follows: 201 Gutturals Brahman! Intellect (buddhi) Palatals Mahegvari Ego (ahankara) Cerebrals Kaumarl Mind (manas) Dentals Narayani Hearing Labials Varahi Touch Semivowels Aindri Sight Sibilants Camuntfa Taste Vowels Mahalak§ml Smell The yogi who grasps the true nature of the power of Matfka and its phonemic forces is liberated 202 by recognising that the activity of the senses and the discursive representations of the mind are in fact emanations of universal consciousness. Conversely, when ignorant, he is affected by its power in its multiple negative aspects known as 'Mahdghora' ('greatly terrible') and, unable to rest within himself free of the sense of diversity, he is constantly disturbed by the flux of extroverted perceptions. Abhinava explains: When the [phonemic energies] are not known to be [emanations of the Lord] they conceal the wonder (camatkara) of consciousness which is the one essential non-discursive awareness [present throughout perception] and even in discursive thought. They obscure it with thought-constructs constituted by the diverse configurations of phonemes and syllables which [although also] a form of the deity [are no longer benevolent but] most terrible. Inducing doubt and fear, they engender the fettered soul's state, bound by the shackles of transmigra tion. But once their true nature is understood correctly in this way, they bestow freedom in this very life This knowledge of their intimate being [at one with the absolute] consists of this, namely, that even in the midst of all these fluctuations, free at their inception of discursive representations, thought-constructs do not conjoin [individualised consciousness] with the wheel of energies consisting of the totality of phonemes, even though [these constructs] are coloured by the many diverse words

generated by the aggregate of phonemes. Language has a powerful effect on us. A few words we may hear or read can inspire us with joy, fear or sadness, and the constant inner 200 The Doctrine of Vibration dialogue of thought arouses intense feelings within us. This power hidden in language, which binds us through the thought-constructs it generates, can also be used to free us of them by channeling it through Mantra. Mantric practice begins at the Individual (driava) level where Mantras are recited in consonance with the rising and falling away of the breath. In this way they are charged with the vibration (spandd) of consciousness and, in their turn, make consciousness vibration (Doctrine of Vibration S.G.Dyczkowski) On without knowledge that everything is manifest within consciousness is illusory or unreal in that sense alone. 78 Things are more real or more tangibly experienced according to their own essential nature (svabhava) to the degree in which we recognise that they are appearances (abhasa) within absolute consciousness As Jayaratha says: Just as images manifest in a mirror, for example, are essentially mere appearances, so too are [phenomena] manifest within conscious- ness. Thus, beause they are external, [phenomena] have no being (sattva) of their own. The Lord says this [not with the intention of saying anything about the nature of things] but in order to raise the level of consciousness of those people who are attached to outer things; thus everything in this sense is essentially a mere appearance. [Knowing this], in order to quell the delusion of duality, one should not be attached to anything external. 79 The ultimate experience is the realisation that everything is contained within consciousness. We can discover this in two ways. Either we merge the external world into the inner subject, or we look upon the outer as a gross form of the inner. In these two ways we come to recognise that all things reside within our own consciousness just as consciousness resides within them. This all-embracing inwardness is only possible if there is an essential identity between the universe and consciousness. The events which constitute the universe are always internal events happening within consciousness because their essential nature is consciousness itself. 80 We can only account for the fact that things appear if there is an essential identity between consciousness and the object perceived. If a physical object were really totally material, that is, part of a reality independent of, and external to, consciousness, it could never be experienced. Abhinava says: The existence or non-existence of phenomena within the domain of the empirical (iha) cannot be established unless they rest within consciousness. In fact, phenomena which rest within consciousness are apparent (prakasamana). And the fact of their appearing is itself their oneness {abheda) with consciousness because consciousness is nothing but the fact of appearing {prakasa). If one were to say that they were separate from the light of [that consciousness] and that they appeared [it would be tantamount to saying that] 'blue' is separate from its own nature. However, [insofar as it appears and is known as such] one says: 'this is blue 1 . Thus, in this sense, [phenomena] rest in conscious- ness; they are not separate from consciousness. The Doctrine of Vibration The universe and consciousness are two aspects of the whole, just as quality and substance constitute two aspects of a single entity. The universe is an attribute (dharma) of consciousness which bears (dharmiri) it as its substance. It is said that 'substance' is that resting in which this entire group of categories manifests and is made effective. Now, if you don't get angry [we insist that] this entire class of worlds, entities, elements and categories (tattva) rests in consciousness and [resting in it] is as it is. Thus consciousness contains everything in the sense that it is the ground or basis (adhard) of all things, their very being (satta) and substance from which they are made. But, unlike the Brahman of the Advaita Vedanta, it is not the real basis (adhisthand) of an unreal projection or illusion. Consciousness and its contents are essentially identical and equally real. They are two forms of the same reality. Consciousness is both the substratum and what it supports: The perceiving awareness and its object. In this respect, the Kashmiri Saiva is frankly and without reserve an idealist. Although he does not deny the reality of the object, his position is at odds with most commonly accepted forms of realism. The realist maintains that the content perceived is independent of the act of perception. The content is only accidentally an object of perception and undergoes no change in the process of being perceived. His contention, however, is essentially unverifiable; to verify it, we would have to know an object without perceiving it. This, from the Kashmiri Saiva point of view, is not possible. Objects of which we have no knowledge may indeed exist, but they are knowable as objects only if they are related to subjects who perceive them. In this sense, if there were no subjects, there could be no objects. 86 The subject, however, as opposed to the object is, in terms of the phenomenology of perception, apparent to himself. He is self-luminous (svaprakdsa). Thus, conscious- ness (the essence of subjectivity) is one s own awareness by virtue of which all things exist. The realist maintains that consciousness clearly differs from its object insofar as their properties are contrary to each other. The Saivite

idealist. However, says that the object is a form of awareness (vijnanakara)™ The objective status of the object is cognition itself. Perception manifests its object and renders it immediately apparent (sphuta) to those who perceive it. It does not appear at any other time. If 'blue' were to exist apart from the cognition of k blue\ two things would appear: 'blue' and its cognition, which is not the case. It is the perception of the object which constitutes its manifest nature. An entity becomes an Integral Monism of Kashmiri Saivism object of knowledge not by virtue of the entity itself but by our knowledge of it. If objects had the property of making other objects appear, it would be possible for one object to make another appear in its own likeness. 'Blue' is perceived to be 'blue' because it is manifest as such to the perceiver. As Abhinava points out: The [nature of an] object of knowledge could not be established through a means of knowledge totally unrelated to it — a crow does not become white because a swan [sitting next to it] is white. Perception, on the other hand, is immediately apparent to conscious- ness. It is self-luminous in the sense that it is directly known without need of being known by any ulterior acts of perception and makes its object known at the same time. Adopting the Buddhist Yogacara doctrine that things necessarily perceived together are the same (sahopa- lambhaniyamavada), the Saivite affirms that because the perceived is never found apart from perception, they are in fact identical. Reality (satya) is the point where the intelligible and the sensible meet in the common unity of being; it cannot be said to exist in itself outside, and apart from, knowledge or vision. Bhagavatotpala in his commentary on the Stanzas on Vibration quotes: Once the object is reduced to its authentic nature, one knows [the true nature of] consciousness. What then [remains of] objectivity? What [indeed could be] higher than consciousness? Consciousness is essentially active. Full of the vibration of its own energy engaged in the act of perception, it manifests itself externally as its own object. When the act of perception is over, consciousness reabsorbs the object and turns in on itself to resume its undifferentiated inner nature. Knowledge (jnana) manifests internally and externally as each individual entity.... Once knowledge has assumed that form it falls back [into itself]. The Yogacara Buddhist similarly maintains that consciousness creates its own forms. But, according to him, because the perceived and perception are identical; there is no perceived object at all. The so-called outer world is merely a flux of cognitions, it is not real. He is firmly committed to a doctrine of illusion. The reality of consciousness from The Doctrine of Vibration his point of view is established by proving the unreality of the universe. "All this consists of the act of consciousness alone", says Vasubandhu, "because unreal entities appear, just as a man with defective vision sees unreal hair or a moon, etc."He points to dreams as examples of purely subjective constructs which appear to be objective realities. The apparent reality dreams possess is not derived from any concrete, objective world, but merely from the idea of objectivity. While the Yogacara does not say that an idea has, for example, spatial attributes, it does have a form manifesting them. While he agrees with the Saiva idealist that appearances have no independent existence apart from their appearing to consciousness, he maintains that for this reason they are unreal. The creativity of consciousness consists in its diversification in many modes having apparent externality; it is not a creation of objects. While the Kashmiri Saivite agrees that the world is pure conscious- ness alone, he maintains that it is such because it is a real creation of consciousness. The effect is essentially identical with the cause and shares in its reality. Matter and the entire universe are absolutely real, as 'congealed' (sty ana) or 'contracted' (samkucita) forms of consciousness. "This God of consciousness", writes Ksemaraja, "generates the universe and its form is a condensation of His own essence (rasa)" m By boiling sugarcane juice it condenses to form treacle, brown sugar and candy which retains its sweetness. Similarly, consciousness abides unchanged even though it assumes the concrete material form of the five gross elements. The same reality thus abides equally in gross and subtle forms. Consequently no object is totally insentient. Even stones bear a trace (vasana) of consciousness, although it is not clearly apparent because it is not associated with the vital breath (prana) and other components of a psycho-physical organism. Somananda goes so far as to affirm that physical objects, far from being insentient, can only exist insofar as they are aware of themselves as existing. The jar performs its function because it knows itself to be its agent. Indeed, all things are pervaded by consciousness and at one with it and hence share in its omniscience. Thus, Siva, Who perceives Himself in the form of physical objects, is the one ultimate reality. "The jar knows because it is of my nature", writes Somananda, "and I know it because I am of the jar's nature. I know because I am of Sadasiva's nature and He knows because He is of my nature; Yajnadatta [knows] because he is of Siva's nature and Siva [knows] because He is of Yajnadatta's nature". Integral Monism of Kashmiri Saivism Everything in this sense is directly perceived by absolute conscious- ness, and this direct perception (pratyaksa)

unifies the knowable into a single, undivided whole. This is the central concept behind a doctrine originally expounded by Narasimha called 'the non-dualism of direct perception' (pratyaksadvaita). This states that consciousness is essentially perceptive and that its perception of all things operates throughout the universe. Insofar as phenomena are clearly evident (sphufa) to us, everything is directly perceived by absolute consciousness, with which our individual consciousness is identical. This direct perception unfolds everywhere; the one true reality, it is alone and without companion or rival (nihsapatna). Even though it remains one, it can, by its very nature, perceive distinctions (bheda) between one entity and another, without this engendering any division within it. We distinguish between two entities in empirical terms on the basis of their mutual exclusion (anyonyabhava). The relative distinction {bheda) between them is essentially the perceived difference between their respective characteristics. Despite this difference they are united within the purview of a single cognition insofar as they are equally both manifest appearances. This cognition is the undivided essence (rasa) or 'own nature' (svabhava) of both. Encompassed by the 'fire of consciousness', there is no essential difference between them. Just as when an emerald and ruby reflect each other's light, the ruby is reddish-green and the emerald greenish-red, similarly everything is connected with everything else as part of the single variegated (vicitra) cognition of absolute consciousness. Mahesvarananda writes: The Supreme Lord's unique state of emotivity (asadharanabhava) is the outpouring of pure Being (mahasatta). It is manifest as the brilliance (sphuratta) of the universe which, if we ponder deeply, [is realized to be] the single flavour (ekarasa) of the essence of Beauty which is the vibration of the bliss of one's own nature. In this way all things are in reality one although divided from the one another sharing as they do the 'single flavour' (ekarasa) of the pure vibration of consciousness. Kashmiri Saiva Realism Kashmiri Saivism as a whole has been variously called a form of 'realistic idealism', 'monistic idealism', ‘idealistic monism' and 'concrete monism'. It is easy to understand why Kashmiri Saivism is The Doctrine of Vibration said to be 'idealistic' and 'monistic', but in what sense is it also 'realistic*? The answer to this question is of no small importance in trying to under- stand the central idea behind its metaphysics and the fundamental importance of the concept of Spanda, in this seemingly impossible marriage between monistic idealism and pluralistic realism. The Kashmiri Saiva approach understands the world to be a symbol of the absolute, that is, as the manner in which it presents itself to us. Again we can contrast this view with that of the Advaita Vedanta. The Advaita Vedanta understands the world to be an expression of the absolute insofar as it exists by virtue of the absolute's Being. Being is understood to be the real unity which underlies empirically manifest separateness and as such is never empirically manifest. It is only transcendentally actual as 'being-in-itself. The Kashmiri Saiva position represents, in a sense, a reversal of this point of view. The nature of the absolute, and also that of Being, is onceived as an eternal becoming (satatodita), a dynamic flux or Spanda, 'the agency of the act of being'. It is dentified with the concrete actuality of the fact of appearing, not passive unmanifest Being. Appearance (abhasa) alone is real.ppearing (prakasamanatva) is equivalent to the fact of being (astitva). Ksemaraja writes in his commentary on the Stanzas on Vibration: Indeed, all things are manifest because they are nothing but manifestation. The point being that nothing is manifest apart from manifestation The absolutely unmanifest, from this point of view, can have as little existence as the space in a lattice window of a sky-palace. Nay, even less, because even that space can appear as an imagined mage manifest within consciousness. Everything is real according to the manner in which it appears. Even an illusion is in this sense real, insofar as it appears and is known in the manner in which it appears. The empirical and the real are identical categories of thought. As Abhinava says: Thus this is the supreme doctrine (upanisatf), namely that, when- ever and in whatever form [an entity] appears, that then is its particular nature. Perhaps at this stage a brief comparison with Heidegger's ideas might prove to be enlightening and not altogether out of place. According to Heidegger's phenomenology of Being, reality is intelligible in a two-fold Integral Monism of Kashmiri Saivism manner as 'phenomenon' and 'logos'. Heidegger defines what he means by 'phenomenon' as: "that-which-shows-itself. The manifest . . . phenomena are then the collection of that which lies open in broad daylight or can be brought to the light of day — what the Greeks at times implicitly identified as 'ta onta' (the things-which-are)". 127 In his later writings Heidegger drops the term 'phenomenon' in preference for the verbal form 'phainesthai' in order to emphasize even more the actuality or presentational property of Being. Explaining this new form of the term he writes: "Being disclosed itself to the ancient Greeks as 'physis'. The etymological roots 'phy-' and 'pha-' designate the same thing: 'phyein', the rising-up or upsurge which resides within itself as 'phainesthai', lighting-up, self-showing, coming-out, appearing-forth."

Heidegger contrasted his notion of phenomenon with semblance (Schein) and with appearing (Erscheinung). In the case of semblance a thing can show itself as that which it is not, as when fool's gold shows itself to be gold. The ancients always allied semblance with non-being. Heidegger points out, however, that semblances are grounded in showings, and so does Abhinava. Both Heidegger and Abhinava consequently maintain that all semblances have a real basis and are to be treated as instances of phenomena along with the so-called real showing or manifestation of non-deceptive objects. So Heidegger states that: 'how- ever much seeming, just that much being'. Thus self-showing or appearing defines Being as phenomenon, but this definition of Being is as yet incomplete. Being is not only self-showing but 'logos' which Heidegger explains means 'discourse' (Rede) in the sense of 'apophansis': 'letting-be-seen'. Phenomenology, which according to Heidegger is the only correct study of Being, means 'letting-be-seen-that-which- shows-itself. This is true of Saiva Paramadvaita as well. The reality of the world demands recognition; we are forced to accept the direct presentation of the fact of our daily experience. As Abhinava says: "if practical life, which is useful to all persons at all times, places and conditions were not real, then there would be nothing left which could be said to be real" A thousand proofs could not make 'blue' other than the colour blue. The reality of whatever appears in consciousness cannot be denied. Objects appear; they do not cease to do so by a mere emphatic denial. The manifestation of an entity in its own specific form is a fact at one level of consciousness; it is real. The appearing of the same entity in the same form but recognised to be a direct representation of the absolute is also a fact, but at another level of consciousness. It is no more or less real than the first. 'As is the state of consciousness, so is the experience,' says Abhinava. Although the nature of the absolute is discovered at a higher level of consciousness, The Doctrine of Vibration nonetheless it presents itself to us directly in the specific form in which we perceive things; otherwise there would be no way in which we could penetrate from the level of appearing to that of its source and basis. Abhinava writes: Real is the entity iyastu) that appears in the moment of direct perception (sak$atkara), that is to say, within our experience of it. Once its own specific form has been clearly determined one should, with effort, induce it to penetrate into its pure conscious nature. All things are known to be just as they present themselves. The concrete actuality of being known (pramiti), irrespective of content, is itself the vibrant (spanda) actuality of the absolute. Liberating knowledge is gained not by going beyond appearances but by attending closely to them. "The secret," Mahesvarananda says, "is that liberation while alive (jivanmukti) is the profound contemplation of Maya's nature." No ontological distinction can be drawn between the absolute and its manifestations because both are an appearing (dbhasa), the latter of diversity and the former of 'the true light of consciousness which is beyond Maya and is the category Siva'. Those who have attained the category of Pure Knowledge above Maya and have thus gone beyond the category of Maya, see the entire universe as the light of consciousness . . . Just as the markings [on a feather] are nothing apart from the feather, the feather [is nothing apart from] them, similarly, when the light of consciousness is manifest, the whole group of phenomena is manifest as the light of consciousness itself. Within the sphere of Maya, every entity's 'own nature' (svabhdva) corresponds to its specific manifest form. Accordingly it is defined as that which distinguishes it from all else and from which it never deviates. Above the sphere of Maya, that is, above the level of objectivity, is the domain of the subject. At this level, everything is realised to be part of the fullness of the experience r and hence no longer bound by the conditions which impinge on the object. Here the part is discovered to be the whole, that is, consciousness in toto. In this sphere beyond relative distinctions, the yogi realises that (all) the categories of existence are present in every single category. m The yogi experiences every individual particular as the sum total of everything else. He recognises that all things have one nature and that every particular is all things. This is the 'essence' (sard) or co-extensive unity (samarasya) of all things. Integral Monism of Kashmiri SaivismWe have established that reality is manifest according to how [and the degree in which] the freedom of consciousness reveals it and that [this freedom] is the womb of all forms. Just as 'sweetness' is present in its entirety in every atom of the sugarcane, so each and every atom [of the universe] bears within itself the emanation of all things. This is the level of consciousness in which the absolute reflects on itself realising to its eternal delight and astonishment (camatkara) its own integral nature. The reality of the world of diversity is not denied, but experienced in a new mode of awareness free of time and space in the eternal omnipresence of the Here and Now. [Phenomenal forms of awareness] such as 'this [exists]', born of the colouring [imparted to the absolute] by the limitations engendered by the diversifying power of time {kalakalana) also emanate within the Supreme Principle. There [at that level], Fullness {purriata) is the one nature [of all things]

and so everything is omnipresent; otherwise, associated with division (khancjlana), the Fullness [of the absolute] would not be full. The content of absolute consciousness consists of diverse appearings (abhdsa) which, because they are manifest through it in this way, do not compromise the wholeness of consciousness. Everything we perceive is a momentary collocation of a number of such manifestations which combine together like 4 a row of altar lamps' (dipavalT) to form the single radiant picture of the universe. The individual objects which constitute the universe are specific collocations of such 'atomic' appearings. Together they form a single unified particular which appears according to its own defining features (svalaksand). A jar, for example, consists of a number of appearances such as 'round', 4 fat\ 'earthen', 'red', etc., which together discharge a single function (arthakriya), in this case, that of carrying the appearance 'water'. They unite with each other much as the scattered rays of a lamp come together when focused, or as the various currents of the sea together give rise to waves. Atomic appearings can combine in any number of ways, provided that they are not contrary to one another as established by the dictates of natural law (niyati). An appearance of 'form', for example, cannot combine with that of 'air'. Insofar as they share a common basis (samanyadhikaranya), a given cluster of appearances appears as a single whole. This common basis is the most prominent member of the group; the appearance 'jar' is such in the example quoted above. Any one appearance in a cluster may assume a more important or subordinate role. The result is a specific The Doctrine of Vibration awareness of an object of the form: 'here this is such.' While individual appearances do not lose their separate identity {svarupabheda) when they rest on a common basis, even so the particular object which appears according to its own characteristics (svalak$ana) is an individual reality in its own right. It is a different kind of appearance characterised by its association with the appearance of the specific location and time in which it is made manifest. The form of our experience is thus 'I now see this here'. But when we perceive each particular constituent appearance separately, each assumes a separate fixed function. Abhinava cites the following colourful example to illustrate how the various combinations of appearances account for the variety of experience: Thus even though the appearance of the beloved may manifest externally, it is as if far away in the absence of another appearance, namely, that of 'embracing'. So when the [appearing of the beloved] is associated with another appearance [namely that of 'far away'] the power (arthakriya) it formerly had of giving pleasure appears as its contrary. The form our experience assumes depends, not only on the nature of the object perceived, but also on personal factors entirely peculiar to ourselves. This theory explains this in two ways. In one sense, the object remains the same, but one or other of its constituent appearances comes to the fore according to the inclinations of the perceiver. From another point of view, we can say that the perceived object is different for each perceiver according to the difference in the prominent appearance manifest to him. Abhinava, citing as an example a golden jar, illustrates how the same object appears differently to different perceivers according to the use they wish to make of it and to their state of mind: When a person who is depressed and feels that there is nothing [of value for him in the world] sees the jar, he merely perceives the appearance 'exists' [in the form of the awareness that] 'it is\ He is not conscious of any other [of its constituent appearances] at all. An individual who desires to fetch water [perceives] the appearance 'jar'. The man who simply wants something that can be taken somewhere and then brought back [perceives] the appearance 'thing'. The man who desires money [perceives] the appearance 'gold'. The man who desires a pleasing object [perceives] the appearance 'brightness' while he who wants something solid sees the appearance 'hardness'. These 'atomic events' or appearances emerge from the pure subject's consciousness and combine together to form a total event at each moment. Integral Monism of Kashmiri Saivism Daily life (vyavahara) goes on by virtue of this ever renewed flux of appearances. They are connected together and work towards a single unified experience because they appear within the field of consciousness of the universal subject. The aggregate of appearances arises in the [supreme] subject as do [sprouts in] a rice field. Even though each sprout germinates from its own seed, they are perceived as a collective whole. Appearances rest in this way within the universal subject. 'External- ity' is itself another appearance; it arises from a distinction between appearances and the individual subject. So, although all manifestation always occurs within the subject, it appears to be external due to the power of Maya which separates the individual subject from his object. This split must occur for daily life to be possible. Only externally manifest appearances can perform their functions; when they are merged within the subject and at one with him, they cannot do so. Daily life proceeds on the basis of the operation and withdrawal of the conditions necessary for fruitful action to be possible. Appearance in this sense represents the actualisation of a potential hidden in consciousness made possible by virtue of its

dynamic, Spanda nature which is both the flow from inner to outer and back as well as the power that impels it. The emergence from, and submergence into, pure consciousness of each individual appearance is a particular pulsation (visesaspanda) of differentiated awareness. Together these individual pulsations constitute the universal pulse (samanyaspandd) of cosmic creation and destruction. Thus, every single thing in this way forms a part of the radiant vibration {sphuratta, sphurana) of the light of absolute consciousness. Light and Awareness: The Two Aspects of Consciousness Absolute consciousness understood as the unchanging ontological ground of all appearing is termed 'Prakasa'. As the creative awareness of its own Being, the absolute is called ' Vimarsa '. Prakasa and Vimarsa — the Divine Light of consciousness and the reflective awareness this Light has of its own nature — together constitute the all-embracing fullness (purnata) of consciousness. The Recognition (pratyabhijfia) school of Kashmiri Saivism develops this concept of the absolute which finds its fullest expression in Utpaladeva's Stanzas on the Recognition of God. Even though neither of these two key terms appear in the Stanzas on Vibration or the Aphorisms of Siva, they recur frequently in their commentaries. Thus, although the original formulation of the Doctrine of Vibration differs from the theology of Recognition in this respect, it was extended in the course of its development to accommodate this concept of the absolute as well. This was possible, and quite justified, insofar as the absolute understood in Pratyabhijfia terms does not, as we shall see, differ essentially from that of the Spanda school. We can, as Kashmiri Saivites themselves have done, explain one in terms of the other. The Doctrine of Vibration Prakasa: The Light of Consciousness Prakasa is the pure luminosity' (bhdna) or 'self-showing' that constitutes the essence and ultimate identity (atman) of phenomena. That things appear at all is due to the light of consciousness, and their appearing (avabhasana) is itself this Light which bestows on all things their evident, manifest nature. Established in the light of consciousness everything appears there according to its own specific nature (svabhava). Anything that supposedly does not rest in this Light is as unreal as a sky-flower. 3 Thus, according to Rajanaka Rama, unlike the light of the sun, or any other light, this Light not only makes all things apparent, it is also their ultimate source. 4 Full of its divine vibration the Light makes all things manifest and withdraws them into itself. This supra- temporal activity characterises it most specifically; devoid of it, it would be no better than an inert physical phenomenon. At the same time, this light is the conjunction (slesa) or oneness (aikdtmya) of its countless manifest forms, 6 and the collective whole (sampina'ana) of all the categories of existence. The universe is nothing but the shining of the Light within itself. It is the radiant vibration (sphuratta) of this Light, the state (avast hand) in which consciousness becomes manifest.Although the Light shines as all things at all times and hence also makes their diversity manifest, 9 penetrating each object individually as well as collectively, it is not totally 'merged' (magna) or identified with the object so as to suffer any division within itself. Our experience of any object is of the form: T see this': it is not itself an object, but the manifest form the object assumes as a luminous principle of experience. The Light is ever revealed and can never be obscured; objectivity can never cast a shadow on the light of consciousness. The Stanzas on Vibration declare: That in which all this creation is established and from whence it arises is nowhere obstructed because it is unconditioned by [its very] nature. This Light is the highest reality (paramartha). It is the 'Ancient Light' (puranaprakasa) that makes all things new and fresh every moment. It is 'always new and secret, ancient and known to all'. It is the form of the Present (vartamanarupa), the Eternal Now. Time and space are relations between the contents of consciousness; they cannot impinge on the integrity of the absolute itself. 16 Neither space nor time can divide it, for they are one with the Light that illumines them Light and Awareness: Two Aspects of Consciousness and makes them known as elements of experience. But this Light is the shining of the absolute; it is not an impersonal principle. It is the living Light of God, indeed it is God Himself, the Master Who instructs the entire universe. 18 Siva is this 'auspicious lamp', Who illumines all things. He is the Light of consciousness that reveals the presence of both the real and the unreal, of light' and 'darkness'. 20 Abhinavagupta writes: Thus Bhairava, the Light, is self-evident (svatahsiddha); without beginning, He is the first and last of all things, the Eternal Present. And so what else can be said of Him? The unfolding of the categories of existence (tattva) and creation, which are the expansion of His own Self, He illumines, luminous with His own Light, in identity with Himself, and because He illumines Himself, so too He reflects on His own nature, without His wonder (camatkara) being in any way diminished. Although It Is Possible To Catch Glimpses Of The Highest Reality In Advanced States Of Contemplation Before Attaining Perfect Enlightenment, These States, However Long They Last, Are Transitory (Kadacitka) And When They End The Vision Of The Absolute Ceases With Them. The

Highest Realisation, However, Persists In All States Of Consciousness. It Happens Once And Need Never Occur Again. A Passage From A Lost Tantra Declares: "The Self Shines Forth But Once, It Is Full [Of All Things] And Can Nowhere Be Unmanifest." All Spiritual Discipline Culminates In This Moment Of Realisation. Accordingly, Abhinava Stresses That The Goal Of All The Means To Realisation, Even The Individual Means, Is This Absolute Consciousness. Finally, It Is Worth Noting That Although Abhinava Affirms That The Teachings Concerning Anupaya Are Found In The Siddhayogesvarimata And The Malinxvijaya, Both Of Which, According To Abhinava, Are Major Tantras Of The Trika School, It Is In The Theology Of The School Of Recognition That It Is Best Exemplified. Abhinava Himself Refers To Somananda, The Founder Of This School, As Teaching It And Alludes To The Following Passage In The Vision Of Siva To Support His Own Exposition: When Siva, Who Is Everywhere Present, Is Known Just Once Through The Firm Insight Born Of Right Knowledge (Pramana), The Scripture And The Master's Words, No Means [To Realisation] Serves Any Purpose And Even Contemplation {Bhavana) [Is Of No Further Use]. Anupaya Is Therefore, According To Abhinavagupta, The Recognition Of One's Own Authentic Siva-Nature, Which All The Higher Tantric Traditions Teach Is The Ultimate Realisation. This Is Also True Of The Doctrine Of Vibration Whose Precedents Are Clearly Traceable To These Same Traditions. Thus, Although The Stanzas Themselves Never Refer Directly To Enlighten- Ment As An Experience Of Recognition, There Can Be Little Doubt That Spanda Practice Leads To This Same Realisation. Accordingly, Commentators Stress That We Realise The Vibration Of Consciousness By Recognising Its Activity And That Liberation Depends On The Recognition Of This As One's Own Nature. Ksemaraja Describes What Happens In This Moment Of Recognition According To The Doctrine Of Vibration Thus: At The End Of Countless Rebirths, The Yogi's [Psycho-Physical] Activity [Which Issues From Ignorance] Is Suddenly Interrupted By The Recognition Of His Own Transcendent Nature, Full Of A Novel And Supreme Bliss. He Is Like One Struck With Awe. And In This Attitude Of Astonishment (Vismaya- Mudra) Achieves The Great Expansion [Of Consciousness] {Mahavikasa). Thus He, The Best Of Yogis, Whose True Nature Has Been Revealed [To Him] Is Well Established [At The Highest Level Of Consciousness], Which He Grasps Firmly And His Hold Upon It Never Slackens. Thus He Is No Longer Subject To Profane Existence (Pravftti), The Abhorrent And Continuing Round Of Birth And Death, Which Inspires Fear In All Living Beings, Because Its Cause, His Own Impurity, No Longer Exists. The Divine Means (Sambhavopava) In Anupaya The Yogi Does Not Need To Deal With The World Of Diversity At All; Only Paramasiva Exists There. Beyond Both Immanence And Transcendence, He Has Nothing To Do With The World Of Practice And Realisation. Anupaya Is The Experience Of The Undefinable (Qnakhya) Light Of Consciousness, Which Is The Pure Bliss Beyond Even The Supreme State (Paratlta) Of Sivatattva. At A Slightly Lower Level, Corresponding To The Divine Means, A Subtle Distinction Emerges Between The Goal And The Path. The Yogi Now Practises Within The Domain Of The Outpouring Of The Power Of Consciousness. From This Level He Penetrates Directly Into The Universal Egoity Of Pure Consciousness By The Subtle Exertion (Udyama) Of Its Freedom (Svatantrya) And Reflective Awareness. The Yogi Who Practises The Divine Means Is Not Concerned With Any Partial Aspect Of Reality But Centers His Attention Directly On Its Abounding Plenitude. Hence This Means Is Based On Siva's Own State (Sambhavavastha) In Which Only The Power Of Freedom Operates As The Pure Being (Satta) Or Essence Of All The Other Powers. This State Is The Light Of Consciousness Which, Free Of All Thought- Forms, Is The Basis Of All Practice." The Yogi Who Recognises That Pure Consciousness, Free Of Thought-Constructs (Nirvikalpa), Is His Basic State, Can Practice In Any Way He Chooses; Even The Most Common Mantra Will Lead Him Directly To The Highest State. Thus The Forms Of Contemplative Absorption, Empowered (Sakta) And Individual (Anava), Which Are The Fruits Of The Other Means To Realisation Both, Attain Maturity In This Same Undifferentiated Awareness. This Awareness Is The Pure Ego Manifest At The Initial Moment Of Perception (Prathamikalocana), When The Power Of The Will To Perceive Is Activated. It Is The Subtle State Of Consciousness That Reveals The Presence And Nature Of Its Object Directly: That Which Shines And Is Directly Grasped In The First Moment Of Perception While It Is Still Free Of Differentiated Representations And Reflects Upon Itself Is [The Basis Of The Divine Means] Said To Be The Will. Just As An Object Appears Directly To One Whose Eyes Are Open Without The Intervention Of Any Mental Cogitation (Anusaipdhana), So, For Some, Does Siva's Nature. The Movement Of Awareness At This Level Of Practice Attains Its Goal Quickly. While Consciousness Is Heightened Progressively In The Other Means, Here It Expands Freely To The Higher Levels, Unconfined By Any Intruding Thought-

Constructs. The Divine Means Is A 'Thoughtless Thought', A 'Processless Process', That Occurs At The Juncture Between Being And Becoming. Abhinava Explains: When The Heart [Of Consciousness] Is Pure And [Free Of Thought- Constructs], It Harbours The Light Which Illumines The Radiant, Primordial Plane Ipragrabhumi) Together With All The Categories Of Existence. [The Yogi] Then Realises Through It His Identity With Siva Who Is Pure Consciousness The Yogi Must Catch The Initial Moment Of Awareness (Adiparamarsa) Just When Perception Begins^ He Must Not Move On From The First Pure Sensation Of The Object But Return To Its Original Source In His Own T Consciousness. Observing In This Way The Objective Field Of Consciousness Without Laboring To Distinguish Particulars, The Yogi Penetrates Into His Own Subjectivity Which, Vacuous And Divested Of All Outer Supports (Niralamba), Is Not Directed Anywhere Outside Itself (Ananyamukha- Preksin). Here He Can Hold Of The Power Inherent In His Own Consciousness Through Which He Discerns The True Nature Of Whatever Appears Before Him. Thus The Stanzas On Vibration Teach: Just As An Object, Which Is Not Seen Clearly At First Even When The Mind Attends To It Carefully, Becomes Later Fully Evident When Observed With The Effort Exerted Through One's Own [Inherent] Strength (Svabala), In The Same Way, When [The Yogi] Lays Hold Of That Same Power, Then Whatever [He Perceives Manifests To Him] Quickly According To Its True Nature, Whatever Be Its Form, Locus, Time Or State. Thus, Although The Practice Of This Divine Means Starts By Catching Hold Of The Will In The First Moment Of Awareness, It Also Concerns The Second And Third Moments In Which The Means Of Knowledge And The Object Are Made Manifest. When Practice At This Level Proceeds Smoothly And Without Interruption, The Three Powers Of Will, Knowledge And Action Fuse Into The Trident (Trisula) Of Power, Which Is The Subject Free Of All Obscuration (Niranjana), M At One With The Power Of Action In Its Most Powerful And Evident Form. The Kaula Schools Call This State The Stainless (Niranjanatattva). Equated In The Spanda Tradition With The Dawning Of The Vibration Of Consciousness {Spandodaya), It Is The Enlightenment The Spanda Yogi Seeks. Spanda Practice Is Based On The Experience Of Spanda Which, As We Have Seen, Is Defined As The Intent (Aunmukhya) Of Consciousness, Unrestricted To Any Specific Object And Hence Free Of Thought-Constructs. Spanda Can Therefore Be Experienced Directly When A Powerful Intention Develops Within Consciousness, Whatever Is Its Ultimate Goal Or Cause. We Have Already Noted That Intense Anger, Joy, Grief Or Confusion Is Such Occasions. Similarly, The Yogi Can Make Contact With The Omnipotent Will, Which He As Siva Possesses, Through Intense Prayer. Directing His Entire Attention To Siva, The Benefactor Of The World, Entreating Him Fervently And Without Break, His Will Merges With Siva's Universal Will, Which Is The Source Of Every Impulse And Perception. As He Looks About Him, The Yogi Realises That It Is Siva Himself, The Universal Consciousness And The Yogi's Authentic Identity, Who Ordains His Every Action, Thought And Perception. Thus The Yogi's Cognitive Intent On His Object Coincides With The Universal Will To Make That Object Known To Him, Whether The Yogi Is Awake Or Dreaming. He Is Thus No Longer Like The Worldly Man Who Cannot Dream As He Wishes And Is Forced To Experience Whatever Spontaneously Happens In These States Of Consciousness. Ultimately The Yogi Manages, By Siva's Grace, To Maintain A Constant Awareness Of His Own Pure Perceptive Consciousness (Upalabdhrta) Divested Of All Obscuring Thought-Constructs In Deep Sleep As Well As In The Contemplative State (Turiya) Beyond It. When He Rises To The Higher Levels Of Contemplation In Which The Breath Is Suspended And All Sensory And Mental Activity Ceases, The Yogi Who Manages To Sustain This Pure, Undifferentiated Awareness Does Not Succumb To Sleep As Do Less Developed Yogis. Perfection In The Practice Of The Divine Means Thus Coincides With The Goal Of Spanda Practice, Namely, A Constant, Alert Attention To The Perceiving Subjectivity Which Persists Unchanged In Every State Of Consciousness Both As The Perceiver And Agent Of All That It Experiences. Another Important Spanda Practice Belonging To This Means Is Centering. The Spanda Yogi Seeks To Find The Centre (Madhya) Between One Cognition And The Next; For It Is There That He Discovers The Expansion (Unmesd) Of Consciousness Free Of Thought-Constructs From Whence All Differentiated Perceptions (Yikalpa) Emerge. 107 Abhinava Explains That This Pure Awareness Is Called: . . . The Expansion (Unme$A) Of [Consciousness] Or The Creative Intuition {Pratibha) [Experienced] In The Interval Which Divides Two [Moments] Of Differentiated Perception (Vikalpa). It Is Here That They Arise And Disappear. The Sastras And Agamas Proclaim With Reasoned Argument That It Is Free Of Thought-Constructs {Nirvikalpa) And Precedes All Mental Representations Of Any Object. None Can Deny That A Gap Exists Between Perceptions Insofar As Two Moments Of Thought Are Invariably Divided. This

[Gap] Is The Undifferentiated Unity Of All The Countless Manifestations. Similarly, In The Outer More Objective Sphere, Where Change Consists Of The Alterations In The Configurations Of Manifest Appearances (Abhasa\ The Transition From One To Another Corresponds To A Phase Of Pure Luminosity That Marks The Beginning Of One Form And The End Of Another. The World Of Manifestation And Differentiated Perceptions (Yikalpa) Thus Extends From One Centre To The Next. Although It Is Never In Fact Divorced From The Subject Who Resides There, The Ignorant Fail To Grasp This Fact And So, Cut Off From The Centre, The World Of Objectivity Becomes For Them The Sphere Of Maya. Bhagavatotpala Quotes The Light Of Consciousness (Samvitprakdsa): This Ever Pure Experience (Suddhanubhava) Is Variegated By Each Form [Revealed Within It]; Even So It Remains Unstained (Nirmala) When Moving To Another. Just As A Cloth Which Is Naturally White, Once Dyed, Cannot Change Colour Without [First] Becoming White Again, Similarly The Pure Power Of Awareness, (Citi) Once Coloured By Form, Is Pure [Again] At The Centre Where That Form Is Abandoned And From Whence It Proceeds To Another. In His Essence Of Vibration (Spandasarpdoha), K§Emaraja Explains That The Rise And Fall Of Every Individual Perception In The Field Of Awareness Is A Specific Pulsation Of Consciousness. From The Point Of View Of The Object, The Expansion (Unmesa) Of This Pulse Is Represented By The Initial Desire To Perceive (Didrksa) A Particular Object, While The Contracted (Nimesa) Phase Is The Withdrawal Of Attention From The Object Previously Perceived. From The Point Of View Of The Perceiving Subjectivity, The Phases Are Reversed, So That The Initial Desire To Perceive Marks The Contraction (Nimesa) Of Subjective Consciousness While The Falling Away Of The Previous Perception Is Its Expansion (Unmesa). At The Higher Level, Where These Two Phases Are Experienced Within Consciousness, They Represent The State Of The Categories Of Isvara ('This Universe Is Me') And Sadasiva ('I Am This Universe'). Utpaladeva Says: Expansion (Unme$A\ Which Is In The External Manifestation [Of Objectivity], Is Kvaratattva While Contraction (Nime$A), Which Is In The Internal Manifestation [Of Subjectivity], Is Sadasiva. At This Level All The Powers Of Consciousness Fuse And Both Phases Are Manifest As Part Of One Reality. This Unity Is In Fact Apparent To Everybody At Each Moment. However, Within The Domain Of Maya, Which Is The Sphere Of Differentiated Perceptions (Yikalpa), It Is Clearly Manifest Only At The Juncture (Madhya) Between Two Cognitions. In This Centre Resides The Void (Kha) Of Consciousness (Free Of Thought-Constructs) Which, Divested Of Diversity, Digests Into Itself All The Psycho-Physical Processes That Give Life To The Multiplicity Of Perceptions. The Yogi Moves From The Particular Vibrations Of Consciousness At Its Periphery To The Universal Throb Of The Heart In The Centre. As Abhinava Explains: The Self-Reflective Awareness In The Heart Of Pure Consciousness, Present At The Beginning And End Of Each Perception, Within Which The Entire Universe Is Dissolved Away Without Residue, Is Called In The Scriptures, The Universal Vibration Of Consciousness (Samanyaspanda) And Is The Outpouring (Uccalana) [Of Awareness] Within One's Own Nature. All The Categories Of Existence (Tattvas) Are United In The Heart Of The Centre Where The Life-Giving Elixir Of Siva's Consciousness Floods One's Own Inner Nature. To Reside In The Centre Is To Abide By The Law Of Totality (Gramadharma) In A State Which Transcends The Workings Of The Mind (Unmana). Consciousness (Jnana) With Light As Its Support, Residing In The Centre Between Being And Non-Being Is Known As The Act Of Abiding In One's Own Abode As The Perceiving Subjectivity (Dratfrtva) Free Of All Obscuration. That Which Has Been Purified By Pure Awareness (Suddhavijnana) Is Called The Transcendent (Viviktavastu), Said To Be The Mode Of Being (V/7//) Of The Law Of Totality (Gramadharma) Through Which Everything Is Easily Attainable. The Power In The Centre (Madhyasakti) Is The Eternal Present. Beyond Time It Is The Source Of Both Past And Future. To Be Established There Is To Abide Without A Break In Rama, The Supreme Enjoyer, In Every Action Of One's Life. Rama Is Siva, The Supreme Cause Who Pervades The Fourteen Aspects Which Embrace The Entire Universe Of Experience, Namely, Moving, Standing, Dreaming, Waking, The Opening And Closing Of The Eyes, Running, Jumping, Exertion, Knowledge [Born] Of The Power Of The Senses, The [Three] Aspects Of The Mind, Living Beings, Names And All Kinds Of Actions. By Developing An Awareness Of The Centre, The Yogi Experiences The Bliss Of Consciousness. Through This Gap He Plunges Into Introverted Absorption (Nimilanasamadhi) And Then Emerges Again To Pervade The Field Of Awareness Between Centres And So Experience The Cosmic Bliss (Jagadananda) Of The Universal Vibration Of Consciousness. 1 I8 He Then Recognises That This State Pervades Every Aspect Of Experience. In This Way The Yogi's Consciousness Is No Longer Afflicted By The Power Which Obscures It, Hemming The Centre In On Both

Sides With Thought-Constructs That Seemingly Deprive It Of Its Fullness. As He Realises Directly His Pure Conscious Nature As The Universal Ego Free Of All Mental Representations, It Expands Out To Embrace All Things Within Itself. Thus The Realisation The Divine Means Leads To, And Is Directly Based Upon, Is That This Pure Ego Is In All Things Just As All Things Are Within It. In The Spanda Tradition, As Recorded In The Stanzas On Vibration, No Such Ego Is Recognised. ' I9 Man's Authentic Nature Is, However, Understood In Personal Terms As Every Individual's Own 'Own Nature* (Svasvabhava) Which Is Siva, The Universal Vibration Of Pure Subjectivity (Upalabdhfta). It Is Not Surprising, Therefore, That Later Commentators Found These Two Conceptions To Be Essentially The Same And Accordingly Identified One's Own Inner Nature With The Pure Ego. This Came As A Natural Development In Spanda Doctrine Not Only For This Reason But Also Because The Universal Ego Is Experienced As The Inner Dynamics Of Absolute Consciousness. To Conclude Our Summarial Exposition Of The Divine Means, Which Is Centred On The Direct Experience Of This Pure Ego (And Hence On Spanda In This Form), We Turn Now To A Brief Description Of Its Inner, Cyclic Activity. We Shall Do This By Examining Abhinava' S Esoteric Exegesis Of The Symbolic Significance Of The Word 'A HAM', Which In Sanskrit Means T, And Symbolises By Its Form The Ego's Dynamic Nature. The Objective World Of Perceptions Is, As We Have Seen, Essentially A Chain Of Thought-Constructs (Prapanca) Closely Linked To One Another And Woven Into The Fabric Of Diversity (Vicitrata). This Thought (Vikalpa) Is A Form Of Speech (Vac) Uttered Internally By The Mind (Citta), Which Is Itself An Outpouring Of Consciousness. Consciousness Also, In Its Turn, Resounds With The Silent, Supreme Form Of Speech {Para Vac) Which Is The Reflective Awareness Through Which It Expresses Itself To Itself. Consequently, The Fifty Letters Of The Sanskrit Alphabet, Which Are The Smallest Phonemic Units Into Which Speech Can Be Analysed, Are Symbolic Of The Principal Elements Of The Activity Of Consciousness. Letters Come Together To Generate Words And Words Go On To Form Sentences. In The Same Way The Fifty Phases In The Cycle Of Consciousness Represent, In The Realms Of Denoted Meaning (Vacya), The Sum Total Of Its Universal Activity (Kriya) Corresponding To The Principal Forces (Kala) Which Come Together To Form The Metaphysical Categories Of Experience, Which In Their Turn Appear In The Grossest, Most Explicitly 'Articulate' Form As The One Hundred And Eighteen World-Systems (Bhuvana). 'A', The First Letter Of Both AHAM And The Sanskrit Alphabet, Is The Point Of Departure Or Initial Emergence Of All The Other Letters And Hence Denotes Anuttara — The Absolute. 'Ha', Is The Final Letter Of The Alphabet And Represents The Point Of Completion When Ail The Letters Have Emerged. It Represents The State In Which All The Elements Of Experience, In The Domains Of Both Inner Consciousness And Outer Unconsciousness, Are Fully Displayed. It Is Also The Generative, Emission (Visarga) Which, Like The Breath, Casts The Inner Into The Outer, And Draws What Is Outside Inward. The Two Letters 'A' And 'Ha' Thus Represent Siva, The Transcendental Source And Sakti, His Cosmic Outpouring That Flows Back Into Him. The Combined 'A-Ha' Contains Within Itself All The Letters Of The Alphabet — Every Phase Of Consciousness, Both Transcendental And Universal. (For A Graphic Representation Of This Analysis, See Figure 1.) M, The Final Letter Of AHAM, Is Written As A Dot Placed Above The Letter Which Precedes It. It Comes At The End Of The Vowel Series And Before The Consonants And So Is Called 'Anus Vara '(Lit. 'That Which Follows The Vowels') And Also 'Bindu ' (Lit. 'Dot,' 'Drop,' 'Point' Or 'Zero'). While The Consonant 'M' Symbolises The Individual Soul (Purusa), 'Bindu 1 Rep Resents The Subtle Vibration Of T, Which Is The Life Force (Jivakala) And Essence Of The Soul's Subjectivity Manifest At The Transcendental, Supra-Mental Level (Unmana). 12 ° It Is The Zero-Point In The Centre Between The Series Of Negative Numbers, In This Case The Vowels Which Represent The Processes Happening Internally Within Siva, And The Series Of Positive Numbers — The Consonants Which Symbolise The Processes Happening Externally Within Sakti. Bindu, As A Point Without Area, Symbolises The Non-Finite Nature Of The Pure Awareness (Pramitibhava) Of AHAM. It Is The Pivot Around Which The Cycle Of Energies From 'A' To 'Ha' Rotates, The Void In The Centre From Which All The Powers Emanate And Into Which They Collapse. As Such, It Is The Supreme Power Of Action Which Holds Subject, Object And Means Of Knowledge Together In A Potential State In The One Light That Shines As All Three Containing Them In Its Repose (Visrdnti). Bindu Is The 'Knower' (Jndtr), Who Is Essentially Consciousness That, Though Omniscient, Does Not Manifest Its Intelligence, Like A Man Who Knows The Scriptures But Having No Occasion To Explain Them To Others Silently Bears This Knowledge Within Himself. As Such, It Symbolises The Union Of Siva And Sakti (Sivasakti- Mithunapina'a) In A State Of Heightened Potency

In Which They Have Not Yet Divided To Generate The World Of Diversity. It Stands, In Other Words, At The Threshold Of Differentiation In The Stream Of Emanation Still Contained Within Siva. The Absolute. Expansion Commences Bindu — The Individual Soul Withdrawal Commences Then, To The Degree In Which That Which Is To Be Accomplished By The Power Of Action Residing Within It [As A Potential] Penetrates Into The Absolute, It Appears Initially As Bindu, Which Is The Light Of Pure Consciousness. When Outer Objectivity Is Reabsorbed Into Its Transcendent Source, Bindu Is The Point Into Which All The Manifest Powers Of Consciousness Are Gathered And Fused Together. The Universal Potency Of All The Letters Is Thus Contained In Bindu Which, As The Reflective Awareness Of Supreme T Consciousness, Gives Them All Life. Thus Bindu Also Marks The Beginning Of Siva's Internal Movement Back To The Undifferentiated Absolute And So Stands At The Threshold Of Both Emission And Absorption Without Being Involved In Either. The Three Aspects Of AH AM Together Constitute A Movement From The Undifferentiated Source Of Transcendental Consciousness — 'A' — Through The Expansion Or Emission Of Its Power — 4 Ha' — To The Subject — 'M 1 —Which Contains And Makes Manifest The Entire Universe Of Experience. The Reverse Of This Movement, That Of Withdrawal (Samhara), Is Represented By M-Ha-A. AHAM And M-Ha-A Alternate In The Rotation (Ghurnana) Of The Reflective Awareness Of T Consciousness As Immanent Sakti Emerges From Transcendental Siva To Then Merge Back Into Him. As Abhinava Says: The Universe Rests Within Sakti And She On The Plane Of The Absolute (Anutiara) And This Again Within Sakti ... For The Universe Shines Within Consciousness And [Consciousness Shines] There [Within The Universe By The Power Of] Consciousness. These Three Poles, Forming A Couple And Merging, Make Up The One Supreme Nature Of Bhairava Whose Essence Is AHAM- At The Microcosmic Level, Fc A' Represents The Initial Moment When The Subject Begins To Rise Out Of Himself To View The Object. The Movement From 'A To 'Ha' Marks The Emergence Of Sensation Within The Field Of Awareness, Which Is Represented By The Fifty Letters Of The Alphabet Symbolic Of The Fifty Aspects Of The Flux Of Consciousness Leading To Objectified Perception. 'Nl' Is Tne Subject Who, Resting Content Within Himself When He Has Perceived His Object Merges Through The Inner Flow Of Awareness Into K A\ The Absolute. Then From The Absolute (A) Its Emission (Ha) Flows Back Into The Pure Subject (M) Set To Perceive His Object. Thus All The Cycles Of Creation And Destruction Are Contained Within AHAM Through Which They Are Experienced Simultaneously As The Spontaneous Play Of The Absolute. The Yogi Who Recognises This Recurrent Pulse Of Awareness To Be The Movement Of His Own Consciousness Merges His Limited Ego With The Universal Ego. Thus He Realises That Its Power To Create, Sustain And Destroy All Things Is His Own Inner Strength (Svabaia) That He Exerts Effortlessly In The Same State Of Mystical Absorption (Turlya) In Universal Consciousness That The Absolute Itself Enjoys. In This Way He Shares In The Three-Fold Awareness Siva Himself Has Of His Own Nature Which Abhinava Describes As Follows: I Make The Universe Manifest Within Myself In The Sky Of Consciousness. I, Who Am The Universe, Am Its Creator! ' — This Awareness Is The Way In Which One Becomes Bhairava. 'AH Of Manifest Creation(Sadadhvari) Is Reflected Within Me, I Cause It To Persist 1 — This Awareness Is The Way In Which One Becomes The Universe. The Universe Dissolves Within Me. I Who Am The Flame Of The [One] Great And Eternal Fire Of Consciousness' — Seeing Thus One Achieves Peace. The Experience Of The Liberated Thus Coincides With The Realisation Of Their Own Divine Nature Which, Through Its Power, Rules And Guides The Cosmic Order. Thus This Attainment (Siddhi), Which Is Liberation Itself, Is In The Doctrlie Of Vibration Technically Called 'Mastery Over The Wheel Of Energies' (Cakresvaratvasiddhi) Because The Liberated Soul, Identified With Siva, Now Governs, As Does Siva, The Cycle Of The Powers That Bring About The Creation And Destruction Of All Things, The Empowered Means (Saktopaya) All The Practices Taught In The Stanzas On Vibration Are Internal. Whenever Ritual Is Mentioned, It Is Invariably Interpreted In Terms Of The Dynamics Of The Inner Processes The Yogi Experiences And Implements In The Course Of His Yogic Practice. The Doctrine Of Vibration, Ksemaraja Affirms, Is Concerned Entirely With These Inner Disciplines Centred, As It Is, In One Way Or Another, On Consciousness Or, At Least, On The Inner Activity Of The Mind. Thus The Empowered Means Which, Like The Other Categories We Have Discussed, Is Entirely Internal Includes An Important Part Of Spanda Practice. Spanda Practice Belonging To The Divine Means Centres On One's Own Inherent Nature (Svasvabhava) As Siva, The Universal Perceiver And Agent, That Belonging To The Empowered Means On His Power Instead Of Arriving Directly At The All-Embracing Emptiness Of Subjective Conscious- Ness, The Yogi Practising The Empowered

Means Realises His True Nature Through The Fullness Of Its Energy. Practising The Divine Means, The Yogi Plunges, As It Were, Straight Into The Fire Of Consciousness; Practising The Empowered Means He Merges With Its Rays. Either Way The Yogi Is Centred Equally On Ultimate Reality. The Power Of Consciousness Is No Less Absolute Than Its Possessor. To Make This Point Abhinava Quotes The Matanga- Tantra: This Reality Consists Of The Rays Of [Siva's] Power And Is Variously Said To Be The Abode Of The Lord's Manifestation . . . That Same [Power] Illumined [By Siva] Is Itself Also Luminous, Unshaken And Unmoving. That Very [Power] Is The Supreme State, Subtle, Omnipresent, The Nectar Of Immortality, Free Of Obscuration, Peaceful, Yearning For Pure Being Alone {Vastumatra) And Devoid Of Beginning And End. Perfectly Pure, It Is Said To Be The Body [Of Ultimate Reality]. The Yogi Concentrates On The Powers Operating In All Of Life's Activities As Particular Pulsations (Visesaspanda) In The Universal Rhythm (Samanyaspanda) Of The Power Of Consciousness. In This Way He Rises Progressively From The Particular To The Universal Until He Reaches Pure Being (Satta), The Greatest Of All Universals (Mahasamanyd) And The Highest Form Of Siva's Power. Thus The Creative Power Of Maya, Manifest Through Countless Lesser Powers, No Longer Causes The Yogi To Stray From Siva's Consciousness But Becomes The Means Through Which It Can Be Realised In The Illuminating Brilliance (Sphuratta) Which Is Siva's Pure Being. Thus By Discovering The True Nature Of Sakti, The Yogi Realises Himself To Be Siva, Its Possessor Who Consists Of All Its Countless Powers. Thus Practise Belonging To This Means Leads To The Same Pure Consciousness Free Of Thought-Constructs Realised Through The Divine Means. Although The Ultimate Realisation Is Instantaneous, The Yogi Rises To It Gradually By Freeing His Consciousness Of The Limitations Imposed Upon It By Thought. Abhinava Explains: The Same Occurs In The Empowered Means [As Does In The Divine]. At The Discursive Level Of Consciousness {Yaikalpikibhumi) [Where The Empowered Means Functions] Knowledge And Action, Although Evident, Are, For The Reasons Explained Previously, Contracted. A Blazing Energy [Is Revealed Within] The One Who Dedicates Himself To Removing The Burden Of This Contraction. [This Energy Eventually] Brings About The Inner Manifestation {Antarabhasa) Of Pure Consciousness He Seeks. Consciousness Is Individualized And Its Power Of Knowledge And Action Contracted By The Thought-Constructs Born Of Ignorance. The Arising Of These Mental Representations, As The Stanzas On Vibration Say, Deprives The Soul Of Its Freedom And Immortal Life. The Practise Of The Empowered Means Is Meant To Free The Fettered Soul Of This Constriction On His Consciousness. It Operates Within The Mental Sphere (Cetasy™ And Is Designed To Purify Thought (Vikalpasamskara) In Order To. Reveal The Pure Consciousness Which Is Its Ground And Ultimate Source. Thus, The Empowered Means Is Concerned With The Second Instant Of Perception, During Which The Subject Forms Mental Representations Of His Object. Thought Functions On The Basis Of An Awareness Of Relative Distinctions Between Specific Particulars, Distinguishing Them From One Another And Thus Seemingly Fragmenting The Essential Unity Of Reality. The Vibrant Vitality Of Consciousness, Universally Manifest, Is Clouded Like A Mirror By A Child's Breath And The Soul Is Deprived Of The Liberating Intuition Of The One Reality Free Of Thought-Constructs (Nirvikalpa). Abhinava Writes: The [Fettered Soul] Is Like A Dancing Girl Who Although Wishing To Leave The Dancehall Is Collared By The Doorkeeper Of Thought And Thrown Back Onto The Stage Of Maya All Thought Is Centred On Objectivity And Hence Dislodges Awareness From The Plenitude Of Pure Subjective Consciousness. Thus, To Regain The Original State Of Rest (Visranti) Consciousness Enjoys, The Yogi Must Rid Himself Of Thought. As Thought-Forms Decrease, Pure, Thought-Free Awareness Is Strengthened Until The Yogi Is Fully Established In A State In Which The Relative Distinctions (Bheda) Conceived Between Entities Dissolve Away. Everything Appears To Him As Pure Being (Sattdmdtra) L} And The Entire Universe Shines Before Him Pervaded By Siva's Radiance. His Intuitive Faculty (Mati) Thus Purified, The Yogi Gains Both The Perfections (Siddhi) Of Yogic Practice And Liberation (Mukti). His Consciousness Is Now Like A Well-Polished Mirror Which Reflects Everything He Desires And Grants It To Him. Abhinava Writes: Just As A Man Who Has Been Ill For A Long Time Forgets His Past Pain Completely When He Regains His Health, Absorbed As He Is In The Ease Of His Present Condition, So Too Those Who Are Grounded In Pure Awareness Free Of Thought-Constructs Are No Longer Conscious Of Their Previous [Fettered] State. Consciousness, The Sole Truly Existent Reality, Free Of Thought- Constructs Is Made Fully And Evidently Manifest By Eliminating These Differentiated Perceptions. The Wise Man Should Therefore Exert Himself To Attend Closely To This [State Of Awareness]. The Thought-Constructs Generated Within

Consciousness Do Not In Reality Affect It At All. They Can Neither Break Up Nor Add Anything To The Light Which Shines As All Things. They Are In Fact Nothing But Consciousness Itself Which Perceives, Through Its Power Of Reflective Awareness (Yimarsa), The Multitude Of Objects In Diverse Ways, And So Assumes This Form. Although Thought-Constructs Are Mental Representations Of Objects Once Seen Or Present, They Are Products Of The Power Of Consciousness And Not Of The Objects They Represent. Thought Is Both Analytic And Synthetic; It Serves The Useful Purpose Of Separating Individual Elements Of Experience From Others And Linking Together Those That Appear To Be Distinct From One Another So That They Can Be Better Understood. It Does Not Consist Merely Of False Mental Constructs Projected Onto Reality That Need To Be Wholly Rejected. Thought Obscures Consciousness And Distracts It Only When It Appears In The Form Of Doubt, Vacillating Between Alternatives. Once This Conflicting Duality (Dvaitddhivasa) Is Eliminated, Thought Is Purified And Rests In Itself As The 'Thought-Less Thought' Of Pure Consciousness. By Gradually Eliminating The Multitude Of Conflicting Notions That Agitate Him, The Yogi Ultimately Achieves The Certainty (Niscaya) Corresponding To A Direct Awareness Of His Own Divine Nature. Abhinava Explains: Thought Is In Reality None Other Than Pure Consciousness. Even So, It Serves As A Means To Liberation For The Individual Soul (Anu) Only When It Takes The Form Of Certainty (Niscaya). This surprising result--that information capacity depends on surface area—has (e) a natural explanation if the holographic principle (proposed in 1993 by Nobelist Gerard't Hooft of the University of Utrecht in the Netherlands and elaborated by Susskind) is true. In the everyday world, a hologram is a special kind of photograph that generates a full three-dimensional image when it is illuminated in the right manner. When it com,es to structures,one cannot bypass Deleuze and his work. Levi Strauss Has Also Noticed That Signs Always Offer An Excess. The System Of Language, 'The Order Of The Known' Exceeds Actual Speech, Even Attempts At Totalization (48). Laws Pre-Exist Actual Cases. [So We Were Getting Close To A Role For Social Life, But Then It Gets Metaphysical Again]. As LS Put It, The Universe Signified Long Before Human Beings Knew What It Was Signifying. By Contrast, The Domination Of Nature Proceeds Partially And Progressively, Step By Step Unlike Social Life Where All Its Goals And Possibilities Given At Once. We're Back With Two Series, This Time Conceived As Rhythms, Social And Natural. Both Technocrats And Dictators Attempt A False Synthesis Of These Two Rhythms. Levi Strauss Referred To 'The Floating Signifier' As A Creative Force And Deleuze Wants To Say It's 'The Promise Of All Revolutions' (49). There Are Also 'Floated Signifieds', Which Seem To Be Possibilities Which Have Not Yet Been Realized. These Can Fill The Gap Between Signifier And Signified [And Are Found In Common Sense Expressions Like 'Gadgets' Or 'Whatnot' – Maybe Connected To The Idea Of A Bricoleur?]. It Implies A Symbolic Content, But Does Not Attempt To Fill It With Specifics. Together, These Possibilities Constitute A Structure, Two Heterogeneous Series, One Signifying, One Signified, Interdependent, And Including Particular Events, Singularities, Emitted By A Differentiator. The Singularities Belong To Neither Series Exclusively And Thus Have No Coherent Identity –Each Is An Excess In One Series And A Lack In The Other. The Singularities Can React Back On The Series, So Structures And Events Are Interdependent [So Structures Need A Dynamic Element, And, D Argues, An Excess, An Empty Square Instead Of Total Systematic Closure. Addresses The Old Issue Of The Static Nature Of Structuralism]. The Signifying Series Contains A Series Of Ideal Events, An Internal History. Differentiators Articulate Series And This Produces A 'Tangled Tale' Overall (51). **Sense Can Be Found In Either Series. It Is Not Just Signification But The Relation That Produces Signifier And Signified**, [The Operation Of The Whole Structure In This Expanded Sense]. Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules.All the information describing the 3-D scene is encoded into the pattern of light and dark areas on the two-dimensional piece of film, ready to be regenerated. The holographic principle contends that an analogue of this visual magic applies to the full physical description of any system occupying a 3-D region It proposes that another physical theory defined only on the 2-D boundary of the region completely describes the 3-D physics. If a 3-D system can be fully described by a physical theory operating solely on its 2-D boundary, one would expect the information content of the system not to exceed that of the description on the boundary. Can we apply the holographic principle to the universe at large? The real universe is a

4-D system It has volume and extends in time. If the physics of our universe is holographic, there would be an alternative set of physical laws, operating on a 3-D boundary of spacetime somewhere, which would be equivalent to our known 4-D physics. We do not yet know of any such 3-D theory that works in that way. Indeed, what surface should we use as the boundary of the universe? One step toward realizing these ideas is to study models that are simpler than our real universe. A class of concrete examples of the holographic principle at work involves so-called anti-de Sitter spacetimes **.** The original de Sitter spacetime is a model universe first obtained by Dutch astronomer Willem de Sitter in 1917 as a solution of Einstein's equations, including the repulsive force known as the cosmological constant. De Sitter's spacetime is empty, expands at an accelerating rate and is very highly symmetrical. In 1997 astronomers studying distant supernova explosions concluded that our universe now expands in an accelerated fashion and will probably become increasingly like a de Sitter spacetime in the future. Now, if the repulsion in Einstein's equations is changed to attraction, de Sitter's solution turns into the anti-de Sitter spacetime, which has equally as much symmetry. More important for the holographic concept, it possesses a boundary, which is located "at infinity" and is a lot like our everyday spacetime. Using anti-de Sitter spacetime, theorists have devised a concrete example of the holographic principle at work: a universe described by superstring theory functioning in an anti-de Sitter spacetime is completely equivalent to a quantum field theory operating on the boundary of that spacetime Thus, the full majesty of superstring theory in an anti-de Sitter universe is painted on the boundary of the universe. Juan Maldacena, then at Harvard University, first conjectured such a relation in 1997 for the 5-D anti-de Sitter case, and it was later confirmed for many situations by Edward Witten of the Institute for Advanced Study in Princeton, N.J., and Steven S. Gubser, Igor R. Klebanov and Alexander M. Polyakov of Princeton University. Examples of this holographic correspondence are now known for spacetimes with a variety of dimensions. This result means that two ostensibly very different theories--not even acting in spaces of the same dimension--are equivalent. Creatures living in one of these universes would be incapable of determining if they inhabited a 5-D universe described by string theory or a 4-D one described by a quantum field theory of point particles. (Of course, the structures of their brains might give them an overwhelming "commonsense" prejudice in favor of one description or another **Holographic Space-Time (Crystal Links, Tom Hanks and Wikipedia Francisco Di Biase** propose a quantum-informational holographic model of brain-consciousness-universe interactions based in the holonomic neural networks of Karl Pribram, in the holographic quantum theory developed by David Bohm, and in the non-locality property of the quantum field described by Hiroomi Umezawa. He considers this model an extension of the interactive dualism of Sir John Eccles, of an interconnection between brain and spirit by means of quantum microsites named dendrons and psychons. **Francisco Di Biase** proposes a dynamic concept of consciousness seen as a holoinformational flux interconnecting the holonomic informational quantum brain dynamics, with the quantum informational holographic nature of the universe. This self-organizing flux is generated by the holographic mode of treatment of neuronal information and can be optimized through practices of deep meditation, prayer, and others states of higher consciousness that underlie the coherence of cerebral waves. In brain mapping studies performed during the occurrence of these harmonic states we can see the spectral array of brain waves highly synchronized and perfectly ordered like a unique harmonic wave, as if all frequencies of all neurons from all cerebral centers played the same symphony. This highly coherent brain state generates the non-local holographic informational cortical field of consciousness that interconnect the human brain and the holographic cosmos. The comprehension of this holonomic quantum informational nature of brain-consciousness-universe interconnectedness allows us to solve the old mind-matter Cartesian hard problem, unifying science, philosophy, and spiritual traditions in a more transdisciplinary, holistic, integrated paradigm. In this new arrangement cosmovision, consciousness and transpersonal phenomena becomes part of Science and of the very holoinformational nature of the Holographic Conscious Multiverse. **DOI: 10.14704/nq.2009.7.4.259 Quantum-Holographic Informational Consciousness Francisco Di Biase NeuroQuantology** Holography suggests a considerable reduction of degrees of freedom in theories with gravity. However it seems to be difficult to understand how holography could be realized in a closed re-contracting universe. In this Letter we claim that a scenario which achieves that goal will eliminate all spatial degrees of freedom. This would require a different concept of quantum mechanics and would imply an intriguing increase of power for the natural laws. **Physics Letters B Volume 451, Issues 1–2, 1 April 1999, Pages 19–26 Richard Dawid doi:10.1016/S0370-2693(99)00205-1Vijay**

Balasubramanian, Per Kraus, Albion Lawrence, and Sandip P. Trivedi describe probes of anti–de Sitter spacetimes in terms of conformal field theories on the AdS boundary. Basic tool is a formula that relates bulk and boundary states—classical bulk field configurations are dual to expectation values of operators on the boundary. At the quantum level they relate the operator expansions of bulk and boundary fields. Using our methods, we discuss the CFT description of local bulk probes including normalizable wave packets, fundamental and D-strings, and D-instantons. Radial motions of probes in the bulk spacetime are related to motions in scale on the boundary, demonstrating a scale-radius duality. They also discuss the implications of these results for the holographic description of black hole horizons in the boundary field theory.DOI: http://dx.doi.org/10.1103/PhysRevD.59.104021 **Holographic probes of anti–de Sitter spacetimes Phys. Rev. D 59, 104021 – Published 26 April 1999 Vijay Balasubramanian, Per Kraus, Albion Lawrence, and Sandip P. Trivedi** The existence of a fundamental scale, a lower bound to any output of a position measurement, seems to be a model-independent feature of quantum gravity. In fact, different approaches to this theory lead to this result. The key ingredients for the appearance of this minimum length are quantum mechanics, special relativity and general relativity. As a consequence, classical notions such as causality or distance between events cannot be expected to be applicable at this scale. They must be replaced by some other, yet unknown, structure. **LUIS J. GARAY, Int. J. Mod. Phys A, 10, 145 (1995) DOI: 10.1142/S0217751X95000085 QUANTUM GRAVITY AND MINIMUM LENGTH** A digital computer is generally believed to be an efficient universal computing device; that is, it is believed able to simulate any physical computing device with an increase in computation time by at most a polynomial factor. This may not be true when quantum mechanics is taken into consideration. This paper considers factoring integers and finding discrete logarithms, two problems which are generally thought to be hard on a classical computer and which have been used as the basis of several proposed cryptosystems. Efficient randomized algorithms are given for these two problems on a hypothetical quantum computer. These algorithms take a number of steps polynomial in the input size, e.g., the number of digits of the integer to be factored. Copyright © 1997 Society for Industrial and Applied Mathematics **SIAM J. Comput., 26(5), 1484–1509. (26 pages) Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a Quantum Computer ISSN (print): 0097-5397 ISSN (online): 1095-7111 Publisher: Society for Industrial and Applied Mathematics Peter W. Shor DOI: 10.1137/S0097539795293172** The classical dynamics of M-dimensional extended objects arising from stationary points of the world volume swept out in space time is discussed from various points of view. A introduction to the Hamiltonian mechanics of bosonic compact M (em) branes is given, emphasing the diversity of the different formulations and gauge choices. For moving hypersurfaces, a graph description—including its nonlinear realization of Lorentz invariance—and hydrodynamic formulations (in light-cone coordinates as well as when choosing the time coordinate of a Lorentz observer as the dependent variable) are presented. A matrix regularization for M = 2 (existing for all topologies) is explained in detail for the 2-sphere, as well as multilinear formulations for M > 2. The recently found dynamical symmetry that exists for all M and related reconstruction algebras are covered, just as some explicit solutions of the level-set equations. **Jens Hoppe 2013 J. Phys A: Math. Theor. 46 023001 doi:10.1088/1751-8113/46/2/023001 Relativistic membranes N Bodendorfer et al** rederive the results of our companion paper, for matching space–time and internal signature, by applying in detail the Dirac algorithm to the Palatini action. While the constraint set of the Palatini action contains second class constraints, by an appeal to the method of gauge unfixing, we map the second class system to an equivalent first class system which turns out to be identical to the first class constraint system obtained via the extension of the ADM phase space performed in our companion paper. Central to analysis is again the appropriate treatment of the simplicity constraint. Remarkably, the simplicity constraint invariant extension of the Hamiltonian constraint, that is a necessary step in the gauge unfixing procedure, involves a correction term which is precisely the one found in the companion paper and which makes sure that the Hamiltonian constraint derived from the Palatini Lagrangian coincides with the ADM Hamiltonian constraint when Gauß and simplicity constraints are satisfied. **N Bodendorfer et al** therefore have rederived new connection formulation of general relativity from an independent starting point, thus confirming the consistency of this framework. **N Bodendorfer et al 2013 Class. Quantum Grav 30 045002 doi:10.1088/0264-9381/30/4/045002 new variables for classical and quantum gravity in all dimensions: II. Lagrangian analysis** As K§emaraja points out, none of the practices taught in the Stanzas on Vibration belong to the Individual Means and so it does not, strictly speaking,

concern Spanda doctrine, if that is, we consider the Stanzas to be the basic text of the Spanda school. From K§emaraja's point of view, however, the third section of the Aphorisms of Siva (Sivasutra), which is both the last and most extensive, is largely an exposition of this category of practice. The Stanzas and Aphorisms have been traditionally linked together and so, even though we feel that they should be distinguished sofar as the Stanzas rather than the Aphorisms teach the Doctrine of vibration as such, we are nonetheless justified in referring to the Aphorisms as its major source. Our exposition of the Individual Means will therefore be largely based on K§emaraja's interpretation of the third section of Aphorisms and we will present it, as he does, as an exposition of a possible mystical journey of individualised (anava) consciousness to realisation. We follow K§emaraja because he understood the practise taught in the Aphorisms in these terms, thereby not only illustrating for us how it fits into this scheme but also how he understood the basic categories of practice and their relationship to one another. According to K§emaraja, the first Aphorism of each section of the Sivasutra characterises the condition and nature of the Self at the corresponding three levels of practice. In other words, they indicate the yogi's basic state at each level in terms of his self-identification. This identification corresponds to his existential condition as a degree of self- realisation in the process leading to the authentic self-awareness of the liberated. The very first Aphorism starts directly with this, the highest state, by declaring that the Self is pure, dynamic and universal consciousness (caitanya).This is true for the yogi who has awakened to his authentic nature at the Divine (sambhava) level of being. At the Individual (anava) level, however, the situation has changed. In this sphere of consciousness the intermediate processes of discernment, analysis and classification of perceptions, which bridge the gap in the flow of awareness from the universal subject to a specific object of knowledge, appear to take over the status of the perceiving subjectivity which underlies them. The universal Self recedes into the background as a pure, undefinable awareness, and the individual ego, consisting of the perceptions, thoughts and emotions generated by the contact between the universal perceiver and the perceived, emerges in the juncture between them. Thus at this level, as the Aphorisms say, the Self is the mind. This is the Self which moves (atati) from one state of being to another, from one body to the next carrying with it subtle traces left behind by its sensory and mental activity. Together these are said to constitute, and be caused by, the subtle body technically called the 'City of Eight' (puryasfaka) with which consciousness is identified and due to which it is subject to the constant alterations of pleasure, pain and inertia. The Stanzas teach: [The soul] is bound by the City of Eight (puryaffaka) that resides in the mind, intellect and ego and consists of the arising of the [five] subtle elements [of sensory perception]. He helplessly suffers worldly pleasure and pain (bhoga) which consists of the arising of mental representations born of that [City of Eight] and so its existence subjects him to transmigration.**Whereas consciousness itself is the subject who practises the Divine Means (Sambhavopayd), the subject who practises the Individual Means is the mind. Unlike the Empowered Means, however, the mind is not directed inwards onto itself. At the Empowered level, enlivened by the direct intuition (pratibha) consciousness has of its own nature, mind ceases to function merely in the paradigmatic, formative manner which gives rise to mental representations, but operates instead as the subtle introverted activity of reflective awareness (vimarsa), the power of consciousness (sakti).** This activity, as we have seen, is the essence of Mantra which, independent of the senses, is no longer restricted in any way. At the Individual lew, however, the creative powers of consciousness reflected through the eAtroverted mind are greatly attenuated. All that remains is the power to form thought-constructs and make determined resolutions (sankalpa) which go on to issue through the body into outer action to make the private creations of the mind apparent to others. The Individual Means, therefore, deals with the objectively perceived contents of consciousness and hence with the individual subject as a composite aggregate of objective elements, ranging from the subtle life force (prdrta) to the physical body and its outer environment. The practices belonging to this Means are thus of two types. One is concerned with the individual subject who resides in, and as, the psycho-physical organism; the other with external reality. What this implies essentially is that practice at this level is not concerned as much with the will or cognitive consciousness ***(refer hiedigger who proposes the same philosophy: italics mine***) as are the other two Means, but with the power of action applied, in the context of the practice taught in the Aphorisms, to the spiritual activity of Yoga. According to Ksemaraja, the Individual Means culminates in the Empowered state and hence leads to the levels of practice beyond it. 251 This is possible because, despite their differences, there is an essential similarity between them. The aim of both the Individual and Empowered Means is to purify the discursive representa- tions of differentiated

perceptions (vikalpasamskara) 252 and so lead the yogi to the expanded (yikasita) consciousness of the Divine (sambhava) state. The other levels of practice therefore both sustain and complement it. The activity of individual consciousness can be fully perfected only when it operates through the flow of the conative and cognitive powers which together constitute the pure activity of universal consciousness beyond all means (anupaya). In fact, according to Ksemaraja, all three soteriological types function together in various ways, their corresponding states representing dimensions of the same experience. For example, the upsurge of consciousness (udyama) which is the supreme, **illuminating intuition (parapralibha)** of the Divine state (sambhavavastha) is concomitant with the gathering together of all the powers of consciousness in the Empowered state. The Divine Means, in other words, leads to the experience of Power (sakti) which in its turn, when fully affirmed, marks the attainment of a permanent contemplative consciousness (turiyatita) at the Divine level which persists unaltered in every state of consciousness. Consequently, Ksemaraja concludes his exposition of the first section of the Aphorisms which exemplifies, according to him, the Divine Means, by saying: Thus we have explained the first expansion which starts with [the Aphorism] 'the Self is pure dynamic consciousness' ***(note this happens in evolutionary pocess and in normal human being it remains as witness consciousness italics mine)*** and expounds the nature of the realisation (prathana) attained through the Divine Means. It is the intuitive insight (samapatti) of Bhairava's nature which is, as we have said, the upsurge of consciousness that quells all bondage, namely, the ignorance of that freedom which makes it manifest. Transforming all things into the nectar of one's own innate bliss, it bestows every yogic accomplishment (siddhi) including mystic absorption in the vitality of Mantra, the highest of them all. Accordingly, we have, in the course of this exposition, explained the nature of Sakti in order to show that the Divine nature (Sambhavarupa) possesses [every] power. Another way in which the Means are related to one another is illustrated by the recurrence of the same Aphorism in different sections of the Sivasutra which indicates, according to Ksemaraja, that the same practice belongs to more than one Means. Both times this happens, the Aphorism appears first in the section dealing with the Divine Means and then recurs in that concerned with the Individual Means. In one case, Ksemaraja tells us this is because practice at the Divine level requires no effort whereas at the Individual level, the yogi must exert himself to achieve the same state that at the Divine level dawns spontaneously. At the Empowered level also, as the Sivasutra says, 'effort achieves the goal'. Here, however, because as the Empowered Means is, according to Ksemaraja, predominantly concerned with the contemplation (anusamdhi) of the vitality of Mantra, the effort exerted is that required to bring the practice of Mantra to fulfillment. It is, as Ksemaraja says, 'the spontaneous effort exerted to grasp the initial expansion of intention to apply oneself to the contemplation [of Mantra]. It is this exertion which wins the favour of the gods of Mantra and identifies the adept with them.' The second case of the same practice being taught in different sections of the Aphorisms concerns the realisation of the Fourth State of contemplative consciousness (turiya) in the other three states of waking, dreaming and deep sleep. At the Divine level this takes place by **'violently digesting' (hafhapaka)** the three states in the Fourth. At the Individual level the Fourth state is first experienced at the junctures between the other three states and then induced gradually to spread out from these Centres to pervade the other states like oil extending slowly through a piece of cloth. The difference in this case between the levels of practice is not only that at the Divine level it reaches fulfillment spontaneously, but it is also sudden and complete, leading directly to the liberated state of consciousness Beyond the Fourth (turiyatita) At the Individual level, however, practice is gradual and even when the yogi manages to rise to states of contemplation, he must take care not to fall to lower levels of consciousness. Indeed, until the yogi attains the sudden and direct realisation of perfect enlightenment, whatever be his state of consciousness or level of practice, he is bound to rise and fall because his contemplative state is necessarily transitory (kadacitka) however long it may last. The yogi is more prone to these ups and downs the lower his basic state of consciousness. Consequently, the last section of the Sivasutra repeatedly instructs the yogi not only how to rise to higher levels of consciousness and maintain them, but also in what way he is liable to fall from them and how to regain them. Ksemaraja stresses that the rise from one level of consciousness to another is marked by the transition from a lower Means to a higher. Conversely, a fall from the higher level to the lower entails practice of a lower Means. The measure of the yogi's level of consciousness and that which sustains him in it allowing him to progress further, is his attentiveness (avadhana) to the higher realities he experiences in the more elevated states. Thus the last Aphorism of the second section of the Sivasutra warns the yogi that if his pure awareness (suddhavidya) of his oneness with all

things slackens, he will fall from his awakened state to dream the dream of thought-constructs. From Ksemaraja's point of view this means that the negligent yogi must now resort to the Individual Means described in the next section to return to his former, higher Empowered practice in which he experiences this oneness. Ksemaraja expounds practice at the Individual level, as he sees it in the Aphorisms, as extending from one Means to the next. For example, practice at the Individual level diverts the flow of the vital breath (prana) from its more usual course and induces it to enter the Central Channel (susumna) along which it rises as a pure conscious energy (technically called 'kuritfalini'). This leads the yogi to the Empowered state in which he enjoys the pure awareness of unity. If he manages to make it truly his own and it becomes his basic state of being, he enters the Divine plane (sambhavapadd) of identity with Siva. The Individual Means is both a point of departure to higher levels of practice and the level to which the yogi returns if he falls. Thus although the practices taught in the last section of the Aphorisms may belong to any one of the three Means, they are collectively treated as part of the Individual Means because they start from it and because it is the yogi's abiding standby if he falls. Let us turn now to the basic practice at the Individual level, as Ksemaraja understands it. This is essentially Yoga. According to the Classical Yoga system taught by Patanjali in the Aphorisms of Yoga (Yogasutra), Yoga is defined as 'the quelling of the fluctuations of the mind' (cittavrttinirodha). The aim is to sever the spiritual essence of the Person (purusa) from the defiling materiality of Nature (prakfti), even though the word 'Yoga' means to 'unite' or 'yoke together.' Here, however, Yoga combines both union and cessation. It is the act (kriya) of removing the latent traces (yasana) of differentiated perceptions (yikalpa) born of the impurities (mala) which contract consciousness ***(like pravrutti and Nivrutti; italcis mine***). This is achieved by uniting all the elements of experience (tattva) together in the wholeness of the activity of consciousness. As Jayaratha explains: The [wise] consider Yoga to be the union of one thing with another/ thus, in accord with this dictum, Yoga is the [act] of uniting [all] the metaphysical principles together within consciousness . . . , Ksemaraja seeks initially to establish the best form of Yoga for the yogi to practice at the Individual level. His sources are two Tantras he knew well and considered to be amongst the most important, namely, The Tantra of (Siva's Third) Eye (Netratantra) and The Tantra of the Liberated Bhairava (Svacchandabhairavatantra). The basic model is that of the Eight-limbed Yoga (asfariga) taught by Patanjali which consists of: 1) The five restraints (yama\ namely, abstention from violence (ahirrisa), falsehood (satya), dishonesty (asteya), sexual intercourse (brahmacarya) and desire for more than the essential (aparigrahd) 2) The five disciplines (niyama), namely, cleanliness (sauca), contentment (santo$a), austerity (tapas\ study (svadhyaya) and reverence for God (Uvarapranidhand) 3) Posturing of the body (asana) in a manner conducive to the practice of meditation and physical health. 4) Regulation of the breath (pranayama) 5) Withdrawal of the senses from their objects (pratyahara) 6) Focusing of attention (dharana) 7) Meditation (dhyana), that is, steady, uninterrupted concentration. 8) Contemplation (samadhi). Ksemaraja **rejects** Patanjali's system because he believes it to be a form of Yoga that can, at best, lead only to limited yogic attainments (mitasiddhi). In the Netratantra, however, Siva teaches a different, higher form of the Eight Limbs of Yoga which lead to perfect penetration into the supreme, transcendental principle 280 of which the Netratantra says: Speech cannot express, nor the eye see, the ears hear, or the nose smell, the tongue taste, the skin touch or the mind conceive that which is eternal. Free of all colour and flavour, endowed with all colours and flavours, it is beyond the senses and cannot be objectively perceived. O goddess, those yogis who attain it become immortal gods! By great practice and supreme dispassion . . . one attains Siva, the supreme imperishable, eternal and unchanging reality. A necessary preliminary of all Tantric Yoga is a process technically called the 'purification of the elements' (bhutasuddhi), through which the body is homologized with the macrocosm and so made a fit vessel for the pure, conscious presence of the Deity within it. Ksemaraja equates this with the meditation (dhyana) which, according to the Mdlinivijayatantra, characterises the Individual Means. In order to practice this meditation the yogi must visualise the dissolving away of all the forces in the body. There are two ways in which this can be done. The first is called 'the contemplation of dissolution' (layabhdvana). Through it the progressive differentiation of consciousness from its causal, pre-cosmic form to its phenomenal manifestation is reversed As the Vijnanabhairava teaches: "One should meditate on the All in the form of the Paths of the world-orders etc. considered in their gross, subtle and supreme forms until, at the end, the mind dissolves away." Mediated by consciousness, the macrocosm rests in the microcosm which is emitted along with it successively in the emptiness of the individual subject, vital breaths, mind, psychic nerves (ndtfi), senses and external body. The yogi

reproduces this process by visualising the totality of reality including the world-systems, metaphysical principles and cosmic forces along with the Mantras, letters and syllables which represent them, as arising successively throughout the psycho-physical body so as to constitute it. Deployed in this way they form the Cosmic Path along which the yogi ascends, absorbing as he does so, the lower elements into the higher, thus strengthening and extending his unifying awareness (anusamdhdnd) of the configuration of the Path. Thus, moving from the gross elements constituting the outer physical body, to pure sensations (tanmdtra), then to the senses and mind back to their primordial source, the yogi rises from the **embodied subjectivity** of the waking state to the Fourth State (turiya) of contemplation where he is one with the pervasive intent which initiates the creative vision of consciousness. Abhinava writes: Once [the yogi] has known [this] Path in its completeness, he must then dissolve it into the deities who sustain it and these successively into the body, breath, mind [and emptiness] as before, and all these into his own consciousness. Once this is full and an object of constant worship, it destroys, like the fire at the end of time, the ocean of transmigration.Thus, the second method Ksemaraja teaches to dissolve away the diversity of sensory, mental and physical energies into the unity of consciousness is a meditation on the Fire of Consciousness (dahacintd) which the yogi visualises as burning away all division. At the Divine level (sambhavopaya) the yogi witnesses the sudden and violent withdrawal of all objectivity into the pure ego (aharri), like the pouring of fuel into a raging fire. He does not need to visualise this process but merely attend to it with a passive, receptive attitude. At the Individual level the yogi must exert his imagination to induce this process and so rise to the Divine level through the Empowered. The Vijnanabhairava teaches: Visualise the fortress [of your body] burning with the Fire of Time (kalagni) risen from the Abode of Time; then at the end peace manifests. The Fire of Time {kalagni) resides underneath the hell worlds at the bottom of the Cosmic Egg (Brahman^a). It issues from Ananta — a form of Siva who presides over the lower regions. He floats on a boat in the causal waters supporting the Egg, his mind all the while fixed on Bhairava. The flames of the Fire of Time rise up to the hell-worlds heating them intensely and radiate its energy throughout the universe. At the end of each period of creation the flames rise higher and destroy the old cosmic order to make room for a new one. At the microcosmic level the yogi reproduces this process by mentally placing the letters of the alphabet, in the prescribed order, on the limbs of his body starting from the left toe to the top of the head. As his attention progresses upwards, he visualises the Fire of Time moving with it in such a way that his bodily consciousness, together with the universe of differentiated perceptions, is gradually burnt away leaving in its place the white ashes of the undivided light of consciousness. Ksemaraja considers this meditation (dhyana) to be a limb of a programme of yogic practice at the Individual level 291 of which the remaining limbs are as follows: Posture (A sana). The yogi fixes his attention on the centre between the inhaled and exhaled breath, absorbing in this way the flux of his awareness into the unfolding power of knowledge which rises initially as the upward flowing breath (udanaprana) in the Central Channel (su§umna) between the other two breaths. The Prank aspect of this flow disappears as it moves upward and the yogi experiences the spontaneous rise of the omniscience of consciousness within himself. The mind reverts back to its original, pervasive conscious nature and understands the infinite fact of Siva's omnipresence. This is the firm seat (asana) upon which the yogi sits to practice. Regulation of the Breath (Pranayama) .To regulate the movement of the breath, the yogi must first cleanse the right and left channels of the ascending and descending breath by blocking the left nostril while exhaling and the right while inhaling a few times. This ensures that the movement of the breath is firm and evenly distributed. Next, without attempting to control it in any way, he attends to the flow of his breathing. As the mind becomes steadier and in closer harmony with the rhythm of its movement, the duration of each inhalation and exhalation gradually alters until they become equal. At this stage they unite and merge in the upward flowing current of vitality in the Central Channel (Susumna). This is when true Pranayama begins. The yogi's mind pure and tranquil, he returns, as it were, to a prenatal state and the external breathing cycle is internalized, so that it no longer moves through the lungs but passes directly to the universal source of vitality. The yogi, now at the Empowered level of practice, experiences this movement as travelling from the Heart centre upwards to a point distant twelve fingers above the head where it merges in the void of consciousness. Free of its outer gross form, the breath moves freely through the Central Channel and soon transcends even this subtle movement to become one with the supreme vibration of consciousness. In this way, the yogi's breathing becomes one with the spontaneous rise and fall of energy from the bosom of the absolute. Abhinava quotes the Tantra of the Line of Heroes (Viravalitantra) as saying: When, by

constantly merging the mind in Siva, Who is the pure conscious nature, the Sun and Moon [of the two breaths] have dissolved away and the Sun of Life, which is one's own consciousness, has reached the twelve-finger space, this is termed liberation. Breath control [at this stage] serves no useful purpose. Breath control which merely inflicts pain on the body is not to be practised. He who knows this secret is both themselves liberated and liberates others. **Focusing of Attention (Dhara nA):** Attention is fixed on the psychic centres in the body corresponding to the five gross elements In this way the vital breath is successively directed to these centres from the Heart of consciousness to refresh and stimulate their activity. First it moves to the Earth centre in the throat which regulates the firmness of the bones and flesh of the body; then to the Water centre in the glottis responsible for the balance of the bodily fluids. After this it travels to the navel which is the Fire centre dealing with digestion and anabolisms and catabolesis in general. It then moves to the Wind centre in the toe of the left foot which governs the movement of gases to and from the cells via the circulatory system. When the yogi has thus achieved control over these forces, the breath rises from the Heart to the top of the head and he becomes master of the Ether element and so attains every yogic power. **Meditation (Dhyana)** The highest form of meditation stills the flux of the qualities (guna) and induces the mind into a state of contemplative absorption. The object of this meditation is the supreme and pervasive divinity of the pure subject whose true nature is known to none but himself alone (svasatpvedya). The yogi attains him by merging into the constant flow of awareness that streams into the Light which illumines his own nature. Contemplation (Samadhi) .The yogi rises to the level of contemplation when the awareness he has of himself and the things around him become one and he realises his own identity with Siva, ***(or Rama the omniscient: cosmic general ledger)*** the sole reality. The aim of this Yoga in all its phases is to achieve the Fourth State of consciousness (turiya) beyond the three states of waking, dreaming and deep sleep and to then ultimately reach the liberated state Beyond the Fourth (turiyatita). These five states correspond to: (a) Siva's activity (vy apara), that is, His power of action; (b) Siva's Lordship (adhipatya), which is His power of knowledge; (c) the absence of these two, which corresponds to Siva's power of will; (d) His exertion (prerakatva), which contains all the cycles of creation and destruction and, (e) the rest Siva enjoys in His own nature, which is His power of consciousness. The first three states, when divorced from the last two, belong to the sphere of transmigratory existence. The Fourth and Beyond the Fourth on the other hand are higher, supramundane (alaukika) states of consciousness in which the yogi enjoys bliss and repose (visranti) in his own nature by penetrating (samavesa) into the universal consciousness of the Self, through which he ultimately becomes liberated (jivanmukta). Beyond the Fourth is the state of awareness ParamaSiva Himself enjoys when duality has entirely disappeared and everything is realised to be one with consciousness. The Fourth is the state of awareness of the yogi who, catching hold of the pure subjectivity (upalabdhrta) flowing through the lower three states, is still actively eliminating his sense of duality. While the former is the supreme subject as T consciousness (aham), the latter is the pure awareness (prama) or I-nesses (ahanta) of the subject which encompasses the lower states, giving them life and uniting them together As such, the Fourth State is the reflective awareness of one's own nature shining in all three states at one with them. The fact that we recall that we slept well is proof that this state of consciousness persists even in deep sleep. Indeed, if the flow of Turiya could somehow be brought to a halt, all the other states of consciousness would come to an end in the absence of the pure subjectivity which makes them, and their contents, manifest. The states of waking, dreaming and deep sleep correspond to the form of awareness consciousness assumes when it predominantly manifest as the object, means of knowledge and individual subject, respectively. Turiya is the pure awareness (prama) that both transcends them and merges them all into itself. 300 As such, it appears as the triad of deed, means and agent in the pure act (vy apara) of consciousness unsullied by any outer reality. Abhinava explains: {Turiya) transcends the three aspects of *form\ 'sight' and T consisting as it does of the pure act of 'seeing'; therefore any means [by which this state could be realised] has merely a [provisional] instrumental value. It is, in other words, pure subjectivity of the nature of absolute freedom, independent of all external means. This is the state of consciousness called Turiya, luminous with its own light. Turiya is thus not just a psychological state but the supreme creative power (para sakti) of consciousness, the Goddess (samviddevi) who generates and withdraws the entire universe of subject, object and means of knowledge. In the Heart of Recognition Ksemaraja explains: Whenever the extroverted [conscious] nature rests within itself, external objectivity is withdrawn [and consciousness] is established in the inner abode of peace which threads through the flux of awareness in every [externally] emanated [state]. Thus Turiya, the Goddess of Consciousness, is the union of

creation, persistence and destruction. ***(Read as cosmic consciousness; Kshemaraja does not distinguish like we have done, individual, collective and cosmic and of three states)*** She emanates every individual [cycle] of creation and withdraws it. Eternally full [of all things] and [yet] void [of diversity] She is both and yet neither, shining radiantly as non-successive [consciousness] alone. The yogi is fully absorbed in this state of consciousness and takes possession of its power when he is able to rise from contemplation (samadhi) carrying with him the abiding awareness of Turiya throughout his waking, dreaming and deep sleep. When he achieves this constantly, he continues to experience these states individually, but they no longer obscure the insight (pratibha) he has acquired because he realises that they are all aspects of the bliss of Turiya. Thus, while the common man calls this state the 'Fourth' (turiya) because he cannot experience it directly and knows only that it is beyond the other three, the yogi calls it 'Beyond Form' (rupatita) because it transcends the detachment of the state of deep sleep which, devoid of objective content, is the naked form' of the individual subject tending towards the fullness of consciousness. Those who are on the path of knowledge (jnaniri) call it the 'Whole* (pracaya) because, in this state, they see the entire universe gathered together in one place. 'Supra-mental Awareness' (manonmana) is the name given to the experience of Turiya in the waking state. The yogi in this state moves and lives in the world of waking experience free of all disturbing thoughts while abiding in the transcendental silence beyond the activities of the mind. 'Infinite' is the name of the experience of Turiya while dreaming because, free of the limitations imposed upon the body by time and space, the yogi enjoys the unlimited expanse of the Self. When Turiya is experienced in deep sleep, the yogi's state is called 'All things' (sarvdrtha) because in it he discovers his freedom from limitations in this, the most contracted state of human consciousness. The yogi who manages to maintain Turiya- consciousness comes to experience the three states of waking, dreaming and deep sleep as the constant flow of the bliss of consciousness in which all traces of the relative distinction between these states and their contents is eradicated. Following the stream of Turiya to its highest level (para katfha), he reaches the state Beyond the Fourth (turiydtita), which is the universal consciousness (caitanya) of the Self. Here the yogi comes to rest within his own nature. Plunged in the vast, waveless ocean of the consciousness and bliss (ciddnanda) of the state Beyond the Fourth, the yogi becomes Siva, the Free One (svacchanda), and thus wanders freely, practising the Yoga of Freedom. K§emaraja equates the Fourth State with the pure (suddha), innate (sahaja) knowledge that one's own conscious nature is all things. It is the Supra-mental State (unmana) in which Siva's pervasive presence is experienced once the Yoga practised at the Individual level attains fruition at the Empowered. What the yogi must do, once consciousness is elevated to grasp the Fourth State, is make it constant. He must forcefully lay hold of it within himself and not release his grip until it becomes permanent. Then he travels 'Beyond the Fourth' to enlighten- ment. Before this ultimate attainment the yogi inevitably falls. The forces operating within consciousness that limit and obscure it throw him down whenever they possibly can. The only way the yogi can defend himself against them is to maintain a constant attentive awareness of the Fourth State. He falls when he is distracted but when he attends carefully to his pure conscious nature, he realises that every aspect of his state of being, including the forces that lead him astray, are one with the pulsing flux of his own consciousness and so cannot affect him. These powers, which are the energies of Matrka we have already discussed, are not the only obstacles the yogi must overcome. He must, for example, also resist the temptation to rest content with the miraculous yogic powers (siddhi) he acquires in the course of his spiritual develop- ment. Again to do this he must practice Yoga. Similarly, in order to pervade the Fourth State gradually through the other states in the manner proper to practice at the Individual level, the method is the same. He must practise the higher yoga of the Tantras which, turning his mind inwards and freeing it from discursive representations, allows him to penetrate into the Supreme Principle. Once the yogi has attained this contemplative state, his main problem is to make it permanent. In the introvered state the gross external movement of the breath is suspended and with it the activity of the intellect, mind, individualised consciousness, powers of the senses and the ego. 313 When the yogi rises out of this state, he is liable to fall again into the lower order of creation generated by Maya if he does not maintain his awareness of the higher reality he has experienced and allows his awakened, illumined insight to be obscured by the dream-like vision of thought-constructs. 314 Naturally, the yogi must rise out of the introverted condition of suspension. It is inherent in the very nature of reality that it should move out of itself. 315 Pure, universal consciousness initially transforms itself into the vital breath 316 charged with the impression iyasana) of the power of awareness attained through introversion. By attending to the pulse (spanda) of the breath as it moves out of the

absolute, the yogi can develop an intuitive sense of the inherent unity of all he will perceive in the mental and physical spheres created by the outpouring of consciousness. In this way he realises that his own nature is everywhere present in all he perceives and that all things thus reside within him. Blessed with this insight his consciousness remains free and unlimited even at the individual level where the breath, mind, senses and body are active. If the yogi fails to do this, he finds himself once again beset by the strictures of his embodied existence and must, as before, try to pervade all his other states of consciousness with the aesthetic delight (rasa) and wonder (camatkara) of the Fourth State he experienced in contemplation. Again this means that he must strengthen his pure, empowered awareness that his universal nature manifests as all things. In this way he discovers Siva's presence in every sphere of individualised consciousness ranging from the breath to outer objectivity. The yogi's mind then becomes tranquil and undistracted because wherever it may wander, the yogi perceives only Siva, his authentic nature. Consciousness is thus freed of all external referents and the yogi's subjectivity is purified of all identification with the body or anything else that belongs to the objective sphere. The yogi then becomes detached from the opposites of pleasure and pain and is transcendentally free (kevalin). The yogi is again, however, liable to fall if he allows himself to get entangled in the play of opposites. This fall is more serious than the others because, although he is caught by the confining restrictions of individual- ised consciousness as before, he is now also affected by karma. Fleeing from pain in the pursuit of pleasure he is bound to act (karma) to minimise one and maximise the (a ***maximaization problem is mentioned elsewhere about the svabhava of human beings to enhance pleasure reduce deprivation)*** other and so is thrown down to the lowest level of embodied subjectivity (sakala). In order to regain his lost state, he must ascend gradually, by Siva's grace, from one order of subjectivity to the next and so free himself progressively of the limitations of the lower levels to gain the greater freedom and expansion of the higher. As he progresses, the objective sphere also evolves from the grossest perceptions of physical objects outside the lowest order to subjectivity, through to the subtler inner, mental perceptions to finally reach the order of subjectivity that contains objectivity within itself and is free to externalise it at will. The degree to which this process develops depends, as before, on the yogi's awareness of the Fourth State. In consonance with the general principle that the remedy should suit the defect, the yogi is instructed to seek this higher state of consciousness in the wonder (camatkara) or delight (ananda) he feels in moments of intense physical pleasure. At first he experiences this subtle consciousness for an instant in the subjective sphere. If he manages to catch hold of it, it becomes more intense as the cognitive and objective spheres are also gradually pervaded and vitalised by it. Occasions for this practice are, for example, the sense of satisfaction one feels after a good meal or the aesthetic delight one experiences when listening to good music or the pleasure of sexual union with the Tantric consort or even solitary sexual excitation. In these moments of delight the yogi can penetrate momentarily into his own authentic Siva-nature (sambhavavesa) through the empowered contact (saktasparsa) m he makes with it in the freedom of the pure subjectivity of the Fourth State. If the yogi develops his awareness of this higher level of consciousness and maintains it, he eventually experiences it constantly. Clearly, what prevents the yogi from attending to his state of consciousness rather than the circumstances which induce it is the craving for pleasure (abhila§a) born of ignorance — the source of every impurity which clouds consciousness. Craving directs the yogi's attention towards outer, worldly things and so he is caught in the **net of thought- constructs**. To free himself of his worldly desires and reverse this binding extroversion, the yogi must eradicate its cause. To be freed of all the ups and downs of the path and no longer be tormented by the possibility of a fall, the yogi must see reality perfectly and completely. This insight is itself liberation and the moment it dawns the yogi is instantly freed. This sudden realisation is the goal of Tantric Yoga. Accordingly the Tantra declares: "He, who perceives reality directly, even for the brief moment it takes to blink, is liberated that very instant and never reborn again." Although the yogi's body and mind continue to function as before, they are like mere outer coverings which contain, but do not obscure, the mighty, universal consciousness which operates through them. The yogi's body is the universe, the senses the energies that vitalise it, his mind Mantra, the rhythm of his breath the pulse of time and his inner nature pure, dynamic consciousness. Raised above all practice, and hence all possibility of falling to lower levels, the yogi realises that he has always been free and that his journey through the dark land of Maya was nothing but a dream, a construct of his own imagination. **(Models are given in one of the papers of the series due to spatial constraints) Relationship betwixt string theory (note the spade remarks above) and holographic principle has been enucleated and expatiated in**

literature. Some exemplar work is sententiously mentioned in the following:The ratio of shear viscosity to volume density of entropy can be used to characterize how close a given fluid is to being perfect. Using string theory methods, we show that this ratio is equal to a universal value of $\hbar/4\pi kB$ for a large class of strongly interacting quantum field theories whose dual description involves black holes in anti–de Sitter space. **P K. Kovtun, D. T. Son, and A. O. Starinets** provide evidence that this value may serve as a lower bound for a wide class of systems, thus suggesting that black hole horizons are dual to the most ideal fluid **Viscosity in Strongly Interacting Quantum Field Theories from Black Hole Physics Phys. Rev. Lett. 94, 111601 – Published 22 March 2005 P K. Kovtun, D. T. Son, and A. O. Starinets** According to't Hooft the combination of quantum mechanics and gravity requires the three-dimensional world to be an image of data that can be stored on a two-dimensional projection much like a holographic image. The two-dimensional description only requires one discrete degree of freedom per Planck area and yet it is rich enough to describe all three-dimensional phenomena. After outlining't Hooft's proposal we give a preliminary informal description of how it may be implemented. One finds a basic requirement that particles must grow in size as their momenta are increased far above the Planck scale. The consequences for high-energy particle collisions are described. The phenomenon of particle growth with momentum was previously discussed in the context of string theory and was related to information spreading near black hole horizons. The considerations of this paper indicate that the effect is much more rapid at all but the earliest times. In fact the rate of spreading is found to saturate the bound from causality. Perception is not reality. It needs some hidden variables to posit the reality. (see earlier models).We Make Sense Of Objects, Events Or State Of Affairs When We Pin Them Down And Describe Them, State Propositions About Them. We Can Then Develop These Propositions Into Logical Forms To Draw Conclusions, Make Predictions Or Whatever. The Trouble Is, Neither Object As We See Them, Nor Propositions As We Commonly Use Them, Are At All Simple. Objects As We Perceive Them, For Example, Clearly Have All Sorts Of Characteristics As Well That We Don't Perceive. They Have A History, And A Potential For The Future, For Example. The Worst Problem Is That Objects As We See Them Are Actually Mixtures Of Things, The Affects Of Complex Causes: They Are 'Overdetermined', To Use The Phrase Associated With Both Freud And Marx. Deleuze Actually Draws On Resources In Classical Philosophy To Argue This, Especially The Work Of The Stoics, Who Saw Objects And Events As A 'Mixture Of Bodies', This Mixture Emanating From Somewhere In The Depths, Below Conscious Sense Making. Incidentally, This Reliance On The Stoics Also Makes Deleuze A Bit Stoical, In The Sense That He Thinks That These Mixed Bodies Are Part Of Some Unified Whole (He Was Going To Call This Being), And The Role Of Destiny Is Associated With The Whole. It Follows That Ethically And Conceptually, All Human Beings Can Do This Discover The Ways In Which Being Works. There's No Point Complaining About It Either. This Is the Basis of Badiou's Critique Of Deleuze. It Is Also The Basis Of Deleuze's Anti-Humanism: We Don't Understand Reality By Trying To Develop A Conscious Synthesis, Tracking Analogies Based On Some Notion Of A Shared Essence, Or Seeing God As An Ultimate Synthesiser For That Matter. Events And States Of Affairs Are Also More Complex Than They Look, And Deleuze Illustrates This Discussion With The Extraordinary Notion Of 'Compossibility' In Leibniz. Basically, The Idea Is That Singularities Can Produce Events In All Sorts Of Divergent Ways, All Of Them Equally Possible, And This Potential Is Shrunk If We Just Study The Actual Course Of Events. Objects Are More Complex Than They Look, With All Sorts Of Bits Of Their Being Unavailable To Us, But Language Is Equally Complex. Here, Deleuze Tries To Demonstrate This By Looking At People Who Use All Kinds Of Linguistic Anomalies Such As Paradoxes, Or, In The Case Of Lewis Carroll Especially, Portmanteau Words Or Esoteric Words. Although These Constructions Offend Logic, And Are Strictly Speaking Nonsense, They Still Make Sense. This Also Points To Some Powers Of Language That Are Not Immediately Available To Inspection. More Conventionally, We Know That The Use Of Particular Words Can Have Different Functions In Any Language System—They Can Denote, They Can Manifest Inner Thoughts, And They Can Signify (In The French Sense, That Is Indicate The Existence Of A Structured Language System Which Enables Us To Communicate With Each Other. Deleuze Proposes To Modify Classical Saussurian Semiotics By Adding A Moveable Element, An 'Empty Space' Or Floating Signifier'). Again Wordplay Can Deliberately Confuse These Different Functions, As When I Use The Term 'It' To Denote A Specific Object, Only To Refer To A Whole Process As 'It', To Express Myself By Saying Something About 'It' And So On (Actual Easy Examples Are Thin On The Ground In Deleuze!). Then There Is One Of Those (Many) Philosophical

Diversions Into Various Types Of Explanation For The Extra Elements That Are Not Available To Actual Objects Or Speech Acts. Platonists Thought The Extra Bits Referred To Some Universal Form Or Idea, Other Philosophers Like Husserl Or Kant Suggest There Was Some Transcendental Realm Beyond The Immediate. Deleuze Offers Objections To Both For Those Approaches, And Suggests That The Real Is Actually Divided Into Virtual And Actual Levels (Delanda's Commentary Is Invaluable Here). The Objects And Words Of Our Immediate Perception Are Actualised Are Condensed Out From Much More Complex Objects And Events At The Virtual Level. In This Book, About The Only Candidate For These Complex Objects Is The Singularity, Which Actualizes Itself In Various Partial Ways. Deleuze Goes On To Develop This Idea By Saying That Singularities Themselves, And The Events And Objects They Actualize, Are Produced In A Random Or Arbitrary Fashion, The Chance Is At The Heart Of Actualisation. We Learn That The Virtual Also Has Its Own Sense Of Time, Which Is Not The Usual One Which Is Highly Limited By The Insistence On The Present Tense In Human Operations. It Is A Very Abstract Discussion, Which Finally Comes To An Example I Could Understand—The Emergence Of Language In Freud. As Infants Develop, Their Biological Urges Produce Certain Infantile Concepts Like Part Objects. These Are Classically Mixed, Deriving From Contradictory Drives In The Depths Of The Unconscious. The Sounds The Infant Makes Our Initially Just Bodily Emanations, But They Gradually Come To Take On The Form Of Linguistic Units, Like Phonemes. There Are Elements Of Adult Language Available Too, Of Course, But These Are Initially Unintelligible. Then A Process Described As The Phantasm Manages To Combine Various Infantile Concepts Together Into A'disjunctive Unity', A Mixture Of Heterogeneous Elements Which Deleuze Thinks Characterises Most Objects. A Kind Of Primitive Narrative Develops In The Phantasm, Partly Driven By Biological Drives, And Partly Driven By Emerging Linguistic Competencies (Such As The Oedipal Scene). The Key To This Is The Freudian Notion Of The Phallus, Which Both Refers To Biological Organs Like Penises, And To Linguistic And Cultural Functions To Do With Authority And Value. The Phallus Is The Ideal Ambiguous Object (Or Empty Object As Deleuze Insists On Calling It), Able To Zigzag Between Bodies And Language, And Thus Make Divergent Series 'Resonate'. The Phantasm Therefore Develops An Energy Of Its Own, Which Permits A Relative Disengagement From Sexual Energy (Sublimation) And, In The Final Stages, The Proper Development Of Language In The Form Of Symbolisation. At Last We Have Language And Events Brought Together On The Surface, Which Is Of The Level Of Consciousness. If I Have Read The Appendices Correctly, This Phantasmic Form Is Then Generalised To Include All The Operations Of Making Sense As Describes Right At The Beginning. I Have Lots Of Reservations About This Whole Schema, In Fact, Ranging From The General Structure Of The Argument, Where Philosophical Issues Are Introduced As If They Were Necessary, Whereas Their Role Is Been Predetermined All Along (For Example, Deleuze Has Always Admired Stoics, And Here They Are As Offering The Best Account), Right Down To The Curious Insistence On The Phantasm As The Adult Form Of Making Sense—In My View, Dreams Are A Lot More Like A General Model Of Thinking, And They Include The More Sophisticated Linguistic Operations Of Metonymy (Condensation), And Metaphor (Displacement). Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules. Finally **Leonard Susskind** considers string theory as a possible realization of't Hooft's idea. The light front lattice string model of Klebanov and Susskind is reviewed and its similarities with the holographic theory are demonstrated. The agreement between the two requires unproven but plausible assumptions about the nonperturbative behavior of string theory. Very similar ideas to those in this paper have long been held by Charles Thorn. © 1995 American Institute of Physics **The world as a hologram Leonard Susskind J. Math Phys. 36, 6377 (1995); http://dx.doi.org/10.1063/1.531249 Ofer Aharonya, , Steven S. Gubserb, , Juan Maldacenab, c, , Hirosi Oogurid, e, , Yaron Ozf,** review the holographic correspondence between field theories and string/M theory, focusing on the relation between compactifications of string/M theory on Anti-de Sitter spaces and conformal field theories. They review the background for this correspondence and discuss its motivations and the evidence for its correctness. They describe the main results that have been derived from the correspondence in the regime that the field theory is approximated by classical or semiclassical gravity. We focus on the case of the supersymmetric gauge

theory in four dimensions, but discuss also field theories in other dimensions, conformal and non-conformal, with or without supersymmetry, and in particular the relation to QCD. We also discuss some implications for black hole physics. **Physics Reports Volume 323, Issues 3–4, January 2000, Pages 183–386 Large N field theories, string theory and gravity Ofer Aharonya, , Steven S. Gubserb, , Juan Maldacenab, c, , Hirosi Oogurid, e, , Yaron Ozf, , doi:10.1016/S0370-1573(99)00083-6 T. Banks, W. Fischler, S. H. Shenker, and L. Susskind** suggest and motivate a precise equivalence between uncompactified 11-dimensional M theory and the N=∞ limit of the supersymmetric matrix quantum mechanics describing D0 branes. The evidence for the conjecture consists of several correspondences between the two theories. As a consequence of supersymmetry the simple matrix model is rich enough to describe the properties of the entire Fock space of massless well separated particles of the supergravity theory. In one particular kinematic situation the leading large distance interaction of these particles is exactly described by supergravity. The model appears to be a nonperturbative realization of the holographic principle. The membrane states required by M theory are contained as excitations of the matrix model. The membrane world volume is a noncommutative geometry embedded in a noncommutative spacetime. DOI: http://dx.doi.org/10.1103/PhysRevD.55.5112 **M theory as a matrix model: A conjecture Phys. Rev. D 55, 5112 – Published 15 April 1997 T. Banks, W. Fischler, S. H. Shenker, and L. Susskind** These TASI lectures review the Holographic principle. The first lecture describes the puzzle of black hole information loss that led to the idea of Black Hole Complementarity and subsequently to the Holographic Principle itself. The second lecture discusses the holographic entropy bound in general space-times. The final two lectures are devoted to the ADS/CFT duality as a special case of the principle. The presentation is self contained and emphasizes the physical principles. Very little technical knowledge of string theory or supergravity is assumed. Report number: SU-ITP 99-14, KUL-TF-2000/03 Cite as: arXiv: hep-th/0002044 **TASI lectures on the Holographic Principle Daniela Bigatti, Leonard Susskind** The notion of a space-time uncertainty principle in string theory is clarified and further developed. The motivation and the derivation of the principle are first reviewed in a reasonably self-contained way. It is then shown that the nonperturbative (Borel summed) high-energy and high-momentum transfer behavior of string scattering is consistent with the space-time uncertainty principle. It is also shown that, as a consequence of this principle, string theories in 10 dimensions generically exhibit a characteristic length scale which is equal to the well-known 11 dimensional Planck length gs1/3 ℓs of M-theory as the scale at which stringy effects take over the effects of classical supergravity, even without involving D-branes directly. The implications of the space-time uncertainty relation in connection with D-branes and black holes are discussed and reinterpreted. Finally, we present a novel interpretation of the Schild-gauge action for strings from the viewpoint of noncommutative geometry. This conforms to the space-time uncertainty relation by manifestly exhibiting a noncommutativity of quantized string coordinates between, dominantly, space and time. We also discuss the consistency of the space-time uncertainty relation with S and T dualities. Copyright (c) 2000 Progress of Theoretical Physics **String Theory and the Space-Time Uncertainty Principle Tamiaki Yoneya Oxford Journals Science & Mathematics Progress of Theoretical Physics Volume 103, Issue 6Pp. 1081-112** In this Letter a recently proposed gravity dual of noncommutative Yang-Mills theory is derived from the relations between closed string moduli and open string moduli recently suggested by Seiberg and Witten. The only new input one needs is a simple form of the running string tension as a function of energy. This derivation provides convincing evidence that string theory integrates with the holographical principle and demonstrates a direct link between noncommutative Yang-Mills theory and holography.DOI: http://dx.doi.org/10.1103/PhysRevLett.84.2084 **Holography and Noncommutative Yang-Mills Theory Phys. Rev. Lett. 84, 2084 – Published 6 March 2000 Miao Li and Yong-Shi Wu Petr Hořava** suggests that M theory could be nonperturbatively equivalent to a local quantum field theory. More precisely, we present a "renormalizable" gauge theory in eleven dimensions, and show that it exhibits various properties expected of quantum M theory, most notably the holographic principle of't Hooft and Susskind. The theory also satisfies Mach's principle: A macroscopically large space-time (and the inertia of low-energy excitations) is generated by a large number of "partons" in the microscopic theory. **Petr Hořava** argues that at low energies in large eleven dimensions, the theory should be effectively described by eleven-dimensional supergravity. This effective description breaks down at much lower energies than naively expected, precisely when the system saturates the Bekenstein bound on energy density. He shows that the number of partons scales like the area of the surface surrounding the system, and

discuss how this holographic reduction of degrees of freedom affects the cosmological constant problem. proposal is put forth for the holographic field theory as a candidate for a covariant, nonperturbative formulation of quantum M theory. DOI: http://dx.doi.org/10.1103/PhysRevD.59.046004 **M theory as a holographic field theory Phys. Rev. D 59, 046004 – Published 26 January 1999 Petr Hořava** The recent proposal on the correspondence between the super-Yang–Mills theory and string theory in the Penrose limit of the AdS5×S5 geometry involves a few puzzles from the viewpoint of holographic principle, especially in connection with the interpretation of times. To resolve these puzzles, we propose to interpret the PP-wave strings on the basis of tunneling null geodesics connecting boundaries of the AdS geometry. Our approach predicts a direct and systematic identification of the S-matrix of Euclidean string theory in the bulk with the short-distance structure of correlation functions of super-Yang–Mills theory on the AdS boundary, as an extension of the ordinary relation in supergravity–CFT correspondence. Holography requires an infinite number of contact terms for interaction vertices of string field theory and constrains their forms in a way consistent with supersymmetry. Copyright © 2003 Elsevier B.V. All rights reserved. **Nuclear Physics B Volume 665, 18 August 2003, Pages 94–128 Holographic reformulation of string theory on AdS5×S5 background in the PP-wave limit Suguru Dobashi, Hidehiko Shimada, Tamiaki Yoneya doi:10.1016/S0550-3213(03)00460-7** The strongest adversary in quantum information science is decoherence, which arises owing to the coupling of a system with its environment1. The induced dissipation tends to destroy and wash out the interesting quantum effects that give rise to the power of quantum computation2, cryptography2 and simulation3. Whereas such a statement is true for many forms of dissipation, **Frank Verstraete1, Michael M. Wolf2 & J. Ignacio Cirac3** show here that dissipation can also have exactly the opposite effect: it can be a fully fledged resource for universal quantum computation without any coherent dynamics needed to complement it. The coupling to the environment drives the system to a steady state where the outcome of the computation is encoded. In a similar vein, they show that dissipation can be used to engineer a large variety of strongly correlated states in steady state, including all stabilizer codes, matrix product states4, and their generalization to higher dimensions5. **Letter abstract Nature Physics 5, 633 - 636 (2009) Published online: 20 July 2009 | doi: 10.1038/nphys1342 Quantum computation and quantum-state engineering driven by dissipation Frank Verstraete1, Michael M. Wolf2 & J. Ignacio Cirac3** Nonlocal twist operators are introduced for the O (n) and Q-state Potts models in two dimensions which count the numbers of self-avoiding loops (respectively, percolation clusters) surrounding a given point. Their scaling dimensions are computed exactly. This yields many results: for example, the number of percolation clusters which must be crossed to connect a given point to an infinitely distant boundary. Its mean behaves as $(1/33\sqrt{\pi})\,|\ln(p_c-p)|$ as $p\to p_c-$. As an application **John Cardy** computes the exact value $3\sqrt{/2}$ for the conductivity at the spin Hall transition, as well as the shape dependence of the mean conductance in an arbitrary simply connected geometry with two extended edge contacts. DOI: http://dx.doi.org/10.1103/PhysRevLett.84.3507 **Linking Numbers for Self-Avoiding Loops and Percolation: Application to the Spin Quantum Hall Transition Phys. Rev. Lett 84, 3507 – Published 17 April 2000 John Cardy Ferenc Iglói, Loïc Turban, Dragi Karevski, and Ferenc Szalma** consider the Ising model and the directed walk on two-dimensional layered lattices and show that the two problems are inherently related: zero-field thermodynamical properties of the Ising model are contained in the spectrum of the transfer matrix of the directed walk. The critical properties of the two models are connected to the scaling behavior of the eigenvalue spectrum of the transfer matrix which is studied exactly through renormalization for different self-similar distributions of the couplings. The models show very rich bulk and surface critical behaviors with nonuniversal critical exponents, coupling-dependent anisotropic scaling, first-order surface transition, and stretched exponential critical correlations. It is shown that all the nonuniversal critical exponents obtained for the aperiodic Ising models satisfy scaling relations and can be expressed as functions of varying surface magnetic exponents. DOI: http://dx.doi.org/10.1103/PhysRevB.56.11031 **exact renormalization-group study of aperiodic Ising quantum chains and directed walks Phys. Rev. B 56, 11031 – Published 1 November 1997 Ferenc Iglói, Loïc Turban, Dragi Karevski, and Ferenc Szalma** Using exact expressions for the persistence probability and for the leading eigenvalue of the Fokker-Planck operator of a random walk in a random environment, we establish a fundamental relation between the statistical properties of anomalous diffusion and the critical and off-critical behavior of random quantum spin chains. Many exact results are obtained from this correspondence, including the space and time

correlations of surviving random walks and the distribution of the gaps of the corresponding Fokker-Planck operator. In turn **Ferenc Iglói and Heiko Rieger** derives analytically the dynamical exponent of the random transverse-field Ising spin chain in the Griffiths-McCoy region. DOI: http://dx.doi.org/10.1103/PhysRevE.58.4238 **Anomalous diffusion in disordered media and random quantum spin chains Phys. Rev. E 58, 4238 – Published 1 October 1998 Ferenc Iglói and Heiko Rieger** To gain deeper insight into the dynamics of complex quantum systems we need a quantum leap in computer simulations. We cannot translate quantum behaviour arising from superposition states or entanglement efficiently into the classical language of conventional computers. The solution to this problem, proposed in 1982 (ref. 1), is simulating the quantum behaviour of interest in a different quantum system where the interactions can be controlled and the outcome detected sufficiently well. **A. Friedenauer1, H. Schmitz1, J. T. Glueckert, D. Porras & T. Schaetz** study the building blocks for simulating quantum spin Hamiltonians with trapped ions2. They experimentally simulate the adiabatic evolution of the smallest non-trivial spin system from paramagnetic into ferromagnetic order with a quantum magnetization for two spins of 98%. We prove that the transition is not driven by thermal fluctuations but is of quantum-mechanical origin (analogous to quantum fluctuations in quantum phase transitions3). We observe a final superposition state of the two degenerate spin configurations for the ferromagnetic order (|↑↑right fence+|↓↓right fence), corresponding to deterministic entanglement achieved with 88% fidelity. This method should allow for scaling to a higher number of coupled spins2, enabling implementation of simulations that are intractable on conventional computers. **Letter Nature Physics 4, 757 - 761 (2008) Published online: 27 July 2008 | doi:10.1038/nphys1032 Subject Categories: Quantum physics | Atomic and molecular physics Simulating a quantum magnet with trapped ions A. Friedenauer1, H. Schmitz1, J. T. Glueckert, D. Porras & T. Schaetz** This article gives a comprehensive description of the fractal geometry of conformally-invariant (CI) scaling curves, in the plane or half-plane. It focuses on deriving critical exponents associated with interacting random paths, by exploiting an underlying quantum gravity (QG) structure, which uses KPZ maps relating exponents in the plane to those on a random lattice, i.e., in a fluctuating metric. This is applied to critical models, like O(N) and Potts models, and to the Stochastic L\"owner Evolution (SLE). The multifractal (MF) function f(alpha, c) of the harmonic measure near any CI fractal boundary, is given as a function of the central charge c of the associated CFT. The Hausdorff dimensions D_{H} of a non-simple scaling curve or cluster hull, and D_{EP} of its external perimeter or frontier, are shown to obey the duality equation (D_{H}-1)(D_{EP}-1)=1/4, valid for any c. The universal mixed MF spectrum f(alpha,lambda;c) describing the local spiralling rate lambda and singularity exponent alpha of the potential near any CI scaling curve is given. The duality between simple and non-simple random paths is established via symmetry of the KPZ quantum gravity map. An extended dual KPZ relation is introduced for the SLE_{kappa}, which commutes with the kappa to kappa'=16/kappa duality. This gives the SLE exponents from simple QG rules, established from the general structure of correlation functions of arbitrary interacting random sets on a random lattice. Journal reference: {\it Fractal Geometry and Applications: A Jubilee of Beno\^{i}t Mandelbrot} (M. L. Lapidus and M. van Frankenhuysen, eds.), Proc. Symposia Pure Math. vol. 72, Part 2, 365-482 (AMS, Providence, R.I., 2004) arXiv: math-ph/0303034(or arXiv: math-ph/0303034v2 for this version) **Conformal Fractal Geometry and Boundary Quantum Gravity Bertrand Duplantier** Hume, For Deleuze, Considers The Mind To Be A System Of Associations Alone, A Network Of Tendencies (ES 25): “We Are Habits, Nothing But Habits – The Habit Of Saying ‘I’. Perhaps There Is No More Striking Answer To The Problem Of The Self.” (ES X.) The Mind, Affected By The Natural Principle Of Association, Becomes Human Nature, From The Ground Up: Empirical Subjectivity Is Constituted In The Mind Under The Influence Of The Principles Affecting It; The Mind Therefore Does Not Have The Characteristics Of A Preexisting Subject. (ES 29) These Associations Account Not Only for Experience in the Basic Sense, But up To the Highest Level of Social and Cultural Life: This Is The Basis For Hume’s Rejection (E) Of A Social Contract Model Of Society (Such As Hobbes’), In Favour Of Convention Alone. Morals, Feelings, Bodily Comportment, All Of These Elements Of Subjectivity Are Explained, Not By Transcendental Structures, Such As Kant Will Propose, But By The Immanent Activity Of Association. Once This Habitual Structure Of The Self Is In Place, Deleuze Suggests, The Humean Concept Of Belief Comes Into Play , Which Is Resolutely A Central Part Of Human Nature. It Describes The Particularly Human Way Of Going Beyond The Given. When We Expect The Sun To Come Up Tomorrow, We Do Not Do So Because We Know That It Will, But Because Of A

Belief Based On A Habit. This In Turn Reverses The Hierarchy Of Knowledge And Belief, And Results, For Deleuze, In A, "Great Conversion Of Theory To Practice." (PI 36) Every Act Of Belief Is A Practical Application Of Habit, Without Any Reference To An A Priori Ability To Judge. Not Only Is The Human Being Thus Habitual, On Deleuze's Reading, But Also Creative Even In The Most Mundane Moments Of Life. Finally, Deleuze Insists That One Of Hume's Greatest Contributions To Modern Philosophy Is His Insistence That All Relations Are External To Their Terms: This Is The Essence Of Hume's Anti-Transcendental Stance. Human Nature Cannot Unite Itself, There Is No 'I' Which Stands Before Experience, But Only Moments Of Experience Themselves, Unattached And Meaningless Without Any Necessary Relation To Each Other. A Flash Of Red, A Movement, A Gust Of Wind, Dashing Of A Woman Against You, Obscene Gestures, Spitting At You, Talking Extempore About You (My Addition) And These Elements Must Be Externally Related To Each Other To Create The Sensation Of A Tree In Autumn. In The Social World, This Externality Attests To The Always-Already Interested Nature Of Life: No Relation Is Necessary, Or Governed By Neutral Laws, So Every Relation Has A Localised And Passional Motive. The Ways In Which Habits Are Formed Attests To The Desires At The Heart Of Our Social Milieu. Subjectivity, As Deleuze Describes It Through His Reading Of Hume, Is A Practical, Passional, And Empiricist Concept, Immediately Located At The Heart Of The Conventional, Which Is To Say The Social. (Stanford Encyclopedia of Philosophy). The Buddha Always Used The Terms Void, No Rising And Falling, Calmness And Extinction To Explain The Profound Meaning Of Sunyata And Cessation. For Example, Sunyata And The State Of Nirvana Where There Is No Rising Nor Falling, Are Interpreted By Most People As A State Of Non-Existence And Gloom. They Fail To Realise That Quite The Opposite, Sunyata Is Of Substantial And Positive Significance. The Sutras Often Use The Word "Great Void" To Explain The Significance Of Sunyata. In General, We Understand The "Great Void" As Something That Contains Absolutely Nothing. However, From A Buddhist Perspective, The Nature Of The "Great Void" Implies Something Which Does Not Obstruct Other Things, In Which All Matters Perform Their Own Functions. Materials Are Form, Which By Their Nature, Imply Obstruction. The Special Characteristic Of The "Great Void" Is Non-Obstruction. The "Great Void" Therefore, Does Not Serve As An Obstacle To Them. Since The "Great Void" Exhibits No Obstructive Tendencies, It Serves As The Foundation For Matter To Function. In Other Words, If There Was Neither "Great Void" Nor Characteristic Of Non-Obstruction, It Would Be Impossible For The Material World To Exist And Function. The "Great Void" Is Not Separated From The Material World. The Latter Depends On The Former. We Can State That the Profound Significance of Sunyata and the Nature of Sunyata in Buddhism Highlight The "Great Void's" Non-Obstructive Nature. Sunyata Does Not Imply The "Great Void". Instead, It Is The Foundation Of All Phenomena (Form And Mind). It Is The True Nature Of All Phenomena, And It Is The Basic Principle Of All Existence. In Other Words, If The Universe's Existence Was Not Empty Nor Impermanent, Then All Resulting Phenomena Could Not Have Arisen Due To The Co-Existence Of Various Causes And There Would Be No Rising Nor Falling. The Nature Of Sunyata Is Of Positive Significance. Calmness And Extinction Are The Opposite Of Rising And Falling. They Are Another Way To Express That There Is No Rising And Falling. Rising And Falling Are The Common Characteristics Of Worldly Existence. All Phenomena Are Always In The Cycle Of Rising And Falling. However, Most People Concentrate On Living (Rising). They Think That The Universe And Life Are The Reality Of A Continuous Existence. Buddhism On The Other Hand, Promotes The Value Of A Continuous Cessation (Falling). This Cessation Does Not Imply That It Ceases To Exist Altogether. Instead, It Is Just A State In The Continuous Process Of Phenomena. In This Material World, Or What We May Call This "State Of Existence", Everything Eventually Ceases To Exist. Cessation Is Definitely The Home Of All Existences. Since Cessation Is The Calm State Of Existence And The Eventual Refuge Of All Phenomena, It Is Also The Foundation For All Activities And Functions. Teachings In Chinese Buddhism (6) Sunyata (Emptiness) In The Mahayana Context (Wikipedia) Studies by Eric V. Linder about geometric dark energy bring out the quintessential prerequisites about the extreme physics ad dark matter**.** The acceleration of the expansion of the universe arises from unknown physical processes involving either new fields in high energy physics or modifications of gravitation theory. It is crucial for our understanding to characterize the properties of the dark energy or gravity through cosmological observations and compare and distinguish between them. In fact, close consistencies exist between a dark energy equation of state function w(z) and changes to the framework of the Friedmann cosmological equations as well as direct spacetime geometry quantities involving the acceleration, such as "geometric dark energy" from the

Ricci scalar. We investigate these interrelationships, including for the case of super acceleration or phantom energy where the fate of the universe may be gentler than the Big Rip. DOI: http://dx.doi.org/10.1103/PhysRevD.70.023511 **Probing gravitation, dark energy, and acceleration Phys. Rev. D 70, 023511 – Published 28 July 2004 Eric V. Linder. In the same vein he continues to question the number of parameters that exists in dark energy** .How many dark energy parameters? For exploring the physics behind the accelerating Universe a crucial question is how much we can learn about the dynamics through next generation cosmological experiments. For example, in defining the dark energy behavior through an effective equation of state, how many parameters can we realistically expect to tightly constrain? Through both general and specific examples (including new parametrizations and principal component analysis) we argue that the answer is two. Cosmological parameter analyses involving a measure of the equation of state value at some epoch (e.g. w0) and a measure of the change in equation of state (e.g. w′) are therefore realistic in projecting dark energy parameter constraints. More elaborate parametrizations could have some uses (e.g. testing for bias or comparison with model features), but do not lead to accurately measured dark energy parameters. DOI: http://dx.doi.org/10.1103/PhysRevD.72.043509 **Phys. Rev. D 72, 043509 – Published 11 August 2005 Eric V. Linder and Dragan Huterer D.F. Mota, C. van de Bruck Oxford Univ** study the spherical collapse model in dark energy cosmologies, in which dark energy is modelled as a minimally coupled scalar field. We first follow the standard assumption that dark energy does not cluster on the scales of interest. Investigating four different popular potentials in detail, we show that the predictions of the spherical collapse model depend on the potential used. We also investigate the dependence on the initial conditions. Secondly, they investigate in how far perturbations in the quintessence field affect the predictions of the spherical collapse model. In doing so, we assume that the field collapses along with the dark matter. Although the field is still subdominant at the time of virialisation, the predictions are different from the case of a homogeneous dark energy component. This will in particular be true if the field is non--minimally coupled. **D.F. Mota, C. van de Bruck** conclude that a better understanding of the evolution of dark energy in the highly non--linear regime is needed in order to make predictions using the spherical collapse model in models with dark energy. Astrophys 421 (2004) 71-81DOI:10.1051/0004-6361:20041090arXiv: astro-ph/0401504 **on the spherical collapse model in dark energy cosmologies D.F. Mota, C. van de Bruck (Oxford Univ)** Studies by Eric V. Linder about geometric dark energy bring out the quintessential prerequisites about the extreme physics ad dark matter**.** The acceleration of the expansion of the universe arises from unknown physical processes involving either new fields in high energy physics or modifications of gravitation theory. It is crucial for our understanding to characterize the properties of the dark energy or gravity through cosmological observations and compare and distinguish between them. In fact, close consistencies exist between a dark energy equation of state function w(z) and changes to the framework of the Friedmann cosmological equations as well as direct spacetime geometry quantities involving the acceleration, such as "geometric dark energy" from the Ricci scalar. We investigate these interrelationships, including for the case of super acceleration or phantom energy where the fate of the universe may be gentler than the Big Rip. DOI: http://dx.doi.org/10.1103/PhysRevD.70.023511 **Probing gravitation, dark energy, and acceleration Phys. Rev. D 70, 023511 – Published 28 July 2004 Eric V. Linder. In the same vein he continues to question the number of parameters that exists in dark energy** .How many dark energy parameters? For exploring the physics behind the accelerating Universe a crucial question is how much we can learn about the dynamics through next generation cosmological experiments. For example, in defining the dark energy behavior through an effective equation of state, how many parameters can we realistically expect to tightly constrain? Through both general and specific examples (including new parametrizations and principal component analysis) we argue that the answer is two. All Existences Exhibit Void-Nature And Nirvana-Nature. These Natures Are The Reality Of All Existence. To Realise The Truth, We Have To Contemplate And Observe Our Worldly Existence. We Cannot Realise The Former Without Observing The Latter. Consider This Heart Sutra Extract, "Only When Avalokiteshvara Bodhisattva Practised The Deep Course Of Wisdom Of Prajna Paramita Did He Come To Realise That The Five Skandhas (Aggregates, And Material And Mental Objects) Were Void." Profound Wisdom Leads Us To The Realisation That All Existences Are Of Void-Nature. The Sutras Demonstrate That The Profound Principle Can Be Understood By Contemplating And Observing The Five Skandhas. We Cannot Realise The Truth By Seeking Something Beyond The Material And Mental World. The Buddha, Using His Perfect

Wisdom, Observed Worldly Existence From Various Implications And Aspects, And Came To Understand All Existences. Teachings In Chinese Buddhism (6) Sunyata (Emptiness) In The Mahayana Context (Wikipedia) .But From The Profound Contemplation And Wisdom Of The Buddha And Mahabodhisattvas, We Know There Is No Such Reality. Instead, Egolessness (Non-Self) Is The Only Path To Understand The Reality Of The Deluded Life. All Existences Are Subject To The Law Of Causes And Conditions. These Include The Smallest Particles, The Relationship Between The Particles, The Planets, And The Relationship Between Them, Up To And Including The Whole Universe! From The Smallest Particles To The Biggest Matter, There Exists No Absolute Independent Identity. Egolessness (Non-Self) Implies The Void Characteristics Of All Existence. Egolessness (Non-Self) Signifies The Non-Existence Of Permanent Identity For Self And Existence (Dharma). Sunyata Stresses The Voidness Characteristic Of Self And Existence (Dharma). Sunyata And Egolessness Possess Similar Attributes. As We Have Discussed Before, We Can Observe The Profound Significance Of Sunyata From The Perspective Of Inter-Dependent Relationships. Considering Dharma-Nature And The Condition Of Nirvana, All Existences Are Immaterial And Of A Void-Nature. Then We See Each Existence As Independent Of Each Other. But Then We Cannot Find Any Material That Does Exist Independent Of Everything Else. So Egolessness Also Implies Void-Nature! From The Observation Of All Existences, We Can Infer The Theory Of Nirvana And The Complete Cessation Of All Phenomena. From The Viewpoint Of Phenomena, All Existences Are So Different From Each Other, That They May Contradict Each Other. They Are So Chaotic. In Reality, Their Existence Is Illusionary And Arises From Conditional Causation. They Seem To Exist On One Hand, And Yet Do Not Exist On The Other. They Seem To Be United, But Yet They Are So Different To One Another. They Seem To Exist And Yet They Do Cease! Ultimately Everything Will Return To Harmony And Complete Calmness. This Is The Nature Of All Existence. It Is The Final Resting Place For All. If We Can Understand This Reality And Remove Our Illusions, We Can Find This State Of Harmony And Complete Calmness. Teachings In Chinese Buddhism (6) Sunyata (Emptiness) In The Mahayana Context (Wikipedia) All Our Contradictions, Impediments And Confusion Will Be Converted To Equanimity. Free From Illusion, Complete Calmness Will Be The Result Of Attaining Nirvana. The Buddha Emphasised The Significance Of This Attainment And Encouraged The Direct And Profound Contemplation On Void-Nature. He Said, "Since There Is No Absolute Self-Nature Thus Every Existence Exhibits Void-Nature. Because It Is Void, There Is No Rising Nor Falling. Since There Is No Rising Nor Falling, Thus Everything Was Originally In Complete Calmness. Its Self-Nature Is Nirvana." Teachings In Chinese Buddhism (6) Sunyata (Emptiness) In The Mahayana Context (Wikipedia).The Scholars Of Tien Tai Called It The "Embodied Nature". (This Is The Buddha-Nature That Includes Both Good And Evil.) The Scholars Of Xian Shou Say, "It Is Arising From Primal Nature", And The Scholars Of Chan (Zen) Say, "It Is Nature That Causes The Rising Of Things". All Dharma Is Dharma-Nature. It Is Not Different From Dharma-Nature. Dharma And Dharma-Nature Are Not Two Separate Identities, "Phenomena" And "Nature" Is Also Not Distinguishable Either. In Other Words, There Is No Difference Between Principle (Absolute) And Practice (Relative). Teachings In Chinese Buddhism (6) Sunyata (Emptiness) In The Mahayana Context (Wikipedia). In Other Words, All Dharma Arises From Causes And Conditions, And All Dharma Is Empty In Nature. The Law of Dependent Origination (Existence) And The Nature Of Emptiness Is Neither The Same Nor Different. They Exist Mutually. The Truth Of "Sunyata" And "Existence", And "Nature" And "Phenomena" Are Not In Conflict With Each Other. Unlike The Scholars Of The Dharmalaksana Sect Who Explain The Dharma Only From The Aspect Of Dependent Origination, Or The Scholars Of Dharma-Nature That Explain The Existence Of Dharma Only From The Aspect Of Dharma-Nature, The Scholars Of Madhyamika Explain The Truth Of The Dharma From Both Aspects. Hence This Is Called The Middle Path Which Does Not Incline To Either Side. Teachings In Chinese Buddhism (6) Sunyata (Emptiness) In The Mahayana Context (Wikipedia) **C S Lent, P D Tougaw, W Porod and G H Bernstein** formulate a new paradigm for computing with cellular automata (CAS) composed of arrays of quantum devices-quantum cellular automata. Computing in such a paradigm is edge driven. Input, output, and power are delivered at the edge of the CA array only; no direct flow of information or energy to internal cells is required. Computing in this paradigm is also computing with the ground state. The architecture is so designed that the ground-state configuration of the array, subject to boundary conditions determined by the input, yields the computational result. proposal is put forth for a specific realization of these ideas using two-electron cells composed of quantum dots. The charge density in the cell

is very highly polarized (aligned) along one of the two cell axes, suggestive of a two-state CA. The polarization of one cell induces a polarization in a neighboring cell through the Coulomb interaction in a very non-linear fashion. Quantum cellular automata can perform useful computing. The authors show that AND gates, OR gates and inverters can be constructed and interconnected. **C S Lent et al 1993 Nanotechnology 4 49 doi:10.1088/0957-4484/4/1/004 Quantum cellular automata C S Lent, P D Tougaw, W Porod and G H Bernstein** The basic features of the dynamics of open quantum systems, such as the dissipation of energy, the decay of coherences, the relaxation to equilibrium or non-equilibrium stationary state, and the transport of excitations in complex structures are of central importance in many applications of quantum mechanics. The theoretical description, analysis and control of non-Markovian quantum processes play an important role in this context. While in a Markovian process an open system irretrievably loses information to its surroundings, non-Markovian processes feature a flow of information from the environment back to the open system, which implies the presence of memory effects and represents the key property of non-Markovian quantum behaviour. **Heinz-Peter Breuer** review recent ideas developing a general mathematical definition for non-Markovianity in the quantum regime and a measure for the degree of memory effects in the dynamics of open systems, which are based on the exchange of information between system and environment. **Heinz-Peter Breuer** further study the dynamical effects induced by the presence of system–environment correlations in the total initial state and design suitable methods to detect such correlations through local measurements on the open system. **Heinz-Peter Breuer 2012 J. Phys. B: At. Mol. Opt. Phys. 45 154001 doi:10.1088/0953-4075/45/15/154001 Foundations and measures of quantum non-Markovianity** A fundamental step towards atomic- or molecular-scale spintronic devices has recently been made by demonstrating that the spin of an individual atom deposited on a surface1, or of a small paramagnetic molecule embedded in a nanojunction2, can be externally controlled. An appealing next step is the extension of such a capability to the field of information storage, by taking advantage of the magnetic bistability and rich quantum behaviour of single-molecule magnets3, 4, 5, 6 (SMMs). Recently, a proof of concept that the magnetic memory effect is retained when SMMs are chemically anchored to a metallic surface7 was provided. However, control of the nanoscale organization of these complex systems is required for SMMs to be integrated into molecular spintronic devices8, 9. Here we show that a preferential orientation of Fe4 complexes on a gold surface can be achieved by chemical tailoring. As a result, the most striking quantum feature of SMMs—their stepped hysteresis loop, which results from resonant quantum tunnelling of the magnetization5, 6—can be clearly detected using synchrotron-based spectroscopic techniques. With the aid of multiple theoretical approaches, **M. Mannini et al** relate the angular dependence of the quantum tunnelling resonances to the adsorption geometry, and demonstrate that molecules predominantly lie with their easy axes close to the surface normal. Findings of **M. Mannini et al prove** that the quantum spin dynamics can be observed in SMMs chemically grafted to surfaces, and offer a tool to reveal the organization of matter at the nanoscale. **Quantum tunnelling of the magnetization in a monolayer of oriented single-molecule magnets M. Mannini et al** Nature 468, 417–421 (18 November 2010) doi: 10.1038/nature09478 Supposing That There Were Indeed An "Energy Of Contraction" Constant In All Centers Of Force Of The Universe, It Remains To Be Explained Where Any Difference Would Ever Originate. It Would Be Necessary For The Whole To Dissolve Into An Infinite Number Of Perfectly Identical Existential Rings And Spheres, And We Would Therefore Behold Innumerable And Perfectly Identical Worlds COEXISTING [Nietzsche Underlines This Word Twice] Alongside Each Other. Is It Necessary For Me To Admit This? Is It Necessary To Posit An Eternal Coexistence On Top Of The Eternal Succession Of Identical Worlds? The Eternal Return: Genesis And Interpretation BY PAUL D'IORIO"The Cyclical Hypothesis, So Heavily Criticized By Nietzsche (VP II 325 And 334), Arises In This Way." In Fact, Nietzsche Was Not Criticizing The Cyclical Hypothesis But Only The Particular Form Of That Hypothesis Presented In Vogt's Work. All Of Nietzsche's Texts Without Exception Speak Of The Eternal Return As The Repetition Of The Same Events Within A Cycle Which Repeats Itself Eternally. If Deleuze's Interpretation Holds That The Eternal Return Is Not A Circle, Then What Is It? A Wheel Moving Centrifugally, Operating A "Creative Selection," "Nietzsche's Secret Is That The Eternal Return Is Selective" Says Deleuze: The Eternal Return Produces Becoming-Active. It Is Sufficient To Relate The Will To Nothingness To The Eternal Return In Order To Realize That Reactive Forces Do Not Return. However Far They Go, However Deep The Becoming-Reactive Of Forces, Reactive Forces Will Not Return. The Small, Petty, Reactive Man Will Not Return. Affirmation Alone Returns,

This That Can Be Affirmed Alone Returns, Joy Alone Returns. Everything That Can Be Denied, Everything That Is Negation, Is Expelled Due To The Very Movement Of The Eternal Return. We Were Entitled To Dread That The Combinations Of Nihilism And Reactivity Would Eternally Return Too. The Eternal Return Must Be Compared To A Wheel; Yet, The Movement Of The Wheel Is Endowed With Centrifugal Powers That Drive Away The Entire Negative. Because Being Imposes Itself On Becoming, It Expels From Itself Everything That Contradicts Affirmation, All Forms Of Nihilism And Reactivity: Bad Conscience, Ressentiment..., We Shall Witness Them Only Once. [...] The Eternal Return Is The Repetition, But The Repetition That Selects, The Repetition That Saves. Here Is The Marvelous Secret Of A Selective And Liberating Repetition. There Is No Need To Remind The Reader That Neither The Image Of A Centrifugal Movement Nor The Concept Of A Negativity-Rejecting Repetition Appears Anywhere In Nietzsche's Writings, And Indeed Deleuze Does Not Refer To Any Text In Support Of This Interpretation. Further, One Could Highlight That Nietzsche Never Formulates The Opposition Between Active And Reactive Forces, Which Constitutes The Broader Framework Of Deleuze's Interpretation. For Some Years, Marco Brusotti Has Called Attention To The Fact That Deleuze Introduced A Dualism That Does Not Exist In Nietzsche's Writings. To Be Sure, The German Philosopher Describes A Certain Number Of "Reactive" Phenomena (For Example, In The Second Essay Of The Genealogy Of Morality, § 11, He Talks About "Reactive Affects" [Reaktive Affekte], "Reactive Feelings" [Reaktive Gefühlen], Reactive Men [Reaktive Menschen]); But These Are Nonetheless The Result Of Complex Ensembles Of Configurations Of Centers Of Forces That Remain In Themselves Active. Neither The Word Nor The Concept Of "Reactive Forces" Ever Appears In Nietzsche's Philosophy. The Eternal Return: Genesis And Interpretation BY PAUL D'IORIO Some Parts Are Deleted Due To Spatial Constraints. Please Pardon Me For The Deletion. Deleuze Opposes The Historical Course Of The Hegelian Notion That Confronts Struggles And Finally Dialectizes The Negative And Results In A Consoling Teleology Leading To The Triumph Of The Idea Or The Liberation Of The Masses With The Centrifugal Movement Of The Wheel, Which Simply Ejects The Negative. It Is Still A Case Of A Consoling And Optimistic Teleology, Which, Instead Of Confronting The Weight Of History, The Grief And The Negative, Makes It Disappear In One Centrifugal Stroke Of A Magic Wand. There Is Reason To Worry That This Be A Case Of Repression, Which, Unable To Dialectize Or Accept The Negative, Simply Seeks To Exorcise It In One Gesture Of "Creative Selection." But Exorcism Is A Feat Of Magic And Not Of Philosophy: It Is Unfortunately Not Enough To Make The Negative Disappear. In All Probability, The Negative Will Come Back With A Vengeance. In Contrast To Deleuze's "Affirmation Of Affirmation", Which Affirms Only Affirmation, Nietzsche Conceives Of The Eternal Return From A Rigorously Non-Teleological Perspective As The Accomplishment Of A Philosophy Strong Enough To Accept Existence In All Its Aspects, Even The Most Negative, Without Any Need To Dialecticize Them, Without Any Need To Exclude Them By Way Of Some Centrifugal Movement Of Repression. It Denies Nothing And Incarnates Itself In A Figure Similar To The One Nietzsche, In Twilight Of The Idols, Draws Of Goethe: Such A Spirit, Who Has Become Free Stands In The Middle Of The World With A Cheerful And Trusting Fatalism In The Belief That Only The Individual Is Reprehensible, That Everything Is Redeemed And Affirmed In The Whole-He Does Not Negate Anymore. Such A Faith However, Is The Highest Of All Possible Faiths: I Have Baptized It With The Name Of Dionysus. The Eternal Return: Genesis and Interpretation BY PAUL D'IORIO The Most Complete Theory Of Mind That Comports With The Holographic Principle Of Mind, As Developed Here, Is The Microgenetic Theory Of Neurologist Jason Brown. Brown (2005) Presents An Evolutionary Theory Of Values, Morals, And Ethics, Building On The Microgenetic Theory And Process Theory Of His Earlier Works (1988, 1991, 1996, 1998, And 2000). Brown Is Work Has Been A Progression That Began With The Study Of Brain Processes In Neurological Subjects With Brain Injuries Or Lesions. In His Earlier Work (Brown, 1988; 1991) He Described The Process Of Microgenesis As A Process Of Elaboration Of Mental Contents In An Evolutionary And Developmental Hierarchy Of Brain Structures Within The Process Of The Genesis Of The Mental State, And Related The Hierarchy Of The Genesis Of The Mental State To Disturbances Of Language Comprehension Or Expression (Aphasia), Of Knowledge (Agnosia), And Or Purposeful Movement (Apraxia). The Fundamental Neurology Of These Disturbances Had Been Described In Brownís Previous Work (1972, 1988), And In The Seminal Work Of The Russian Neurologist And Neuropsychologist, Alexander Luria (1966). Brown (1988, 1991, 1996, 1998) Created A Process Theory Of Mind On The Basis Of His Neurological Observations, And Went On From His Neurological

Work To Incorporate Process Theory In Philosophy, As Developed Principally By Henri Bergson (1911/1998). And Alfred North Whitehead (1925/1953, 1929/1978). Bergson Had Pioneered The Concept Of An Irreducible Duration Of Experience. This Concept Was Elaborated By Whitehead, In Keeping With Discontinuities Of Particles, As "Actual Entities" Of A Rudimentary Sort, As They Make "Quantum Leaps" Along Their Trajectories, Halting Or Persisting At A Given Location For A Short Period Of Time Or Duration Before Leaping To The Next. Whitehead Employed These "Quantum Leaps" To The Process By Which Experience Arises In "Actual Entities" At The Most Fundamental Level Of Actuality, Through The Epochal Or Halting Duration Of A Discontinuous Process (Whitehead, 1925/1953, 1929/1978). This Halting Allows Actual Entities To Participate In A Process Of Internal Relations, Relations On The "Inside" Of Things. According The Holographic Principle, This "Inside" Is In The Holographic Surface Of The Relevant Region Of Spacetime Of The Actual Entity, And Within Other Surfaces That Are Non-Locally Connected With That Surface. The Holographic Surface Is Thus The Locus Of Non-Local Information And Experience, In Our Interpretations Of Whitehead Is Metaphysics. Internal Relations Occur Through Feeling, Or "Prehension," Which Is Nothing More Than The Subjective Quality Of Non-Local Experience. The Internal Relations Of The Becoming Entity Occur In A Seriality Of Prior Becomings Of The Actual Entity, With The Entity Inheriting All Of Its Causal Past As Well As Its Relevant Feelings Or Prehensions From Prior "Occasions" Or Quanta Of Experience. This Process Is, In The Holographic Model, The Process By Which Former Surfaces, Now-Pasts, Are Elaborated On To The Now-Present. Prehensions Also Occur With Other Actual Entities Throughout The Universe, With Which They Are Non-Locally Connected, And With Neo-Platonic "Eternal Objects," Existing In An Eternal Heaven. The "Ingression" Of Eternal Objects Is Fundamental To Each Duration Of Becoming Of The Entity, And, On This Basis, Whitehead Formulated The Notion Of An Eternal Heaven In Constant Intercourse All Of Its Creation. There Are Clear Parallels Between Whitehead's Cosmology And The Cosmology Introduced Here According To The Holographic Principle, In That The "Heaven" May Be Regarded As The Holographic Boundary, In Constant Intercourse With The Relevant Volume Of Spacetime. This Interconnection Of Feelings Or Prehensions, According To Their Relevance, Produces Intensities And Contrasts Which Give Rise To The Creative Advance, Which, For Whitehead (1929/1978) Is The Process Which Impels The Movement From The Physical To The Mental Pole Of Process. With Each Halting, Internally-Timeless Duration There Is A Concrescence Or Integration Of These Feelings, Which, When Complete, Constitutes A New Actual Entity Or Actual Occasion (They Are The Same Thing) At A New Locus In Spacetime. This Movement To A New Locus, Or Transition, Is, Fundamentally, The Quantum Leap. MICROGENESIS AND THE HOLOGRAPHIC PRINCIPLE The Holographic Principle Theory Of Mind MARK GERMINE Institute For Psycho Science If the Holographic Principle is true, than it must be the fundamental principle of mind. The brain has no way around the Holographic Principle. For the reductionist, the Holographic Principle is the ultimate reduction. It applies to the most minuscule level of what we can observe, and beyond. For the Universalist, the Holographic Principle gives us the ultimate universal. It extends to the limit of our Universe, the Universal holographic boundary, and beyond. For the Phenomenalist, the Holographic Principle gives us the ultimate ground of our phenomenal perception, our grounding in the Universal "now," in the now-present of the Universal holographic boundary, moving outward from now-pasts to now-futures. Wheeler (1988) has said that all of reality is information, and that other physical quantities are "mere incidentals." Information monism is gaining popularity in physics. By breaking the dichotomy of between information and experience, we find a deep connection between Wheeler is monism and the experiential monism of Whitehead, sometimes called panexperientialism, which relates to the later theological concept of his student, Charles Hartshorne, panentheism, God inside of everything (Hartshorne, 1964). There thus seems to be a convergence of the concepts of information, experience, and spirituality. There is not one universe, but many parallel universes, or, more accurately, a vast superposition of universes called the multiverse (Penrose, 2004). However, as explained in our treatment of the double-slit experiment, in the observation of events on the quantum level, the individual observer sees only one of these vast superpositions, not a summation of a vast number of potentials. This has been explained in terms of the many worlds or many minds theories, in which there are multiple copies of the same observing individual, but this particular issue remains unresolved, leaving physics ungrounded in reality (Penrose, 2004). The concept of One Mind is not only more parsimonious than that of many minds, but also lets us out of the bizarre and counterintuitive idea that there are multiple copies of our own selves, which exist as

mere potential, and are thus not actual. There are many possible universes, but there is only One Mind, which determines events on the quantum level, and thus creates our Universe. As we had discussed previously, the quantum level provides the essential ground of the Holographic Principle, such that quantum-level holographic surfaces are elaborated at higher levels, manifesting higher orders of information from the quantum "world." In this process, there is a reduction of the wave function or potentialities of quantum fields. At the level of consciousness, this entails freedom to choose which observations we make (Stapp, 1997), which gives us the capacity to think and to make decisions. These capacities are the basis of individuality, self-determination, judgment, and values. The progressive evolution of the manifestation of Mind through higher orders of experience, leading to higher orders of consciousness, is entailed by the Holographic Principle. Again, we are dealing here with levels of description, with the multiverse of all potentials fundamentally supporting the single Universe we collectively observe. The multiverse is the wave function of the Universe (Penrose, 2004). The recursive integration of nested hierarchies of holographic surfaces brings out a single actuality in consciousness from a wave function that is unconscious. Consciousness thus gives us information at a level of experience that is causal. In this sense, we partake in the creation of the Universe. We participate in creation, and this participation, when fully realized, leads us to higher levels of consciousness and of realization. As a system, the biosphere that we live in has a holographic surface, creating a deep sense of ecology as we collectively move toward a planetary consciousness. It is only when the Universal Holographic Boundary reaches the information storage and processing capacity needed for the requisite biological and biochemical complexity that consciousness evolves in living things. This evolution is natural and spontaneous, since consciousness gives rise to what is actual, as opposed to what is merely potential. In this sense, consciousness is still in the process of creating our Universe, and levels of higher consciousness will continue to evolve. We are all in the same "now," and that now is defined by the present Universal Boundary. This assures us that our experience is Universal, and does not pass with time. This is the fundamental basis of memory and of cognition. The identity of mind and brain is a myth. We have a continual, internal or non-local relation with the Universe, as it has with us. Once this mystery is resolved, the myth is no longer needed, and there is a confluence of science and spirituality.Experience is primary, information is secondary. We can only gather information from experience, whether it is in the laboratory or in life. We cannot measure the information on the surfaces of systems. The physicists that have formulated the Holographic Principle for all systems are quite aware of this, or else the principle would have been established or discredited. We cannot measure what we experience. It is intangible, yet it is all we know to be actual. Everything else is inferred. Because it cannot be measured, it has been fundamentally disregarded by mainstream science. Materialism is considered scientific, while idealism is considered unscientific. But arenít ideas, fundamentally, made of information? Brain science has mistaken the representation of information for information itself, and has tied those representations to matter and energy. Consciousness, the highest order of information, has generally been regarded as superfluous, something that needs to explained away, or altogether ignored. Yet it is the only "thing" that reaches our awareness. Consciousness comes at a price. For everything that becomes conscious, there must be something that becomes conscious. Consciousness is certainty, and its complement, unconsciousness uncertainty. Consciousness is the particle nature of experience. It has definiteness about it. The unconscious is the wave nature of experience, it is like the metaphysical cloud of unknowing. If we are the most conscious of animals, then we must also be the most unconscious. Perhaps this is the predicament of humankind. The Holographic Principle of Mind leads us naturally from the most fundamental experiences, existing as quantum potentials from the conformations of proteins down through the fields of electrons, through their manifestation upward through a recursion or successive applications of the same holographic process, through higher levels of experience, to the emergence of consciousness as higher and higher orders of experience. The quantum Holographic Principle Mind does not require anything more quantum than is obviously present at the microscopic and submicroscopic levels, as it represents successive orders of manifestations from these levels. Recursion also applies here in the sense that it is used in computer science, in which the function of the part depends on the function of the whole. A program, as a part, cannot work without a functioning whole, the operating system. Recursive wholes which are, for us, supra-conscious, are on the group, species, planetary, and universal levels. As individuals, our consciousness cannot function without recursion to the Universal Consciousness, even though we may be unaware that such Universal Consciousness exists. Reaching upward to these supra-conscious levels is a

spiritual process, making transcendence the ultimate solution to the unconscious human predicament. What was once supra-conscious becomes unconscious through a process of conditioning? We are born as creatures of the Earth and of the Universe, as evidenced by the beliefs and practices of "primitive peoples." There is evidence from cave paintings that our hominid ancestors experienced a kind of holographic perception (Combs, 1996), which could constitute our early connection with the holographic subtext of reality, and which we might then propose existed in our animal ancestors, and in extant animal species. If this is the case, than microgenesis would entail the recapitulation of this holographic experience as it progresses through our ancestral past. The supra-conscious mind seems to envelope perinatal experiences, and Stanislav Grof (1994) has developed techniques to gain access to these experiences, as well as to the earlier experiences of our human and animal ancestral lineage, and of our universal history. Grof (1994) concludes: "Our consciousness seems to have the amazing capacity to directly access the earliest history of the universe ñ witnessing dramatic sequences of the Big Bang, the formation of the galaxies, the birth of the solar system, and the early geophysical processes on this planet billions of years ago." The Holographic Principle Theory Of Mind MARK GERMINE Institute For Psycho Science. There Are Similarities Between The Patterns Of Holography And Of Psychological Transference, Where Holography Is The Process Of Recording And Reconstructing Holograms. Employing A Theoretical Perspective Using A Hermeneutic Method, This Dissertation Parallels Holography With Transference, Offering Another Way To Encounter Transference By Showing Similarities Between The Processes Of Each And The Results Of Each. Though Complex, Infinitely Varying, And Unique, Their Patterns Are Clearly Identifiable. Thus They Are A Metaphorical Fit To The Concept Of Strange Attractors In Physics And A More Literal Fit To The Concept Of Archetypes In Depth Psychology Or Dynamic Psychology, Psychology Which Attends To The Living, Autonomous Unconscious. This Study Explains How Holography Models Transference, What A Hologram Is And How It Works, And How Depth Psychology Understands Of The Interaction Between Consciousness And The Unconscious Is Related To The Hologram. It Describes Transference And Related Psychological Processes As Understood In Six Different Schools Of Depth Psychological Thought. It Shows That The Underlying Pattern Or Strange Attraction Between Transference And Holography Extends To Other Processes Both Within And Outside The Field Of Psychology, Processes Such As Projection, Projective Identification, Splitting, Memory, Biology, Creative Discovery, Theology, Synchronicity, Chaos, And Nonlocality. By Identifying The Similar Patterns Of These Processes, This Study Demonstrates The Existence Of An Underlying Holographic Archetype In Which Essential Qualities Of The Whole Are Present In Each Of The Parts Of The Whole: The Visual Image Of The Overall Hologram Is Present In Each Component Part Of The Hologram, The Autonomy Of The Overall Human Is Present In Each Conscious And Unconscious Component Part Of The Human Psyche. By Noting Differences As Well As Similarities In These Processes, It Suggests An Inventory Of The Qualities Of The Holographic Archetype. This Study Furthers Understanding Of The Pervasiveness, Force, And Autonomy Of The Unconscious Acting Through Transference And Projection By Identifying A Group Projection Of Domestic Violence Lying At The Core Of The Christian Myth. This Study Also Furthers Understanding Of The Concept Of Transference By Providing A Reflection Hologram Of The Human Psyche As An Artistic Work And As A Visual Metaphor Of Transference. Strange Attractors: Transference, Holography, And An Archetype Burke, J. (2003). Strange Attractors: Transference, Holography, And An Archetype (Doctoral Dissertation, Pacifica Graduate Institute, 2003). Onticology, Like All Variations Of Object-Oriented Ontology, Is Realist In Its Orientation. In Defending A Realist Ontology Onticology Holds That The Vast Majorities Of Objects, Actants, Beings, Or Entities Are Independent Of Humans And Are What They Are Regardless Of Whether Any Humans Regard Them Or Register Them. In Short, Onticology Rejects Any Anthropomorphic, Idealist, Or Anti-Realist Thesis To The Effect That To Be Is To Be The Correlate Of Mind, Spirit, The Body, The Human, And Language Or Otherwise. While It Is Certainly The Case That Knowledge Is Necessarily Dependent On The Object To Which It Relates, The Converse Does Not Hold True. Objects Are Not Dependent On Being Known, Regarded, Perceived, Or Spoken About. As Such, And To Put It In Aristotelean Terms, Knowledge Is An Accident Of Objects, Not Objects An Accident Of Knowledge. As Althusser So Nicely Puts It, “[N]O Doubt There Is A Relation Between Thought-About-The-Real And This Real, But It Is A Relation Of Knowledge, A Relation Of Adequacy Or Inadequacy Of Knowledge, Not A Real Relation, Meaning By This A Relation Inscribed In That Real Of Which The Thought Is The (Adequate Or Inadequate) Knowledge” (Reading

Capital, 96). Althusser Goes On To Remark That "[T] He Distinction Between A Relation Of Knowledge And A Relation Of The Real Is A Fundamental One: If We Did Not Respect It We Should Fall Irreversibly Into Either Speculative Or Empiricist Idealism" (Ibid.). Onticology Categorically Endorses Althusser's Verdict It Is A Fundamental Necessity To Distinguish Between Those Relations That Belong To The Object And Those That Belong To Knowledge. Contemporary Philosophy Continuously Confuses These Two Very Different Sorts Of Relations. Naturally The Question Arises Of How It Is Possible To Surmount Our Relation To The Object So As To Determine Whether Objects Themselves Possess The Properties We Encounter In Relating To Objects. In Other Words, Given That We Can Only Ever Relate To The Object In Relating To The Object How Is It Possible To Surmount This Relation To Get At The Being Of The Object Itself? Much More Will Have To Be Said About This Later– And The Answers Will Be Surprising With Respect To Standard Prejudices About Realism –However, For The Moment It Can Be Said That Onticology Takes Its Epistemological Inspiration From The Transcendental Realism Of Roy Bhaskar. Among Other Things, Bhaskar Sought To Provide A Transcendental Grounding For The Sciences. Insofar As Onticology Defends The Thesis That The Field Of Being Is Much Faster Than The Field Of Objects Investigated By The Natural Sciences, It Parts Way With The Thesis That The Domain Of Being Is Exhausted By The Domain Of Natural Objects. However, The General Form of Bhaskar's Argument Holds for Our Realist Purposes. A Transcendental Argument Seeks To Elucidate The Conditions Under Which Certain Acknowledged Practices And Forms Of Cognition Are Possible. Kant, For Example, Asked What Must Be The Case For Mathematical Judgments To Be Possible. How Is It Both That We Are Able To Extend Our Knowledge, As If By Magic, Through Mathematical Judgments And, More Significantly, That These Judgments Are Able To Provide Genuine Knowledge Of The World Despite The Fact That These Forms Of Reasoning Are Not Based On Experience? Part of Kant's Argument Consisted in Claiming That Mind Imposes the Forms of Space and Time on the Data of Experience. In Other Words, Space And Time Are Not Attributes Of Being Itself But Rather Of The Mind That Regards Being. Insofar, Kant Argues, As Mathematics Is Ultimately A Rumination On The Nature Of Space And Time Taken In Their Most Abstract Form And Insofar As The Mind Imposes Space And Time On The Manifold Of Sensation, It Thus Follows That A Priori Judgments About The Nature Of Spatio-Temporal Relations Are Possible That Anticipate The Structure Of Actual-Space Times Without Directly Experiencing These Space-Times. Why? Because Any Manifold Of Sensation Must Necessarily Be Structured By These Forms Imposed By Intuition Onticology– A Manifesto For Object-Oriented Ontology Part I Posted By Larval Subjects Under Object-Oriented Philosophy. ……. it is necessary to distinguish the being of objects from the manifestation of objects. While objects are acts, these acts are not identical with their performance in either nature (events where no humans are about to perceive them) or with their performance for humans. Rather, the proper being of the object is not its performance or manifestation, but the generative mechanism that serves as the condition under which these performances or manifestations are possible. As Graham Harman will argue– though in a very different theoretical constellation –the being of objects is essentially withdrawn or hidden. No one has ever perceived a single object, but we do perceive all sorts of effects of objects. Traditional epistemology has confused these effects with the objects themselves. Fortunately we do occasionally manage to cognize objects through a sort of detective work that infers these generative mechanisms from their effects; without, for all this, ever exhausting the infinity of a single object. At any rate, if objects were not withdrawn in this way, the practice of experiment would be unintelligible. This leads to Bhaskar's answer to the first question: What must the world be like for science to be possible? Note, this is not a question about mind or culture, but about the world itself, regardless of whether or not humans exist. Once again, knowledge is an accident of objects, not objects an accident of consciousness or cognition. If science is to be possible– and I would argue, if any human practice is to be possible –then the world must be structured and differentiated. The world must have joints or, as Harman puts it, the world must be composed of "chunks". Why is this case? Let us return to the question of experimentation and the conditions under which experiment is possible. We will adopt two possible hypotheses pertaining to the ontological nature of the world independent of humans: 1) As certain mystics and contemporary crypto-mystics would have it, the world is an undifferentiated One-All that is only subsequently segmented or partitioned into discrete beings by some form of human agency whether this be through cognition or language (in the case of language we might think of Saussure's and Hjelmslev's undifferentiated "sonorous matter"). 2) Entities are the sum totality of their relations to all other

entities in the universe. The first hypothesis is easily dispatched on two grounds: First, this hypothesis fails to register the contradiction in its own utterance. At the level of explicit content, it claims that the world is an undifferentiated One-All that is only subsequently segmented into discrete beings, yet what it misses at the unconscious level of its own utterance is that it registers at least one structured differentiation that is not undifferentiated within this One-All: Namely, the agency through which the One-All subsequently comes to be differentiated. Certain anti-realist, transcendental philosophers will, in a gesture that is all too cute, claim that the agency by which the world is segmented cannot properly be said to exist, thus attempting to resolve this contradiction through a sleight of hand. However, as Meillassoux has shown in After Finitude, all attempts on the part of transcendental anti-realist philosophies to treat the transcendental subject as a non-existing or non-objectile agency that does not itself exist end up, all too clearly, attaching that conception of finitude and the segmentary work with which it is charged to the body (a differentiated being or generative mechanism). Second, suppose the anti-realist transcendental philosophers were to convince us through his appeals to the non-existence of the transcendental, would we still encounter problems? Like Atlas, transcendental subjectivity is charged with the monumental task of segmenting the formless Apeiron of the One-All from out of primal chaos into a supremely segmented world. But this world appears to us to be too slippery for even a titan like Atlas to grasp. Were the world, prior to and independent of humans genuinely a formless Apeiron it would contain no differences providing hand-holds for Atlas to grasp in his segmenting activity. Consequently, no segments could ever come into being. Yet everywhere we encounter segments, so the world must not be a formless One-All, but must rather be structured and differentiated even if structure and differentiation are transformed in their encounter with the human. Onticology– A Manifesto For Object-Oriented Ontology Part I Posted By Larval Subjects Under Object-Oriented Philosophy. The neural networks of the human brain act as very efficient parallel processing computers co-ordinating memory related responses to a multitude of input signals from sensory organs. Information storage, update and appropriate retrieval are controlled at the molecular level by the neuronal cytoskeleton which serves as the internal communication network within neurons. Information flow in the highly ordered parallel networks of the filamentous protein polymers which make up the cytoskeleton may be compared to atmospheric flows which exhibit long-range spatiotemporal correlations, i.e. long-term memory. Such long-range spatiotemporal correlations are ubiquitous to real world dynamical systems and are recently identified as signature of self-organized criticality or chaos. The signatures of self-organized criticality i.e. long-range temporal correlations have recently been identified in the electrical activity of the brain. A recently developed non-deterministic cell dynamical system model for atmospheric flows predicts the observed long-range spatiotemporal correlations as intrinsic to quantum-like mechanics governing flow dynamics. The model visualizes large scale circulations to form as the result of spatial integration of enclosed small scale perturbations with intrinsic two-way ordered energy flow between the scales. Such a concept maybe applied for the collection and integration of a multitude of signals at the cytoskeletal level and manifested in activation of neurons in the macroscale. The cytoskeleton networks inside neurons may be the elementary units of a unified dynamic memory circulation network with intrinsic global response to local stimuli. Cite as: arXiv: chao-dyn/9809003.The neural network of the human brain responds as a unified whole memory bank to a multitude of input signals from the environment and functions with a high degree of robustness and stability. The three aspects of neural networks memory bank are, storage, real-time update and retrieval. The memory is believed to be embedded in the strength of the numerous connections or synapses in the network. Sensory inputs (electrical) produce particular patterns of activity in groups of neurons which then trigger optimal response to the input signal. The cooperative response of millions of neurons to a multitude of input signals has been compared to a very efficient parallel processing computer with neurons and their synaptic connections as fundamental units of information processing, like switches within computers. However, recent studies by Hameroff et al [1,2] and Rasmussen et al [3] show that neurons and synapses are extremely complex and resemble entire computers, rather than switches. The interiors of neurons (and other eucaryotic cells) are now known to contain highly ordered parallel networks of filamentous protein polymers collectively termed the cytoskeleton. Information storage, update and appropriate retrieval are controlled at the molecular level by the neuronal cytoskeleton which serves as the internal communication network within neuron. Organization of information at the molecular level in the cytoskeletal network contributes to the overall response of each neuron and the collective activity pattern of neurons then

governs the response to the environmental stimuli. The general awareness or consciousness of the individual to environment may also be governed by the overall background activity pattern of the neurons and their cytoskeletal networks. Coherent signal flow patterns in neural networks may form the basis for general consciousness and response to stimuli (external or internal). Inputs signals trigger spontaneous appropriate coherent pattern formation in the activity of the neurons with implicit spatial correlations in the activity pattern. The time variation of electrical activity of the brain as recorded by the Electro Encephalogram (EEG) exhibits fluctuations on all scales of time, i.e. a broadband spectrum of periodicities (frequencies) contribute to the observed fluctuations [4]. Power spectral analysis which is used to resolve the component frequencies (f) and their intensities shows that the intensity (power) of the component frequencies follows the inverse power law form 1/fB where B is the exponent. Inverse power law form for power spectra of temporal fluctuations imply long-range temporal correlations, i.e. long - term memory of short - term fluctuations. The signatures of short - term fluctuations are carried as internal structures of long - term fluctuations. Time variation of spatial activity pattern in neural networks therefore has inbuilt long - term memory. Neural network activity patterns therefore exhibit long - range spatial and temporal correlations. Such non-local connections in space and time are ubiquitous to time evolution of spatially extended dynamical system in nature and are recently identified as signature of self-organized criticality [5]. Examples of dynamical systems, i.e. systems which change with time include atmospheric flows, electrical activity of the brain, heart rhythms, stock market price fluctuations, etc. Extended dynamical systems in nature have self similar fractal geometry. Self similarity implies that submits of a system resemble the whole in shape. The world fractal coined by Mandelbrot [6] means fractional or broken Euclidean geometry appropriate for description of non-Euclidean structure generic to natural phenomena. The fractal dimension D is given by dlnM/dlnR where M is the mass contained within a distance R from a point within the extended object. A constant value for D implies uniform stretching on logarithmic scale for length scale range R. Objects in nature exhibit multifractal structure, i.e. the fractal dimension D varies with length scale R. Fractal architectures generic to nature support functions which exhibit fluctuations on all time scale, i.e. the fluctuations are irregular (nonlinear) and apparently chaotic. The association of fractal structures with chaotic dynamics has been identified in all dynamical systems in nature. Fractals, chaos and nonlinear dynamics or Chaos Science is now an area of intensive research in all branches of science [7]. Incidentally, Chaos Science began in 1963 with identification of sensitive dependence on initial conditions resulting in chaotic solution for computer realizations of deterministic nonlinear mathematical model of atmospheric flows and named appropriately deterministic chaos .The computed trajectory of time evolution exhibits fractal geometry. The discipline of nonlinear dynamics and chaos began with investigation of universal characteristics of deterministic chaos in nonlinear mathematical models of dynamical systems in all branches of science. In mathematics, the Cantorian fractal space-time is now associated with reference to quantum mechanical objects [8,9,10]. Further, El Naschie has shown that fractal structures (space-time) incorporate the golden mean equal to (1+Ö 5)/2 =1.618 in their architecture signifying ordered signal /information flow in the fractal network. The golden mean is incorporated in the fractal architecture of the cycloskeleton network [11] which plays a very important role in sub-consciousness to consciousness process integration [12,13]. Surprisingly similar chaotic behavior in space and time was found to be exhibited by all real world dynamical systems. Fractal structure to the spatial pattern concomitant with chaotic (irregular) dynamics has now been identified to be intrinsic to physiological and biological systems [14,15]. The branching interconnecting networks of neurons and intra-neuronal cytoskeleton networks are fractal structures which generate electrical signal pattern with self-similar fluctuations on all scales of time characterised by 1/fB power law behavior for the power spectrum. Such inverse power law form for spectra of temporal fluctuations implies long-range temporal correlations, i. e., long term memory of short term fluctuations or events. Fractal architecture of neural networks supports and coordinates information (fluctuations) flow on all time and space time scales in a state of dynamic equilibrium, now identified as self-organized criticality, is ubiquitous to natural phenomena (living and non-living) and is independent of the exact details of the dynamical processes governing the space-time evolution. The physics of self-organized criticality or deterministic chaos is not yet identified. The physical mechanism governing self-organized criticality should be universally applicable to diverse biological, physical, chemical and other dynamical systems. In this paper a universal cell dynamical system model for self-organized criticality applicable to neural networks of the brain is summarised [16, 17, and 18]. This model was originally developed to explain the observed

self-organized criticality in atmospheric flows [19, 20, and 21]. Therefore a brief description of the model with respect to atmospheric flows in first described followed by application to neural networks. Atmospheric flows exhibit self-organized criticality or long-range spatiotemporal correlations manifested in the self similar fractal geometry to the global cloud cover pattern concomitant with inverse power law form 1/fB for power spectrum of temporal fluctuations in meteorological variables such as temperature, pressure, etc. documented by Tessier et al [22]. The co-operative existence of fluctuations ranging in size (duration) from the turbulence scale of millimetres (seconds) to the planetary scale of thousands of kilometres (years) contribute to coherent weather pattern in atmospheric flows. Townsend [23] postulated that large eddies (waves) form in atmospheric flows as a chance configuration (envelope) of enclosed turbulent (small scale) eddies. A hierarchical continuum of eddies is therefore generated with larger eddies enclosing smaller eddies. Since large eddy is but the integrated mean of enclosed turbulent eddies, atmospheric eddy energy (kinetic) distribution follows normal distribution characteristics according to the Central Limit Theorem in Statistics. The eddy kinetic energy represented by square of eddy amplitude then represents the probability density. Such a result that the additive amplitudes of eddies, when squared, represent probability densities is observed in the subatomic dynamics of quantum system such as the electron or photon. Atmospheric eddy energy spectrum therefore follows quantum-like mechanical laws [19,20,21]. Condensation of water vapour in updraft regions of large eddies give rise to cloud formation while adjacent downdraft regions are associated with evaporation and cloud dissipation, thereby accounting for the discrete cellular structure to cloud geometry.\ Under the head of applications authors' describe the following: Frohlich [24] had described analogous self-organization of vibrational modes of all frequencies triggering coherent activity in biological functions. Insinna [25] has summarized Froehlich's coherent excitation concept as follows. More than 20 years ago Frohlich [24,26-28] introduced the concept of cooperative vibrational modes between proteins. Coherent oscillations in the range of 1010- 1012 Hz involving cell membranes, DNA and cellular proteins could be generated by interaction between vibrating electric dipoles contained in the proteins as a result of nonlinear properties of the system. Through long-range effects proper to Froehlich's nonlinear electrodynamics a temporospatial link, is, in fact, established between all molecules constituting the system. Single molecules may thus act in a synchronized fashion and can no longer be considered individual. New unexpected features arise from such a dynamic system, reacting as a unified whole entity [25]. Coherent Frohlich oscillations may be associated with the dynamical pattern formation of intraneuronal cytoskeletal architecture which coordinates and integrates information flow into the neuron and generates output signal. Hameroff and colleagues [1,29-31] have simulated such interaction in their cellular automata model. Conclusion they draw in is herein below: Fractal architecture to information flow path results in spatiotemporal integration of signals so that the fractal system responds as a unified whole to a multitude of input signals. Two disparate examples for such self-organized information flow networks are atmospheric flows and the neural networks of the human brain. **Cantorian Fractal Spacetime and Quantum-like Chaos in Neural Networks of the Human Brain A. M. Selvam (for reference s please see the original article. Models are given elsewhere in the papers series .** If The Holographic Principle Is True, Than It Must Be The Fundamental Principle Of Mind. The Brain Has No Way Around The Holographic Principle. For The Reductionist, The Holographic Principle Is The Ultimate Reduction. It Applies To The Most Minuscule Level Of What We Can Observe, And Beyond. For The Universalist, The Holographic Principle Gives Us The Ultimate Universal. It Extends To The Limit Of Our Universe, The Universal Holographic Boundary, And Beyond. For The Phenomenalist, The Holographic Principle Gives Us The Ultimate Ground Of Our Phenomenal Perception, Our Grounding In The Universal "Now," In The Now-Present Of The Universal Holographic Boundary, Moving Outward From Now-Pasts To Now-Futures. Wheeler (1988) Has Said That All Of Reality Is Information, And That Other Physical Quantities Are "Mere Incidentals." Information Monism Is Gaining Popularity In Physics. By Breaking The Dichotomy Of Between Information And Experience, We Find A Deep Connection Between Wheeler Is Monism And The Experiential Monism Of Whitehead, Sometimes Called Panexperientialism, Which Relates To The Later Theological Concept Of His Student, Charles Hartshorne, Panentheism, God Inside Of Everything (Hartshorne, 1964). There Thus Seems To Be A Convergence Of The Concepts Of Information, Experience, And Spirituality. There Is Not One Universe, But Many Parallel Universes, Or, More Accurately, A Vast Superposition Of Universes

Called The Multiverse (Penrose, 2004). However, As Explained In Our Treatment Of The Double-Slit Experiment, In The Observation Of Events On The Quantum Level, The Individual Observer Sees Only One Of These Vast Superpositions, Not A Summation Of A Vast Number Of Potentials. This Has Been Explained In Terms Of The Many Worlds Or Many Minds Theories, In Which There Are Multiple Copies Of The Same Observing Individual, But This Particular Issue Remains Unresolved, Leaving Physics Ungrounded In Reality (Penrose, 2004). The Concept Of One Mind Is Not Only More Parsimonious Than That Of Many Minds, But Also Lets Us Out Of The Bizarre And Counterintuitive Idea That There Are Multiple Copies Of Our Own Selves, Which Exist As Mere Potential, And Are Thus Not Actual. There Are Many Possible Universes, But There Is Only One Mind, Which Determines Events On The Quantum Level, And Thus Creates Our Universe. As We Had Discussed Previously, The Quantum Level Provides The Essential Ground Of The Holographic Principle, Such That Quantum-Level Holographic Surfaces Are Elaborated At Higher Levels, Manifesting Higher Orders Of Information From The Quantum "World." In This Process, There Is A Reduction Of The Wave Function Or Potentialities Of Quantum Fields. At The Level Of Consciousness, This Entails Freedom To Choose Which Observations We Make (Stapp, 1997), Which Gives Us The Capacity To Think And To Make Decisions. These Capacities Are The Basis Of Individuality, Self-Determination, Judgment, And Values. The Progressive Evolution Of The Manifestation Of Mind Through Higher Orders Of Experience, Leading To Higher Orders Of Consciousness, Is Entailed By The Holographic Principle. Again, We Are Dealing Here With Levels Of Description, With The Multiverse Of All Potentials Fundamentally Supporting The Single Universe We Collectively Observe. The Multiverse Is The Wave Function Of The Universe (Penrose, 2004). The Recursive Integration Of Nested Hierarchies Of Holographic Surfaces Brings Out A Single Actuality In Consciousness From A Wave Function That Is Unconscious. Consciousness Thus Gives Us Information At A Level Of Experience That Is Causal. In This Sense, We Partake In The Creation Of The Universe. We Participate In Creation, And This Participation, When Fully Realized, Leads Us To Higher Levels Of Consciousness And Of Realization. As A System, The Biosphere That We Live In Has A Holographic Surface, Creating A Deep Sense Of Ecology As We Collectively Move Toward A Planetary Consciousness. It Is Only When The Universal Holographic Boundary Reaches The Information Storage And Processing Capacity Needed For The Requisite Biological And Biochemical Complexity That Consciousness Evolves In Living Things. This Evolution Is Natural And Spontaneous, Since Consciousness Gives Rise To What Is Actual, As Opposed To What Is Merely Potential. In This Sense, Consciousness Is Still In The Process Of Creating Our Universe, And Levels Of Higher Consciousness Will Continue To Evolve. We Are All In The Same "Now," And That Now Is Defined By The Present Universal Boundary. This Assures Us That Our Experience Is Universal, And Does Not Pass With Time. This Is The Fundamental Basis Of Memory And Of Cognition. The Identity Of Mind And Brain Is A Myth. We Have A Continual, Internal Or Non-Local Relation With The Universe, As It Has With Us. Once This Mystery Is Resolved, The Myth Is No Longer Needed, And There Is A Confluence Of Science And Spirituality.Experience Is Primary, Information Is Secondary. We Can Only Gather Information From Experience, Whether It Is In The Laboratory Or In Life. We Cannot Measure The Information On The Surfaces Of Systems. The Physicists That Have Formulated The Holographic Principle For All Systems Are Quite Aware Of This, Or Else The Principle Would Have Been Established Or Discredited. We Cannot Measure What We Experience. It Is Intangible, Yet It Is All We Know To Be Actual. Everything Else Is Inferred. Because It Cannot Be Measured, It Has Been Fundamentally Disregarded By Mainstream Science. Materialism Is Considered Scientific, While Idealism Is Considered Unscientific. But Aren't Ideas, Fundamentally, Made Of Information? Brain Science Has Mistaken The Representation Of Information For Information Itself, And Has Tied Those Representations To Matter And Energy. Consciousness, The Highest Order Of Information, Has Generally Been Regarded As Superfluous, Something That Needs To Explained Away, Or Altogether Ignored. Yet It Is The Only "Thing" That Reaches Our Awareness. Consciousness Comes At A Price. For Everything That Becomes Conscious, There Must Be Something That Becomes Conscious. Consciousness Is Certainty, And Its Complement, Unconsciousness Uncertainty. Consciousness Is The Particle Nature Of Experience. It Has Definiteness About It. The Unconscious Is The Wave Nature Of Experience, It Is Like The Metaphysical Cloud Of Unknowing. If We Are The Most Conscious Of Animals, Then We Must Also Be The Most Unconscious. Perhaps This Is The Predicament Of Humankind. The Holographic Principle Of Mind Leads Us Naturally From The Most Fundamental Experiences,

Existing As Quantum Potentials From The Conformations Of Proteins Down Through The Fields Of Electrons, Through Their Manifestation Upward Through A Recursion Or Successive Applications Of The Same Holographic Process, Through Higher Levels Of Experience, To The Emergence Of Consciousness As Higher And Higher Orders Of Experience. The Quantum Holographic Principle Mind Does Not Require Anything More Quantum Than Is Obviously Present At The Microscopic And Submicroscopic Levels, As It Represents Successive Orders Of Manifestations From These Levels. Recursion Also Applies Here In The Sense That It Is Used In Computer Science, In Which The Function Of The Part Depends On The Function Of The Whole. A Program, As A Part, Cannot Work Without A Functioning Whole, The Operating System Recursive Wholes Which Are, For Us, Supra-Conscious, Are On The Group, Species, Planetary, And Universal Levels. As Individuals, Our Consciousness Cannot Function Without Recursion To The Universal Consciousness, Even Though We May Be Unaware That Such Universal Consciousness Exists. Reaching Upward To These Supra-Conscious Levels Is A Spiritual Process, Making Transcendence The Ultimate Solution To The Unconscious Human Predicament. What Was Once Supra-Conscious Becomes Unconscious Through A Process Of Conditioning? We Are Born As Creatures Of The Earth And Of The Universe, As Evidenced By The Beliefs And Practices Of "Primitive Peoples." There Is Evidence From Cave Paintings That Our Hominid Ancestors Experienced A Kind Of Holographic Perception (Combs, 1996), Which Could Constitute Our Early Connection With The Holographic Subtext Of Reality, And Which We Might Then Propose Existed In Our Animal Ancestors, And In Extant Animal Species. If This Is The Case, Than Microgenesis Would Entail The Recapitulation Of This Holographic Experience As It Progresses Through Our Ancestral Past. The Supra-Conscious Mind Seems To Envelope Perinatal Experiences, And Stanislav Grof (1994) Has Developed Techniques To Gain Access To These Experiences, As Well As To The Earlier Experiences Of Our Human And Animal Ancestral Lineage, And Of Our Universal History. Grof (1994) Concludes: "Our Consciousness Seems To Have The Amazing Capacity To Directly Access The Earliest History Of The Universe Ñ Witnessing Dramatic Sequences Of The Big Bang, The Formation Of The Galaxies, The Birth Of The Solar System, And The Early Geophysical Processes On This Planet Billions Of Years Ago." The Holographic Principle Theory Of Mind MARK GERMINE Institute For Psycho Science There Are Similarities Between The Patterns Of Holography And Of Psychological Transference, Where Holography Is The Process Of Recording And Reconstructing Holograms Employing A Theoretical Perspective Using A Hermeneutic Method, This Dissertation Parallels Holography With Transference, Offering Another Way To Encounter Transference By Showing Similarities Between The Processes Of Each And The Results Of Each. Though Complex, Infinitely Varying, And Unique, Their Patterns Are Clearly Identifiable. Thus They Are A Metaphorical Fit To The Concept Of Strange Attractors In Physics And A More Literal Fit To The Concept Of Archetypes In Depth Psychology Or Dynamic Psychology, Psychology Which Attends To The Living, Autonomous Unconscious. This Study Explains How Holography Models Transference, What A Hologram Is And How It Works, And How Depth Psychology Understands Of The Interaction Between Consciousness And The Unconscious Is Related To The Hologram. It Describes Transference And Related Psychological Processes As Understood In Six Different Schools Of Depth Psychological Thought. It Shows That The Underlying Pattern Or Strange Attraction Between Transference And Holography Extends To Other Processes Both Within And Outside The Field Of Psychology, Processes Such As Projection, Projective Identification, Splitting, Memory, Biology, Creative Discovery, Theology, Synchronicity, Chaos, And Nonlocality. By Identifying The Similar Patterns Of These Processes, This Study Demonstrates The Existence Of An Underlying Holographic Archetype In Which Essential Qualities Of The Whole Are Present In Each Of The Parts Of The Whole: The Visual Image Of The Overall Hologram Is Present In Each Component Part Of The Hologram, The Autonomy Of The Overall Human Is Present In Each Conscious And Unconscious Component Part Of The Human Psyche. By Noting Differences As Well As Similarities In These Processes, It Suggests An Inventory Of The Qualities Of The Holographic Archetype. This Study Furthers Understanding Of The Pervasiveness, Force, And Autonomy Of The Unconscious Acting Through Transference And Projection By Identifying A Group Projection Of Domestic Violence Lying At The Core Of The Christian Myth. This Study Also Furthers Understanding Of The Concept Of Transference By Providing A Reflection Hologram Of The Human Psyche As An Artistic Work And As A Visual Metaphor Of Transference. **Strange Attractors: Transference, Holography, And An Archetype Burke, J. (2003). Strange Attractors: Transference, Holography, And An**

Archetype (Doctoral Dissertation, Pacifica Graduate Institute, 2003). A host of observed, but very basic human phenomena, including consciousness itself,have eluded rigorous scientific description by all disciplines of science. This is true, not because of insufficient evidence for a particular phenomenon's existence, but rather forlack of a theoretical construct, which could fit within the prevailing paradigms of science. For the past century eminent men and women of science have accumulated thousands of pages of data on mind/mind and mind/matter interactions. Many of the most telling experiments have been criticized, perfected and repeated numerous times during the past five decades, using increasingly sophisticated technologies. Meta analysis of these experiments produce accumulated probabilities against chance occurrences exceeding trillions to one (Radin, 1997). It has required, however, that quantum science mature for seventy-five years and during that period, test, validate and synthesize a number of seemingly outrageous physical concepts arising from quantum theory, before testable theories could arise which offer hope that anomalous mind and consciousness data can be explained (Mitchell, 1996).The missing concepts that prevented the earliest investigators of consciousness from succeeding in their quest were 1) a generalized theory of information, and 2) quantum science itself, with the associated phenomena of non-locality, the zero point energy field and the quantum hologram. These associated phenomena are still not well understood butare sufficiently validated today by both theory and experiment to provide a basis for postulating a necessary condition for the existence of consciousness phenomena, as experienced in the observable four dimensional space/time universes. A third concept, chaos theory, is also necessary to understand the nonlinear evolutionary processes that caused consciousness to evolve toward the anthropic consciousness experienced by humans. In particular, chaos theory maps far from equilibrium systems and demonstrates the irreversibility of nonlinear processes and thus the irreversibility of time in the macro-scale universe. Another class of phenomena, including normal sensory perception and evolution, to cite but two, have explanatory theories in classical science, but which in view of current developments in late quantum physics and in chaos theory may be incomplete approximations to the correct theory. Information concepts have been examined by Weiner, von Neuman and Shannon in well-known seminal works and by Frieden more recently (1998) to produce theories useful to physics, to computation and to communications technologies. These theories, although accurate and mathematically useful in their domains, fall short of being sufficiently encompassing when considering the problem of consciousness, its evolution and its associated phenomena. Even relatively simple perceptual organisms utilize patterns of energy, that is, information, not completely described by existing mathematical theories. Theory and experimental evidence for the zero point energy field has been published by many authors, but I shall cite Haisch, Rueda and Puthoff, (1997, 1998), as the most contemporary and relevant work for this paper. Theory and experimental evidence concerning the quantum hologram has been developed by Schempp (1992, 1993) andMarcer (1996, 1997, 1998), separately and jointly, based upon a new understanding ofquantum mechanics. (See previous work by Cramer [1986], Berry [1988], Anandan [1992] and Resta [1997]).Non-locality, although predicted by the earliest work in quantum theory and decisively demonstrated by Aspect in 1982, has been thought to be a curious property of particle physics but of little relevance to macro-scale reality until discovery of the quantum hologram. Further, it is widely believed that non-local quantum information represented by entanglement of particles could not be recovered locally as useable information (Eberhard's theorem). However recent work both in theory and experiment (e.g. see Nature, 1997, 11th December, vol. 390, Sudbury T pp551-552, and Bouwmeester D. et alp 575-579) is in line with the work by Berry, Resta, Schempp and Marcer and makes it clear that this is not the general case for quantum information processing andcommunication.It has been widely accepted in science, until recently, particularly in the field of artificialintelligence, that the brain was likely a complex classical computer, incapable of supporting quantum processes. The work by Hammeroff (1994) and Penrose in isolating and describing microtubules in brain tissue have caused a re-examination of this dogma, and renewed interest in uncovering the quantum processes involved.Based upon this earlier work I postulate and examine the evidence in this paper for thefollowing theories: 1. The basis of subjective experience is rooted in the quantum attribute of naturecalled non-locality. I will use the word "perception" in its most generic sense to denote a basic subjective experience at all levels of complex matter. Thus the non-local quantum correlation between entangled quantum particles is considered the root cause of the phenomenon experienced as perception in more complex matter, but the non-local quantum hologram is the non-local carrier of information for molecular and larger scale matter. Thus, perception is not an object but rather the label for a

nonlinear process involving an object, a percipient and information.2. The experience of humans is that they sometimes, perhaps often, perceive information from or about physical objects that is not available through normal, local, sensory mechanisms, nor classical space/time information. Objective testing data in overwhelming abundance provides evidence that this is true, though an explanatory mechanism has until contemporary times remained elusive. I shall call this intuitive information or intuitive perception. I postulate that a quantum hologram is the source of this intuitive perception and that the percipient is at that time in phase-conjugate-adaptive-resonance (pcar) with the entity or object associated with the quantum hologram.3. The phenomenon of "learning" in humans is a subjective process that involves perception, memory, intentionality, and evaluation of outcome and behavior change. This may be viewed as a classical nonlinear feedback loop. Although we cannot know precisely the subjective experience of another entity, presumably in the successful training of animals, an analogous subjective process is in effect. Sheldrake (1981) has published a successful theory of morphic resonance related to animal learning based upon non-local information. Marcer has published papers (1996, 1997) theorizing a mechanism by which the quantum hologram causes learning to take place in both DNA molecules and prokaryote cells as an adaptation process of environmental resonance, rather than mutation and adaptation solely by random processes. I postulate that Marcer's concept can be generalized to nature alarge and that the quantum hologram is the information structure suitable to explain Sheldrake's morphic resonance. The non-local quantum correlations observed in particles, and the non-local quantum hologram associated with molecular and larger scale objects, serve the purpose of providing information at all scale sizes to guide evolutionary processes. That is to say, that quantum non-locality is the basis of perception, and thus fundamental and necessary to the complex organizations of matter and information in the universe. Further, since learning is an observed property of complex systems such as animals and, via the quantum hologram, is theorized to be a property of simple cells and molecules, one can also postulate the generalization that nature evolves through a learning process rather than because of random mutations.4. Marcer (1997) has proposed that the condition of phase-conjugate-adaptive-resonance (pcar) is a necessary condition for an object in three-dimensional reality to be perceived as it really is. That is, resonance requires a virtual path mathematically equal but opposite to the incoming sensory information about the object. Further, that it is the incoming electromagnetic (space/time) information (visual, acoustic, etc), which decodes the information of the quantum hologram and establishes the condition of pcar so that accurate three-dimensional perception is possible. That is to say, both quantum information and space/time information aroused in the act of perception by organisms having sensory preceptors. I propose that the two equal but opposite paths required by the pcar condition are the mathematical equivalent of perception and attention (or intention). (I shall distinguish between attention and intention in following pages.) Discussion The anecdotal evidence for humans perceiving non-local information dates to prehistory. The data were sufficiently robust that both experiences and philosophers, from Plato and Aristotle forward, accepted that both physical and non-physical realms of reality must exist. Non-physical was thought to explain the subtle, ephemeral and mystical subjective experiences ubiquitously reported in human culture. After Descartes and Newton, however, classical western science rapidly discarded the non-physical hypothesis and systematically began to ignore all evidence for perception of non-local information Field theories and point particles were created to preserve the concept of physical contact between particles and to explain obvious examples of "spooky action at a distance" such as gravitation and electromagnetic interactions. Information, broadly defined as patterns of energy, reemerges however, in non-local form in the mysterious quantum spin correlations of double slit experiments, although it has been widely believed that such non-local information could not be recovered and utilized by sensory systems. With validation of theory and experiments concerned with the non-local quantum hologram, information, including non-local information, suddenly acquires a more important status in physical theory, a status as important as energy itself. This is true because information is the basis of the cognition and knowing by which creatures perceive reality, and non-local information can now be seen as a ubiquitous and useful property of the cosmos, rather than a unique attribute of particles (and human animals). It is likely that most, if not all, subtle, ephemeral and unexplained phenomena associated with subjective experience are connected, directly or indirectly, with the phenomenon of non-locality. The brain is clearly a quantum computer (Schempp & Marcer, 1996) which utilizes both quantum and space/time information. This discovery alone almost certainly sets a necessary, but not sufficient condition, for intelligent life to have arisen in the cosmos, wherever environmental conditions permit. Many volumes have been

written in this century by scientists experimenting with remote viewing, ESP, telepathy, clairvoyance, precognition, etc. Police agencies routinely use “psychics" to assist in criminal cases often with success. Intelligence agencies of governments have clandestinely utilized the findings to successfully gain information about an adversary. Many reports of these activities have been recently declassified and printed in open professional journals, even though no explanatory physical mechanism has yet been reported which is acceptable to mainstream science. The most succinct modern summary of this activity and analysis of results have been published by Radin (1997). Quantum Holography Non-locality and the non-local quantum hologram provide the only testable mechanism discovered to date which offer a possible solution to the host of enigmatic observations and data associated with consciousness and such consciousness phenomena. Schempp (1992) has successfully validated the concept of recovery and utilization of non-local quantum information in the case of functional Magnetic Resonance Imaging (fMRI) using quantum holography. Marcer (1995) has made compelling arguments that a number of other chemical and electromagnetic processes in common use have a deeper quantum explanation that is not revealed by the classical interpretation of these processes. Hammeroff (1994) and Penrose have presented experimental data on microtubules in the brain supporting quantum processes. The absorption/re-emission phenomena associated with all matter is well recognized. That such re-emissions are sufficiently coherent to be considered a source of information about the object is due to the theoretical and experimental work of Schempp and Marcer, based upon the transactional interpretation of quantum mechanics of Cramer (1986), the Berry geometric phase analysis of information (Berry, 1988; Anandan, 1992) and the ability of quantum phase information to be recovered and utilized (Resta, 1997). The mathematical formalism appropriate to these analyses is consistent with standard quantum mechanical formalism and is defined by means of the harmonic analysis on the Heisenberg nilpotent Lie group G, algebra g and nilmanifold (see Schempp (1986) for a full mathematical treatment). The information carried by a quantum hologram encodes the complete event history of the object with respect to its three dimensional environment. It evolves overtime to provide an encoded non-local record of the "experience" of the object in the four-dimensional space/time of the object as to its journey in space/time and the quantum states visited. The question of the brain’s ability, as a massively parallel quantum processor, to decode this information is addressed by Marcer and Schempp in "Model of the Neuron Working by Quantum Holography" (1997) and "The Brain as a Conscious System" (1998). They argue that an organism’s ability to perceive objects as they are and where they actually are in three-dimensional reality requires the phase conjugate relationship provided by quantum holography. It is not sufficient for the incoming electromagnetic illumination (or acoustic signal) carrying object information to present tothe brain a wave front in the manner presented to a flat photographic plate. Rather, a virtual signal as mapped by the phase conjugation of quantum holographic formalism is required to decode the information in order for perception and cognition to exist as we experience it in three dimensional realities. The percipient and the source of information are in a resonant relationship for the information to be accurately perceived. Many investigators have proposed a holography mechanism as a basis for brain functioning, beginning with Pribram, and indeed, others have proposed holography as a construct forth universe itself, but discovery of the non-local quantum hologram created by the absorption/remission phenomenon and characteristic of all physical objects provides the first quantum physical mechanism compatible with macro-scale three dimensional worlds we experience it. The existence of a quantum hologram associated with each physical object provides each physical object with the non-local waveform predicted by quantum theory’s wave/particle duality and extends quantum theory to all physical matter. It allows, for the first time, possible approach for understanding the mysterious world of consciousness. Postulating that this is globally true, we inhabit a quantum world where non-local effects should be expected at all levels of functioning, not just as a curious artifact of the subatomic level of reality. Existence of the non-local quantum hologram suggests that nature has utilized non-local information from the big bang forward, throughout its evolutionary history; and long before planetary environments self organized to permit living matter and complex space/time sensory systems to evolve. The papers of Marcer and Schempp on learning inherent in DNA and prokaryote cells using quantum holography, when generalized, helps explain the ubiquitous appearance in nature across distances, scale sizes and species, of similar processes, organs and sensory systems. This certainly conforms to the fractal geometry of chaos theory. Certainly the similarities, of DNA, cell structure, organs and brains across species are easier to reconcile with a non-local learning process than with a theory of localized random mutation and natural adaptation. It is important to

observe that in standard particle physics experiments the object is to discover the quantum characteristics of the individual types of particles, and the conditions under which they split and recombine. In quantum holography the object is to treat the entire group of re-emitted quanta as a whole, and as in laser holography, to examine the information carried in the interference pattern and phase relationships. These represent two quite different levels of approach to quantum information. In particle experiments, it is considered that the eigenvalues of the applicable matrix represent measurable values; and that information is lost during measurement due to decoherence of the particles and energy exchange. But in the quantum holographic formalism, the information is carried in the phase relationships, which are represented by off-diagonal terms in the matrix, and the information is recoverable under the proper conditions as Berry and Resta have predicted, and as Schempp has demonstrated with fMRI. The quantum mathematics is consistent with standard quantum theory in both cases. In decoding the quantum holographic information, however, the energy exchange is insignificant. The similarity of the mathematical treatment in these various experiments is important tothe thesis of this paper. In examining the quantum non-locality of particles it is spin numbers and/or polarization that are the parameters of interest. A standard technique of analysis is to use the Fourier transform to map the state of the particles into the frequency domain. In the formalism of the quantum hologram, mapping into the frequency domain is also fundamental; however, the requirement for pcar assures that the phase relationships are matched so that the percipient (sensory system) is able to decode the information carried in the phase relationships. It is precisely the pcar requirement that permits the encoded holographic information to be decoded by the percipient. Mathematically, decoding is simply reversing the rotation of the phase vector in phase space. Physically, it is matching the frequencies and phase of the information such that resonance results. Frequency, phase matching and resonance are an operational characteristic of every type receiver technology. Pribram's earliest proposal that the brain stored information encoded as in a hologram and mapped by the Fourier transform is in complete agreement with the evidence presented by quantum holographic mathematical formalism. It is the spin and polarization attributes of particles (both are mapped by wave mathematics) that represent the puzzling non-local property of subatomic matter. It is the phase relationships that carry the information in holography (again mapped by waveform mathematics). And it appears that the brain stores and manages information not as a classical digital machine, but rather as an analog device using non-local properties of thequantum hologram, which can be analyzed by wave form mathematics (harmonic analysis on the Heisenberg Lie group). In the cosmological evolutionary scheme of things this similarity of appropriate mapping techniques is too bizarre a coincidence to be ignored Asa cosmic accident. Thus there is ample evidence that the **non-local attribute of** nature is much more than just a curious artifact of subatomic particle interactions, but rather is a more fundamental phenomenon that appears at all scale sizes and is, in particular, associated with the utilization of information in nature, and associated with the fact that information has a causal effect independent of distance. It is precisely information, however, that is the basis of the phenomena of perception, cognition, memory, learning etc, that is to say, consciousness and the subjective experience. Though the evidence is quite ample to postulate that non-locality is the unique, universal basis for perception and the subjective experience, the evidence though compelling is not sufficient to be conclusive that such is indeed the case. The next steps are to validate more completely with experimental evidence that non-locality plays a major role at all scale sizes and that all physical objects are quantum objects and thus interconnected by information in this strange way. **Non-Locality in Nature**: There is experimental evidence to strongly suggest that simple organisms perceive and respond to information non-locally as well. Cleve Backster was perhaps the earliest to experiment with plants and simple life forms in electromagnetic isolation in late 1960'sand early 1970's. His work was not confirmed through replication by others at that time. Other investigators have had mixed results replicating non-local information perception by simple organisms and living tissues. In the area of human experimentation, results likewise have been mixed and controversial for three-quarters of a century. However Meta analysis by Radin (1997) and independently by Utts (1991) across a large and appropriate spectrum of experiments demonstrates compelling statistics that the **perception of non-local information exists and is real.** Perhaps was there a larger body of experimental evidence for simple life forms, similar Meta patterns would emerge. Failure to replicate results in well constructed experiments does not, in the case of subtle consciousness phenomena, prove that the phenomenon is missing but rather that a **hidden mechanism** below the threshold of classical measurement is operating. For example, the most telling experimental evidence to explain the sometimes

inconsistent results relates to direct non-local observer and/or experimenter effects. Gertrude Schmiedler isolated the **"sheep/goat "**effect in human experimentation decades ago (1972). Experimenters and/or participants in a human telepathy (or similar non-local) experiments exhibited results statistically above or below chance results depending upon their subjective bias toward the experiment. (In other words, 100% wrong answers would be as statistically significant as 100% correct answers in such tests, and in addition betrays the mind set or intention of the subject; whereas only chance results would be inconclusive.) More recently, a series of experiments by Marilyn Schlitz (1997) investigating "intentionality" clearly demonstrated that experimenter bias (intentionality) affected the outcome even of double blind experiments. Thus, in the subtle realms of mind and consciousness studies, bias, belief and intention clearly have an effect. The lack of an existing theoretical structure in classical science to support any type perception of non-local information, much less to support bias, belief or intention as having a non-local effect, when in fact it does have a non-local effect, is quite sufficient to account for anomalous results in many scientific experiments. Further validation and acceptance of the non-local thesis will have strong positive repercussions for the prevailing scientific paradigm and particularly the theory of measurement. **Attention and Intention:** A powerful and telling series of experiments conducted by Dean Radin (1997) at University of Nevada at Las Vegas following a decade long set of equally significant experiments by Brenda Dunne and Robert Jahn at Princeton University (1988) provide insight as to the subtleties involved in this level of mind/brain functioning. Jahn and Dunne provided overwhelming evidence that subjects could intentionally produce statistically skewed results in mechanical processes normally thought to be driven by random processes. Radin went further; he discovered that audiences watching a stage performance would skew the output of nearby random number generators during periods of high emotional content in the stage performance. Further, in a wide-ranging audience participation experiment, he recorded the output of computer random number generators during the television broadcasts of the O.J. Simpson murder trial. Most television media reported this event for weeks on end and tens of millions of humans were watching the results. Again, the results of the random number generators were skewed corresponding to emotional peaks during the trial drama and corresponding to the number of people watching television. The thesis in the Princeton experiments was that participant intentionality created a non-random effect to bias the skewed distribution. In the Radin experiments the results were not intentional, as the participants were unaware of the experiment, but the hypothesis was that attention (in particular, rapt attention) drove the system away from chaos (randomness) and toward greater order (reduced entropy). These results suggest that attention and intention provide closely correlated outcomes, further, that randomness may not be a general property of nature, but that what is perceived as random noise in a system may be information (a pattern of energy) that is not in resonance at that moment with the particular perceptual system. William Tiller, emeritus professor at Stanford also has performed experiments (1997) that are consistent with these results, though his interpretation of the operating mechanism is somewhat different. These different types of mind/mind, mind/matter experiments have been rigorously and routinely conducted for decades with statistically compelling results but just as routinely dismissed or ignored by main stream science because the implications of non-local action are so foreign to the classical paradigm. However, if we consider that the condition of phase-conjugate-adaptive-resonance is necessary to completely specify the act of perception as described in the mathematical formalism of the non-local quantum hologram by Marcer, then we may also consider the perceived object and the percipient's perceptual system as locked in a resonant feedback loop. The incoming wave front carrying information may be labeled as "perception" from the point of view of the percipient, and the return path required by the resonant relationship may be labeled "attention" (or for subsequent discussion, "intention"). It is a well established principle in the meditative practices of esoteric disciplines that prolonged focused attention on a object of meditation causes the percipient and the object to appear to merge so that a deeper level of information about the object is obtained; information such as history or internal functioning, that would not be available through classical space/time information. The concept of the quantum hologram adequately and completely describes how this phenomenon might take place. Further, it is accepted that the mind/brain is a massively parallel processor, capable of performing many tasks simultaneously and subconsciously (in the right, intuitive part of the brain). Attention (meaning conscious, focused attention) is a unique and singular task that must take place sequentially, mostly in the left cognitive part of the brain. The condition of attention deficit disorder (ADD) is precisely the problem of a percipient being unable to maintain a singular focus for sufficient time to

complete a desired task or observation. Thus, the action of focusing attention by apercipient may be construed as a necessary condition for pcar to be established with the perceived object. Non-Locality, Near and Far Marcer has presented the case for the pcar requirement in normal sensory perception (visual and acoustic). A frequent modality used by psychic sensitive individuals to gain information is to physically touch an object. Touching an object satisfies the pcar requirement and presumably allows the percipient access to information about the object not available from space/time information. Police agencies frequently use this modality with psychic sensitives to gain information about a crime scene, much as they utilize bloodhound to track the scent of an individual, often with considerable success. If, as in the theory of the quantum hologram, the object has been in the presence of the individual about whom information is desired, the event history of the object and that of the individual intersect. The Berry phase information of the object contains its journey in three dimensional space and time, as well as the quantum states through which it has passed on this journey. The sensitive individual, with a honed talent, seems often able to decode useful Berry phase information from the object about the individual sought. It may also be the case with the blood hound, which additional non-local information has been gained about the subject, even though the classical explanation is that the animal is operating only with heightened olfactory sensing. Although perception in the three dimensional world requires and utilizes pcar, most humans, however, do not bring to conscious awareness non-local information when weare routinely operating in three-dimensional reality. We perceive objects as presented by space/time information, that is, shape, color, function (tree, chair, table, etc) but are not usually aware of the additional non-local information. It takes training as provided by many of the esoteric traditions and/or certain naturally sensitive individuals to routinely perceive the non-local holographic information associated with a particular object. There is massive evidence to suggest, however, that the brain has these latter capabilities at birth. Suppression by cultural conditioning in childhood and subsequent lack of practice cause the natural ability for conscious, intuitive perceptions to atrophy. Particularly in western tradition, educational interest has been on the left brain, rational functions rather than right brain, intuitive functions. However, mystic adepts and natural psychics' routinely demonstrate that non-local information is perceptible from physical objects by focusing attention, quieting the left brain and allowing intuitive perceptions to appear. It is the left brain cognitive ability in humans that provides canonical labeling of the intuitive and artistic processes taking place in the right brain. The fact that with training and practice, individuals can recover, deepen and label their individual cognitive access to intuitive, on-local information demonstrates that learning is taking place within the whole brain itself and involves enhanced coherence and coordination between the hemispheres. This process is different and distinct from the left brain function of extending and extrapolating factual data and logical deduction to leap to an "intuitive" conclusion, while omitting the intermediate steps leading to that conclusion. Experimental protocols for remote viewing normally provide clues to the location of the object such as adescription, a picture, or location by latitude and longitude, that is to say, an icon representing the object. These clues seem to be sufficient for the percipient to establish a resonance with the object. Normal space/time information (visual, acoustic, tactile) about the object is not being directly perceived by the percipient, nor does the object usually appear at its physical location in space/time like a photograph or map in the mind. Rather, the information is perceived and presented as internal information and the percipient must associate the perceptions with his/her internal data base of experience in order to cognize and to describe the object's perceived attributes. In the case of complex objects being remotely viewed, the perceived information is seldom so unambiguous as to be instantly recognizable as correct. Sketches, metaphors and analogies are usually employed to cognize and communicate the non-local information. A considerable amount of training, teamwork and experience are necessary to reliably and correctly extract complex non-local information from a distant location. The information appears to the percipient as sketchy, often dream-like, and wispy, subtle impressions of the remote reality. Very skilled individuals may report the internal information as frequently vivid, clear and unambiguous. The remote viewing information, being strictly non-local, and in this hypothesis, the information perceived by quantumholography, is missing the normal space/time components of information necessary to completely specify the object. It has been demonstrated that this intuitive mode of perception can be trained in most individuals. Perhaps additional training and greater acceptance of this capability will allow percipients to develop greater detail, accuracy and reliability in their skill. In principle, training will not only increase the skill and accuracy but should cause the appropriate neural circuitry to become more robust as well. In

the absence of space/time (electromagnetic) signals to establish the pcar condition and to provide a basis for decoding the quantum hologram, an icon representing an object seems to be sufficient to allow the brain to focus on the object and to establish the pcar condition. However, a reference signal is also required to provide decoding of the encoded holographic phase dependent information. Marcer (1998) has established, using Huygen's principle of waves and secondary sources, which any waves reverberating through theuniverse remain coherent with the waves at the source, and are thus sufficient to serve as the reference to decode the holographic information of any quantum hologram emanating from remote locations. The Zero Point Field: The results of the Michelson/Morley experiment banished the concept of aether from early twentieth century physics. However, it left a void as the nature of interstellar space and nothing for propagating waves to wave in. Quantum physics reincarnated the aether as the zero point fields, a seething cauldron of quantum potential and unmanifest energy where particles and antiparticles spontaneously arise and then disappear. The very fabricant structure of space/time itself is again in question; its structure and its metric under intense investigation with far more questions than answers having emerged to date. For the purposes of this paper the relevant issues are two: 1) the emission/absorption phenomenon, and 2) the structure and mechanics of non-locality. Zero point (zero degrees Kelvin) emission and absorption of quanta from all physical objects is a well established phenomenon. It is our view that the zero point fields are the plenum (or cauldron) which supports this absorption and re-emission, and makes the phenomenon of the quantum hologram possible at all temperatures. Although particle experiments are carried out under rigid conditions of temperature and pressure, Schempps experimental work with the fMRI requires no such constraints. There are deep and difficult questions yet to be answered about how the information of thequantum hologram maintains its integrity and is propagated, about how resonance takes place at extremely large distances. There is considerable evidence that intuitively perceived information is truly non-local. It does not obey the inverse square law for space/time energy propagation; it is time independent and cannot be shielded by electromagnetic shielding. Such characteristics are the mark of non-locality. But understanding the mechanics of non-locality (or a visual picture) is missing from standard models. Some physicists turn to superluminal speed of propagation, others to the zero point as a zero dimension, which is resonant with all parts of the universe simultaneously. The issue of instantaneous communication (or at least superluminal communication) of non-local effects on a cosmic scale remains a problem, even though the phenomenon itself is well validated. Perhaps it is a problem of topology. What shape can the universe have such that one point can be in simultaneous contact with all other points? In this regard it is clear that certain problems between quantum mechanics, special and general relativity remain inexistence. Haisch, Puthoff and Rueda continue to investigate the **metrics of the zero point fields** with regard to better defining the unanswered questions about, mass, gravitation and inertia. Perhaps these investigations will also bring answers for how phase related information is propagated non-locally, likely within the zero point fields, and thereby unveil the mechanics of the resonance phenomenon. Further, new investigations reported by Van Flandern (1998) on measurements from orbiting Global Positioning System (GPS)clocks indicate the predictions from Lorentzian relativity to be approximately four times more accurate (0.7% opposed to 3%) than predictions of special relativity. If these measurements are further validated, it implies that Lorentzian relativity with a Hubble absolute rest reference frame, an aether (zero point field), and instantaneous propagation of non-local effects may be the preferred one. If this is the case, then many questions about non-locality would be resolved. Intentionality I have argued that by establishing pcar between a percipient and an object, the phase conjugate (equal but opposite) paths connecting the two can be labeled "perception" and "attention". In the case where the object is a simple physical object (rock, flower, etc.), our interest is on the non-local information perceived by the percipient about the object. However, from the point of view of the object, information about the percipient is also available to the object. The pcar condition is a reciprocal relationship, mathematically. Quantum holographic formalism predicts that the history of events of quantum objects is carried in the quantum hologram, thus we must conclude that the **"attention" focused upon the object causes that event to be recorded in that object's quantum hologram.** Although we cannot query the object about its experience perhaps an experiment such as one utilizing the Aharonov-Bohm effect would detect a phase shift in the object's holographic field. (In this discussion I use anthropic labeling as we are discussing human perception. The phenomena however, are rooted in natural (and primitive) non-local physical processes, which are fundamental. The evolved complexities of perception, cognition, etc, associated with a brain obviously, as yet, have no analogous label

to describe the experience of simple objects.)Once the pcar condition is established, the percipient can change its mind state with regard to the object. The perceived information can be operated upon by the brain's function so that cognition occurs with respect to the perceived information and meaning assigned. Cognition and meaning require finding a relationship between the perceived informationand the information residing in the percipient's memory. The percipient can then form intent with respect to the object. In such case the path I have labeled "attention" could Figure 1 In self aware animals (those with a brain) cognition, meaning and intent with respect to an object can often be described in simple terms, for example: enemy, fight or flight; food, eat; friend, greet, etc. The non-local component of information, although present and creating effect, is operating below the level of conscious perception in humans and results in "instinctual" subconscious behaviors in all animals. The brain as a massive parallel computer is simultaneously performing numerous tasks to accomplish the desired intention. Classical modelings of this autonomous activity describe it only in terms of classical information and energy flow in the central nervous system and the brain. However, if non-locality is operating at all levels of activity, as this theory suggests, certainly there are resonances involving non-local information operating throughout the body of an organism in parallel with classical space/time functions. Subsequent experimental work will surely uncover these quantum processes where non-local resonance is involved in the functioning of an animal's internal processes. In the case of non-local effects at a distance, outside the body, simple spin correlations of entangled particles is the most basic. The spin coherence is reciprocal. Action on one particle creates an effect on other entangled particles. The non-local information is causal of affects at large distances. It is no less important for macro-scale objects. Sheldrake (1995) proposed and others conducted experiments with dogs whereby the animals correctly anticipated their owners' departure from work to return home. He proposed other successful experiments where rats learning a new maze benefited non-locally from the experience of others that had previously learned the maze, in the total absence of classical space/time information. It is not surprising then, that human's exhibit an even wider range of reactions to non-local information. The evidence suggests that humans can perceive, cognize, and give meaning to non-local information across a range of complexity, from inanimate objects, simple organisms, animals and other humans. The existence of quantum holography provides an adequate informational structure to permit a theory for the observed results. The case is a classic case in phenomenology, where results are repeatedly observed over time that fall outside the prevailing paradigm, and must await new developments in science before an explanation is forthcoming. The results for intentional effects of non-locality should be no more difficult to accept than the results for perception. The pcar relationship implies symmetry, that is, information flows in both directions between object and percipient such that each is object and each is percipient. Only the complexity of the more ordered perceiving system suggests a non-symmetrical relationship. We humans have great difficulty in accepting that thoughts, specifically intentionality, can cause action at a distance. Yet, it has been observed for centuries and in recent decades subjected to scientific scrutiny. Were not prayer to have produced some positive results, religion would have been abandoned centuries ago. That cause was ascribed to supernatural agency rather than non-locality is simply, again, phenomenology needing to wait while science caught up. Modern studies by Dossey (1993), Byrd (1988) plus many others have attempted to document the efficacy of prayer, particularly healing prayer. The results in most cases are very suggestive of non-local effects, and some claim they establish the case for healing prayer. However, the difficulties of controlling all variables in such clinical studies leave many avenues for valid criticism. The fact that Radin's several studies (1997) demonstrated that attention alone produced non-local results in machines, i.e., reduced randomness (increases order) does confirm that information has non-local effect and may be correctly formulated asnegentropy. These results apply directly to healing prayer as well.The case for pcar conditions to create remote effects by transfer of non-local information between equally complex percipients, humans for example, is not difficult to understand. Indeed, hundreds of successful experiments establish the case. In these cases no energy transfer is required, only non-local information, as each percipient/object has access to its own energy source. The case for intentionally creating remote physical effects in inanimate objects is more puzzling. Even though teleportation of quantum states has been successfully accomplished for particles, and numerous studies (Radin, 1997; Dunne and Dunne, 1988) show that macro-scale objects can also be changed or moved, the energy transfer mechanism by which the classical states of a remote object are affected remains elusive. Conclusions The case for mind/mind and mind/matter interactions is impressively well documented over many decades as studies in

phenomenology, with staggering probabilities against chance having produced the results. The discovery of the non-local quantum hologram, which is theoretically sound and experimentally validated in at least one application, thé fMRI, is sufficient to postulate that the quantum hologram is a solution to the foregoing enigma. Further, recognition that the quantum hologram is a macro-scale, non-local, information structure described by the standard formalism of quantum mechanics extends quantum mechanics to all physical objects including DNA molecules, organic cells, organs, brains and bodies. The discovery of a solution which seems to resolve so many phenomena, and also that points to the fact that in many instances classical theory is incomplete without including the subtle non-local components involved, suggests a major paradigm change must be forthcoming. The papers already published by Marcer and Schempp proposing a learning model both for DNA and prokaryote cells, which uses quantum holography, suggests that evolution in general is driven by a learning feedback loop with the environment, rather than by random mutations. This solution to biological evolution was proposed by Lamarck in 1809 but discarded for the mechanistic solution of random mutations by the colleagues of Darwin. The fact that non-local correlations and non-local quantum information can now be seen as ubiquitous in nature leads to the conclusions that the **quantum hologram can properly be labeled as "nature's mind"** and that the intuitive function we label in humans as the "sixth sense" should properly be called the "first sense". The perception of non-local information certainly preceded and helped to shape, through learning feedback, the sensory systems that evolved in planetary environments, and which we currently label as the five normal senses. We must conclude that evolved, complex organisms, which can form intent, can produce and often **do produce non-local causal effects associated with that intent.** Further, that attention alone produces coherence in nature that in some measure reduces randomness. Finally, I conclude that the cited experiments and current understanding of non-locality in nature is sufficient to postulate that **non-locality is the antecedent attribute of energy and matter which permits perception and is the root of the consciousness which manifests in the evolved organisms existing in three dimensional realities**. ReferencesAnandan, J. "The Geometric Phase"; Nature, 360,26, 307-313 (1992)Berry, M. V. "The Geometric Phase" Scientific American, December, 26–32 (1988)Byrd, R. C. "Positive therapeutic effect of intercessory prayer in a coronary care population"Southern Medical Journal 81 (7): 826–829 (1988)Dossey, L. "Healing Words" Harper San Francisco, (1993)Dunne, B. J., Nelson, R. D., and Jahn, R. G. "Operator-related anomalies in a random mechanicalcascade" JSE 2:155-80 (1988)Frieden, B, Roy "Physics from Fisher Information", Cambridge University Press (1998) Haisch, B., Rueda, A., Puthoff, H.E., "Physics of the Zero-Point Field: Implications for Inertia, Gravity and Mass". Speculations in Science and Technology 20 99–114 (1997) Haisch, B., Rueda, A., Puthoff, H.E. "Advances in the Proposed Zero-Point Field Theory of Inertia", 34th AIAA Joint Propulsion Conference, paper AIAA 98 3143. (1998) Hammeroff, S.R. "Quantum coherence in microtubules: A neural basis for emergent consciousness?" Journal of Consciousness Studies 1:91–118 (1994) Marcer, P. J. "Getting Quantum Theory off the Rocks: Nature as we consciously perceive it, asquantum reality! Proc. 14 International Congress of Cybernetics, Namur, Aug 21–25 (1995) Marcer, P. J. & Schempp, W. "A mathematically specified template for DNA and the genetic code in terms of the physically realizable processes of quantum holography", Proc. The Greenwich Symposium on Living Computers, eds. Fedorec, AM, Marcer, PJ, pp. 45–62 (1996)Marcer, P. J. & Schempp, W. "Model of the Neuron Working by Quantum Holography "Informatica 21:519–534 (1997)Marcer, P. J. & Schempp W. The model of the prokaryote cell as an anticipatory system working by quantum holography, Proc. 1st Intern. Conf. Computing Anticipatory System (1997)Marcer, P. J. "The Jigsaw, the Elephant and the Lighthouse", Proceedings of ANPA 20, (1998) inpress.Marcer P. J. & Schempp, W. "The brain as a conscious system" Inter Journal of General Systems,(1998)Mitchell, E. D. & Williams D. "The Way of the Explorer" Putnam, NY (1996)Puthoff, H. E. & Targ, R. "A perceptual channel for information transfer over kilometer distances:Historical perspective and recent research." Proceedings of the IEEE 64: 329–354 (1976)Puthoff, H. E., "CIA initiated remote viewing program at Stanford Research Institute" JSE 10:63–76 (1996)Radin, Dean; "The Conscious Universe", Harper, San Francisco (1997)Resta, R. "The Berry Phase", Europhysics News, 28, 19 (1997)Sheldrake, R. "A New Science of Life: the Hypothesis of Formative Causation", Tarcher, LosAngeles, (1981) Sheldrake, R. "Seven Experiments That Could Change the World", Riverhead Books, NY (1995)Schempp, W. "Harmonic analysis on the Heisenberg group with application in signal theory",Pitman Research Notes in Mathematics, Series 14, Longman Scientific and Technica, London(1986)Schempp, W. "Quantum Holography and Neurocomputer Architectures", Journal of

MathematicalImaging and Vision 2: 109–164 (1992)Schlitz, M. & Wiserman, R. "Experimenter Effects and the Remote Detection of Staring", Journalof Parapsychology Vol. 61, September, (1997)Schmeidler, G. R., Craig, J. G. "Moods and Esp Scores in Group Testing" Journal of ASPR vol66no. 3, pp. 280–287 (1972)Tiller, A. W. "Science and Human Transformation" Pavoir Publishing, California, (1997)Utts, J. M. "Replication and meta-analysis in parapsychology" Statistical Science 6: 363–382,(1991)Van Flandern, T. "What the Global Positioning System tells us about Relativity", Open Questions in Relativistic Physics, ed. by F. Selleri, Apeiron, Montreal (1998) **http://www.cosmicdreaming.com/pdf2011/Nature's%20Mind%20the%20Quantum%20Hologram%20by%20Edgar%20Mitchell,%20Ph_D.pdf**. **Models given in one of the papers. Some sentences are deleted and reformulated for felicity of reading and spatial restrictions. Kindly pardon me on that count. National Institute for Discovery Science: Nature's Mind: the Quantum Hologram by Edgar Mitchell, Ph.D. National Institute for Discovery Science Home > Resources > Consciousness Studies Nature's Mind: the Quantum Hologram Edgar Mitchell, Sc.D. Institute of Noetic Sciences, Sausalito, Calif. Fax: 561-641-5242, edgarmitchell@msn.comIntroduction** My Answer To Quora Question "If Consciousness Has No Evolutionary Advantage, Doesn't That Imply That It Is An Emergent Property? " While Human Consciousness Has Certainly Been Shaped By Evolution, Which Does Not Mean That Consciousness Itself Could Have Evolved From Non-Consciousness. Whether We Are Talking About Other Species Of Animals Or Cells Or Organic Molecules, The Same Issues Which We Run Into In Explaining Human Consciousness Are Still Present At Any Scale. The Issues Of The Hard Problem Of Consciousness, Explanatory Gap, Binding Problem, And Symbol Grounding Problem Make The Mind-Body Split Just As Relevant Wither The 'Body' Is A Brain, Neuron, Or Subatomic Particle. No Matter What, You Have To Explain How An 'Interior World' Can 'Exist' In A Physical Structure Whose Behavior Is Causally Closed. Whatever Way You Slice It, If We Accept That T-Cells Can Be Effective In Detecting And Neutralizing Threats On A Cellular Level Without Having Consciousness, Or That DNA Can Create Cellular Machines Which Build A Brain Without Consciousness, And Then We Are Admitting That Consciousness Doesn't Make Sense As A Functional Adaptation. The Rest Of The Universe Already Works Too Well Without It. There Is Nothing Especially Interesting about A Hominid's Need for Food and Shelter Which Would Demand Rich Awareness to Develop out Of Blind Reflex. Single Celled Organisms Chase Food, Avoid Danger, Etc Also. We Are Then Left With Considering That Either Consciousness Could Somehow Be An Accident Of Evolution, Or That Consciousness May Be Intrinsic To All Physical Phenomena In Some Sense (Panpsychism, Panexperientialism) Or Even That Consciousness Is The Universal Substrate Upon Which All Phenomena Depends (Idealism, Idealist Monism). If Consciousness Is A Mutation That Has No Functional Role (A Spandrel), We Have To Ask Why It Would Even Be A Possibility. Remember That If Consciousness Is A Mutation, We Are Assuming That There Is A Whole Universe Already In Place Which Is Overflowing With Processes, Biological And Otherwise, Which Are Perfectly Capable Of Directing Themselves Effectively While Being Unconscious. It's Actually A Radically Anthropocentric Cosmology Since We Are Privileging Our Tiny Piece Of History In The Universe As The Only Piece Which Is Not Devoid Of Experience. We Are Saying That Everything That Existed Before Humans Was Unconscious, Therefore An Invisible, Intangible, Silent Void With No Memory Etc. If We Are Not Intending That, And Prefer To Think That The Universe Looked, Sounded, Felt, And Tasted Just Like It Does For Homo Sapiens Since The Dawn Of Time, Then We Would Have To Ask Exactly What We Think Consciousness Is Adding To That Kind Of Eternal-Universal 'Unconsciousness'. If Consciousness Is Intrinsic to Physical Phenomena (As in Penrose-Hameroff's Microtubule-Based Quantum Consciousness) or Is Intrinsic to Information Integration (As in Tononi-Koch's IIT), We Still Have the Same Kind Of Mind-Body Problem. A 'Body' Which Is A Statistical Function Rather Than A Literal Form In Space Is Still Falls Short Of Explaining Why And How There Is Any Such Thing As Consciousness. In My View, Only The Idealist Monist View Or What I Call Pansensitivity Makes Sense Ultimately As The Parent Of Both Physics And Information. Just As We Learn To Count On Our Fingers, All Forms Of Information Are Representations Of Experiences Which Have An Aesthetic Foundation – A Seeing, Feeling, Touching, Thinking, Etc. Without That Sensory-Motive Context From The Start, There Would Nothing To Evolve; Only Abstractions In The Dark (Or Not Even Dark). Once We Can Get Over Ourselves As A Species And Recognize That Consciousness Doesn't Begin And End With Us, I Think That Awareness Will Be Seen As The Container Of Relativity Itself, With Quantum Mechanics And Evolutionary Biology As A Consequence Of Deeper

Stories Rather Than Their Originator. Multisense Realism A Cosmology Of Sense, Essence & Existence For Models See One Of The Papers In The Series Procrastinated Due To Constraints On Space. Events Occur In Time, But There Is No Time Except That Which Has Been Produced By The Big Bang. It Makes No Difference Whether The Big Bang Is The Only Beginning Of The Only Universe Or Just The Beginning Of One Of Many Universes, Either Way It Doesn't Explain The Beginning Of Existence. Some Say That Everything Following The Big Bang Is Random, Not Designed, But Why Is 'Designed' Even An Option? If That Word Means Anything, And Anything Has Ever Been Designed By Anything, Then What Difference Does It Make Whether That Capacity Of Intention Came Early On In The Universe Or More Recently? How Do We Know The Difference Between What We Design And What Is Random? How Do We Know That Randomness Even Exists? We Can't Generate True Randomness Computationally, We Can Only Grab Onto Some Pattern That Seems Nearly Random By Some Arbitrary Duration Of Measurement And Say That It Is Good Enough. Random Is A Concept, and the Difference between Random and Intentional May Ultimately A Matter of Perspective. Some Interesting Possibilities Arise When We Consider The Observation That The More Something Seems Intentional, The More It Is 'Like Me' And The More Unlike Me Something Appears, The More It Seems Mechanistic. In A Way That Is Similar To How Any Object Appears As A Dot Or Smudge If It Is Too Small Or Distant For Us To See, The Notion That Perceptual Relativity Dictates The Quality Of Intention And Unintention Is A Provocative Hypothesis Of MSR. Random Events Cannot Follow A Series Nor Have Results. That Doesn't Mean That There Has To Be An Entity Making Everything Happen, But It Suggests That The Universe Is A Phenomenon Of Appearance, And Part Of Appearance Is Oscillation Between Intentional And Unintentional Attributes. The Universe Speaks In Both Entropy And Significance. Randomness Doesn't Do Anything, Can't Be Anywhere Or Feel Anything. Randomness Is An Abstraction Derived From An Expectation Of The Contrary. By Random, All We Mean Is That It Lacks Pattern And Intent – Both Of Which Must Be Implicitly Present Before They Can Be Hypothetically Absent. There Is Sense And There Is Non-Sense, Order And Dis-Order, It Is Not Non-Randomness And Dis-Chaos. Many People Believe That Physical Law Excludes The Possibility Of Free Will Because Of Strong Causal Closure. By This, What Is Meant Is That There Is No Room In Physics For Any Force That Causes A Physical Effect That Has Not Been Accounted For, Whether Or Not This Is Actually True Or Not Is Questionable In The First Place. The Sudden Addition Of Dark Matter And Dark Energy In The 1990s, Which Together Must Account For 95.1% Of Mass-Energy Of The Universe, Doesn't Seem To Have Encountered Nearly As Much Resistance From The Scientific Community Than Conscious Intention Has. Even If It Were The Case, However, And The Typing And Reading Of These Words Will Be Someday Explained By Purely Neurochemical Mechanisms, The Question Of Why The Feeling Of Intention Exists, And How It Can Be Produced Will Remain Unanswered. My Free Will Demands That Your Free Will Admit That There Is No Free Will. In All Of The Contemporary Debates On Free Will There Seems To Be A Blind Spot, In Which Free Will Does Not Exist, Except Where It Concerns The Application Of The Results Of Scientific Experiments. There Is Always A Call For The Educated And Enlightened To Voluntarily Change Their Own Minds, To Be Persuaded By The Argument Of Their Own Free Will. The Hypocrisy Is Hard To Overlook Once You See It. It Seems That The Anti-Free Will Assertion Makes An Exception For People Who Are 'Right'. Being Right Seems To Give Us A Right To Have Opinions Which Others Cannot, Due To Their Enslavement To Physical Causes. An Issue Which Often Comes Up In Free Will Debates Is How Our Stance On Free Will Impacts Criminal Prosecution. Again, The Blind Spot Of The Anti-Free Will Philosopher Projects That While Criminals May Not Be Held Liable For Their Actions Because Their Neurology Is To Blame, Then We Can't Allow Society To Take Credit Or Blame For Its System Of Justice Either. If Guilt Or Innocence Is Irrational, Then Believing That We Can Modify Our Own Attitudes Toward Guilt And Innocence Must Also Be Irrational. What Seems More Irrational Is To Codify Law Into A Mechanistic Formula Which Diminishes Our Capacity For Thought And Feeling In Consideration Of The Fate Of Others (Or Others Considering Our Fate).Zero-Tolerance Determinism: Another Pillar Of The Anti-Free Will Position Is That It Must Be An All-Or-Nothing Phenomenon. This Seems To Be More Of A Hasty Generalization Than An Honest Look At The Phenomena. In A More Tolerant Analysis, It Should Be Clear That Free Will Doesn't Have To Be Absolutely Free. The Fact That Our Conscious Mind Thinks That It Was Presented With Certain Options Still Requires That We Choose Freely From Among Those Options. It's True That We Are Always, In a Sense, Only Able to Choose What Our Mind Thinks Is the 'Best' Option (Even If That Choice Is Based on Bad Advice from Our Stomach or

Emotions), But Still There Is an Intentional Participation Which Cannot Be Explained by Statistical Functions. No Matter How Constrained We Are By The Rules Of The Road, Our Car, The Limits Of Our Driving Skill, Etc, There Is Still A Difference Between Driving And Being Asleep At The Wheel, Or Between Driving And Being Forced To Drive Somewhere At Gunpoint. We Should Not Look Only At How Our Freedom Seems To Vanish Upon Thorough Inspection; We Must Also Look At How It Appears In The First Place – Unbidden And Self-Evident. Look At The Universal Appeal Of Freedom And The Universal Disparagement Of That Which Is Unfeeling, Robotic, And Mechanistic. One Thing That Is Seldom Mentioned In These Endless Debates Is Creativity. Were The Egyptians Destined To Build Pyramids And Not Megalithic Circles? Was It Inevitable That The Star Spangled Banner Was Written In Reference To The United States? Did Deterministic Processes Have No Choice But To Create The Illusion Of Free Will? Why? The Work of Benjamin Libet Is Frequently (Compulsively?) Cited As Well, Work Which Even Libet He Later Made Clear Did Not Show That Free Will Didn't Exist. The Fact That A Neurological Signal Can Be Detected Before The Various Parts Of Us (The Self Who Makes The Decision, The Self Who Knows They Make The Decision, The Self Who Reports That They Make The Decision) Can Arrive At A Consensus Does Not Mean That The Initial Impulse Doesn't Correspond To Conscious Experience. There Is Also The Matter Of Focusing On Repetitive, Reflex Actions Which Minimize Free Will And Maximize Predictive Expectation. These Kinds Of Experiments Are Like Proving That Chefs Lack Imagination By Studying The Fry Cooks At Mcdonalds. Since Physics And Neuroscience Has No Theory At All As To The Origin Or Utility Of Consciousness, We Cannot Give Inanimate Instruments The Benefit Of The Doubt When It Comes To Our Subjectivity. Because We Are A Single Zygote Which Has Reproduced Itself, In Some Sense, Every Cell In Our Body Is 'Us' Just As Much As Any Organ Or Process In Our Body. We Are Complex, But Not In The Way That A Machine Is Complex. We Are Not Assembled From Specialized Parts; We Are A Single Whole, Divided Into Relative Specialization. Just Because Every Part Of Us Doesn't Know What Every Other Part Of Us Is Doing At All Times, Doesn't Mean That It Isn't All 'Us' Doing It Pansensitive Relativity: If The Perceptual Relativity Hypothesis Is Correct, And Pansensitivity Is The Engine Of Both Physics And Subjectivity, Then Cause Itself Is Relativistic. Intention Is Real And Primitively Creative In The First Person, And Unreal/Recombinant-Derivative In The Third Person…But Third Person Is Only Real Because There Is A First Person Participant Present. **Free Will, Determinism, And the Big Bang: Multi Realism Blog: Wikipedia http://multisenserealism.com/2012/08/27/deleuzes-the-logic-of-sense-part-i/.** From A Whiteheadian Perspective, However, There Are At Least Two Key Sources Of Misunderstanding Latent In Some Of These "Returns" To "The Body", "The Object" And "Materiality". There Is A Rather Too Hasty Dismissal Of The Concept Of Subjectivity As Such And There Is A Related Tendency To "Flatten Out" Any Would-Be Distinctions Between Human And Non-Human Entities. Such Positions Thus Risk A Return To A Bleak Anti-Subjectivism That Mocks Those Who Might Cling To The Idea That "Humans Are Very Different From Knives Or Paper" (Harman, 2002, Cited In Thrift, 2008). With Respect To The Latter, For Example, Nigel Thrift Invokes The ANT Principle Of The Democracy Of Things And States That In His Theory "Things [And Human Beings] Are Given Equal Weight, And I Do Mean Equal" (Thrift, 2008, P. 9). Much Of This Is Done In The Name Of A Whiteheadian Inspiration Since The Book Begins With Strong References To Whitehead, Including Key Concepts Such As The "Actual Occasion". The Point Concerning The Dismissal Of Subjectivity Can Be Brought Out Most Starkly By Contrasting One Of Thrift's Statements About His Non-Representational Theory With A Statement From Whitehead Himself. "Thus Things", Writes Thrift, "Are Not Just Bound By Their Brute Efficacy To The Visible Termini Of Humans In Some Form Of Latent Subjectivism Such As 'Concern' Or 'Care'". Now, It Is Certainly True That For Whitehead A "Thing" Is, By Definition, "Describable Without Reference To Its Entertainment" In An Occasion Of Experience (Whitehead, 1933/1935, P. 226). In This Sense, Things Are Not "Bound By Their Brute Efficacy To The Visible Termini Of Humans". But It Is Certainly Problematic To Imply That His Work Is Based Upon A Move Away From The Subject–Object Relation As The Fundamental Structure Of Experience And Hence From Concepts Such As Concern And Care. On The Contrary, On This Matter Whitehead's Thinking Is Quite Comparable To That Of His Contemporary Martin Heidegger In Being Grounded In The Concept Of Concern. The Centrality Of This Concept Is Due To The Way In Which It Brings Together Subject And Object As Relative Terms In The Unity Of What He Calls An Actual Occasion Of Experience: Thus The Quaker Word "Concern", Divested Of Any Suggestion Of

Knowledge, Is More Fitted To Suggest This Fundamental Structure. The Occasion As Subject Has a "Concern" For the Object. And The "Concern" At Once Places The Object As A Component In The Experience Of The Subject, With An Affective Tone Drawn From This Object And Directed Towards It. With This Interpretation, The Subject–Object Relation Is The Fundamental Structure Of Experience. (Whitehead, 1933/1935, P. 226) Granted, Thrift Does State That He Wishes To "Temper… The More Extreme Manifestations Of This Lineage, Which Can End Up By Positing A Continuity Of And To Experience About Which I Am Sceptical" (2008, P. 6) However, One Wonders What Is Left Of Whiteheadian Cosmology If This Radical Extension Of The Concept Of Experience And Hence Of Empiricism Is Omitted. It Is A Little Like Being A Marxist Without The Dialectical Materialism. It Is Also Difficult Not To Be Sceptical About The Remaining Self-Consciously "Inhuman" Theoretical Framework "In Which Individuals Are Generally Understood As Effects Of The Events To Which Their Body Parts (Broadly Understood) Respond And In Which They Participate" (Thrift, 2008, P. 60). In Recent "Radical" Social Theory, It Seems The Baby Of Subjectivity Is At Risk Of Being Thrown Out With The Bathwater Of Representationalism, Leaving Only The Hollow Remainder Of A Reactive Ensemble Of "Body Parts". Some Clarity Is Therefore Needed If We Are Not Simply Going To Recruit Whitehead Into Concerns Alien To His Own. His Chief Problem Was Not the Notion of the Subject/Object Structure of Experience Itself, But It's Too Rapid Identification with the Difference between Knower and Known (Whitehead, 1933/1935, P. 225). It Is This Conflation Of The Deeper Subject/Object Relation With The More Superficial Distinction Between Knower And Known That Gives Rise To A "Representational" Style Of Thinking And Its Interminable Debates. I Will Argue That It Is Precisely By Way Of The Subject/Object Relation Of The Actual Occasion That We Are Best Positioned To Conceive Of "The Cumulation Of The Universe And Not A Stage-Play About It" (Whitehead, 1927–1928/1985, P. 237). **Palgrave Communications | Palgrave Macmillan Journals Journal Home > Archive > Original Articles > Full Text Original Article Subjectivity (2008) 22, 90–109 Doi:10.1057/Sub.2008.4 A.N. Whitehead And Subjectivity Paul Stenner1** 1University Of Brighton, Brighton, UK Correspondence: Paul Stenner, School Of Applied Social Science, University Of Brighton, Falmer Campus, Mayfield House, Brighton BN9 1PH, UK. E-Mail: P.Stenner@Brighton.Ac.Uk (For Models Please See One Of The Papers In The Series Attributed To Spatial Restraints It Is Accommodated Where It Could Be). In Sum, Whitehead Offers A Relational Process Ontology That Promises To Deepen The Constructivist Insights Associated With The Turn To Textuality, But Without Reducing The Universe To "Discourse" And "Materiality". In This Ontology, Things (Whether Occasions Or Assemblages) Are Definable As Their Relevance To Other Things And In Terms Of The Way Other Things Are Relevant To Them. Things Have Relational Essences. Likewise, Things Do Not Exist Independently Of Temporality But Are Constituted By The History Of Their Specific And Situated Encounters. Every Actual Thing Is Thus "Something By Reason Of Its Activity" (Whitehead, 1927/1985, P. 26). Importantly, This Talk Of "Things" Need Not Incline One Towards Denying The Relevance Of Subjectivity. I Have Thus Taken Issue With A Tendency Illustrated In The Work Of Thrift (2008), Who Appears To Define Cutting-Edge Social Theory As Concerned With "Flow" And "Play" Rather Than With Stability (Since "Non-Foundational Theory Takes The Leitmotif Of Movement", P. 5 And "Privileges Play", P. 7); As "A Means Of Going Beyond Constructivism" (P. 5); And As "Trading" In "Modes Of Perception Which Are Not Subject-Based" (P. 7). I Have Tried To Show That, In Fact, The Concept Of Process Is As Much About Stability As About Change. Stability Is To Be Thought Of As An Achievement Resulting From Particular Ways Of Actualizing Potential And Of Patterning Occasions Into Spatial And Temporal Co-Assemblies. Nevertheless, I Have Also Stressed That Becoming Is An Inherently Self-Creative Process, Albeit A Self-Creation Grounded In The Facticity Of A Concrete Inheritance. Whitehead's Category Of Subjective Unity Is Thus An Exemplary Statement Of Constructivism, Which States That: "Self-Realization Is The Ultimate Fact Of Facts. An Actuality Is Self-Realizing, And Whatever Is Self-Realizing Is An Actuality" (Whitehead, 1927–1928/1985, P. 222). The "Capture Of Intensity" And The "Clutch At Vivid Immediacy" Are Thus The Defining Characteristics Of Life (Whitehead, 1927–1928/1985, P. 105). Finally, I Have Suggested That It Is Only On The Basis Of A Deep Extension Of The Concept Of Experience Throughout Nature That Whitehead Is Able To Resoundingly Affirm His Reformed Notion Of The Subjectivist Principle: "That Apart From The Experiences Of Subjects There Is Nothing, Nothing, Nothing, Bare Nothingness" (1927–1928/1985, P. 167). "Scientific Reasoning Is Completely Dominated By the Presupposition That Mental Functionings Are Not Properly Part of Nature" (Whitehead, 1938/1966, P. 156). (Ibid). Lewis Carroll Is Wonderful

In Exploring Some Of The Paradoxes Of Logic. Following Some Of These Paradoxes Will Lead To The Important Role Of 'Sense' In Understanding. [NB Weird Titles Are Deleuze's Own]. First Series Of Paradoxes Of Pure Becoming: We Need To Explore The Nature Of An Event. Events Assume Becoming, Since They Refer To States In The Past And The Future In A Way Which 'Eludes The Present'. This Is Paradoxical But Still Makes Sense. Plato Tried To Distinguish Between Limited Fixed Things And Pure Becoming, But The Two Cannot Be Separated. Instead, A Dualism Is Hidden In Material Bodies. We Need To Introduce The Notion Of A Simulacrum Which Is Neither Copy Nor Model [Massumi Has A Useful Article On This—Roughly, The Issue Is That What We Take To Be Material Reality Is Actually A Simulacrum Of The Virtual, A Limited Condensation 'Beneath Things' (2)]. It Also Avoids The Problems Of Subsuming Reality Under The Idea. Sometimes, This Is Indicated By A Summer The Peculiarities Of Language, Which Can Seem To Flow Over Specific Referents. This Provides A Clue That There Is Some Dimension To Language Which Serves To Come To The Aid Of More Specific Attempts To Name And Describe. There Are Implications For Identity. Fully Grasping Becoming Means That Identities Are Infinite, Incorporating Future And Past, Active And Passive, And Even Cause And Effect. Language Attempts To Limit This Infinity, But Still Often Alludes To It [With Open-Ended Statements Or Generalisations]. As With The Alice Stories, This Also Disrupts The Conventional Notion Of The Personal Identity. Normally, This Is Maintained By Some Underlying Commonsense Or Knowledge, As When 'The Personal Self Requires God And The World In General' (3). Becoming Threatens This Stability With The Paradox Of Events, Which Can Penetrate Even Commonsense—It Is Not Just A Doubt About Reality, But A Clear Indication Of The 'Objective Structure Of The Event Itself, Insofar As It Moves In Two Directions At Once, And Insofar As It Fragments The Subject Following This Double Direction' (3). Second Series Of Paradoxes Of Surface Effects: Stoics Divided Things Into Bodies And States Of Affairs, 'Actions And Passions' (4). There Is Also Some Cosmic Unifying Quality, Always In The Present. Bodies Can Interact and Cause Effects in Each Other, But These Effects Are Incorporeal, 'Logical or Dialectical Attributes… Not Things or Facts but Events' (5). They Have The Kind Of Subsidiary Existence, Acting As Verbs, And They Are Infinitives—The Example Is A Cut Inflicted On The Body, Which Is Seen As An Incorporeal Surface Effect, Compared To The Actuality Of Bodies And Their Mixtures. This Argument Had Important Implications For Understanding The Causal Relation. Specific Bodily Causes Produce Other Bodies, Linked By Some Cosmic Unity Or Destiny. Similarly, Effects Can Be Seen As Having Bonds Between Them, But Effects Can Never Be Causes In Themselves. They Can Only Be '"Quasi-Causes' Following Laws Which Perhaps Express In Each Case The Relative Unity Or Mixture Of Bodies On Which They Depend For Their Real Causes' (6). These Combinations and Bonds to Provide For Some Emergent Qualities, Which Means That Destiny Can Be Avoided. An Alternative Is Offered By The Epicurean Classification Of Different Kinds Of Causes Which Are Relatively Independent [And So Can Interact], And This Is A Kantian Idea Too. There Is A Reference Back To The Capacities Of Language To Offer 'A Declension Of Causes... [And]... A Conjugation of Effects' (6). Stoic Philosophy Introduces The Notion Of A Something Behind Both Specific Material Beings And Incorporeal Events. The Idea Must Belong To 'This Impassive Extra-Being Which Is Sterile, Inefficacious, And On The Surface Of Things: The Ideational Or The Incorporeal Can No Longer Be Anything Other Than An "Effect"' (7). This In Turn Leads To A Change Of Metaphor From Surface/Depth To Just Surface, To A Series Of Effects Which Are Manifestations And Are Of Different Types. We Have A Notion of Possibilities, Of Ideality Itself, Rather Than the Platonic Idea, But With No 'Causal and Spiritual Efficacy' (7). The Simulacrum Now Appears On The Surface, Rather Than Being Hidden In The Depths. Events As Effects Combine Past And Present, Active And Passive, All Of Which Are Located Elsewhere As Causes. The Relation Between Events Can Only Be Quasi-Causes. Stoics Saw Dialectical Analysis Has Explorations Of These Combinations, Once They Had Been Expressed In Propositions—Dialectics As Conjugation. Language Also Enables Us To Go Beyond Events Into The Possible Or Becoming. The Relation Between Propositions And Specifics Is Itself Still Paradoxical—'Chryssipus Taught "If You Say Something It Passes Through Your Lips, So If You Say "Chariot", A Chariot Passes Through Your Lips' (8). It Is Deliberate Nonsense In The Anglo American Sense, Or Humourous Play On The Surface, As Opposed To An Ironic Exploration Of Depths And Heights. Lewis Carroll Did Something Similar In Alice. [A Commentary On Alice Ensues, Stressing The Surface Rather Than The Underground World, And Picking Up The Disdain Lewis Carroll Felt For Boys Who Did Not Like To Operate At The Surface. Left Handers And Stutterers Can Sometimes

Remind Us Of The Paradoxes Of The Surface, However, Which Can Defeat Commonsense Understandings]. Third Series Of The Proposition: Describing Events As Propositions Raises The Question Of How Best To Analyse Surface Events. There Are Three Possibilities: Denotation [Roughly, A Direct Connection Between Words And Images Which Represent States Of Affairs, As In Indexical Signs. Here, Propositions Are Either True Or False, And May Be True In All Cases]; Manifestation -- A Relation Between The Proposition And The Person Expressing It, Statements Of Desire And Belief. These Are Causal Relations: 'Desire Is The Internal Causality Of An Image With Respect To The Existence Of The Object Or The Corresponding State Of Affairs' (13). Belief Anticipates Production Of An Effect By A Cause. Manifestation Includes Denotation, Makes It Possible. '"I" Is the Basic Manifester' (13). Manifestation Is 'The Domain Of The Personal, Which Functions As The Principle Of All Possible Denotation' (13). The Issues Here Turn On Veracity Other Than Truth And Falsehood, The Avoidance Of Illusion. Signification, The Relation Of A Word To Universal Or General Concepts, And Connections To Implications Which Have To Follow The Rules Of Syntax. Again, Signifying Involves Conceptual Implications Referring To Other Propositions, As In Premises Or Conclusions. This Involves A Certain Indirect Process, Implication Or Assertion, Instead Of Truth Or Veracity, Which Remain As Possible In Certain Conditions. However, It Is Not Just Formal Logical Operations That Are Involved, But Notions Of Probability Or Even Moral Terms Such As Promise Or Commitment. Error Produces Not Falsehood But Absurdity [Looks Really Close To Habermas and the Three Validity Claims Here]. Signification May Not Be Primary, Since All Language Begins From The Standpoint Of The 'I', But There Is An Assumption That Propositions Must Be Understood By Others And Have A General Force. This Implies That Manifestation Has Primacy. But Signification Is Implied, And [In Social Relations] Would Be The Basis Of Manifestation. It Is The Difference Between Langue And Parole. Particular Utterances Only Make Sense Against The Background Of Constant Concepts. This Is Extended To Particular Desires And Beliefs, As Opposed To 'Simple Opinions' Which Would Not Signify (16). Actual Utterances Often Involve Truth Claims And General Signification [And Other Presuppositions, Which Are Technically Infinite]. This Is The Paradox Facing Pure Logic, Solved By A Form Of [Smuggling]: 'Implication Never Succeeds In Grounding Denotation Except By Giving Itself A Ready- Made Denotation, Once In The Premises And Again In The Conclusion' (16). So Actual Propositions Feature Circular Relations between Signification, Manifestation, and Denotation. There Might Even Be A Fourth Dimension—Sense, But Introducing This Will Depend On Making Relations Theoretically Consistent—It Is 'Not Simply A Question Of Fact' (17). To Approach The Issue, We Ask Whether Sense Might Be Located In One Of The Existing Three. Denotation Concerns Itself With Truth And Falsity, Which Is Too Narrow. The Mere Relation Between Words And Denoted Things Is To Paradoxical To Always Makes Sense, As In The Example Of Speaking The Term 'Chariot'. Instead, Denotation Presupposes Sense. Manifestation Does Involve Some Manifesting Subject Which Initiates, So May Be Sense Is Itself A Subjective Matter Of Beliefs And Desires Of Persons—But Subjects Only Possess This Ability To Speak Because Of A General System Of Signification In Language. It Looks Like Sense Must Be Identified With Signification—But Signification Is Linked In A Circular Relation With Denotation And Manifestation. Perhaps it's Necessary to Think Of Different Forms of Possibility of Propositions—Logical, Physical, Syntactic and So On. This Might Serve As Foundations For Sense, But This Would Be An External Foundation, Independent Of Speech [I Think The Problem Is The Connections Between Any Foundations And Actual Act Of Speech, Whether Anything Would Escape The Foundations]. The Concept Of Truth In Particular Implies Independence From Form. This Independence, Separate From Conceptual Possibilities In Signification, Is What Constitutes Sense. This Is The Fourth Dimension. It Is 'An Incorporeal, Complex, And Irreducible Entity, At The Surface Of Things' (19). There Is Philosophers Have Discovered And Rediscovered This Quality. It Is The Idea Of A Something Again, Beyond The Propositions And The Terms And The Objects Which Are Denoted, Beyond The Subjective I And Things Which Are Expressed. Sense Is Irreducible To Propositions, and It Is and Must Be '"Neutral," Altogether Indifferent To Particular and General, Singular and Universal, Personal and Impersonal' (19). There Is Been Little Agreement About This Possible Fourth Dimension, Whether It Exists Simply In The Form Of Some Enquiry. It Is Not Even Immediately Useful Because It Is Neutral. It Can Only Be Inferred Indirectly, By Questioning Characteristics Of Propositions As Above—This Is 'Inspired In Its Entirety By Empiricism… [Avoiding Notions of Essence or Idea, and Knowing]… Have To Track Down, Invoked And Perhaps Produce A Phantom At The Limit

Of A Length Or Unfolded Experience' (20). It Might Be What Husserl Called 'Expression', Lingering In Terms Such As The Noema, As Pure Appearance, Outside Denotation Or Manifestation, Linked In Complex Ways To Appearances. In The Same Way, Sense Does Not Exist Outside Propositions Exactly, But 'Inheres or Subsists' (21). It Is Not Just An Expression, But An Attribute, Not Just Of The Proposition, But 'Of The Thing Or State Of Affairs' (21), [The Potential, 'To Be Able To Be Green' Rather Than Just The Denotation 'Green' Is The Example Here]. It Is Said Of A Thing, So It Depends On Propositions Which Express It And Is Therefore Not Separate From The Proposition. It Is Something Else, Both the Expressible, And the State Of Affairs: 'It Turns One Side towards Things and One Side towards Propositions' (22). It Is What Joins Propositions And Things. [It Is A Becoming]. It Operates On The Surface, Rather As Mathematics Does, Or The Nonsense Of Carroll. It Is The Operation Of Sense That Produces [Meaningful] Paradox. Fourth Series Of Dualities: Important Dualities Exist Between Causes And Effects, And 'Corporeal Things And Incorporeal Events' (23). This Is Extended To A Duality Between Things And Propositions, Bodies And Language. This Is Expressed In Lewis Carroll As A Duality Between Eating And Speaking—The Former Is A Matter Of Bodies Actions And Passions, And The Latter Movements Of The Surface And 'Ideational Attributes Or Incorporeal Events' (23) [Lots Of Examples From Alice About Being Presented To Food And Having Food Presented To You]. The Normal Relationship Can Be Distorted By 'Verbal Hallucinations… Unrestricted Oral Behaviour… And Various Disorders of the Surface' As Bodily Matters Intrude—Stuttering, Left Handedness (24). Sense Is Always Expressed In Propositions, But It Lies In States of Affairs, It Happens To Things. In This Sense, Bodies And Language Are United In The Production Of Sense, Existing 'On The Two Sides Of The Frontier Represented By Sense', Which Constantly Articulates The Differences (24)—Things Include 'Ideational Logical Attributes Which Indicate Incorporeal Events', And Propositions Include Both Denotations And Expressions, Names And Adjectives, And Verbs [The Latter Indicating Becoming And Chains Of Events] (24) [Illustrated With Words By Humpty Dumpty]. This Duality In Propositions Represents Two Dimensions, The 'Denotation Of Things And The Expression Of Sense' (25) [Here, It Is Sense That Is Being Expressed Not Subjectivity, However]. This Means The Duality Is Inherent In Propositions As Well As Between Propositions And Things. [More From Lewis Carroll Page 26, Turning On The Universal Denotation Implied By The Term 'It', And Also The Ability Of The Term To Summarise The Sense Of An Earlier Proposition]. The Two Dimensions Can Emerge In an Esoteric Word, Or In A 'Non Identifiable Aliquid' (26). The Example Given Is The Word 'Snark' [Both A Limited Thing And A Representation Of Anything That Is Too Exotic To Exist?]. [The Section ends with an Extended Quotation from the Gardener's Song in Sylvie And Bruno—Pass] Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable to Constraints on Space Accomodation Model Is Provided in Some Paper in the series Notwithstanding the Generality and Commonalty of the Observation and Its Concatenatability with the Other Modules. The Scholars Of Tien Tai Called It The "Embodied Nature". (This Is The Buddha-Nature That Includes Both Good And Evil.) The Scholars of Xian Shou Say, "It Is Arising From Primal Nature", And the Scholars of Chan (Zen) Say, "It Is Nature That Causes the Rising of Things". All Dharma Is Dharma-Nature. It Is Not Different From Dharma-Nature. Dharma And Dharma-Nature Are Not Two Separate Identities, "Phenomena" And "Nature" Is Also Not Distinguishable Either. In Other Words, There Is No Difference Between Principle (Absolute) And Practice (Relative). Teachings in Chinese Buddhism (6) Sunyata (Emptiness) In the Mahayana Context (Wikipedia). The reason for becoming, according to Whitehead, is always the actual entity itself. He called this the ontological principle. The entity "enjoys" its relations and is "satisfied" by its own becomings. Whiteheads theory of process and relations can be viewed as an experiential alternative view of quantum physics which incorporates a theory of mind. However, Whitehead's master work, Process and Reality (1929/1978), is nearly impossible to fully understand, has its own idiosyncratic vocabulary, and is not entirely self-consistent. This relative opacity of Whitehead's work has impeded the generalization of his quantum process theory, or metaphysics, to other areas of discourse. In his independent formulation of the microgenesis of the mental state, Jason Brown discovered a process theory of mind that had many similarities to Whiteheadian process. He took the concepts of duration and discontinuities of process and applied them to the duration of the mental state as it recapitulates the contents of prior mental states. Through the introduction of novelty, and through the process of

microgenesis during the progression of mental states, the Self gives rise to a percept of self, the ego, in the object world, at the termination of the microgenetic process. This termination is the conscious instant, or instantiation of the microgenetic process. It is all that we consciously see. We see the ego, not the Self, and thus mistake the ego for self, replacing self with selfhood. The process itself is obscured from our consciousness, but can be inferred from those neurological deficit syndromes, which expose lower "layers" in the becoming. This parallels the "layers" of the nested hierarchy of holographic surfaces in the brain itself. Brown is theory of microgenesis involves a progression in the genesis of the mental state in a developmental sequence that recapitulates the organism is evolutionary development (phylogeny), embryologic development (ontogeny), and history of personal and social experience and development, in a movement in an upward direction through structures of increasing phylogenetic and ontogenic recency (Brown, 1988, 1991, 1996, 1998, 2005). He developed a view that considered the evolution of mental states to be a continuum from depth to surface, with leaps or saltation between mental states at intervals that are temporally irreducible units of the duration of microgenetic process (Brown, 1996, 1998). The process of neoteny, through which earlier features of development and evolution are brought to the present, was described (Brown, 1996) as a change in the development of some aspect of the mental state, which is attenuated at some stage of elaboration. In neurological and psychiatric deficit syndromes, this attenuation corresponds with the locus of a lesion or functional deficiency. The signature of the attenuated stage is carried through the remaining process of microgenesis, leading to expression of the attenuated phase at the endpoint of the duration of microgenesis, coupled with full elaboration of other elements of microgenesis. The intellectual pedigree of the theory of microgenesis dates back to the principle of superposition of Sir Charles Lyell, according to which younger strata lie on top of older strata. Lyell, a geologist, was to have great influence on evolutionary theorists, including Darwin and Wallace. The principle of superposition was taken up by Herbert Spencer, a scholar of many fields, who was a contemporary of Charles Darwin, and who had developed an evolutionary theory that perhaps rivaled Darwin is in popularity during the late 1800ís and early 1900ís. An important element of Spencer is theory was his observation of the brain, which he fashioned to be layered, with the newer layers added on to older layer during the course of evolution. Looking at the deeper layers was, for Spencer, like peeling away the layers of an onion. Spencer is idea was taken up by the renowned British neurologist Hughlings Jackson (Kennard and Swash eds., 1989), who developed the notion of hierarchies of brain process and the concept of neurological deficit syndromes as disturbances in the hierarchical structure and function of the brain. The concepts of microgeny and microgenesis, derived from hierarchies of brain function, were later developed by Arieti (1962) and Werner (1956). Later, the concept of a hierarchy of adaptive ego mechanisms, including lower order defenses such as denial and reaction formation, and higher order defenses such as altruism, was developed by Vaillant (1971), with the more mature defense mechanisms bearing on higher order values. The process theory of the evolution of the brain is extraordinarily important to Brown is process theory of mind. We can place these theories in perspective with the process theory of Whitehead, into which the theory of Brown partially merged (Brown, 1996; 1998; 2005). The materialist perspective of evolution, was, according to Whitehead (1925/1953), inconsistent with the process by which organisms become: "...in truth, a thoroughgoing evolutionary philosophy is inconsistent with materialism. The aboriginal stuff, or material, from which a materialistic philosophy starts, is incapable of evolution. This material is in itself the ultimate substance. Evolution, in the materialistic theory, is reduced to the role of being another word for the description of the changes of the external relations between portions of matter. There is nothing to evolve, because one set of external relations is as good as any other set of external relations. There can merely be change, purposeless and unprogressive." Whitehead is (1929/1978) realm of eternal objects, apprehended in the unconscious, can be viewed as a basic element in the evolution of organisms and of mind. Whitehead (1929/1978), in his view of eternal objects, hearkens is back to Plato: "...eternal objects, as in Gods primordial nature, constitute the Platonic world of ideas." Whitehead is realm of eternal objects may include Carl Jung is (1934/1967) archetypes of the collective unconscious (Griffin, ed., 1989), connecting Whitehead is cosmology to the corpus of literature on depth psychology. Jung is archetypes were largely based on mythological, religious, and alchemical constructs. Mythology embodies the spiritual and social history of the species, as well as a variety of metaphysical metaphors (Campbell, 1969/1976; Neumann, 1954/1970). The physical nature of the brain state, including the notion of binding of the mental state into a coherent whole, is also critical to Brown is theory of microgenesis. Recent evidence indicates that the brain is a

chaotic or dynamical system (Freeman, 1987; Combs, 2004; Abraham, 2004). The self-organization of dynamical systems, which include all living organisms as well as the brain, has been described in detail by Ilya Prigogine (1980), who won the Nobel Prize in chemistry in 1977 for his work on dynamical systems. Prigogine defines an "internal time" or duration of the states of dynamical systems. Physical time, he states, is secondary, and internal time is primary (Prigogine, 1986). The dynamical interval of internal time, or duration, is governed by the baker transformation (Prigogine, 1980), which can be envisioned as the time it takes the baker to knead or fold two different colors of dough successively, until it seems to be uniform in color. The brain, as a dynamical system, thus "deposits" time in the sense of Brown (1996). Here we have a parallel with the "layering" of process time according to the Holographic Principle. We can also detect, remotely, the undertones of Lyellís principle of superposition, albeit in a different form, and in a sense that Lyell could never have imagined. The work of Jason Brown in process neurology and psychology stands in sharp contrast to the prevailing scientific and philosophical nihilism in these fields regarding the self, free will, and the process of experience. In the sense of Brown is theory of microgenetic process of the brain, and in the sense of the duration of process discontinuities of Whitehead as it relates to the mind, it is important for the purposes of this paper to explore the scientific basis of the brain state. Mental process involves successive durations of such states, each with a "temporal thickness" (Brown, 1996). In recent years the "brain electrical microstate" has been described in detail as patterns of electrical field potential which endure for a period a period of time, typically around 100 milliseconds or a tenth of a second, and are punctuated by rapid transitions to the next state (Fingelkurts and Fingelkurts, 2001; Koenig and others, 2002). Brain electrical microstates are prime candidates for the brain states of microgenetic process and for the dynamical interval of internal time. The application of the theory of discontinuous information states, with the complete repetition of former states, as applied to the Holographic Principle of Mind, requires an information theory that includes time and the duration of mental states (e.g. Germine, 1993).The process of microgenesis, as it applies to brain evolution, can be seen in purely classical way, related to the recapitulation theory of vertebrate brain evolution (Agoutis, Montiel, and Lopez, 2002), which implies that "sensory projection sites and sites and processing circuits have been conserved [recapitulated] in reptiles and mammals" in the course of evolution. Again, there is an element of predictability in evolution which seems to belie the purposelessness of random natural selection. In keeping with the Holographic Principle, the development of the organism is a product of its evolutionary history, and is in continuous intercourse with that history. Microgenesis is readily related to the Holographic Principle of Mind, whereby the past remains actual, and, in the course of each mental state or mental process, there is a literal recapitulation of the organisms' entire past. The depth to surface principle entails a direction of mental process which is essential to microgenesis, and which is explained naturally by the nested hierarchy of surfaces in the brain according to the Holographic Principle. In the Holographic Principle of Mind we have a universal principle of microgenesis, whereby the entire relevant history of the organism is repeated during the course of each timeless and eternal moment. In this way, microgenesis can extend back to the very inception of the Universe. MICROGENESIS AND THE HOLOGRAPHIC PRINCIPLE the Holographic Principle Theory of Mind MARK GERMINE Institute for Psycho Science. Aristotle and others (including, especially, Leibniz) have argued that time does not exist independently of the events that occur in time. This view is typically called either "Reductionism with Respect to Time" or "Relationism with Respect to Time," since according to this view; all talk that appears to be about time can somehow be reduced to talk about temporal relations among things and events. The opposing view, normally referred to either as "Platonism with Respect to Time" or as "Substantivalism with Respect to Time" or as "Absolutism with Respect to Time," has been defended by Plato, Newton, and others. On this view, time is like an empty container into which things and events may be placed; but it is a container that exists independently of what (if anything) is placed in it. Time First published Mon Nov 25, 2002; substantive revision Fri Jan 24, 2014Markosian, Ned, "Time", The Stanford Encyclopedia of Philosophy (Spring 2014 Edition), Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/spr2014/entries/time/>. The question of whether there could be time without change has traditionally been thought to be closely tied to the question of whether time exists independently of the events that occur in time. For, the thinking goes, if there could be a period of time without change, then it follows that time could exist without any events to fill it; but if, on the other hand, there could not be a period of time without change, then it must be that time exists only if there are some events to fill it. Time First published Mon

Nov 25, 2002; substantive revision Fri Jan 24, 2014Markosian, Ned, "Time", The Stanford Encyclopedia of Philosophy (Spring 2014 Edition), Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/spr2014/entries/time/>."Whoever thou mayest be, beloved stranger, whom I meet here for the first time, avail thyself of this happy hour and of the stillness around us, and above us, and let me tell thee something of the thought which has suddenly risen before me like a star which would fain shed down its rays upon thee and every one, as befits the nature of light. - Fellow man! Your whole life, like a sandglass, will always be reversed and will ever run out again; - a long minute of time will elapse until all those conditions out of which you were evolved return in the wheel of the cosmic process. And then you will find every pain and every pleasure, every friend and every enemy, every hope and every error, every blade of grass and every ray of sunshine once more, and the whole fabric of things which make up your life. This ring in which you are but a grain will glitter afresh forever. And in every one of these cycles of human life there will be one hour where, for the first time one man, and then many, will perceive the mighty thought of the eternal recurrence of all things:- and for mankind this is always the hour of Noon". Notes on the Eternal Recurrence - Vol. 16 of Oscar Levy Edition of Nietzsche's Complete Works (in English). Physical time is public time, the time that clocks are designed to measure. Biological time, by contrast, is indicated by an organism's circadian rhythm or body clock, which is normally regulated by the pattern of sunlight and darkness. Psychological time is different from both physical time and biological time. Psychological time is private time. It is also called phenomenological time, and it is perhaps best understood as awareness of physical time. Psychological time passes relatively swiftly for us while we are enjoying an activity, but it slows dramatically if we are waiting anxiously for the pot of water to boil on the stove. The slowness is probably due to focusing our attention on short intervals of physical time. Meanwhile, the clock by the stove is measuring physical time and is not affected by any person's awareness or by any organism's biological time. When a physicist defines speed to be the rate of change of position with respect to time, the term "time" refers to physical time, not psychological time or biological time. Physical time is more basic or fundamental than psychological time for helping us understand our shared experiences in the world, and so it is more useful for doing physical science, but psychological time is vitally important for understanding many mental experiences. Psychological time is faster for older people than for children, as you notice when your grandmother says, "Oh, it's my birthday again." That is, an older person's psychological time is faster relative to physical time. Psychological time is slower or faster depending upon where we are in the spectrum of conscious experience: awake normally, involved in a daydream, sleeping normally, drugged with anesthetics, or in a coma. Some philosophers claim that psychological time is completely transcended in the mental state called nirvana because psychological time slows to a complete stop. There is general agreement among philosophers that, when we are awake normally, we do not experience time as stopping and starting Time First published Mon Nov 25, 2002; substantive revision Fri Jan 24, 2014Markosian, Ned, "Time", The Stanford Encyclopedia of Philosophy (Spring 2014 Edition), Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/spr2014/entries/time/>. But From The Profound Contemplation And Wisdom Of The Buddha And Mahabodhisattvas, We Know There Is No Such Reality. Instead, Egolessness (Non-Self) Is The Only Path To Understand The Reality Of The Deluded Life. All Existences Are Subject To The Law Of Causes And Conditions. These Include The Smallest Particles, The Relationship Between The Particles, The Planets, And The Relationship Between Them, Up To And Including The Whole Universe! From The Smallest Particles To The Biggest Matter, There Exists No Absolute Independent Identity. Egolessness (Non-Self) Implies The Void Characteristics Of All Existence. Egolessness (Non-Self) Signifies The Non-Existence Of Permanent Identity For Self And Existence (Dharma). Sunyata Stresses The Voidness Characteristic Of Self And Existence (Dharma). Sunyata And Egolessness Possess Similar Attributes. As We Have Discussed Before, We Can Observe The Profound Significance Of Sunyata From The Perspective Of Inter-Dependent Relationships. Considering Dharma-Nature and the Condition of Nirvana, All Existences Are Immaterial and Of A Void-Nature. Then We See Each Existence As Independent Of Each Other. But Then We Cannot Find Any Material That Does Exist Independent Of Everything Else. So Egolessness Also Implies Void-Nature! From The Observation Of All Existences, We Can Infer The Theory Of Nirvana And The Complete Cessation Of All Phenomena. From The Viewpoint Of Phenomena, All Existences Are So Different From Each Other, That They May Contradict Each Other. They Are So Chaotic. In Reality, Their Existence Is Illusionary And Arises From Conditional Causation. They Seem To Exist On One Hand,

And Yet Do Not Exist On The Other. They Seem To Be United, But Yet They Are So Different To One Another. They Seem To Exist And Yet They Do Cease! Ultimately Everything Will Return To Harmony And Complete Calmness. This Is The Nature Of All Existence. It Is The Final Resting Place For All. If We Can Understand This Reality And Remove Our Illusions, We Can Find This State Of Harmony And Complete Calmness. Teachings in Chinese Buddhism (6) Sunyata (Emptiness) In the Mahayana Context (Wikipedia) for models please see one of the papers in the series. "The Difference Between Supermind And Big Mind (If We Take Big Mind To Mean The State Experience Of Nondual Suchness, Or Turiyatita) Is That Big Mind Can Be Experienced Or Recognized At Virtually Any Lower Level Or Rung. Magic to Integral. In Fact, One Can Be At, Say, The Pluralistic Stage, And Experience Several Core Characteristics Of The Entire Sequence Of State-Stages (Gross To Subtle To Causal To Witnessing To Nondual), Although, Of Course, The Entire Sequence, Including Nondual Suchness, Will Be Interpreted In Pluralistic Terms. This Is Unfortunate In Many Ways—Interpreting Dharma In Merely Pluralistic Terms (Or Mythic Terms, Or Rational, And So On)—Because It Is So Ultimately Reductionistic; But It Happens All The Time, Given The Relative Independence Of States And Structures At 1st And 2nd Tier. Supermind, On The Other Hand, As A Basic Structure-Rung (Conjoined With Nondual Suchness) Can Only Be Experienced Once All The Previous Junior Levels Have Emerged And Developed, And As In All Structure Development, Stages Cannot Be Skipped. Therefore, Unlike Big Mind, Supermind Can Only Be Experienced After All 1st-, 2nd-, and 3rd-Tier Junior Stages Have Been Passed Through. While, As Genpo Roshi Has Abundantly Demonstrated, Big Mind State Experience Is Available To Virtually Anybody At Almost Any Age (And Will Be Interpreted According To The View Of Their Current Stage), Supermind Is An Extremely Rare Recognition. Supermind, As The Highest Structure-Rung To Date, Has Access To All Previous Structures, All The Way Back To Archaic—And The Archaic Itself, Of Course, Has Transcended And Included, And Now Embraces, Every Major Structural Evolution Going All The Way Back To The Big Bang. (A Human Being Literally Enfolds And Embraces All The Major Transformative Unfoldings Of The Entire Kosmic History—Strings To Quarks To Subatomic Particles To Atoms To Molecules To Cells, All The Way Through The Tree Of Life Up To Its Latest Evolutionary Emergent, The Triune Brain, The Most Complex Structure In The Known Natural World.) Supermind, In Any Given Individual, Is Experienced As A Type Of "Omniscience"—The Supermind, Since It Transcends And Includes All Of The Previous Structure-Rungs, And Inherently Is Conjoined With The Highest Nondual Suchness State, Has A Full And Complete Knowledge Of All Of The Potentials In That Person. It Literally "Knows All," At Least For The Individual." ― Ken Wilber, The Fourth Turning: imagining the Evolution of an Integral Buddhism (fopr models please see one of the papers in the series). "The Movement Of Descent And Discovery Begins At The Moment You Consciously Become Dissatisfied With Life. Contrary To Most Professional Opinion, This Gnawing Dissatisfaction With Life Is Not A Sign Of "Mental Illness," or An Indication Of Poor Social Adjustment, or A Character Disorder. For Concealed Within This Basic Unhappiness With Life And Existence Is The Embryo Of A Growing Intelligence, A Special Intelligence Usually Buried Under The Immense Weight Of Social Shams. A Person Who Is Beginning To Sense The Suffering Of Life Is, At The Same Time, Beginning To Awaken To Deeper Realities, Truer Realities. For Suffering Smashes To Pieces The Complacency Of Our Normal Fictions About Reality, And Forces Us To Become Alive In A Special Sense—To See Carefully, To Feel Deeply, To Touch Ourselves And Our Worlds In Ways We Have Heretofore Avoided. It Has Been Said, And Truly I Think, That Suffering Is The First Grace. In A Special Sense, Suffering Is Almost A Time Of Rejoicing, For It Marks The Birth Of Creative Insight. But Only In a Special Sense. Some People Cling To Their Suffering As A Mother To Its Child, Carrying It As A Burden They Dare Not Set Down. They Do Not Face Suffering With Awareness, But Rather Clutch At Their Suffering, Secretly Transfixed With The Spasms Of Martyrdom. Suffering Should Neither Be Denied Awareness, Avoided, Despised, Not Glorified, Clung To, And Dramatized. The Emergence Of Suffering Is Not So Much Good As It Is A Good Sign, An Indication That One Is Starting To Realize That Life Lived Outside Unity Consciousness Is Ultimately Painful, Distressing, And Sorrowful. The Life Of Boundaries Is A Life Of Battles—Of Fear, Anxiety, Pain, And Finally Death. It Is Only Through All Manner Of Numbing Compensations, Distractions, And Enchantments That We Agree Not To Question Our Illusory Boundaries, The Root Cause Of The Endless Wheel Of Agony. But Sooner Or Later, If We Are Not Rendered Totally Insensitive, Our Defensive Compensations Begin To Fail Their Soothing And Concealing Purpose. As A Consequence, We Begin To Suffer In One Way Or Another, Because Our Awareness Is Finally

Directed Toward The Conflict-Ridden Nature Of Our False Boundaries And The Fragmented Life Supported By Them." ― Ken Wilber, No Boundary: eastern and Western Approaches to Personal Growth for models please see one of the papers in the series. "To The Extent That You Actually Realize That You Are Not, For Example, Your Anxieties, Then Your Anxieties No Longer Threaten You. Even If Anxiety Is Present, It No Longer Overwhelms You Because You Are No Longer Exclusively Tied To It. You Are No Longer Courting It, Fighting It, Resisting It, Or Running From It. In The Most Radical Fashion, Anxiety Is Thoroughly Accepted As It Is And Allowed To Move As It Will. You Have Nothing To Lose, Nothing To Gain, By Its Presence Or Absence, For You Are Simply Watching It Pass By. Thus, Any Emotion, Sensation, Thought, Memory, Or Experience That Disturbs You Is Simply One With Which You Have Exclusively Identified Yourself, And The Ultimate Resolution Of The Disturbance Is Simply To Dis-Identify With It. You Cleanly Let All Of Them Drop Away By Realizing That They Are Not You--Since You Can See Them, They Cannot Be The True Seer And Subject. Since They Are Not Your Real Self, There Is No Reason Whatsoever For You To Identify With Them, Hold On To Them, Or Allow Yourself To Be Bound By Them. Slowly, Gently, As You Pursue This Dis-Identification "Therapy," You May Find That You is Entire Individual Self (Persona, Ego, and Centaur), Which Heretofore You Have Fought to Defend and Protect, Begins to Go Transparent and Drop Away. Not That It Literally Falls Off And You Find Yourself Floating, Disembodied, Through Space. Rather, You Begin To Feel That What Happens To Your Personal Self—Your Wishes, Hopes, Desires, Hurts—Is Not A Matter Of Life-Or-Death Seriousness, Because There Is Within You A Deeper And More Basic Self Which Is Not Touched By These Peripheral Fluctuations, These Surface Waves Of Grand Commotion But Feeble Substance. Thus, Your Personal Mind-And-Body May Be In Pain, Or Humiliation, Or Fear, But As Long As You Abide As The Witness Of These Affairs, As If From On High, They No Longer Threaten You, And Thus You Are No Longer Moved To Manipulate Them, Wrestle With Them, Or Subdue Them. Because You Are Willing To Witness Them, To Look At Them Impartially, You Are Able To Transcend Them. As St. Thomas Put It, "Whatever Knows Certain Things Cannot Have Any Of Them In Its Own Nature." Thus, If The Eye Were Colored Red, It Wouldn't Be Able To Perceive Red Objects. It Can See Red Because It Is Clear, Or "Redless." Likewise, If We Can But Watch Or Witness Our Distresses, We Prove Ourselves Thereby To Be "Distress-Less," Free Of The Witnessed Turmoil. That Within Which Feels Pain Is Itself Pain-Less; That Which Feels Fear Is Fear-Less; That Which Perceives Tension Is Tensionless. To Witness These States Is To Transcend Them. They No Longer Seize You From Behind Because You Look At Them Up Front." ― Ken Wilber, No Boundary: eastern and Western Approaches to Personal Growth (for models please see one of the papers in the series). "The Difference Between Supermind And Big Mind (If We Take Big Mind To Mean The State Experience Of Nondual Suchness, Or Turiyatita) Is That Big Mind Can Be Experienced Or Recognized At Virtually Any Lower Level Or Rung. ……In Difference and Repetition (1968, English 1994), Deleuze Develops His Project in Multiple Directions. His Work, He Says, Stems From The Convergence Of Two Lines Of Research: The Concept Of Difference Without Negation, And The Concept Of Repetition, In Which Physical And Mechanical Repetitions Are Masks For A Hidden Differential That Is Disguised And Displaced. His Major Focus Is A Thoroughgoing Critique Of Representational Thinking, Including Identity, Opposition, Analogy, And Resemblance (Deleuze 1994, 132). For Deleuze, "Appearances Of" Are Not Representations, But Sensory Intensities Free Of Subjective Or Objective Identities (Deleuze 1994, 144). Without These Identities, Appearances Are Simulacra of A Non-Apparent Differential He Calls the "Dark Precursor" Or "The In-Itself of Difference" (Deleuze 1994, 119). This Differential Is The Non-Sensible Being Of The Sensible, A Being Not Identical To The Sensible, Or To Itself, But Irreducibly Problematic Insofar As It Forces Us To Encounter The Sensible As "Given." Furthermore, Any Move Against Representational Thinking Impinges Upon The Identity Of The Subject. Where Kant Founds The Representational Unity Of Space And Time Upon The Formal Unity Of Consciousness (Kant 1964, 135-137), Difference Re-Distributes Intuitions Of Past, Present, And Future, Fracturing Consciousness Into Multiple States Not Predicable Of A Single Subject. Intensive Qualities Are Individuating By Themselves, Says Deleuze, And Individuality Is Not Characteristic Of A Self Or An Ego, But Of A Differential Forever Dividing Itself And Changing Its Configuration (Deleuze 1994, 246, 254, 257). In Nietzschean Fashion, The "I" Refers Not To The Unity Of Consciousness, But To A Multitude Of Simulacra Without An Identical Subject For Whom This Multitude Appears. Instead, Subjects Arise And Multiply As "Effects" Of The Intensive Qualities Saturating Space And Time.

This Leads Deleuze to Postulate Multiple Faculties for Subjectivity, Which Are Correlates of the Sensible Insofar As It Gives Rise to Feeling, Thought, and Action. "Each Faculty, Including Thought, Has Only Involuntary Adventures," He Says, And "Involuntary Operation Remains Embedded In The Empirical" (Deleuze 1994, 145). Subjectively, The Paradox Of The Differential Breaks Up The Faculties' Common Function And Places Them Before Their Own Limits: Thought Before The Unthinkable, Memory Before The Immemorial, Sensibility Before The Imperceptible, Etc. (Deleuze 1994, 227). This Fracturing And Multiplying Of The Subject, He Notes, Leads To The Realization That "Schizophrenia Is Not Only A Human Fact But Also A Possibility For Thought" (Deleuze 1994, 148), Thus Expanding The Term Into A Philosophical Concept, Beyond Its Clinical Application. Postmodernism First Published Fri Sep 30, 2005 Copyright © 2005 By Gary Aylesworth Geaylesworth@Eiu.Edu Open Access To The SEP Is Made Possible By A World-Wide Funding Initiative for models kindly see one of the papers in the series, procrastinated due to restrictions on space. The Dissolution Of The Subject And Its Implications For Society Is The Theme Of Anti-Oedipus: Capitalism And Schizophrenia, Which Deleuze Published With Félix Guattari In 1972 (English 1983). The Book, In Large Part, Is Written Against An Established Intellectual Orthodoxy Of The Political Left In France During The 1950s And 1960s, An Orthodoxy Consisting Of Marx, Freud, And Structuralist Concepts Applied To Them By Louis Althusser And Jacques Lacan. Deleuze And Guattari Argue That This Mixture Is Still Limited By Representational Thinking, Including Concepts Of Production Based Upon Lack, And Concepts Of Alienation Based Upon Identity And Negation. Furthermore, The Oedipus Concept In Psychoanalysis, They Say, Institutes A Theater Of Desire In Which The Psyche Is Embedded In A Family Drama Closed Off From The Extra-Familial And Extra-Psychic Forces At Work In Society. They Characterize These Forces As "Desiring Machines" Whose Function Is To Connect, Disconnect, And Reconnect With One Another Without Meaning Or Intention. The Authors Portray Society As A Series Of "Territorializations" Or Inscriptions Upon The "Body Without Organs, "Or The Free-Flowing Matter Of Intensive Qualities Filling Space In Their Varying Degrees. The First Inscriptions Are Relations Of Kinship And Filiation Structuring Primitive Societies, Often Involving The Marking And Scarring Of Human Bodies. As An Interruption and Encoding of "Flows," The Primitive Inscriptions Constitute a Nexus of Desiring Machines, Both Technical and Social, Who's Elements Are Humans and Their Organs. The Full Body Of Society Is The Sacred Earth, Which Appropriates To Itself All Social Products As Their Natural Or Divine Precondition, And To Whom All Members Of Society Are Bound By Direct Filiation (Deleuze 1983b, 141-42). These First Inscriptions Are Then De-Territorialized And Re-Coded By The "Despotic Machine," Establishing New Relations Of Alliance And Filiation Through The Body Of The Ruler Or Emperor, Who Alone Stands In Direct Filiation To The Deity (Deleuze 1983b, 192) And Who Institutes The Mechanism Of The State Upon Pre-Existing Social Arrangements. Finally, Capitalism De-Territorializes The Inscriptions Of The Despotic Machine And Re-Codes All Relations Of Alliance And Filiation Into Flows Of Money (Deleuze 1983b, 224-27). The Organs Of Society And The State Are Appropriated Into The Functioning Of Capital, And Humans Become Secondary To The Filiation Of Money With Itself. Postmodernism First Published Fri Sep 30, 2005 Copyright © 2005 By Gary Aylesworth Geaylesworth@Eiu.Edu Open Access To The SEP Is Made Possible By A World-Wide Funding Initiative For Models Kindly See One Of The Papers In The Series, Procrastinated Due To Restrictions On Space. Deleuze And Guattari See In The Capitalist Money System "An Axiomatic Of Abstract Quantities That Keeps Moving Further And Further In The Direction Of The Deterritorialization Of The Socius" (Deleuze 1983a, 33), Which Is To Say That Capital Is Inherently Schizophrenic. However, Because Capital Also Re-Territorializes All Flows Into Money, Schizophrenia Remains Capitalism's External Limit. Nevertheless, It Is Precisely That Limit Against Which Thinking Can Subject Capitalism To Philosophical Critique. Psychoanalysis, They Say, Is Part Of The Reign Of Capital Because It Re-Territorializes The Subject As "Private" And "Individual," Instituting Psychic Identity Through Images Of The Oedipal Family. However, The Oedipal Triangle Is Merely A Representational Simulacrum Of Kinship And Filiation, Re-Coded Within A System Of Debt And Payment. In This System, They Insist, Flows Of Desire Have Become Mere Representations Of Desire, Cut Off From The Body Without Organs And The Extra-Familial Mechanisms Of Society. A Radical Critique Of Capital Cannot Therefore Be Accomplished By Psychoanalysis, But Requires A Schizoanalysis "To Overturn The Theater Of Representation Into The Order Of Desiring-Production" (Deleuze 1983b, 271). Here, The Authors See A Revolutionary Potential In Modern Art And Science, Where, In Bringing About The "New," They Circulate De-Coded And De-Territorialized

Flows Within Society Without Automatically Re-Coding Them Into Money (Deleuze 1983a, 379). In This Revolutionary Aspect, Anti-Oedipus Reads As A Statement Of The Desire That Took To The Streets Of Paris In May Of 1968, And Which Continues, Even Now, To Make Itself Felt In Intellectual Life. Postmodernism First Published Fri Sep 30, 2005 Copyright © 2005 By Gary Aylesworth Geaylesworth@Eiu.Edu Open Access To The SEP Is Made Possible By A World-Wide Funding Initiative For Models Kindly See One Of The Papers In The Series, Procrastinated Due To Restrictions On Space. The Term "Deconstruction," Like "Postmodernism," Has Taken On Many Meanings In The Popular Imagination. However, In Philosophy, It Signifies Certain Strategies For Reading And Writing Texts. The Term Was Introduced Into Philosophical Literature In 1967, With The Publication Of Three Texts By Jacques Derrida: Of Grammatology (English 1974), Writing And Difference (English 1978), And Speech And Phenomena (English 1973). This So-Called "Publication Blitz" Immediately Established Derrida As A Major Figure In The New Movement In Philosophy And The Human Sciences Centered In Paris, And Brought The Idiom "Deconstruction" Into Its Vocabulary. Derrida And Deconstruction Are Routinely Associated With Postmodernism, Although Like Deleuze And Foucault, He Does Not Use The Term And Would Resist Affiliation With "-Isms" Of Any Sort. Of The Three Books From 1967, Of Grammatology Is The More Comprehensive In Laying Out The Background For Deconstruction As A Way Of Reading Modern Theories Of Language, Especially Structuralism, And Heidegger's Meditations On The Non-Presence Of Being. It Also Sets Out Derrida's Difference With Heidegger Over Nietzsche. Where Heidegger Places Nietzsche Within The Metaphysics Of Presence, Derrida Insists That "Reading, And Therefore Writing, The Text Were For Nietzsche 'Originary' Operations," (Derrida 1974, 19), And This Puts Him At The Closure Of Metaphysics (Not The End), A Closure That Liberates Writing From The Traditional Logos, Which Takes Writing To Be A Sign (A Visible Mark) For Another Sign (Speech), Whose "Signified" Is A Fully Present Meaning. This Closure Has Emerged, Says Derrida, With The Latest Developments In Linguistics, The Human Sciences, Mathematics, And Cybernetics, Where The Written Mark Or Signifier Is Purely Technical, That Is, A Matter Of Function Rather Than Meaning. Precisely The Liberation Of Function Over Meaning Indicates That The Epoch Of What Heidegger Calls The Metaphysics Of Presence Has Come To Closure, Although This Closure Does Not Mean Its Termination. Just As In The Essay "On The Question Of Being" (Heidegger 1998, 291-322) Heidegger Sees Fit To Cross Out The Word "Being," Leaving It Visible, Nevertheless, Under The Mark, Derrida Takes The Closure Of Metaphysics To Be Its "Erasure," Where It Does Not Entirely Disappear, But Remains Inscribed As One Side Of A Difference, And Where The Mark Of Deletion Is Itself A Trace Of The Difference That Joins And Separates This Mark And What It Crosses Out. Derrida Calls This Joining And Separating Of Signs Différance (Derrida 1974, 23), A Device That Can Only Be Read And Not Heard When Différance And Différence Are Pronounced In French. The "A" Is A Written Mark That Differentiates Independently Of The Voice, The Privileged Medium Of Metaphysics. In This Sense, Différance As The Spacing Of Difference, As Archi-Writing, Would Be The Gram Of Grammatology. However, As Derrida Remarks: "There Cannot Be A Science Of Difference Itself In Its Operation, As It Is Impossible To Have A Science Of The Origin Of Presence Itself, That Is To Say Of A Certain Non-Origin" (Derrida 1974, 63). Instead, There Is Only The Marking Of The Trace Of Difference, That Is, Deconstruction. Because At Its Functional Level All Language Is A System Of Differences, Says Derrida, All Language, Even When Spoken, Is Writing, And This Truth Is Suppressed When Meaning Is Taken As An Origin, Present And Complete Unto Itself. Texts That Take Meaning Or Being As Their Theme Are Therefore Particularly Susceptible To Deconstruction, As Are All Other Texts Insofar As They Are Conjoined With These. For Derrida, Written Marks Or Signifiers Do Not Arrange Themselves Within Natural Limits, But Form Chains Of Signification That Radiate In All Directions. As Derrida Famously Remarks, "There Is No Outside-Text" (Derrida 1974, 158), That Is, The Text Includes The Difference Between Any "Inside" Or "Outside." A Text, Then, Is Not A Book, And Does Not, Strictly Speaking, Have An Author. On The Contrary, The Name Of The Author Is A Signifier Linked With Others, And There Is No Master Signifier (Such As The Phallus In Lacan) Present Or Even Absent In A Text. This Goes For The Term "Différance" As Well, Which Can Only Serve As A Supplement For The Productive Spacing Between Signs. Therefore, Derrida Insists That "Différance Is Literally Neither A Word Nor A Concept" (Derrida 1982, 3). Instead, It Can Only Be Marked As A Wandering Play Of Differences That Is Both A Spacing Of Signifiers In Relation To One Another And A Deferral Of Meaning Or Presence When They Are Read. How, Then, Can Différance Be Characterized? Derrida Refuses To Answer

Questions As To "Who" Or "What" Differs, Because To Do So Would Suggest There Is A Proper Name For Difference Instead Of Endless Supplements, Of Which "Différance" Is But One. Structurally, This Supplemental Displacement Functions Just As, For Heidegger, All Names For Being Reduce Being To The Presence Of Beings, Thus Ignoring The "Ontological Difference" Between Them. However, Derrida Takes The Ontological Difference As One Difference Among Others, As A Product Of What The Idiom "Différance" Supplements. As He Remarks: "Différance, In A Certain And Very Strange Way, (Is) 'Older' Than The Ontological Difference Or Than The Truth Of Being" (Derrida 1982, 22). Deconstruction, Then, Traces The Repetitions Of The Supplement. It Is Not So Much A Theory About Texts As A Practice Of Reading And Transforming Texts, Where Tracing The Movements Of Différance Produces Other Texts Interwoven With The First. While There Is Certain Arbitrariness In The Play Of Differences That Result, It Is Not The Arbitrariness Of A Reader Getting The Text To Mean Whatever He Or She Wants. It Is A Question Of Function Rather Than Meaning, If Meaning Is Understood As A Terminal Presence, And The Signifying Connections Traced In Deconstruction Are First Offered By The Text Itself. A Deconstructive Reading, Then, Does Not Assert Or Impose Meaning, But Marks Out Places Where The Function Of The Text Works Against Its Apparent Meaning, Or Against The History Of Its Interpretation. Hyperreality is closely related to the concept of the simulacrum: a copy or image without reference to an original. In postmodernism, hyperreality is the result of the technological mediation of experience, where what passes for reality is a network of images and signs without an external referent, such that what is represented is representation itself. In Symbolic Exchange and Death (1976) (English 1993), Jean Baudrillard uses Lacan's concepts of the symbolic, the imaginary, and the real to develop this concept while attacking orthodoxies of the political Left, beginning with the assumed reality of power, production, desire, society, and political legitimacy. Baudrillard argues that all of these realities have become simulations, that is, signs without any referent, because the real and the imaginary have been absorbed into the symbolic Postmodernism First Published Fri Sep 30, 2005 Copyright © 2005 By Gary Aylesworth Geaylesworth@Eiu.Edu Open Access To The SEP Is Made Possible By A World-Wide Funding Initiative For Models Kindly See One Of The Papers In The Series, Procrastinated Due To Restrictions On Space. Magic to Integral. In Fact, One Can Be At, Say, The Pluralistic Stage, And Experience Several Core Characteristics Of The Entire Sequence Of State-Stages (Gross To Subtle To Causal To Witnessing To Nondual), Although, Of Course, The Entire Sequence, Including Nondual Suchness, Will Be Interpreted In Pluralistic Terms. This Is Unfortunate In Many Ways—Interpreting Dharma In Merely Pluralistic Terms (Or Mythic Terms, Or Rational, And So On)—Because It Is So Ultimately Reductionistic; But It Happens All The Time, Given The Relative Independence Of States And Structures At 1st And 2nd Tier. Supermind, On The Other Hand, As A Basic Structure-Rung (Conjoined With Nondual Suchness) Can Only Be Experienced Once All The Previous Junior Levels Have Emerged And Developed, And As In All Structure Development, Stages Cannot Be Skipped. Therefore, Unlike Big Mind, Supermind Can Only Be Experienced After All 1st-, 2nd-, and 3rd-Tier Junior Stages Have Been Passed Through. While, As Genpo Roshi Has Abundantly Demonstrated, Big Mind State Experience Is Available To Virtually Anybody At Almost Any Age (And Will Be Interpreted According To The View Of Their Current Stage), Supermind Is An Extremely Rare Recognition. Supermind, As The Highest Structure-Rung To Date, Has Access To All Previous Structures, All The Way Back To Archaic—And The Archaic Itself, Of Course, Has Transcended And Included, And Now Embraces, Every Major Structural Evolution Going All The Way Back To The Big Bang. (A Human Being Literally Enfolds And Embraces All The Major Transformative Unfoldings Of The Entire Kosmic History—Strings To Quarks To Subatomic Particles To Atoms To Molecules To Cells, All The Way Through The Tree Of Life Up To Its Latest Evolutionary Emergent, The Triune Brain, The Most Complex Structure In The Known Natural World.) Supermind, In Any Given Individual, Is Experienced As A Type Of "Omniscience"—The Supermind, Since It Transcends And Includes All Of The Previous Structure-Rungs, And Inherently Is Conjoined With The Highest Nondual Suchness State, Has A Full And Complete Knowledge Of All Of The Potentials In That Person. It Literally "Knows All," At Least For The Individual." ― Ken Wilber, The Fourth Turning: imagining the Evolution of an Integral Buddhism (fopr models please see one of the papers in the series). As Lyotard Argues, Aesthetic Judgment Is The Appropriate Model For The Problem Of Justice In Postmodern Experience Because We Are Confronted With A Plurality Of Games And Rules Without A Concept Under Which To Unify Them. Judgment Must Therefore Be Reflective Rather Than Determining. Furthermore, Judgment Must Be Aesthetic Insofar As It

Does Not Produce Denotative Knowledge About A Determinable State Of Affairs, But Refers To The Way Our Faculties Interact With Each Other As We Move From One Mode Of Phrasing To Another, I.E. The Denotative, the Prescriptive, the Performative, the Political, the Cognitive, the Artistic, Etc. In Kantian Terms, This Interaction Registers As An Aesthetic Feeling. Where Kant Emphasizes The Feeling Of The Beautiful As A Harmonious Interaction Between Imagination And Understanding, Lyotard Stresses The Mode In Which Faculties (Imagination And Reason,) Are In Disharmony, I.E. The Feeling of the Sublime. For Kant, The Sublime Occurs When Our Faculties Of Sensible Presentation Are Overwhelmed By Impressions Of Absolute Power And Magnitude, And Reason Is Thrown Back Upon Its Own Power To Conceive Ideas (Such As The Moral Law) Which Surpass The Sensible World. For Lyotard, However, The Postmodern Sublime Occurs When We Are Affected By A Multitude Of Unpresentables Without Reference To Reason As Their Unifying Origin. Justice, Then, Would Not Be A Definable Rule, But Ability To Move And Judge Among Rules In Their Heterogeneity And Multiplicity. In This Respect, It Would Be More Akin To The Production Of Art Than A Moral Judgment In Kant's Sense. Postmodernism First Published Fri Sep 30, 2005 Copyright © 2005 By Gary Aylesworth Geaylesworth@Eiu.Edu Open Access To The SEP Is Made Possible By A World-Wide Funding Initiative. The Scholars Of Tien Tai Called It The "Embodied Nature". (This Is The Buddha-Nature That Includes Both Good And Evil.) The Scholars of Xian Shou Say, "It Is Arising From Primal Nature", And the Scholars of Chan (Zen) Say, "It Is Nature That Causes the Rising of Things". All Dharma Is Dharma-Nature. It Is Not Different From Dharma-Nature. Dharma And Dharma-Nature Are Not Two Separate Identities, "Phenomena" And "Nature" Is Also Not Distinguishable Either. In Other Words, There Is No Difference Between Principle (Absolute) And Practice (Relative). Teachings in Chinese Buddhism (6) Sunyata (Emptiness) In the Mahayana Context (Wikipedia). In Other Words, All Dharma Arises From Causes and Conditions, and All Dharma Is Empty In Nature. The Law of Dependent Origination (Existence) and the Nature of Emptiness Is Neither the Same nor Different. They Exist Mutually. The Truth Of "Sunyata" And "Existence", And "Nature" And "Phenomena" Are Not In Conflict With Each Other. Unlike The Scholars Of The Dharmalaksana Sect Who Explain The Dharma Only From The Aspect Of Dependent Origination, Or The Scholars Of Dharma-Nature That Explain The Existence Of Dharma Only From The Aspect Of Dharma-Nature, The Scholars Of Madhyamika Explain The Truth Of The Dharma From Both Aspects. Hence This Is Called The Middle Path Which Does Not Incline To Either Side. Teachings in Chinese Buddhism (6) Sunyata (Emptiness) In the Mahayana Context (Wikipedia). For Stoics, Causes Involved Reference To The Depths, But Effects At The Surface Could Also Have Relations Among Themselves. This Permitted The Distinction To Be Drawn Between Destiny And Necessity—The Stoics Wanted To Affirm Destiny And Deny Necessity. In The First Place, Effects Express Causes, But Expressions Of Relations [Rather Than Necessity] Connects Those Effects. Those Relations May Be Described As Compatibility Or Incompatibility, Conjunction Or Disjunction. These Are Not Causal Relations Themselves, But Represent 'An Aggregate Of Noncausal Correspondences Which Form A System Of Echoes, Of Presumptions And Resonances, A System Of Signs—In Short, An Expressive Quasi Causality And Not At All A Necessitating Causality' (170). This Need Not Involve Contradiction, Which Is Applying To Events Rules That Really Only Applied To Logic And Argument. There Can Be Incompatability Without Contradiction, A Noncausal Correspondence. Leibniz Described Incompossible Worlds [Delanda Is Very Clear On This Too]. Only Impossible Events Contradict Possible Ones. Events Can Be Compossible [Roughly, They Have Predictable and Predicative Future and Past Events]. For Deleuze, It Is A Matter Of 'The Convergence Of Series Which Singularities Of Events Form As They Stretch Themselves Out Over Lines Of Ordinary Points. Incompossibility Must Be Defined By the Divergence of Such Series' (171). Such A Notion Is Essential To Any Theory of Sense. We Should Not See Divergence As A Matter Of Exclusion, As Leibniz Did [Since God Chose Actual Events]. Divergent And Disjunction Can Both Be Positive, While Preserving Differences. In Fact, Differences Are Crucial, Preserving The Distance Between Objects While Affirming That They Are Related. This 'Permits The Measuring Of Contraries Through Their Finite Difference Instead Of Equating Difference With A Measureless Contrariety' (173). It Is Contradiction Which Is The Special Case. Difference Here Is A Topological Term Relating To Distance On Surfaces Rather Than Depths. It Is Not Just A Matter Of Suggesting 'Some Unknown Identity Of Contraries (As In Commonplace In Spiritualist And Colorist Philosophy)' (173) [Take That St Pierre!]. An Example Is Nietzsche Arguing That Health And Sickness Can Both Inform Each Other; Act As Points Of View, Remembering That

'Things, Beings, Are Themselves Points Of View' (173). Divergence Does Not Mean Exclusion, And Disjunction Does Not Mean Separation. Connective Syntheses '(If…, Then)' Construct A Single Series; Conjunctive Series '(And)' Produces Convergent Series, But Disjunctive Series '(Or)' Produces A Divergence Series. Normally, Disjunction Helps Us To Criticise Synthesis, But Disjunction Can Still Be A Synthesis Itself, Despite Its Use In Logical Analysis [I Think What Is Going On Here Is Arguing That There Is A Difference Between 'Either/Or' In A Logical Sense, And 'One Or Two' In The Real Sense—The Latter Can Mean That Both Are Compossible. This Is The 'Communication Of Events' Rather Than The Logical Business Of Analysing Predicates (174)]. The Synthetic Disjunction Expresses The Paradox, With Divergence At The Centre. The Discussion of Paradoxes and Esoteric Words Above Are Examples: They Contract 'The Multitude of Divergent Series in the Successive Appearance of A Single One' (175). There Is A Difference Again Between the Subversion of the Present and Simple Identity by Depths, And Operations at the Surface. By Considering The Depths, We Encounter Infinite Identities [As Events Become Examples Of Deeper Categories, Wholes?]. At The Surface Events Communicate With Each Other Directly Through Maintaining Distance And By Affirming Disjunctions. Disjunction Threatens The Identity Of The Self, And Helps Us To See The Self As 'So Many Impersonal And Pre-Individual Singularities' [Connected Through Disjunctive Synthesis. Hence the Importance of Heterogeneity] (175). The Normal Concept Of The Self Implies Some Connected Series, 'But When Disjunction Accedes To The Principle Which Gives To It A Synthetic And Affirmative Value, The Self, The World, And God Share In A Common Death' (176). [There Is Also A Point That Divergent Series Explain And Also Exceed Normal Conjunctive And Connective Series]. In The Usual Conception, 'The Self Is the Principle of Manifestation, In Relation To the Proposition, the World Is the Principle of Denotation, And God the Principle of Signification' (176). But The Theory Of Sense Here Says That It Emanates From Nonsense, From Paradox, And From The 'Eternally Decentred Ex-Centric Centre' (176). This Position 'Does Not Tolerate the Subsistence of God as an Original Individuality, Nor The Self As The Person, Nor The World As An Element Of The Self And As God's Product. The Divergence Of The Affirmed Series Forms A "Chaosmos" And No Longer A World; The Aleatory Point Which Traverses Them Forms A Counter self, And No Longer A Self' (176). There Is No Centre But Only 'Pure Events Which The Instant, Displaced Over The Line [Of Aion], Goes On Dividing Into Already Past And Yet To Come. Nothing Other Than The Event Subsists… This Communicates With Itself through Its Own Distance and Resonates Across All of Its Disjuncts' (176). Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules Sixteenth Series Of The Static Ontological Genesis Twenty-fourth series of the communication of events. The Buddha Always Used The Terms Void, No Rising And Falling, Calmness And Extinction To Explain The Profound Meaning Of Sunyata And Cessation. For Example, Sunyata And The State Of Nirvana Where There Is No Rising Nor Falling, Are Interpreted By Most People As A State Of Non-Existence And Gloom. They Fail To Realise That Quite The Opposite, Sunyata Is Of Substantial And Positive Significance. The Sutras Often Use The Word "Great Void" To Explain The Significance Of Sunyata. In General, We Understand The "Great Void" As Something That Contains Absolutely Nothing. However, From A Buddhist Perspective, The Nature Of The "Great Void" Implies Something Which Does Not Obstruct Other Things, In Which All Matters Perform Their Own Functions. Materials Are Form, Which By Their Nature, Imply Obstruction. The Special Characteristic Of The "Great Void" Is Non-Obstruction. The "Great Void" Therefore, Does Not Serve As An Obstacle To Them. Since The "Great Void" Exhibits No Obstructive Tendencies, It Serves As The Foundation For Matter To Function. In Other Words, If There Was Neither "Great Void" Nor Characteristic Of Non-Obstruction, It Would Be Impossible For The Material World To Exist And Function. The "Great Void" Is Not Separated From The Material World. The Latter Depends On The Former. We Can State That the Profound Significance of Sunyata and the Nature of Sunyata in Buddhism highlight The "Great Void's" Non-Obstructive Nature. Sunyata Does Not Imply The "Great Void". Instead, It Is The Foundation Of All Phenomena (Form And Mind). It Is The True Nature Of All Phenomena, And It Is The Basic Principle Of All Existence. In Other Words, If The Universe's Existence Was Not Empty Nor Impermanent, Then All Resulting Phenomena Could Not Have Arisen Due To The Co-Existence Of Various Causes

And There Would Be No Rising Nor Falling. The Nature Of Sunyata Is Of Positive Significance. Calmness And Extinction Are The Opposite Of Rising And Falling. They Are Another Way To Express That There Is No Rising And Falling. Rising And Falling Are The Common Characteristics Of Worldly Existence. All Phenomena Are Always In The Cycle Of Rising And Falling. However, Most People Concentrate On Living (Rising). They Think That The Universe And Life Are The Reality Of A Continuous Existence. Buddhism On The Other Hand, Promotes The Value Of A Continuous Cessation (Falling). This Cessation Does Not Imply That It Ceases To Exist Altogether. Instead, It Is Just A State In The Continuous Process Of Phenomena. In This Material World, Or What We May Call This "State Of Existence", Everything Eventually Ceases To Exist. Cessation Is Definitely The Home Of All Existences. Since Cessation Is The Calm State Of Existence And The Eventual Refuge Of All Phenomena, It Is Also The Foundation For All Activities And Functions. Teachings in Chinese Buddhism (6) Sunyata (Emptiness) In the Mahayana Context (Wikipedia). You Are In Your Own Film A Subjective Experience Where You Are Both Actor And Director Writing Your Script As You Go Along. You Are Both The Audience And The Actor A Near State Of Pure Consciousness. You Are An Individual General Ledger With A Place In Collective General Ledger And Cosmic General Ledger With Celestial Brahman Anti Brahman Simulating The Events Which Appear To You On The Screen Of Consciousness. Sensations Are Disruptions Within Actual Identifications; They Are Signs Of The Expression Of Ideas And Intensities In Actual Structures. There Is No Denying the Difficulty of Deleuze's Account of Reality – Necessary for an Understanding of the Role Played by Kantian Arguments. It May Therefore Be Helpful To Think In Terms Of An Example. You Have Waited Years To See A Loved One Again, Tarrying With Dimming Memories And Shifts In Longings And Needs. A Meeting Has Been Arranged. The Opening Gestures, Words And Images Bring Together Fixed Memories And Preconceptions (Actual Identifications) With Feelings That Destroy those (Actual Sensations Of Disappointment, Excitement, Bemusement, And Renewed Passion). The Power Of These Sensations Cannot Come Solely From The Disrupted Identities; Rather, They Are Expressions Of Deeper Charges (Virtual Intensities). But, Beyond Local Identifications And Their More And More Distant Actual Effects, Imagined And Even Undreamed Worlds Of Ideas Are Changed Through The Meeting. According To Deleuze, These Are Not Possible Worlds, But Virtual Ones That Ground Actual Identifications. The Actual Meeting Is Accompanied By A Virtual One, Where Ideas Take Shape And Acquire Significance Through Shifts In Relations Between Intensities. These Shifts Light Up Actual Situations In New Ways And Determine Different Ideas To Come Into Play. The Meeting Is A Failure. Intensities Push Feelings of Disappointment to the Fore, Destroying One World of Ideas and Perhaps Determining a Colder One with Greater Clarity. The Transversal Thought of Gilles Deleuze: Encounters and Influences James Williams. For models please see one of the papers in the series. Models are procrastinated due to spatial restrictions. You Can Stir The Conscious State And Consequentially Connect All The Three States. As Lyotard Argues, Aesthetic Judgment Is The Appropriate Model For The Problem Of Justice In Postmodern Experience Because We Are Confronted With A Plurality Of Games And Rules Without A Concept Under Which To Unify Them. Judgment Must Therefore Be Reflective Rather Than Determining. Furthermore, Judgment Must Be Aesthetic Insofar As It Does Not Produce Denotative Knowledge About A Determinable State Of Affairs, But Refers To The Way Our Faculties Interact With Each Other As We Move From One Mode Of Phrasing To Another, I.E. The Denotative, the Prescriptive, the Performative, the Political, the Cognitive, the Artistic, Etc. In Kantian Terms, This Interaction Registers As An Aesthetic Feeling. Where Kant Emphasizes The Feeling Of The Beautiful As A Harmonious Interaction Between Imagination And Understanding, Lyotard Stresses The Mode In Which Faculties (Imagination And Reason,) Are In Disharmony, I.E. The Feeling of the Sublime. For Kant, The Sublime Occurs When Our Faculties Of Sensible Presentation Are Overwhelmed By Impressions Of Absolute Power And Magnitude, And Reason Is Thrown Back Upon Its Own Power To Conceive Ideas (Such As The Moral Law) Which Surpass The Sensible World. For Lyotard, However, The Postmodern Sublime Occurs When We Are Affected By A Multitude Of Unpresentables Without Reference To Reason As Their Unifying Origin. Justice, Then, Would Not Be A Definable Rule, But Ability To Move And Judge Among Rules In Their Heterogeneity And Multiplicity. In This Respect, It Would Be More Akin To The Production Of Art Than A Moral Judgment In Kant's Sense. Postmodernism First Published Fri Sep 30, 2005 Copyright © 2005 By Gary Aylesworth Geaylesworth@Eiu.Edu Open Access To The SEP Is Made Possible By A World-Wide Funding Initiative. In Dissolving All Social Phenomena In The Acid Bath Of Power And Domination, Foucault Prevents Critical Theory

From Drawing Crucial Distinctions, Such As Those "Between Just And Unjust Social Arrangements, Legitimate And Illegitimate Uses Of Political Power, Strategic And Cooperative Interpersonal Relations, Coercive And Consensual Measures" (McCarthy 1991: 54). One Cannot Say, For Example, That One Regime Of Power Is Any Better Or Worse Than Another, Only That They Are Different -- "Another Power, Another Knowledge" (Foucault 1979: 226). Since Ruling Powers Attempt To Erase Such Distinctions, Or To Present Injustice As Justice, Falsehood As Truth, And Domination As Freedom, Foucault's Position Unwittingly Supports The Mystifications Of Orwellian Doublespeak, Now More Rife Than Ever (See Kellner 2001), And Blocks The Discriminations Necessary For Social Critique. If There Are No Standards Or Right, Then, With Thrasymacus And Hobbes, We Can Conclude Might Is As Right As Anything. There Can Be No Ideology Critique Where There Is No Distinction Between True And False, And any Social Or Moral Critique Without A Distinction Between Right And Wrong. The Evaluative Character Of Foucault's Own Work Is Not Any Less Normative For His Refusal To Explicitly Confront It. The Problem Becomes Glaring In His Later Work Where He Employs Normative Terms Such As Liberty And Autonomy, But Fails To State What We Should Be Free For. Foucault's Anti-Normative Stance Therefore Forces Him into Self-Defeating Value Neutrality. Richard Rorty, The Attack on Theory, And Renunciation of Radical Politics (for models see one of the papers in the series). Our Environment Can Either Open Up or Constrict Possibilities for Spiritual and Intellectual Growth. In The Politics Of Experience Laing Offers A Fine Passage From Erving Goffman’s Studies Encounters: There Seems To Be No Agent More Effective Than Another Person In Bringing A World For Oneself Alive, Or, By A Glance, A Gesture, Or A Remark, Shriveling Up The Reality In Which One Is Lodged. "The Anxiety Of Meaninglessness Is Anxiety About The Loss Of An Ultimate Concern, Of A Meaning Which Gives Meaning To All Meanings. This Anxiety Is Aroused By The Loss Of A Spiritual Center, Of An Answer, However Symbolic And Indirect, To The Question Of The Meaning Of Existence." Philosophy And Depression By Tim Ruggiero -- Paul Tillich, The Courage To Be (1952) As Quoted In Depression And The Threat Of Nonbeing Tim Ruggiero, September 4, 2001 (Philosophical Society.Com).The Dissolution Of The Subject And Its Implications For Society Is The Theme Of Anti-Oedipus: Capitalism And Schizophrenia, Which Deleuze Published With Félix Guattari In 1972 (English 1983). The Book, In Large Part, Is Written Against An Established Intellectual Orthodoxy Of The Political Left In France During The 1950s And 1960s, An Orthodoxy Consisting Of Marx, Freud, And Structuralist Concepts Applied To Them By Louis Althusser And Jacques Lacan. Deleuze And Guattari Argue That This Mixture Is Still Limited By Representational Thinking, Including Concepts Of Production Based Upon Lack, And Concepts Of Alienation Based Upon Identity And Negation. Furthermore, The Oedipus Concept In Psychoanalysis, They Say, Institutes A Theater Of Desire In Which The Psyche Is Embedded In A Family Drama Closed Off From The Extra-Familial And Extra-Psychic Forces At Work In Society. They Characterize These Forces As “Desiring Machines” Whose Function Is To Connect, Disconnect, And Reconnect With One Another Without Meaning Or Intention. The Authors Portray Society As A Series Of “Territorializations” Or Inscriptions Upon The “Body Without Organs, “Or The Free-Flowing Matter Of Intensive Qualities Filling Space In Their Varying Degrees. The First Inscriptions Are Relations Of Kinship And Filiation Structuring Primitive Societies, Often Involving The Marking And Scarring Of Human Bodies. As An Interruption and Encoding of “Flows,” The Primitive Inscriptions Constitute a Nexus of Desiring Machines, Both Technical and Social, Who’s Elements Are Humans and Their Organs. The Full Body Of Society Is The Sacred Earth, Which Appropriates To Itself All Social Products As Their Natural Or Divine Precondition, And To Whom All Members Of Society Are Bound By Direct Filiation (Deleuze 1983b, 141-42). These First Inscriptions Are Then De-Territorialized And Re-Coded By The “Despotic Machine,” Establishing New Relations Of Alliance And Filiation Through The Body Of The Ruler Or Emperor, Who Alone Stands In Direct Filiation To The Deity (Deleuze 1983b, 192) And Who Institutes The Mechanism Of The State Upon Pre-Existing Social Arrangements. Finally, Capitalism De-Territorializes The Inscriptions Of The Despotic Machine And Re-Codes All Relations Of Alliance And Filiation Into Flows Of Money (Deleuze 1983b, 224-27). The Organs Of Society And The State Are Appropriated Into The Functioning Of Capital, And Humans Become Secondary To The Filiation Of Money With Itself. Postmodernism First Published Fri Sep 30, 2005 Copyright © 2005 By Gary Aylesworth Geaylesworth@Eiu.Edu Open Access To The SEP Is Made Possible By A World-Wide Funding Initiative For Models Kindly See One Of The Papers In The Series, Procrastinated Due To Restrictions On Space. "The Vanity Of Existence Is Revealed In

The Whole Form Existence Assumes: In The Infiniteness Of Time And Space Contrasted With The Finiteness Of The Individual In Both; In The Fleeting Present As The Sole Form In Which Actuality Exists; In The Contingency And Relativity Of All Things; In Continual Becoming Without Being; In Continual Desire Without Satisfaction; In The Continual Frustration Of Striving Of Which Life Consists." (For Models See One of the Papers in the Series, Which Are Procrastinated Due to Spatial Constraints) -- Arthur Schopenhauer, "On the Vanity of Existence" as quoted in Philosophy and Depression by Tim Ruggiero. "There Is Little Conjunction Of Truth And Social 'Reality'. Around Us Are Pseudo-Events, To Which We Adjust With A False Consciousness Adapted To See These Events As True And Real, And Even As Beautiful. In The Society Of Men The Truth Resides Now Less In What Things Are Than In What They Are Not. Our Social Realities Are So Ugly If Seen In The Light Of Exiled Truth, And Beauty Is Almost No Longer Possible If It Is Not A Lie." (For Models See One of the Papers in the Series, Which Are Procrastinated Due to Spatial Constraints) -- R.D. Laing, the Politics of Experience. [We're going to Use the Idea of a Game to Further Illustrate How the Virtual Is Imminent in the Actual. Why This Example? To Have A Go At The Metaphor Of Games In Philosophy? To Instate Lewis Carroll As Some Great Philosopher?] The Actual Games Have A Series Of Rules About Playing And Outcomes That Limit The Influence Of Chance. It's Possible to Invert These Rules to arrive at an Ideal Game, With No Rules, And with the Full Operation of Chance [Which Describes the Virtual, I Suppose]. Carroll Describe Some. We Also Learn About Time And How It Operates. In Each Throw Of The Dice, Singularities Are Distributed As A 'Nomadic And Non Sedentary Distribution, Wherein Each System Of Singularities Communicate And Resonates With The Others, Being At Once Implicated By The Others And Implicating Them In The Most Important Cast' (60). This Game Can Only Be Played In Thought [Since Any Empirical Reality Limits Chance. Chance Is Fixed By The Mechanisms Of Say The Roulette Wheel]. Infinite Time Can Be Compared With Finite [Or Metric] Time. They Are Linked Together, Although It Is Useful To Consider Both. In Finite Time, The Present Exists And Features Mixtures Of Matter Which Are Temporally Realized And Can Take On A [Empirical] Depth. Any Other Kind of Time, Only the Past and Future Exist, and Colonise Moments in the Presence. It Offers An Unlimited Kind Of Time, Since The Gap Between Past And Future Is Endlessly Subdivided And Never Closed. It Is Neutral And Incorporeal. Here, the Present Is A 'Pure Mathematical Instant' (62). It Is The Time Of Events Which Are Always Things Which Have Just Happened And Things Which Are About To Happen 'Never Something Which Is Happening' (63). As With The Operations Of Chance In A Game, Specific Events Or Throws Occupy A Time Which Is Smaller Than That Which Can Be Thought [That Is The Unfolding Of Singularities Which Happen Instantly?], And Also The Maximum Notion Of Time [I Can Only Grasped This By Thinking Of Specific Chances Being Understood By Their Location In Whole Sequences Of The Infinite Throws Of The Dice]. [There Is Some Stuff About The Eternal Return Here Too Which I Don't Understand—63 – 64]. 'The Aion Is The Straight Line Traced By The Aleatory [Unpredictable, Accidental] Point' [Search Me] Which Means That Events The Can Communicate With All The Others, As In Events Of The Aion Itself. This Sort Of Unity Is Not The Same Is An Empirical Corporeal Unity. The Aion Underpins or Circulates Within the Series of Events. This Also Helps Explain The Connection Between The Sense Of Propositions And The Sense Of States Of Affairs. [Weird. Making The Strongest Possible Case For Chance, I Assume. Denying Essence or Subjective Intent. Makes Empirical {Including Social} Science As Limited To The Finite And Metric, But Requiring Thought Experiments To Move To The 'Pure' Level? So Says Delanda Anyway, I Think]. Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Please See One Of The papers In The Series Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable and Ascribable To Constraints on Space Accomodation Model Is Provided In Some Paper, Notwithstanding the Generality and Commonalty of the Observation and Its Concatenatability with the Other Modules. [Particularly Dense And Baffling Stuff, Attempting To Show How The Subjective World Is A Derivative Of The Objective Play Of Singularities. Makes More Sense With Concrete And Familiar Examples As In Delanda's Stuff On Biological Membranes And Their Importance In Embryology, Instead Of The Abstract 'Surfaces' Of This Account]] Individuals Are Capable Of Infinite Description, Limited Only By Their Bodies To Express. Persons [Human Beings Based the More Abstract

Notion of an Individual?] Can Only Produce Propositions To Describe The World To A Limited Extent. Both, However, Are 'Ontological Propositions' [That Is They Create A Kind Of Reality?]. Multiple Classes And Systems Of Categories Are Not Produced By Propositions. Instead We Have To Look At Something That Now Produces Human Propositions Themselves, As 'Material Instances' (118). [So Human Ontological Activity Only Realises Sense Making In The Form Of Denotations, Manifestations, And Signification As Above?]. Denotations And The Others Are Interconnected As We Saw Above. There Is Also No Simple Connection Between, Say, 'The Individual And Denotation, The Person And Manifestation, Multiple Classes Or Variable Properties And Signification' (119) [For Similar Reasons—For Example, Signification Depends On The Good Sense Already Established By Individuation]. The Whole Complex Structure Is Produced By Both Ontological And Logical Genesis. Sense Operates On The Whole Structure. [We Can See This Both In The Fragility Of Sense And Its Tendency To Be Threatened By Nonsense, And Because The Alternative Is Unpalatable—Language And Sense-Making Would Be Based On Nothing But A 'Undifferentiated Abyss' (120)]. So Sense, 'In Its Organization of Aleatory and Singular Points, Problems and Questions, Series and Displacement' (120) Generates both Logical Propositions and Also 'the Objective Correlates' Of Propositions… The Denoted, the Manifested, and the Signified' (120). The Notion Of An Error Suggests This Although In A Confused Way. Normally We Think Of Error As A Matter Of Truth Or Falsity, When Propositions Are Formed And Tested. However, When We Consider Problems Instead Of Propositions That Offer Solutions, The Category Of Sense Emerges Strongly. [We Have Seen Above the Problem Is an Objective Matter of Structured Possibilities]. We Can See How Both Knowledge And The Known Are Produced By This Structure. [Problematics Are Further Discussed As Involving Particular Distributions Of Singularities In Space And In Time. As Problems Condense Out, So Do Solutions—'The Synthesis Of The Problem With Its Conditions Engenders Propositions, Their Dimensions, And Their Correlates' (121)]. Sense Is Produced Or Expressed When Solutions, Expressed As Propositions, Correspond To Problems [Act As 'Instances Of A General Solution' (121)] It Is Common To Express Sense In An Interrogative Form [Although The Interrogative Also Includes A More Limited Operation As In The Closed Question]. Specific Questions And Solutions Are Already Determined By The Problematic, However—'The Problem In Itself Is The Reality Of A Genetic Element, The Complex Theme Which Does Not Allow It To Be Reduced To Any Propositional Thesis' (122). It Is A Mistake To Define A Problem In Terms Of Possible Solutions [Which Arise From Human Consciousness]. This Would Mean Us 'Confuse Sense with Signification, and… Conceive Of the Condition Only In the Image of the Conditioned' (122). [This Autonomous Constitution Of Problems Shows The Inadequacy Of Subjective Conceptions. Seeing Problems As Derived From Propositions Expressing Solutions Would Also Infringe The Neutrality Of Sense]. Problems Are Neutral Insofar As [Modes Of] Propositions Are Concerned. For Example 'A Circle Qua Circle Is Neither A Particular Circle, Nor A Concept Represented In An Equation… It Is Rather A Differential System To Which An Emission Of Singularities Corresponds' (123). Problems In This Sense Exist In Propositions But Also Allude To None Being, As Above. As A Result, Problems Are Neutral, That Is 'Independent of both the Negative and the Affirmative' (123). The Neutrality of Sense Means It Is Never Just the Echo ['the Double'] Of Propositions. Working With Propositions Can Only Lead To A Partial Understanding Of Sense. We Have To Develop Another Conception, Not Based On Propositions Or On Images Of Conventional Logical Thinking. Philosophy Must 'Purge The Transcendental Field Of All Resemblance' In Order To Avoid Trap Of Consciousness As The Origin Of The Transcendental (123). However, The Earlier Discussions Defined The Neutrality Of Sense As An Effect Produced By Corporeal Causes. Here, We Are Implying That It Arises From Its Genetic Power [To Produce Problematics] And This Relates To A Quasi Cause. Sense Is Produced By Bodies In A Way Which Presupposes This More General Kind Of Sense. The More General Kind Operates In A Different Way, Not Through Concepts Or Describing Mixtures. This Time It Is a Matter of Depth [Of Bodies] and the Effects of a Surface. The Pulsations Of Bodies Produce Surfaces In Particular Ways, Sometimes As A Minimum Energy Conserving Form [Delanda's Soap Bubble], Sometimes As A More Complex Structure Of Multiplied Differentiated Surfaces [Stretched, Emulsified, Absorbing Etc Are The Examples Given, P. 124]. Surfaces Are Produced By the 'Actions and Passions of Mixed Bodies' (124). Surfaces Have No Thickness Of Their Own, Which Permits Contact Between The Internal And The External. The Quasi Causes Play On These Surfaces, As A Kind Of 'Fictitious Surface Tension… A Force Exerting Itself On The Plane Of The Surface' (125). Singularities Are Condensed Extended And Reshuffled. These Surfaces

Have An Existence In Actual Physics, And Also In Metaphysics—The Surface Becomes The Transcendental Field, The Border Between Bodies And Propositions. As Such, The Surface Becomes The 'Locus Of Sense And Expression' (125). Propositions And Bodies Are Actually Articulated On Surfaces, 'So That Sense Is Presented Both As That Which Happens To Bodies And That To Which Insists [Sic] In Propositions' (125). It Is In This Process Of 'Doubling Up' That Neutrality Arises [I Think The Argument Here Is That Neutrality Is Not Some Disembodied Quality But A Function Of Surfaces Which Enable 'The Continuity Of Reverse And Right Sides' (125). It Is The Indifference Of The Objective, Immune To Appeals From Human Subjectivity?]. The Surface Enables Sense To Be Distributed, As Both The Expressed In Propositions, And The Event In Bodies [Depends On This Very General Definition Of 'Expression' Again - More General Than The Use Above Where It Means Human Expression As In Manifestation Etc]. If The Surface Is Destroyed, Bodies' Fall Back Into The Depth, The 'Primary Order [Some Sort Of Natural Being Which Cannot Be Named Or Expressed?] Which Grumbles Beneath the Secondary Organisation of Sense' (125). But On The Surface, Sense Is Unfolded And Is Also Affected By Quasi Causes. This Sense In Turn Individuates and Determines Bodies, and Signification, and All the Propositions 'The Entire Tertiary Arrangement or the Object of the Static Genesis' (126). Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules Sixteenth Series Of The Static Ontological Genesis. Chronos And Aion Are Compared In Terms Of The Importance Of The Present As Opposed To The Past And The Future. For Chronos, The Present Represents The Main Interest, Although There Is 'The Relativity Of Presents Themselves In Relation To Each Other. God Experiences As Present That Which for Me Is Future or Past, Since I Live Inside More Limited Presents' (162). The Present Is Corporeal, A Matter of Mixtures [Imperfect for Humans and Perfect in the Divine Present for Stoics as Above]. The Present Limits The Action Of Bodies [Compare This With Bergsonian Stuff About The Present As A Cone—Here We Are Talking About Contraction And Dilation To Connect The Empirical With A Cosmic Present]. The Partial Mixtures Of Bodies In The Present Threaten To Subvert The Sufficiency Of The Notion. Stoics Had To Distinguish Between Good Or Bad Mixtures, For Example, Which Lead To Their Notion Of Cosmic Perfection, And To Aion. This Implies That Bodies Are Actually Nothing But Simulacra, And The Present Less Important Than The Future Or The Past. However, Chronos Represents the Only Kind Of Empirical Understanding [?] (164). For Aion, The Present Is A Mere Instant, Always Divided Into Past And Future. This Is A Different Way Of Subverting The Present As In Depth Metaphors [Some Of Which Appear To Linger In The Argument About Partial Mixtures Of Bodies In The Present?]. [Somehow], Aion Operates At The Surface, Since The Present Is Evaded In Favour Of The Instant Rather Than The Fathomless Depths. Aion Operates With Incorporeal Events, And Their Effects, Which Are Limitless And Infinite. Thus Aion Is 'The Eternal Truth Of Time: Pure Empty Form Of Time, Which Is Freed Itself Of Its Present Corporeal Content' (165) [I Think Delanda Is Much Clearer On The Differences Between Metric And Intensive Time]. Aion Is An Essential Element In The Development Of Language, Allowing Language To Escape Corporeal Determinations, Alluding To An Existence Outside Of The Present, Allowing Signification And Manifestation. [Once Having Escaped, Language Can Persist In Itself]. The Instant Demonstrates The Aleatory Point, Nonsense, And Quasi-Cause. It Is A Pure Abstraction And It 'Extracts Singularities From The Present, And From Individuals And Persons Which Occupy This Present' (166) [Because It Alludes To Future And Past, And Thereby Constitutes The Pure Event?]. Without The Notion Of Aion, We Would Only Be Left With Bodies And States Of Affairs, Not Language And Propositions. [The Argument Here Seems To Be That Language Necessarily Has A Future Element?]. The Surface Divides The Two Series Of States Of Affairs And Propositions, And Sense Can Now Relate To Propositions And Events [In The Form Of The Commentary On Events Mentioned Wayback At The Beginning?]. [There Is Some Sort Of Topological Connection Between Points, Lines And Surfaces, Presumably In Terms Of The Way In Which One Can Be 'Cut' From The One Above]. Actualizations Occur When Bodies, States Of Affairs And Mixtures Intersect At The Surface. This Is A Matter Of 'Imprisoning First Their Singularities Within The Limits Of Worlds, Individuals, And Persons' (167). However, There Is Always Something In Excess Of Actualizations, Which Alludes To The Quasi- Cause [Which Apparently Can Be Identified By Sages].

This Involves Analysing the 'Pure Perverse "Moment"', the Pure Operation—It Is Graspable At and As A Moment of Counter Actualization. Twenty-Third Series Of The Aionalexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules Sixteenth Series Of The Static Ontological Genesis. Wild And Wacky Stuff Here Relating To Artaud's Work. I Would Normally Chop It Out Altogether, Except It Provides Some Context For The Emergence Of The Phrase 'Body Without Organs' That Crops Up So Prominently In Anti Oedipus Sense Is Fragile And Is Threatened By Nonsense. Sometimes Such Nonsense Can Destroy Everything, And We See This If We Switch From The Playful Portmanteaux Of Carroll To The Schizophrenic Writings Of Artaud. These Are Real Examples Of Nonsense, Instead Of The Artificial Ones Discussed By Philosophers. Artaud Apparently Disliked Carroll, and Rendered Jabberwocky in A Much More Challenging Schizophrenic Language. Carroll Is Too Superficial, Whereas Schizophrenia Reveals The Real Problems With Language. Another Work Is Discussed, By Wolfson, To Focus On The Duality Between Things And Words. This Takes The Form Of Someone [Schizophrenic?] Experiencing A Problem In Translating From One Language To The Other, Which Somehow Becomes Transposed Into An Anxiety About Eating. A Schizophrenic Interlude Ensues Involving Associations Between Consonants As The Basis For Translation, And The Paradoxes That Emerge (85). The Same Basic 'Oral Duality (To Eat/To Speak)' Is Also Found In Carroll's Work, And In Artaud 's. However, In Caroll, Some Sense Is Retained Since This Duality Is Explored 'At The Surface' (86) [This Also Indicates, Apparently That The Operation Of Sense Of The Surface Shows Only A Quasi-Causal Relation Between Its Elements, Since There Are Incorporeal Elements Driving It]. For Artaud, The Classic Schizophrenic Symptoms Included The Absence Of Surface, Especially With Bodies. Apparently Freud Also Noticed This Tendency For Schizophrenics To See Their Body As 'Punctured By An Infinite Number Of Little Holes' (87). The Body Therefore Incorporates Everything Into Its Depths, Everything Becomes Corporeal And Physical. The Surface No Longer Limits The Extension Of The Body. 'Hence The Schizophrenic Manner Of Living The Contradiction: Either In The Deep Fissure Which Traverses The Body, Or In The Fragmented Parts Which Encase One Another And Spin About' (87). The World Loses Its Meaning And Sense [Because It Can No Longer Split Sensation Into A Signifying And Signified Separated By A Surface?] Words Become Physical And Affect Bodies, Or They Burst Into Components [Which Relates Back To The Wolfson Example]. Schizophrenics Experience 'A Pure Language–Affect' (88) [Sic --Affect Not Effect]. Schizophrenics Manage This By Overcoming The Effects Of Language, As In The Strange Translation Activity In The Wolfson Example. In Artaud's Case The Solution Was To Create Special Words Expressing 'Values Which Are Exclusively Tonic And Not Written' (88). 'To These Values A Glorious Body Corresponds, Being A New Dimension Of The Schizophrenic Body, An Organism About Parts Which Operates Entirely By Insufflations, Respiration, Evaporation, And Fluid Transmission (The Superior Body Or Body Without Organs Of Antonin Artaud)' (88). This Solution Can Never Be Complete Because There Can Never Be A Total Separation Between Sufferings ['Passion'] And [Remedial] Action, And Passion Can Be Reintroduced, And The Body Corrupted-- A Schizophrenic Body Is Therefore A Constant Mixture Of Two Actions Or Principles. Artaud Tries To Invent A New Language Which Cannot Be Decomposed And Thus Cannot Be Colonised, A Language Of 'Consonantal Guttural And Aspirated Overloads' (89). The Words Are Joined By Some Invented Principle, In This Case A 'Palatalized' One (89) Which Blurs The Consonants Together And Prevents Them Being Written Down. The Result Is 'So Many Active Howls In One Continuous Breath' (89) [Sounds Very Much Like Tara's Dadaist Tone Poems --Artaud's Example On P.83]]. These Words Are Often The Equivalent To Portmanteaux [Some Examples Are Given On Page 90]. Using These Words Can 'Enact A Chain Of Associations… In A Region Of Infra Sense, According To A Fluid And Burning Principle Which Absorbs And Reabsorbs Effectively The Sense As Soon As It Is Produced' (90). So Two Sorts Of Words Related To Two Sorts Of Bodies, One Fragmented And One Without Organs. There Are Also Two Theatres Or Two Types Of Nonsense Implied Here: One Where Ordinary Words Are Decomposed Into Nonsense, And One Where Tonic Elements Alone Form Nonsensical Words. They Are Produced By Things Happening Beneath The Surface, Unlike Carroll's Playful Superficiality. The Two Signifying and Signified Series Disappear, and Non Sense

Engulfs Signifiers and Signified. There Is No Surface Division To Separate The Expressivity Of Words And The Attributes Of Actual Bodies [Which Regulates Ordinary Language]. In Schizophrenic Language There Is No Grammar Or Syntax Either, Although Both Are Preserved In Carroll. Nevertheless, It Is Artaud Who Has 'Discovered A Vital Body And The Prodigious Language Of This Body… He Explored The Infra Sense Which Is Still Unknown Today' (93). However, Caroll Has Explored Those Important Surfaces, On Which 'The Entire Logic Of Sense Is Located' (93). We Can Still Find Schizoid Fragments In Ordinary Speech, But These Are Normally Reorganized. Similarly, Carroll Can Be Retranslated As A Schizophrenic Piece (92). But It Is Wrong To Generalise Here, 'Believing To Have Discovered Analogous Forms Which Create False Differences' (92). Psychoanalysis Should Operate With A Surface/Depth Structure Rather Than With Analogies—'It Is Geographical Before It Is Historical' (93). [While We Are Here, Note That 'It Is Hardly Acceptable… To Run Together a Child's Nursery Rhymes, Poetic Experimentations, And Experiences of Madness… [And] Justify the Grotesque Trinity of Child, Poet, and Madmen' (82-83). This Must Be A Problem For Those Who Think That Deleuze Is Arguing That Children Are Philosophers?] Thirteenth Series Of The Schizophrenic And The Little Girl Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable and Ascribable To Constraints on Space Accomodation Model Is Provided In Some Paper, Notwithstanding the Generality and Commonalty of the Observation and Its Concatenatability with the Other Modules. Singularities Constitute The Transcendental Field. The First Stage Of Actualization Involves The Derivation Of Individuals. Singularities Spread Out 'In a Determined Direction over a Line of Ordinary Points', Up To the Vicinity of another Singularity (109). When Series Converge, A World Is Constituted [And Other Worlds Arise From Diverging Series]. The Infinite System Of Singularities Are Selected And Rendered Finite By Individuals [Does He Mean Human Individuals Throughout? I Think So] Who Combine Them, 'Spread Them Out Over Their Ordinary Lines' (109) And Even Form Them Again On Various Membranes. 'An Individual Is Therefore Always In A World As A Circle Of Convergence, And A World May Be Formed And Thought Only In The Vicinity Of The Individuals Which Occupy Or Fill It' (110). [Deleuze Is Suggesting That This Role For Individuals Is The Only Way For Worlds To Persist, Since Entropy Prevents The Endless Renewal Of Singularities At The Virtual Level—'The Power Of Renewal Is Conceded Only To Individuals In The World, And Only For A Time—The Time Of Their Living Present' (110). This Is Static Genesis, The First Level Of Actualization. Singularities Are Actualized In The World And In Individuals. Actualization Means Being Extended, Selected, Incarnated In A Body, And Renewed Locally. These Qualities Require Individuals. Actualization Also Means Being Expressed, But It Would Be Wrong To Assume That Only The Expressed World Exists. There Are, For Example Incompossible Worlds, Produced By Diverging Series [As Real Events, And Not Just Matters Of Contradictory Consciousness]. This Implies That There Is A Continuum Of Singularities Outside Of [Conscious] Individuals, Worlds In Which Other Possibilities Arise. [So Grasping These Other Possibilities Again Distinguishes Sense From Mere Logic, Based On The Truth And Falsity Of Propositions Expressed By Individuals]. Because Individuals Express Worlds, However, The World Looks As If It Is Merely Subjective, And Objective Events Appear To Be 'The Analytic Predicate Of A Subject' (112). [These Analytic Predicates Appear To Be Properties Of People Or Objects]. [Much Of This Argument Is Referring To Leibniz]. However, The Problem Is To Explain Logical Hierarchies Of Properties And General Categories. Subjective Analytic Predicates Have Immediacy, But Do Not Have Any Kind of Order; They Get Elaborated But Only As Mixtures. They Are Descriptive Of Actual Structures And Diversities. They Are 'Intuitions... Immediate Representations' (113). The Second Level Of Actualization Involves The Transcendental Again, Originally Conceived As The Development Of Transcendency In The Individual [See The Critique Of Husserl Above]. The Transcendental Ego Is Constituted Just As Other Individuals Are, As A Circle Of Convergences, So The Problem Is To Escape This And Develop Knowledge Of The Whole Continuum, Of Incompossible Worlds And Divergent Series, A New Sense Of World [Deleuze Uses The German Terms Welt For This New World And Umwelt For The Subjective World. I Am Grateful To Wikipedia For Explaining That: In The Semiotic Theories Of Jakob Von Uexküll And Thomas A. Sebeok, Umwelt (Plural: Umwelten; The German Word Umwelt Means "Environment" Or "Surrounding World") Is The "Biological Foundations That Lie At The Very Epicenter Of The Study Of Both Communication And Signification In The

Human [And Non-Human] Animal."[Citation Needed] The Term Is Usually Translated As "Self-Centered World". [1] Uexküll Theorised That Organisms Can Have Different Umwelten, Even Though They Share The Same Environment']. Proper Transcendence Involves Transcending Individuated Worlds. Singularities Come With Their Own 'Perfectly Objective Determination Which Is The Open Space Of Its Nomadic Distribution' (113), And This Is How Deleuze Wants To Define A Problematic [Above]. Leibniz Apparently Half Grasped This Argument, Suggesting That Conic Sections, For Example, All Referred Back To The Same Event, Subdivided By 'Ambiguous Signs' (114). Similarly, Incompossible Worlds Also Have 'Something Objectively In Common', And 'Several Worlds Appear As Instances Of Solutions For One And The Same Problem... Variants of the Same Story... There Is Thus A"Vague Adam", That Is A Vagabond, A Nomad… Common to Several Worlds' (114). In Other Words, All The Objective And Subjective Worlds Can Be Explained By A Series Of Singularities. It Is Not Just The Analytic Predicates Of Individuals That Actualize Worlds. 'On The Contrary, [There] Are Predicates Which Define Persons Synthetically And Open Different Worlds And Individualities To Them As So Many Variables Or Possibilities' (115). It Is Incompossible Worlds Which Synthesise 'Primary Possibilities Or Categories' (115). These Possibilities or Categories 'Necessarily Signify Classes and Properties', And These Are Distinct From Individual Categories at the First Level. They Still Seen Grounded In Persons, But That Is 'Because Persons Themselves Are Primarily Classes Having One Single Member, And Their Predicates Are Properties Having One Constant' (115). There Is Thus A System Involving Persons, Classes With One Single Member, Extensive Classes And Variable Properties '-- That Is The General Concepts Which Derive From Them' (115). If There Is A Universal Ego, It Is Something That Corresponds To Common Elements In All Worlds. So, Firstly, Sense Gets Actualized In A Field Of Singularities, Then The Umwelt Develops Around Individuals Which Express Or Describe This World, Then, Secondly, A Whole Objective World Develops From Common Elements, A Welt. Persons Can Then Define This Common Element And Develop Classes And Properties Derived From It. At The First Level We Find Good Sense, 'An Already Fixed And Sedentary Organization Of Differences'. At The Second Level, Commonsense Serves The Function Of Identification. Neither Term Explains How These Activities Are Derived [In Fact, Deleuze Wants To Suggest That The Second Level 'Is The Work Of Non Sense Which Is Always Copresent To Sense (Aleatory Point Or Ambiguous Sign)... Productive Nonsense Which Animates The Ideal Game And The Impersonal Transcendental Field' (116). There Is No Conventional Transcendental Movement, Especially One Driven By A Version Of The Ego. The Person Is A 'Produced Form, Derived From This Impersonal Transcendental Field… Always an Individual in General, Born… From The Singularity Which Extends Itself Over A Line of Ordinary Points and Starts from the Preindividual Transcendental Field' (116). Persons And Their Varieties Of Sense Are All Produced 'On The Basis Of Sense And Nonsense Which Do Not Resemble Them' (117). This Explains The Paradoxes And Limits Of Good Sense And Common Sense. [So The Existence Of Nonsense -- Paradox And Impossible Sentences Etc -- In The Ordinary Subjective World Shows The Existence Of Non Sense -- Something That Produces Ordinary Sense In The First Place, Something Objective? Individuals Can Only Describe But As Soon As They Generalise And Order Concepts They Necessarily Assume Some Non-Subjective World? The Man oeuvre seems Rather like the Illicit Transcendental Ego Move by Husserl, Generalising from an Ordinary Ego: Here, Explaining Ordinary Experience Is Used to Explain Some Deeper Virtual World as an Extension?] Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules Sixteenth Series Of The Static Ontological Genesis. The Issue Is Of The Validity Or Truth Of Propositions And How This Might Be Grounded. It Is As Difficult To Root Propositions In The Idea As It Is Actual Events. Significations Cannot Be Just Seen As Exhausted By Their Denoted Examples. A List Of Denoted Examples Would Be Endless And Groundless Anyway. Classic Philosophers Often Refuse To Answer Questions Denoted Examples Any Way, And Replied With A Blow From The Staff, Or A Mute Demonstration. After All, 'There Is No Resemblance (Nor Should There Be One) Between What One Points Out And What One Has Been Asked' (135) [Typical Philosophical Elitism]. The Issue Is Often Covered By Humour, Especially Irony or the Absurd. We Must Refuse Ascent and Descent and Occupy the Surface 'Where Pure Sense Is Produced' (136). At

The Surface, 'One Finds Pure Singularities, An Emission Of Singularities Considered From This Perspective Of The Aleatory Element, Independent Of The Individuals And Persons Which Embody Or Actualise Them' (136). Getting To The Surface Involves Humour And Adventure, Seen In The Routines Of Zen Masters, Who Can Allude To The Void Which Is Present In Events [Events Are 'Not The Object As Denoted, But The Object As Expressed Or Expressible, Never Present, But Always Already In The Past And Yet To Come' (136), And The Abolition Of The Object Alludes To The Void]. Negating Objects Indicates The Expressible. [Examples From Zen Follow, Page 137. What They Seem To Be Suggesting Is That Sense Combines Actual Objects With The Notion Of The Void, Non Sense—An Actual Brushstroke And Also The White Space—'Language Becomes Possible And, By Becoming Possible, It Inspires Only A Silent And Immediate Communication' Somehow Outside All The Significations And Denotations]. The Same Reasoning Affects The Question Of Who Speaks—Individuals Or Language Itself In The Form Of Ideal Forms [And We Are Back With Greek Philosophy Examples. Socratic irony apparently Refers to the Fact That Individuals Both Initiate Speech and Are the Products of Speech—138. This In Turn Permits The Idea Of A 'Pure Rational Language… [Enabling]… Natural Communication between A Supremely Individuated God and the Derived Individuals Which He Created' (138)]. Further Irony Awaits The Romantic Notion Of The Person, 'The Finite Synthetic Unity Of The Person... [Not Just]... The Analytic Identity of the Individual' (138). Here, The Person And Representations Are Linked Ironically, Implying Both A Universal Idea And 'Sensible Particularity' (138), Both Universal And Particular Characteristics Of The Person—Hence The Earlier Argument That Persons Should Really Be Seen As A Class Which Has Only One Member. All These Formulations Assume That Singularities Are Located In The Individual Or The Person, Influenced Only By Some Groundless Abyss, Or Chaos, Threatening Both Classical And Romantic Discourse With The Lack Of Articulation. There Is Also A Threefold Division Of Language—The Ordinary Or Real; The Ideal Language, Like The Purely Rational One; Esoteric Language Which Subverts The Ideal Language And The Individuality Of The Speaker Of Ordinary Language [For Example Social Scientific Explanations, Or Structural Linguistic Ones?] [Weird Examples of Esoteric Languages Which Mean Nothing to Me on Page 140, With a Reference Back To Portmanteau Words]. So Sometimes Individual Speaks, Sometimes The Person, And Sometimes 'The Ground Which Dissolves Both' (140). The Only Way Out Is To See That Singularities Are Not Coterminous With Individuals, And There Is Not Just A Groundless Abyss Beneath Them. There Are Other Impersonal And Pre-Individual Singularities. Nonsense And Sense Collaborate At The Surface. Irony Gives Way To Humour—'The Co extensiveness Off Sense with Nonsense', A Matter Of 'Surfaces… Doubles… Nomad Singularities and Of the Always Displaced Aleatory Point' (141). Normal Significations, Denotations and Manifestations Are Suspended, and 'All Height and Depth Abolished' (141). Nineteenth Series of Humour. Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules Sixteenth Series Of The Static Ontological Genesis.Doctrines Come From 'Wounds And Vital Aphorisms' (148), And Some Writers See Themselves As Embodying Events [Hints Of The Stuff On Nietzsche And Illness Earlier]. Our Will Can Act As A Quasi Cause Of Bodily Events, As We Live Them. This Is An Ethical Stance—'Not To Be Unworthy Of What Happens To Us' (149). The Alternative Is Ressentiment. The Normal Moral Notions Such As Just Or Unjust Are Themselves Immoral. It Is Not Just A Matter Of Resignation To Events, Which Can Still Be Ressentiment, More An 'Apotheosis Of The Will' (149) [Citing Bousquet]. The Organic Is Exchanged For The Spiritual Will [Isn't This Just Making The Best Of Things, Finding Some Deep Meaning In Personal Tragedy?]. In This Sense, Freedom Is The Same As Submitting To Fate, Actualizing Events, Making Sense Of Events, Seeing Events As Something Expressed. One Becomes 'The Offspring of One's Events Are Not of One's Actions, For the Action Is Itself produced By the Offspring Of The Event' (150). In This Way, Actors Somehow Communicate With The Aion, Instead Of Being Dominated By Chronos. [Lots Of Implicit Christianity Here, Surely?]. All The Components Of An Event, Future As Well As Past, Are United, So That One Realises The Impact Of Singularities, Including Preindividual Components. [In This Way, Some Sort Of Agency Seems To Remain? One Can Become 'The Actor of One's Own Events – A Counter Actualization' (150)]. Ressentiment Arises From Not Realizing That Our

Particular Part Of Experience, Which May Well Be Unjust, Is Part Of A More Perfect Whole. It Is Incorrect To Judge Everything From The Perspective Of The Present. Some Humourous Recognition of the Futility of It All Is Valuable—In the Great Scheme of Things, States of Affairs Are Always 'Impersonal and Pre-Individual, Neutral, neither General nor Particular' (151). In These Circumstances, Either Life Can Seem 'Too Weak For Me' [Not Vivid Enough, Not Enough Focused In The Present?], Or 'It Is I Who Am Too Weak For Life, It Is Life Which Overwhelms Me, Scattering Its Singularities All About, In No Relation To Me, Nor To A Moment Determinable As The Present Except An Impersonal Instant' (151). This Is Apparent When One Considers Death Or The Mortal Wound, An Event Which Is Indifferent To Me, 'Incorporeal And Infinitive, Impersonal, Grounded Only In It' (151), Although It Is I Who Has To Actualize The Event. However I Can Also Counter actualizes In the Sense Above. It Follows That The Events Of Reality Are Quite Different From Personal Experience, Shown In The 'Splendour Of The "They"… The Splendour of the Event Itself Order Of the Fourth Person' (152) [This Fourth Person Is the 'It' As In Phrases Like 'It Is Raining']. The Old Distinctions Between Private And Collective Refer Only To Personal Experience, Whereas 'Everything Is Singular, And Thus Both Collective And Private… Which Private Event Does Not Have All Its Coordinates, That Is, All Its Impersonal Social Singularities?' (152). Freedom Exists Only In Recognising The Impersonal Nature Of The Event, Seeing All Events As 'A Single Event Which No Longer Makes Room For The Accident, And Which Denounces And Removes The Power Of Ressentiment Within The Individual As Well As The Power Of Oppression Within Society' (152) [Ridiculous Philosophical Notion Of Freedom Overcoming Oppression By An Act Of Philosophy]. Ressentiment Ties One to an Oppressive Order. [Then A Really Sentimental And Pathos Ridden Bit: 'It Is At This Mobile And Precise Point, Where All Events Gather Together In One That Transmutation Happens: This Is The Point At Which Death Turns Against Death; Where Dying Is The Negation Of Death And The Impersonal At He Of Dying No Longer Indicates Only The Moment When I Disappear Outside Of Myself, But Rather The Moment When Death Loses Itself In Itself, And Also The Figure Which The Most Singular Life Takes On In Order To Substitute Itself For Me' (153)] Ethics Links Logic and the Body. The Stoic Notion Of Bodies Included The Passions, And Good And Evil Intents: Particular Bodies Might Have Evil Mixtures, But The Aggregate Of Bodies Is Perfect Or Good, The Unity Of Causes Themselves. In Principle, Each Event Can Be Linked To A Particular Cause And Thus The Unity Of Causes, And This Could Lead To The Activity Of Divination [Grasping The Divine Unity Of Causes] As A Basis For Ethics. The Effects, Lines Have To Be Traced Back From Events To Pure Events And Then To Actions And Passions [With A Lot Of Poetic Stuff As Examples, 143]. Stoics Took Another Route To Ethics Through Logic [Kind Of Working Out Which Events Will Actualize]. Stoic Philosophy Then Saw In Representations Of The Limited Event A Connection With Pure Events [And Ethical Conduct Seems To Be To Work Towards Actualizing Such Events?]. [Stoic Accounts Of Representations Ensue, 144-5.] There Is A Difference [For Stoics] Between Representations Of Sense And Of Logic, Denotations And Significations, Representations And Expressions. Representation Is Not Just Resemblance, But Includes A Notion of Adequate Expression. Concepts Have To Be Actualised In Representation, And Also Expressed If They Are To Be Comprehensive. Thus Our Knowledge Of Death Is Abstract, Despite The Number Of Deaths We Witness, Until It Becomes Personal, Not Indifferent, But Concrete. This Is Where Expression Is Needed. In Stoic Philosophy, Moral Conduct Involves A Relation with Pure Events Unites Those Events with One's Self. [The Examples Again Come From Zen: 'The Bowman Must Reach The Point Where The Aim Is Also Not The Aim, That Is To Say, The Bowman Himself' (146).] An Understanding Of The Pure Event Is Required, As Something Which Is 'Eternally Yet To Come And Always Already Past', But Which Has To Be Willed Into Actualization (146). The Stoic Embodies Incorporeal Effects, Aligning Themselves With The Quasi Cause. It Is Necessary Because Quasi Causes Cannot Embody Themselves, Except To The Immediate Instant, The Present. [There Is An Equally Baffling Aside About The Difference Between Actors And Characters—Actors Represent By Occupying The Instant, While The Character Also 'Portrays Hopes Or Fears In The Future And Remembers Or Repents In The Past' (147). Stoics Therefore See Themselves As Uniting The Instant With The Unlimited Future And Past, Willing The Event And Also Representing It. [Largely Incomprehensible, Although I Am Starting To See How The Normal Notion Of Stoicism Might Fit—The Patient Resignation Of One's Self To One's Fate—And Also Seeing How Deleuze Thinks That Human Beings Should Reconcile Themselves To Solving The Problems That Reality Creates In The Form Of A Problematic]. Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES

ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules Sixteenth Series Of The Static Ontological Genesis. Deleuze And Guattari See In The Capitalist Money System "An Axiomatic Of Abstract Quantities That Keeps Moving Further And Further In The Direction Of The Deterritorialization Of The Socius" (Deleuze 1983a, 33), Which Is To Say That Capital Is Inherently Schizophrenic. However, Because Capital Also Re-Territorializes All Flows Into Money, Schizophrenia Remains Capitalism's External Limit. Nevertheless, It Is Precisely That Limit Against Which Thinking Can Subject Capitalism To Philosophical Critique. Psychoanalysis, They Say, Is Part Of The Reign Of Capital Because It Re-Territorializes The Subject As "Private" And "Individual," Instituting Psychic Identity Through Images Of The Oedipal Family. However, The Oedipal Triangle Is Merely A Representational Simulacrum Of Kinship And Filiation, Re-Coded Within A System Of Debt And Payment. In This System, They Insist, Flows Of Desire Have Become Mere Representations Of Desire, Cut Off From The Body Without Organs And The Extra-Familial Mechanisms Of Society. A Radical Critique Of Capital Cannot Therefore Be Accomplished By Psychoanalysis, But Requires A Schizoanalysis "To Overturn The Theater Of Representation Into The Order Of Desiring-Production" (Deleuze 1983b, 271). Here, The Authors See A Revolutionary Potential In Modern Art And Science, Where, In Bringing About The "New," They Circulate De-Coded And De-Territorialized Flows Within Society Without Automatically Re-Coding Them Into Money (Deleuze 1983a, 379). In This Revolutionary Aspect, Anti-Oedipus Reads As A Statement Of The Desire That Took To The Streets Of Paris In May Of 1968, And Which Continues, Even Now, To Make Itself Felt In Intellectual Life. Postmodernism First Published Fri Sep 30, 2005 Copyright © 2005 By Gary Aylesworth Geaylesworth@Eiu.Edu Open Access To The SEP Is Made Possible By A World-Wide Funding Initiative For Models Kindly See One Of The Papers In The Series, Procrastinated Due To Restrictions On Space. The Term "Deconstruction," Like "Postmodernism," Has Taken On Many Meanings In The Popular Imagination. However, In Philosophy, It Signifies Certain Strategies For Reading And Writing Texts. The Term Was Introduced Into Philosophical Literature In 1967, With The Publication Of Three Texts By Jacques Derrida: Of Grammatology (English 1974), Writing And Difference (English 1978), And Speech And Phenomena (English 1973). This So-Called "Publication Blitz" Immediately Established Derrida As A Major Figure In The New Movement In Philosophy And The Human Sciences Centered In Paris, And Brought The Idiom "Deconstruction" Into Its Vocabulary. Derrida And Deconstruction Are Routinely Associated With Postmodernism, Although Like Deleuze And Foucault, He Does Not Use The Term And Would Resist Affiliation With "-Isms" Of Any Sort. Of The Three Books From 1967, Of Grammatology Is The More Comprehensive In Laying Out The Background For Deconstruction As A Way Of Reading Modern Theories Of Language, Especially Structuralism, And Heidegger's Meditations On The Non-Presence Of Being. It Also Sets Out Derrida's Difference With Heidegger Over Nietzsche. Where Heidegger Places Nietzsche Within The Metaphysics Of Presence, Derrida Insists That "Reading, And Therefore Writing, The Text Were For Nietzsche 'Originary' Operations," (Derrida 1974, 19), And This Puts Him At The Closure Of Metaphysics (Not The End), A Closure That Liberates Writing From The Traditional Logos, Which Takes Writing To Be A Sign (A Visible Mark) For Another Sign (Speech), Whose "Signified" Is A Fully Present Meaning. This Closure Has Emerged, Says Derrida, With The Latest Developments In Linguistics, The Human Sciences, Mathematics, And Cybernetics, Where The Written Mark Or Signifier Is Purely Technical, That Is, A Matter Of Function Rather Than Meaning. Precisely The Liberation Of Function Over Meaning Indicates That The Epoch Of What Heidegger Calls The Metaphysics Of Presence Has Come To Closure, Although This Closure Does Not Mean Its Termination. Just As In The Essay "On The Question Of Being" (Heidegger 1998, 291-322) Heidegger Sees Fit To Cross Out The Word "Being," Leaving It Visible, Nevertheless, Under The Mark, Derrida Takes The Closure Of Metaphysics To Be Its "Erasure," Where It Does Not Entirely Disappear, But Remains Inscribed As One Side Of A Difference, And Where The Mark Of Deletion Is Itself A Trace Of The Difference That Joins And Separates This Mark And What It Crosses Out. Derrida Calls This Joining And Separating Of Signs Différance (Derrida 1974, 23), A Device That Can Only Be Read And Not Heard When Différance And Différence Are Pronounced In French. The "A" Is A Written Mark That Differentiates Independently Of The Voice, The Privileged Medium Of Metaphysics. In This Sense, Différance As The Spacing Of

Difference, As Archi-Writing, Would Be The Gram Of Grammatology. However, As Derrida Remarks: "There Cannot Be A Science Of Difference Itself In Its Operation, As It Is Impossible To Have A Science Of The Origin Of Presence Itself, That Is To Say Of A Certain Non-Origin" (Derrida 1974, 63). Instead, There Is Only The Marking Of The Trace Of Difference, That Is, Deconstruction. Because At Its Functional Level All Language Is A System Of Differences, Says Derrida, All Language, Even When Spoken, Is Writing, And This Truth Is Suppressed When Meaning Is Taken As An Origin, Present And Complete Unto Itself. Texts That Take Meaning Or Being As Their Theme Are Therefore Particularly Susceptible To Deconstruction, As Are All Other Texts Insofar As They Are Conjoined With These. For Derrida, Written Marks Or Signifiers Do Not Arrange Themselves Within Natural Limits, But Form Chains Of Signification That Radiate In All Directions. As Derrida Famously Remarks, "There Is No Outside-Text" (Derrida 1974, 158), That Is, The Text Includes The Difference Between Any "Inside" Or "Outside." A Text, Then, Is Not A Book, And Does Not, Strictly Speaking, Have An Author. On The Contrary, The Name Of The Author Is A Signifier Linked With Others, And There Is No Master Signifier (Such As The Phallus In Lacan) Present Or Even Absent In A Text. This Goes For The Term "Différance" As Well, Which Can Only Serve As A Supplement For The Productive Spacing Between Signs. Therefore, Derrida Insists That "Différance Is Literally Neither A Word Nor A Concept" (Derrida 1982, 3). Instead, It Can Only Be Marked As A Wandering Play Of Differences That Is Both A Spacing Of Signifiers In Relation To One Another And A Deferral Of Meaning Or Presence When They Are Read. How, Then, Can Différance Be Characterized? Derrida Refuses To Answer Questions As To "Who" Or "What" Differs, Because To Do So Would Suggest There Is A Proper Name For Difference Instead Of Endless Supplements, Of Which "Différance" Is But One. Structurally, This Supplemental Displacement Functions Just As, For Heidegger, All Names For Being Reduce Being To The Presence Of Beings, Thus Ignoring The "Ontological Difference" Between Them. However, Derrida Takes The Ontological Difference As One Difference Among Others, As A Product Of What The Idiom "Différance" Supplements. As He Remarks: "Différance, In A Certain And Very Strange Way, (Is) 'Older' Than The Ontological Difference Or Than The Truth Of Being" (Derrida 1982, 22). Deconstruction, Then, Traces The Repetitions Of The Supplement. It Is Not So Much A Theory About Texts As A Practice Of Reading And Transforming Texts, Where Tracing The Movements Of Différance Produces Other Texts Interwoven With The First. While There Is A Certain Arbitrariness In The Play Of Differences That Result, It Is Not The Arbitrariness Of A Reader Getting The Text To Mean Whatever He Or She Wants. It Is A Question Of Function Rather Than Meaning, If Meaning Is Understood As A Terminal Presence, And The Signifying Connections Traced In Deconstruction Are First Offered By The Text Itself. A Deconstructive Reading, Then, Does Not Assert Or Impose Meaning, But Marks Out Places Where The Function Of The Text Works Against Its Apparent Meaning, Or Against The History Of Its Interpretation. Postmodernism First Published Fri Sep 30, 2005 Copyright © 2005 By Gary Aylesworth Geaylesworth@Eiu.Edu Open Access To The SEP Is Made Possible By A World-Wide Funding Initiative For Models Kindly See One Of The Papers In The Series, Procrastinated Due To Restrictions On Space. "Anything Which Is Just Born, Which Has Just Come Into Existence, Has No Past Behind It. Birth, In Other Words, Is The Condition Of Having No Past. And Likewise, Anything Which Now Dies, Which Has Just Ceased To Be, Has No Future Left In Front Of It. Death Is The Condition Of Having No Future. But We Have Already Seen That This Present Moment Has Both No Past And No Future Simultaneously. That Is, Birth And Death Are One In This Present Moment. This Moment Is Just Now Being Born—You Can Never Find A Past To This Present Moment, You Can Never Find Something Before It. Yet Also, This Moment Is Just Now Dying — You Can Never Find A Future To This Moment, Never Find Something After It. This Present, Then, Is A Coincidence Of Opposites, A Unity Of Birth And Death, Being And Non-Being, Living And Dying. As Ippen Put It, "Every Moment Is The Last Moment And Every Moment Is A Rebirth." Ken Wilber, No Boundary: eastern and Western Approaches to Personal Growth for models please see one of the papers in the series. "Because We Demand A Future, We Live Each Moment In Expectation And Unfulfillment. We Live Each Moment In Passing. In Just This Way The Real Nunc Stans, The Timeless Present, Is Reduced To The Nunc Fluens, The Fleeting Present, The Passing Present Of A Mere One Or Two Seconds. We Expect Each Moment To Pass On To A Future Moment, For In This Fashion We Pretend To Avoid Death By Always Rushing Toward An Imagined Future. We Want To Meet Ourselves In The Future. We Don't Want Just Now—We Want Another Now, And Another, And Another, Tomorrow And Tomorrow And Tomorrow. And Thus, Paradoxically, Our Impoverished

Present Is Fleeting Precisely Because We Demand That It End! We Want It To End So That It Can Thereby Pass On To Yet Another Moment, A Future Moment, Which Will In Turn Live Only To Pass." ― Ken Wilber, No Boundary: Eastern And Western Approaches To Personal Growth. For Stoics, Causes Involved Reference To The Depths, But Effects At The Surface Could Also Have Relations Among Themselves. This Permitted The Distinction To Be Drawn Between Destiny And Necessity—The Stoics Wanted To Affirm Destiny And Deny Necessity. In The First Place, Effects Express Causes, But Expressions Of Relations [Rather Than Necessity] Connects Those Effects. Those Relations May Be Described As Compatibility Or Incompatibility, Conjunction Or Disjunction. These Are Not Causal Relations Themselves, But Represent 'An Aggregate Of Noncausal Correspondences Which Form A System Of Echoes, Of Presumptions And Resonances, A System Of Signs—In Short, An Expressive Quasi Causality And Not At All A Necessitating Causality' (170). This Need Not Involve Contradiction, Which Is Applying To Events Rules That Really Only Applied To Logic And Argument. There Can Be Incompatability Without Contradiction, A Noncausal Correspondence. Leibniz Described Incompossible Worlds [Delanda Is Very Clear On This Too]. Only Impossible Events Contradict Possible Ones. Events Can Be Compossible [Roughly, They Have Predictable And Predicative Future And Past Events]. For Deleuze, It Is A Matter Of 'The Convergence Of Series Which Singularities Of Events Form As They Stretch Themselves Out Over Lines Of Ordinary Points. Incompossibility Must Be Defined By The Divergence Of Such Series' (171). Such A Notion Is Essential To Any Theory Of Sense. We Should Not See Divergence As A Matter Of Exclusion, As Leibniz Did [Since God Chose Actual Events]. Divergent And Disjunction Can Both Be Positive, While Preserving Differences. In Fact, Differences Are Crucial, Preserving The Distance Between Objects While Affirming That They Are Related. This 'Permits The Measuring Of Contraries Through Their Finite Difference Instead Of Equating Difference With A Measureless Contrariety' (173). It Is Contradiction Which Is The Special Case. Difference Here Is A Topological Term Relating To Distance On Surfaces Rather Than Depths. It Is Not Just A Matter Of Suggesting 'Some Unknown Identity Of Contraries (As In Commonplace In Spiritualist And Dolorist Philosophy)' (173) [Take That St Pierre!]. An Example Is Nietzsche Arguing That Health And Sickness Can Both Inform Each Other, Act As Points Of View, Remembering That 'Things, Beings, Are Themselves Points Of View' (173). Divergence Does Not Mean Exclusion, And Disjunction Does Not Mean Separation. Connective Syntheses '(If…, Then)' Construct A Single Series; Conjunctive Series '(And)' Produces Convergent Series, But Disjunctive Series '(Or)' Produces A Divergence Series. Normally, Disjunction Helps Us To Criticise Synthesis, But Disjunction Can Still Be A Synthesis Itself, Despite Its Use In Logical Analysis [I Think What Is Going On Here Is Arguing That There Is A Difference Between 'Either/Or' In A Logical Sense, And 'One Or Two' In The Real Sense—The Latter Can Mean That Both Are Compossible. This Is The 'Communication Of Events' Rather Than The Logical Business Of Analysing Predicates (174)]. The Synthetic Disjunction Expresses The Paradox, With Divergence At The Centre. The Discussion Of Paradoxes And Esoteric Words Above Are Examples: They Contract 'The Multitude Of Divergent Series In The Successive Appearance Of A Single One' (175). There Is A Difference Again Between The Subversion Of The Present And Simple Identity By Depths, And Operations At The Surface. By Considering The Depths, We Encounter Infinite Identities [As Events Become Examples Of Deeper Categories, Wholes?]. At The Surface Events Communicate With Each Other Directly Through Maintaining Distance And By Affirming Disjunctions. Disjunction Threatens The Identity Of The Self, And Helps Us To See The Self As 'So Many Impersonal And Pre-Individual Singularities' [Connected Through Disjunctive Synthesis. Hence The Importance Of Heterogeneity] (175). The Normal Concept Of The Self Implies Some Connected Series, 'But When Disjunction Accedes To The Principle Which Gives To It A Synthetic And Affirmative Value, The Self, The World, And God Share In A Common Death' (176). [There Is Also A Point That Divergent Series Explain And Also Exceed Normal Conjunctive And Connective Series]. In The Usual Conception, 'The Self Is The Principle Of Manifestation, In Relation To The Proposition, The World Is The Principle Of Denotation, And God The Principle Of Signification' (176). But The Theory Of Sense Here Says That It Emanates From Nonsense, From Paradox, And From The 'Eternally Decentred Ex-Centric Centre' (176). This Position 'Does Not Tolerate The Subsistence Of God As An Original Individuality, Nor The Self As The Person, Nor The World As An Element Of The Self And As God's Product. The Divergence Of The Affirmed Series Forms A "Chaosmos" And No Longer A World; The Aleatory Point Which Traverses Them Forms A Counterself, And No Longer A Self' (176). There Is No Centre But

Only 'Pure Events Which The Instant, Displaced Over The Line[Of Aion] , Goes On Dividing Into Already Past And Yet To Come. Nothing Other Than The Event Subsists… This Communicates With Itself Through Its Own Distance And Resonates Across All Of Its Disjuncts' (176). Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable To Constraints On Space Accomodation Model Is Provided In Some Paper of the series, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules Sixteenth Series Of The Static Ontological Genesis Twenty-fourth series of the communication of events . "As Simple As That Sounds, It Is Nevertheless Extremely Difficult To Adequately Discuss No-Boundary Awareness Or Nondual Consciousness. This Is Because Our Language — The Medium In Which All Verbal Discussion Must Float — Is A Language Of Boundaries. As We Have Seen, Words And Symbols And Thoughts Themselves Are Actually Nothing But Boundaries, For Whenever You Think Or Use A Word Or Name, You Are Already Creating Boundaries. Even To Say "Reality Is No-Boundary Awareness" Is Still To Create A Distinction Between Boundaries And No-Boundary! So We Have To Keep In Mind The Great Difficulty Involved With Dualistic Language. That "Reality Is No-Boundary" Is True Enough, Provided We Remember That No-Boundary Awareness Is A Direct, Immediate, And Nonverbal Awareness, And Not A Mere Philosophical Theory. It Is For These Reasons That The Mystic-Sages Stress That Reality Lies Beyond Names And Forms, Words And Thoughts, Divisions And Boundaries. Beyond All Boundaries Lies The Real World Of Suchness, The Void, The Dharmakaya, Tao, Brahman, The Godhead. And In The World Of Suchness, There Is Neither Good Nor Bad, Saint Nor Sinner, Birth Nor Death, For In The World Of Suchness There Are No Boundaries." ― Ken Wilber, No Boundary: Eastern And Western Approaches To Personal Growth. It Has Just Been Argued That Divergence Can Produce A Positive Synthesis, And That Ultimately, Events And States Of Affairs Are Compatible. Incompatibility Arises Only With Actualizations. It Is Individuals, For Example, Who Actualize Divergent Events. Even These Divergences Are Not Just Logical Contradictions But 'Alogical 'Incompatibilities. Persons Can Enjoy The Paradoxes That Ensue, But The Point Is How To Get To The Universal Communication Of Events, The Disjunctive Syntheses. Individuals Must Grasp Themselves As Events, And See What They Normally Regard As Themselves As An Actualization. Normal Individuality Is Only 'Fortuitous' (178). It Follows That The Same Thinking Must Be Extended To All Other Events And Individuals. 'Each Individual Would Be Like A Mirror For The Condensation Of Singularities… The Ultimate Sense Of Counteractualization' (178) [Undertaken For Cognitive Reasons Here Not Ethical Ones, Although Deleuze Says That This Is A Route To 'The Universal Freedom' (178)]. The Normal Sense Of The Individual As Having A Coherent Identity Must Be Replaced By The Notion Of A Series Of Individualities [With Nomadic Lines Connecting Them?]. We Then Get To The Realm Of Pure Events, And The Universal Connections Between Them, Seen As Disjunctive Syntheses Of Series, And The Knowledge That Actualized Events Are Fortuitous. [The Example Here Is That A Friend In One World Could Be An Enemy In Another Equally Possible World]. 'Philosophy Merges With Ontology, But Ontology Merges With The Univocity Of Being (Analogy Has Always Been A Theological Vision, Not A Philosophical One, Adapted To The Forms Of God, The World, And The Self)' (179). This Does Not Mean That There Is Only One And The Same Being [And Certainly Not Identical Beings, Because These Are Always Heterogeneous]. Instead, 'Being Is Voice That Is Said, And It Is Said In One And The Same "Sense" Of Everything About Which It Is Said' (179). It Is The Ultimate Form. All The Other Forms Remain 'Disjointed In It', But Being Joins Them Into Series And Disjunctions. 'The Positive Use Of The Disjunctive Synthesis… Is The Highest Affirmation... A Single Voice For Every Hum Of Voices And Every Drop Of Water In The Sea' (180). It Is Not Just That There Is A Connection At The Level Of Language—'Being Cannot Be Said Without Also Occurring' (180). In This Way, Event And Sense Are Identical. 'Univocal Being Is Neutral. It Is Extra Being, That Is The Minimum Of Being Common To The Real, The Possible And The Impossible… The Pure Form Of The Aion… One Single Event For All Events; One And The Same Aliquid [Something] For That Which Happens And That Which Is Said; And One And The Same Being For The Impossible, The Possible, And The Real' (180). Alexander I. Stingl's Blog A Nomadic Scholarship Entity NOTES ON: Deleuze, G (1990) The Logic Of Sense, Trans Mark Lester, Edited By Constantin Boundas, New York: Columbia University Press For Models Kindly See One Of The Papers In The Series. Attributable And Ascribable To Constraints On Space

Accomodation Model Is Provided In Some Paper, Notwithstanding The Generality And Commonalty Of The Observation And Its Concatenatability With The Other Modules. Sāṅkhya (Often Spelled Sāṁkhya) Is One Of The Major "Orthodox" (Or Hindu) Indian Philosophies. Two Millennia Ago It Was The Representative Hindu Philosophy. Its Classical Formulation Is Found In Īśvarakṛṣṇa's Sāṅkhya-Kārikā (Ca. 350 C.E.), A Condensed Account In Seventy-Two Verses. It Is A Strong Indian Example Of Metaphysical Dualism, But Unlike Many Western Counterparts It Is Atheistic. The Two Types Of Entities Of Sāṅkhya Are Prakṛti And Puruṣa-S, Namely Nature And Persons. Nature Is Singular, And Persons Are Numerous. Both Are Eternal And Independent Of Each Other. Persons (Puruṣa-S) Are Essentially Unchangeable, Inactive, Conscious Entities, Who Nonetheless Gain Something From Contact With Nature. Creation As We Know It Comes About By A Conjunction Of Nature And Persons. Prakṛti, Or Nature, Is Comprised Of Three Guṇa-S Or Qualities. The Highest Of The Three Is Sattva (Essence), The Principle Of Light, Goodness And Intelligence. Rajas (Dust) Is The Principle Of Change, Energy And Passion, While Tamas (Darkness) Appears As Inactivity, Dullness, Heaviness And Despair. Nature, Though Unconscious, Is Purposeful And Is Said To Function For The Purpose Of The Individual Puruṣa-S. Aside From Comprising The Physical Universe, It Comprises The Gross Body And "Sign-Body" Of A Puruṣa. The Latter Contains Among Other Things The Epistemological Apparati Of Embodied Beings (Such As The Mind, Intellect, And Senses). The Sign Body Of A Puruṣa Transmigrates: After The Death Of The Gross Body, The Sign-Body Is Reborn Into Another Gross Body According To Past Merit, And The Puruṣa Continues To Be A Witness Through Its Various Bodies. An Escape From This Endless Circle Is Possible Only Through The Realization Of The Fundamental Difference Between Nature And Persons, Whereby An Individual Puruṣa Loses Interest In Nature And Is Thereby Liberated Forever From All Bodies, Subtle And Gross. Much Of The Sāṅkhya System Became Widely Accepted In India: Especially The Theory Of The Three Guṇa-S; And It Was Incorporated Into Much Latter Indian Philosophy, Especially Vedānta. Internet Encyclopedia Of Philosophy Sankhya Author Information Ferenc Ruzsa Email: Ferenc.Ruzsa@Elte.Hu Eötvös Loránd University Hungary. The Amitabha Buddha Who Was, And Is, Revered And Praised By Buddhists Around The World, Radiates Indefinite Light And Life From This "State Of Cessation". This State Is A Continuous Process Of Calmness. It Will Be The Eventual Refuge For Us All. If We Think Carefully About The Definitions Of Calmness And Extinction, Then We Can Deduce That They Are The True Natural End-Points Of Rising And Falling. The True Nature Of The Cycle Of Rising And Falling Is Calmness And Extinction. Because Of This Nature, All Chaos And Conflicts In The State Of Rising And Falling Will Eventually Cease. This Is Attainable By The Realisation Of Prajna. Teachings In Chinese Buddhism (6) Sunyata (Emptiness) In The Mahayana Context (Wikipedia)

@@@

I wake to sleep, and take my waking slow.

Of those so close beside me, which are you?

God bless the Ground! I shall walk softly there,

And learn by going where I have to go.

Light takes the Tree; but who can tell us how?

The lowly worm climbs up a winding stair;

I wake to sleep, and take my waking slow.

Great Nature has another thing to do

To you and me, so take the lively air,

And, lovely, learn by going where to go.

This shaking keeps me steady. I should know.

What falls away is always. And is near.

I wake to sleep, and take my waking slow.

I learn by going where I have to go."

— Theodore Roethke, The Collected Poems (Wikipedia)

Dairy Notes: In Excited Reveries:

Intractable disaffect, Mutinous obdurate, Rebellious recalcitrant, Seditious refractory, Insubordinate insurgent, antagonistically antipodean, discrepant incompatible, Contumacious coadjutant, Conflicting converse, Unyielding obdurate, the antagonistic and anachronistic dispositional ties of Anti Brahman are in fact the sine qua non of Brahman only (See Vishwaroopa of Mahavishnu, which contains the polarised positives on RHS and negatives on LHS)……Adversely antagonistic, inexorably irreconcilable, antithetically antipodal, diametrically opposed, repugnantly retrograde, inimically inverse, violatively unsimilar, diversely dissimilar are the views of epistemologists in the analysis of raison d'etre of knowledge……..Autheticative determination, unimpeachable validation, incontrovertible establishment, apodictic evidence, reliable roll out, unfailing cinquecento **quattrocento trecento,** incontrovertible indication.. of unconnected thoughts and disconnected discursion…….. abhorrent, abominable, accursed, atrocious, awful, bad, beastly, crying, cursed, evil, execrable, flagitious, flagrant, frightful, god-awful, grave, gross, hateful, hideous, horrendous, infamous, iniquitous, nefarious, odious, offensive, outrageous,

raunchy, revolting, scandalous, shocking, stinking, unspeakable, vicious, villainous Abominable, accursed, atrocious, damnable, devilish, disgusting, execrable, fiendish…… explanations that revolts one sensibilities and susceptibilities and sentimentalities……. But why did he say movement is object?..... what has emotional change got to do with it?..... ……. The image of an intra-cranially self-stimulating rat…… Its degraded frenzy of lever-pressing is eventually followed by death from inanition and self-neglect (Brave New World: Aldous Huxley) ……"And if from the point of view of the human eye, montage is undoubtedly a construction, from the point of view of another eye, it ceases to be one; it is the purest vision of a non-human eye, of an eye which would be in things." (p81) (Gilles Deleuze Cinema 1 Wikipedia)……Is it a being or an object?..... It is confounding and confusing……..… Deleuze conceives the movement-image as a homogeneous structure. This clarifies how images refer to each other and to the whole: the story being told as an open structure that encompasses the perception of every action. Unlike the movement-image the time-image does not proceed by rational incisions and references. Italian neo-realism and French nouvelle vague introduced new forms of cinematographic reality…..Onticology or Object oriented ontology……... The time-image no longer follows the chronological order and narratological representation of actions and reactions. Time-image makes past, future and present indistinguishable. Crucial for the time-image is the coexistence of several levels of duration. As such the time-image breaks with the Aristotelian principium contradiction is that defines something as not being simultaneously its own opposite: it is to be or not to be. According to Deleuze, their coexistence defines the imaginary. (In between (of) cinema and literature Richard de Brabander)…….. Deleuze phrases this as 'rhizomatic interaction of heterogeneous media'. This mediamatic heterogenesis does not follow rules that prescribe the interaction in advance. It proceeds in an unpredictable way and produces new realities on a reception level. Media do no longer represent nor solely mediate. Interactions have become 'autonomous' to the extent that they produce their own 'virtual' reality. The mediamatic heterogenesis brings forth an endless multiplicity that branches off in every direction. It offers multiple entrances and has neither beginning nor end. According to Deleuze entering this rhizomatic dynamics posits the spectator in the middle or the in-between. This middle or in-between has nothing to do with the Aristotelian middle, known as the Doctrine of the Mean, and can neither be represented nor located between two media. The middle or in-between is not a place: it is a continuous becoming. It cannot be identified, because it is duration and a sensation or – as it is phrased in What is Philosophy? (1991) – "a compound of percepts and affects"6. The becoming in-between is neither an imitation nor a simulation of the in-between. The in-between is not a third element that combines two or more media, rather it affirms the lacking of this third element in producing new realities. (Ibid) (In between (of) cinema and literature Richard de Brabander)…….. "Life can only be understood backwards; but it must be lived forwards." ……Anxiety is the dizziness of freedom……there are two ways to be fooled. One is to believe what isn't true; the other is to refuse to believe what is true. (— Søren Kierkegaard)…….. Death of space time?.....actions are not performed within the jurisdiction of space and time?.....are there any such actions?.....that destabilizes the gratification deprivation complex by raising continuous and simultaneous debits and credits like in a bank……Balancing error!.....it can happen……continuous appearance and disappearance before recordial evidence……..TRS is responsible for all the mess caused …….. Tams, rajas and sattva……Nature does not forget anything that it has experienced(see the model below)….how can human being forget it?.....repetitive thoughts are creating paranoid schizophrenia everywhere……there is just nothing……

Such a creation and destruction is only contributory to cosmic space time (it certainly differs: like the one at Planck's scale) and not substitutional!

You must know mensura tion techniques!

I am given a refresher course in me**********ion!

As are denizens so are their dons!

They thought you are innocent!

So the tutelage!

Befuddling is that some people think everyone is interest what they do in bathroom!

Absolute subjectivity!

What about the perceived!

Conditional reflexes!

Conditional consciousness is both progressive and regressive!

From the point of higher consciousness I could see the process of simulation/dissimulation is a deliberate and advertent act!

Chit Shakti is different from Maya Shakti!

Creation is essentially feminine and subjective!

Feminine is subjective!

No mind condition for pure consciousness!

Coincidence is not coordination!

Correlations are not causations!

Layers and layers of thought constructs piled over one another!

Perception is not reality!

Chit Shakti is not Maya Shakti!

Where ever I go they build stories, strategically so that they can place a room exactly against my study desk!

You are the centre of gravity of the universe!

They are shooting you!

Land ladies come in transparent night gowns to check the progress!

They are shooting you!

This is order!

This is chaos!

Order is chaos!

Cessation of all dualities is pure consciousness!

Co creation is important!

Repetitive thinking is madness!

You start seeing and hearing things!

Thinking is repetitive like Mandelbrot

Nature is also a fractal!

Everything is mad!

More repetitive thinking is pathological fit a case for study by Noetic and neural sciences!

Participatory consciousness eludes you with witness consciousness!

No mind condition is the only solution!

Prediction models are also based on repetitive mathematics!

Unconscious and subconscious adds to your conscious repetitiveness!

It is conscious learning and experiences that are stores in unconscious and subconscious!

Such a quintessentiality and necessity contributes to repetitive action!

Music is also fractal!

$Z \leftrightarrow Z^2 + C$

Accentuation Attrion Model could be applied!

Algebra can be changed in to pictures and so are sentences certainly!

Visualization!

Mandelbrot saw a fractal in noise data!

I also see a fractal in the functioning or dysfunction of my computer!

Legendary appellations read obnoxiously inconsistent and irrelevant to the speaker!

Human Simulation and dissimulation techniques!

Order =Chaos!

Chaotic order!

Riding horse man in some film (Benhur) I think looked like a flame from distance until it became clear!

Minds are self mapping and non scaling fractals (Mind anticipation ad chaos: Mihai Nadin)

Property of self similarity is connected to mind dimension! (Ibid)

Minds are not composites of elementary minds but infinite processes displaying (eb) properties of non regularity and fragmentation! (Ibid)

Minds have infinitely granular structure! (Ibid)

There is nothing like ultimate reality?

Nothing is the ultimate reality!

Come on Buck up!

Mind works in expectation (Ibid)

Recreations of memories are different for different persons! (**Ibid means** Mind anticipation ad chaos: Mihai Nadin**)**

That lady vomited blood because her husband asked her polish his shoes hundredth time in a day!

Mind exists only in Plural! (Ibid)

Reconfigurations are in anticipation of (e) Problems not in reaction to them!

I think questioner asks me something and I become more elegant, eloquent and expansive!

And you are missing woods for trees!

Missing substance for shadow!

AA model could be applied!

Simulation and dissimulation all the way!

Human beings live that way!

Contexts defy domination and absolutism of representation! (Ibid)

They are space and time of our interaction! (Ibid)

It is within which our presence projects previous experiences projects new experiences (Ibid)

That is what I am telling all the way!

You have a theory and tend to have a jaundiced eye!

Double accentuation attrition coefficients are required to represent reality which is not perception!

Probably what is “Yes” is “No”!

Connectional paradigm substitutes parallel distribution processing under the assumption that it will have access cognition parallel distribution for central serial (hierarchal) at sub symbolic levels (Ibid)

Dynamics of such neural net works evidences non linearity and chaotic behaviour!

Mind is anagrammatic you see!

Condition of mind is plurality and interaction of minds is the concrete form of plurality! (Ibid)

So also the fact of observer dependent reality!

Mind depends on the interaction of other minds! (Ibid)

Mind is anagrammatic!

Interaction of minds must be doubly anagrammatic!

It requires a different computation by itself!

I think so by the way people behave however highly placed and erudite they are in putting forth their point of view!

Everything is stored in the unconscious and it needs a conscious churning to electrocute a person and create Prakruti Kshobha or for that matter Prakruti Siddhi!

Nor do star stuff!

Nor do stars!

Non locality if achieved at classical level is instantaneous communication!

Quantum hologram is the first energy phenomenon that directly links all macro-scale matter with (e&eb) the quantum world! (Ibid)

Space time is discrete!

Quantum histories stay with the object itself!

That is exactly what the individual general ledger proposition means!

Individual general ledger, collective general ledger and cosmic general ledger are involuted and convoluted!

Live involution and evolution of consciousness!

The second image of qualitative multiplicity is the color spectrum. We saw in the first image of the spools that there is constant difference or heterogeneity. The color spectrum helps us understand this, since a color spectrum has a multiplicity of different shades or nuances of color. Here we have heterogeneity, but there is a drawback to this image as well. We lose the characteristic of continuity or unity since the spectrum has colors juxtaposed. As Bergson says, “Pure duration excludes all idea of juxtaposition, reciprocal exteriority and extension” (The Creative Mind, p. 164) (SEP)

TBAB and CBAB make use of it phenomenally and monumentally!

To the detriment, damnation and discomfiture!

Observer dependent universe!

Pure consciousness calls for the coincidence of perceiver, perception and the perceived!

Interference science changes the very method of perception!

Particle-wave duality is essential for description of the phenomenon!

Movement Image Has The Tendency, Thanks To The Habitual Experience Of Movement As Normal And Centered, To Justify Itself In Relation To (e&eb) Truth (Gilles Deleuze: Wikipedia)

A complete elucidation of one and the same object may require diverse points of view which defy a unique description. Indeed, strictly speaking, the conscious analysis of any concept stands in a relation of exclusion to its immediate application. This foreshadowing of postmodernist epistemology is by no means coincidental. The profound connections between complementarity and deconstruction have recently been elucidated by Froula17 and Honner18, and, in great depth, by Plotnitsky.19 20 21 A third aspect of quantum physics is discontinuity or rupture: as Bohr explained, [the] essence [of the quantum theory] may be expressed in the so-called quantum postulate, which attributes to any atomic process an essential discontinuity, or rather individuality, completely foreign to the classical theories and symbolized by Planck's quantum of action.22 A half-century later, the expression ``quantum leap'' has so entered our everyday vocabulary that we are likely to use it without any consciousness of its origins in physical theory. Finally, Bell's theorem and its recent generalizations24 show that an act of observation here and now can affect not only the object being observed -- as Heisenberg told us -- but also an object arbitrarily far away (say, on Andromeda galaxy). This phenomenon -- which Einstein termed ``spooky'' -- imposes a radical reevaluation of the traditional mechanistic concepts of space, object and causality25, and suggests an alternative worldview in which the universe is characterized by interconnectedness and (w)holism: what physicist David Bohm has called ``implicate order''.26 New Age interpretations of these insights from quantum physics have often gone overboard in unwarranted speculation, but the general soundness of the argument is undeniable.27 In Bohr's words, ``Planck's discovery of the elementary quantum of action ... revealed a feature of wholeness inherent in atomic physics, going far beyond the ancient idea of the limited divisibility of matter.'' Hermeneutics of Classical General Relativity In the Newtonian mechanistic worldview, space and time are distinct and absolute.29 In Einstein's special theory of relativity (1905), the distinction between space and time dissolves: there is only a new unity, four-dimensional space-time, and the observer's perception of ``space'' and ``time'' depends on her state of motion.30 In Hermann Minkowski's famous words (1908): Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.31 (Bohr quoted in Transgressing the Boundaries: Towards a Transformative Hermeneutics of Quantum Gravity Alan D. Sokal Department of Physics New York University 4 Washington Place New York, NY 10003 USA Internet: SOKAL@NYU.EDU Telephone: (212) 998-7729 Fax: (212) 995-4016 November 28, 1994 revised May 13, 1995 Note: This article was published in Social Text #46/47, pp. 217-252 (spring/summer 1996). Biographical Information: The author is a Professor of Physics at New York University. He has lectured widely in Europe and Latin America, including at the Università di Roma ``La Sapienza'' and, during the Sandinista government, at the Universidad Nacional Autónoma de Nicaragua. He is co-author with Roberto Fernández and Jürg Fröhlich of Random Walks, Critical Phenomena, and Triviality in Quantum Field)

Come on hit a Six!

That is the way!

It is definite you are in a hologram projected on the screen of consciousness!

Life is same as that of a projection beamed on screen!

Donot enter in to dialectic deliberation, polemical conversation, argumentative confabulation, invective harangue, pugnacious remonstration with me!

Contentious scrappy!

Self revocation and self abnegation is good!

It is not self, but an anagrammatic mind acting on consciousness (information)!

Idealism and romanticism is the codicil of morality!

Subjective experience is as important as objective reality!

Ugh!

No repetition please!

Everything That Happens Is Conditioned By The Moment In Which It Happens! Jung In The 21st Century: Synchronicity And Science By John Ryan Haule Mind Control Countermeasures

Condition is being looked as conditioned!

Theory-laden and self-referential counter-hegemonic representation!

Operating system needs updates to keep you go on!

You can never see your birth or death!

I never emphasised any thing!

Photons that travel in free space slower than the speed of light Daniel Giovannini et al

The subject is —Idealism and Realism Reconciled in the Light of the Vedanta. In other words, the subject is - The Vedantic Theory of Perception, - a most important subject for philosophers. You ought to be told a little about what Idealism and Realism are. We have no time to enter into the details of these topics.

(1) In brief, Realism means a belief or theory which looks upon this world as it seems to us to be, a mere phenomenon; while to Idealism the world is not as it appears to us; the world is, but it is not what it appears and according to Realism things are just as they seem to us, real in themselves.

(2) Idealism has several branches. We have Subjective Idealism, the idealism of Berkeley and Fichte; we have Objective Idealism, the idealism of Plato and Kant; we have Absolute Idealism, the idealism of Hegel and Shelley, and many others of the same type. Realism has many philosophers like Bain and Mill to support it. We shall not describe these several branches of Idealism or Realism.

(3) We shall not criticise in tonight's discourse the Subjective Idealism of Berkeley or the Objective Idealism of Plato and Kant, or the Absolute Idealism of Hegel or Shelley.

(4) We shall just allude to these to such an extent that the Vedantic theory about this matter may be easily comprehended by each and all.

(5) Before beginning with the subject two words ought to be explained, the words 'subject' and 'object.' You know these words - 'subject' and 'object,' - are taken in different senses. In Grammar they are used in one sense, in ordinary language they are used in a different sense, and in philosophical language they have a meaning of their own.

(6) The word ' subject' in the language of Philosophy means the knowing one, and the 'object' means the thing known.

(7) When you see this pencil, the pencil is the object, and you that perceive the pencil are the subject: the perceiving one is called the subject, and the thing perceived the object.

(8) Thus in ordinary language, the word subject means the understanding or the intellect; but,, according to the Vedanta, this subject, understanding, intellect, or reason should not be called subject; this also is an object.

(9) You know anything that can be perceived becomes an object, and you can perceive the intellect, you can think and reason about it and lay down its laws; in so far as you ran reason about it and conceive it, in so far is the intellect an 'object' and not a 'subject.' The real subject cannot be conceived; the real subject cannot be perceived.

(10) How can the knower be known? You know the real subject should either be the knower or the known; the very moment it becomes the known, it becomes an object, it no longer remains the subject. But in ordinary language the word subject implies the understanding, intellect, or reason.

(11) The real subject or the real knower is, according to the Vedanta, the true Atma, the only Infinity, one and the - same in all bodies!

(12) It would be useful to remember a Sanskrit word also in connection with this. The word 'subject‘ is called in Sanskrit Drashta, the word 'object‘ is called in Sanskrit Drishya, and the real subject in Sanskrit is the Brahman or Atma! (***Soul is what we have taken as self the witness consciousness the various general ledgers at various states)***

(14) So the real subject is the true Atma, but the practical subject is the Atma as shining in the intellect or in the understanding, so the real Atma with the intellect as an agent is called the subject.

(15) What are the arguments the Realists advance on their side and what are the chief arguments advanced by the Idealists?

(16) That is a long subject, but we shall discuss it very briefly. We have no time to criticise Berkeley. He is one of the principal idealists.

(17) How briskly he starts in his philosophy, and how he soars high so long as he is exactly hand in hand with the Vedanta philosophy, and how he loses his way and falls into a meandering zigzag the very moment he departs from it.

(18) Amongst the many arguments advanced by the Idealists, the following two or three are important.

(19) The first is: you cannot see or perceive anything without your own activity. It is the subject's activity alone which makes you perceive anything in this world.!

(20) You are writing something, your mind is intent on the subject, and there passes before you a snake, but you do not perceive it, for you the snake is not a snake, it is not there!

(21). Now, the Idealists say if the activity of your mind or the subject activity is wanting, there is no object there. When you are asleep, the subject is not active, and all the sounds that may be made around are not heard. There are some people whose eyes do not close when they are asleep.

(22) Now before their eyes all the objects are present, all the objects are being reflected on their retina, but they see them not. The Idealists say your mind is inactive, the subject is not asserting its activity and you do not see the objects.

(23) Can you see anything in this world without mental activity?!

(24) No. Just try to see this table or that wall; try to hear my words, try to perceive anything without the mind being active. Can you do that? Can you see anything without thinking, without your mind's thought?

(25) You cannot. Thus the Idealists say this entire world is nothing else but thought, this entire world is simply a projection of thought. How do you know that the world exists? Through your senses

(26) . But the senses by themselves cannot perceive. It is only when the mind is connected with the senses that they perceive, in other words the senses do not perceive, the mind perceives through them. Now the mind or understanding is the subject.

(27) You cannot hear anything without mental activity; you cannot see anything, you cannot do anything, you cannot perceive anything without mental activity.

(28) So the Idealists say, "O people of this world who call it real and look upon these objects as true by themselves, forget not yourselves, be not mistaken.

(29) All these objects are created by you, or projected by your thought, in fact you make them." This is what the Idealists say, and it appears that Idealists are something like Vedantins.

(30) But Rama tells you that all these Idealists, Berkeley, Plato, Hegel, Kant, Fichte, Shelley, Schopenhauer, have the principles of the Vedanta, but the Vedantic theory of perception far transcends all this.

(31) These people have disputes with each other, they have quibbles and quarrels, but Vedanta philosophy reconciles each and all of them. These people glorify and make much of the self, but the Vedanta does not deify and lionize the subjective self, as most of these philosophers do. We have to take the truth for its own sake.

(32) Another argument advanced by the Idealists is that this world, which people ordinarily take to be real, should not be regarded as such, because the world appears to be as it is through the senses only, and we depend upon the senses in calling the world true in reality as it seems to us.

(33) Now the senses are not trustworthy witnesses.

(34) Take the case of the eye, for instance.

(35) The eyes of the ant see differently from the eyes of man; to the eyes of the elephant things appear to be much bigger than what they appear to the eyes of man; to the eyes of the frog things are clear when seen in the water, and in air things are all hazy, dim, covered with a kind of mist.

(36) Now whose eyes fire to be relied upon, the eyes of man or the eyes of ants?

(37) If things are to be decided by majority, ants do not stand in a small minority; they have the majority on their side. If your eyes be formed upon the microscopic principle, if the lens in the eyes be fixed in a different way to the retina, to you the world will be entirely different. If the retina of the eyes be adjusted on the telescopic principle, all the world is entirely altered.

(38) You may have seen a toy called a Look and Laugh," or the ludicrous glass, which consists of two convex lenses. If we look through it, all the objects in this world become ludicrous, ridiculous.

(39) A most beautiful face when seen through "Look and Laugh" becomes elongated, so that the chin touches the ground and the head touches the star Saturn.

(40) If you look at it in a different way, then the length of the face remains the same, but one ear moves up to East India and the other moves up - to China.

(41) Well, if the eyes be adjusted upon that principle, the world is entirely changed. So is the case with the ears and other organs of sense.

(42) If the nerves and muscles be differently adjusted, the whole world is different, the whole world is changed, and you will say that if the nerves and muscles and sense organs are adjusted in this way they must remain in this way.

(43) It is not so; the law of Evolution tells you that they are undergoing a change. Thus the Idealists say that the world is not what it seems, the world, as it appears to us, is false, the world as it seems to us is unreal, illusory, a delusion.

(44) They have many other arguments on their side, but if we enter into details, many nights would be taken up by Idealism alone.

(45) We shall now pass on to Realism. The Realists say, ―You are wrong, O Idealists! You are altogether wrong. If your statement that everything we see is the creation of our own imagination be true, then, O Idealists, please create a horse where the wall is.

(46) Let that wall appear to be a horse. O Idealists, if the whole world is simply the result of this small subject's understanding or mind's creation, then turn this handkerchief into a lion or make this pencil a big house‖.

(47) The Realists say, ―O Idealists, you cannot be right; the world is real. The wall is a wall, and for that reason it always impresses itself upon your senses as a wall, it does not appear to you a horse tomorrow.

(48) These objections of the Realists are met by the Idealists; they have answers to these objections, but we shall not take up all the questions on both sides.

(49) The Idealists say that it is a question of time; you can create anything you like by your imagination. When you begin to think of ghosts, ghosts appear to us, when we begin to imagine anything, it comes to us. They say, in dreams do we not create things?

(50) Our imagination realises these things.

(51) They have answers and these answers have rejoinders from the Realists. We are not going to enter into detail upon these questions and answers.

(52) The Vedanta also looks upon the world as my idea, as my creation but even when it looks upon the world as my idea or my creation, you cannot call it Idealism.

(53) That seems to be something very strange from the lips of Rama

(54). It will be repeated again.

(55) The people in Europe and America think that the Vedanta is a kind of Idealism, and almost all the books written by Europeans that have passed through Rama's hands, represent the Vedanta as Idealism, but Rama tells you that these people have not understood it.

(56) The Vedanta is not Idealism in the same sense as the Idealism of Berkeley or Plato. It is far higher, far superior.

(57) The Idealists make the world depend upon the little subject, the little understanding, the little mind, but when the Vedanta says that the world is My Idea, that does not mean that the world is the idea of the little subject, the little understanding, and the little mind.

(58) This is something variable; this is something in itself a creation, and Berkeley made a terrible blunder when he said that dreams are the creations of the subject.

(59) The mistake made by him was that he looked upon the subject of the dreamland object to be identical with the subject of the wakeful state, and you know, as it was shown last night, the subject in dreamland is different from the subject in the wakeful state; the subject in the dreamland is a tiling of the same sort as the objects in the dreamland are.

(60) When you wake up, the subject of the wakeful state is of the same sort as the objects of that state and so Berkeley took the subject of the wakeful state to be the same as the subject of dreamland.

(61) The world is not a creation of the subject of the wakeful state or the subject of dreamland; the world is a creation of My Self, the Real God, and the Real Atma.

(62) We come now to the subject, the Vedantic Theory of Perception. The Vedanta says to the Idealists, —O Idealists, you are right in saying that all the names and forms of this world, all the attributes and qualities of objects could not come about without the action of the subject." It will be repeated again.

(63) The subject is very abstruse, and you ought to follow it closely.

(64) Vedanta says to the Idealists, — You are right in saying that all the names and forms of this world could not come about without the action of the subject; all the qualities, attributes and properties of things depend upon the activity and action of the understanding or mind, or the subject.

(65) You are right in so far; but you are not right in saying that there is nothing outside this small subject, this small mind of yours.

(66) The Vedanta says to the Realists, —You are right in saying that this phenomenal world could not appear with the sole action of any outside reality‖.

(67) You know the Realists say that this phenomenal world is due to some action upon our senses from outside. The objects act upon the senses, and thus we perceive things. Photons that travel in free space slower than the speed of light Daniel Giovannini et al

(68) The Vedanta says, —Yes, without some sort of action from without we could not perceive things. So far is Realism right, but, according to the Vedanta, Realism is wrong when it says that all our perception is due solely and wholly to outside action and to the subject's activity.

(69) Let us make this clearer. In this world, take up any subject; take up any object, this pencil for example. To what is the colour of this pencil due? It is due only to the action of the subject together with a reaction from outside, you might say. **Photons that travel in free space slower than the speed of light Daniel Giovannini et al**

(70) If your eyes are colour-blind, you will not see this colour in the pencil. The colour of the pencil is a quality or attribute. Again take the weight of the pencil; Photons that travel in free space slower than the speed of light Daniel Giovannini et al

(71) It is changeable, and so is its colour. If our eyes are jaundiced, we might see the pencil to be of a different colour, and if we do not weigh it here, but at a great height, or in the moon, or in a deep mine, its weight will be different, and you know the weight of every object when weighed in London is different from what it is when weighed in India, the weight will become different. The weight is changeable, the colour is changeable. Photons that travel in free space slower than the speed of light Daniel Giovannini et al

(72) You know the same water, when you touch it in winter appears to be warm, and when you touch it in the summer, it appears to be cold. Why? Photons that travel in free space slower than the speed of light Daniel Giovannini et al

(73) Because the observer or the perceiver is at different temperatures when he touches the water, and the water retains about the same temperature; the difference in its apparent temperature is due to the difference in the temperature of our hands. Photons that travel in free space slower than the speed of light Daniel Giovannini et al

(74) So according to the differences in the subject, there will be differences in the qualities of the object.

(75) And of what is this pencil made? According to Berkeley and some others, it is nothing else but a bundle of attributes and qualities.

(76) Take away these qualities and there is nothing left, but according to Kant there is the thing itself behind it, and according to Plato there is the thing in itself behind it, the idea, as he calls it. So here there are qualities.

(77) All these qualities are due to the action of the subject, but we say that before these qualities were produced in the pencil by this reaction, some reality was there.

(78) This will be made clearer, and it will be repeated again if you ask Rama to do so. It is true that the Vedanta says that all these qualities in the pencil are due to the action of the subject but why was the action of the subject excited? Photons that travel in free space slower than the speed of light Daniel Giovannini et al

(79) This is the question. There must be something outside which acted upon the subject and excited a reaction in the subject, and then these qualities were produced or deposited. Photons that travel in free space slower than the speed of light Daniel Giovannini et al

(80) We cannot say that before this subject acted, these qualities themselves acted upon the mind and excited an action or reaction of the mind; we cannot say that, because these qualities make their appearance after the action or reaction of the mind, so there must be something outside, there must be some reality in the pencil which acted upon your eyes, which acted upon your ears when the sound was heard, which acted upon your taste when you touched it with the tongue, which acted upon your hand when you touched it. There must be something outside which acts upon the eyes, the ears, and the nose. Eat this pencil and it will tell upon your health. How can you say that there is no reality outside? Photons that travel in free space slower than the speed of light Daniel Giovannini et al

(81) There is some reality outside too, and when it acts upon the senses of a man, they report it to his mind and the mind reacts; then are the attributes or qualities of the object projected on the scene. It is just like that. Here is one hand; there is another; one hand alone could not make any sound. Here is the sound produced (clapping the hands together). Photons that travel in free space slower than the speed of light Daniel Giovannini et al

(82) Here was action on one side and reaction on the other, and the result was sound. Here is a violin string; you strike it with your finger, and then the sound is produced. There was action from your finger and reaction from the string or you may say action from the string and reaction from your fingers, and then the sound was produced. In the same, way, one wave came from this side and another from the other side, the two collided, and foam was produced. Photons that travel in free space slower than the speed of light Daniel Giovannini et al

(83) Action and reaction from both sides produced foam. Here is a match and here is a piece of sand - paper. Strike the match on the sand - paper and then a flame is produced. Photons that travel in free space slower than the speed of light Daniel Giovannini et al

(84) Action and reaction from both sides. Here is one positive pole of electricity and there is a negative pole. If they approach each other, we see the electric spark or hear the report; this action and reaction from both sides bring about this phenomenon. Photons that travel in free space slower than the speed of light Daniel Giovannini et al

(85) So according to the Vedanta, in your intellect the thing in itself is present, what we call the Atma. The real self is living in your intellect; there is the thing in itself or the reality in every object in this world.

(86) In this pencil there is the reality or you might say the thing in itself which cannot be known, something which is beyond all attributes or properties. Photons that travel in free space slower than the speed of light Daniel Giovannini et al

(87) The reality outside, the divinity or the absolute in the pencil and the absolute in the intellect are like the two hands, as it were. Photons that travel in free space slower than the speed of light Daniel Giovannini et al

(88) The moment they collide, the attributes of the pencil are posited, they make their appearance like foam, one wave from one side, the other from the other side, and foam is produced, that is, these qualities Photons that travel in free space slower than the speed of light Daniel Giovannini et al.

(89) You might say the positive pole being in the intellect, the negative pole being in the pencil, the two poles approach each other and we see the attributes, qualities, or the phenomena of this universe. In the language of the Vedanta, the very moment the Drashta and the Drishya unite, we see the objects. Drashta and Drishya are the true self or Atma in the pencil and the true self or Atma in the intellect, and action and reaction between the two produces the phenomena. (To be concluded in next part ....) " Om Shanti Shanti Shanti " The Vedantic Theory of Perception : by Swami Rama Tirtha ======== Reconciliation between Idealism and Realism ======== (Adopted from 'In The Woods of God-Realization', volume - 3) [Part - 1] : The subject is —Idealism and Realism Reconciled in the Light of the Vedanta. In other words, the subject is - The Vedantic Theory of Perception, - a most important subject for philosophers. Photons that travel in free space slower than the speed of light Daniel Giovannini et al

(90) You ought to be told a little about what Idealism and Realism are. We have no time to enter into the details of these topics. In brief, Realism means a belief or theory which looks upon this world as it seems to us to be, a mere phenomenon; while to Idealism the world is not as it appears to us; the world is, but it is not what it appears and according to Realism things are just as they seem to us, real in themselves. Idealism has several branches. We have

Subjective Idealism, the idealism of Berkeley and Fichte; we have Objective Idealism, the idealism of Plato and Kant; we have Absolute Idealism, the idealism of Hegel and Shelley, and many others of the same type. Realism has many philosophers like Bain and Mill to support it. We shall not describe these several branches of Idealism or Realism. We shall not criticise in tonight's discourse the Subjective Idealism of Berkeley or the Objective Idealism of Plato and Kant, or the Absolute Idealism of Hegel or Shelley. We shall just allude to these to such an extent that the Vedantic theory about this matter may be easily comprehended by each and all. Photons that travel in free space slower than the speed of light Daniel Giovannini et al

It also can explain Karl Jung's concept of the collective unconscious and the reason why archetypal symbols recur in dreams, regression therapy, and shamanistic rituals, plus many other nonlocal events (Ibid)

Time and Free Will has to be seen as an attack on Kant, for whom freedom belongs to a realm outside of space and time. Bergson thinks that Kant has confused **space and time in a mixture**, with the result that we must conceive human action as determined by natural causality. Bergson offers a twofold response. On the one hand, in order to define consciousness and therefore freedom, Bergson proposes to differentiate between time and space, "to un-mix" them, we might say. On the other hand, through the differentiation, he defines the immediate data of consciousness as being temporal, in other words, as the duration (la durée). In the duration, there is no juxtaposition of events; therefore there is no mechanistic causality. It is in the duration that we can speak of the experience of freedom.(SEP)

"… reality is such that it can be described in various alternative, incompatible ways, using descriptions which cannot be combined or compared." R. B. Griffiths, Consistent Quantum Theory Cambridge University Press, Cambridge, 2002, Chapter 27.5 as quoted in shadow interpretation versus quantum paradoxes Warren Leffler

Vacuum doth create dissolution of one's identity or atleast reduction of intensity of identification one got from people!

The shadow interpretation versus quantum paradoxes Warren Leffler

This paper explores the consequences of denying the "emptiness of paths not taken," EPNT, premise of Bernstein, Greenberger, Horne, and Zeilinger (BGHZ) in their paper titled, Bell theorem without inequalities.[1] Carrying out the negation of EPNT leads to the concept of a "shadow stream." Streams are essentially particle implementations of the paths in Feynman path-integrals, resulting in a simple and consistent extension of the standard axioms for quantum mechanics. The construct provides elegant resolutions of single- and multi-particle interference paradoxes. Moreover, combining the argument of this paper with that of BGHZ shows that there are just two choices for quantum foundations: interpretations closely similar to the present one or those that harbor instantaneous action at a distance.

The physical significance of the result in Sec. IV is interesting. The BGHZ argument can be viewed as a carefully reasoned and detailed proof of the following implication:

EPNT ⇒ (e&eb) non-local causality.

But our above argument shows, both conceptually and mathematically,

not-EPNT ⇒ (e&eb) local causality

Feynman famously emphasized that these are just computational rules, and that "No one has found any machinery behind the law. No one can 'explain' any more than we have just 'explained.' No one will give you any deeper representation of the situation." Nevertheless, as we shall see in Sec. III, the hypothetical entities referred to by BGHZ as "some things," and by Deutsch as "shadow particles," do indeed provide machinery behind the law for single particle interference (which is the kind of interference phenomena Feynman had in mind). (Ibid)

That means there is something wrong with mensura tion!

Certainly there is no reason why 2X2=4!

Double slit also defies commonsense!

We are thoroughly confused with systems!

Recurrent thoughts can be expressed as: Psi of RT in Dirac Brakets = C (T (1)) E(T(1)) +…….+ C(T(n) E(T(n)) in Dirac Brakets !

Accentuation attrition models could be applied!

Identity crisis for CBAB and TBAB!

(1) Identity Is (=) A Characteristic Of The Primitive Mentality And The Real Foundation Of Participation Mystique, Which Is (=) Nothing But A Relic Of (e)The Infantile Non-Differentiation Of Subject And Object, The Primordial Unconscious State.
(2) The Further We Go Back Into History, The More We See (e) Personality Disappearing Beneath The Wrappings Of Collectivity. ***This is questionable. Rather it is the future that holds such a possibility because of technological developments which have crossed the figment of imagination***. Italics mine
(3) Any Individual's Existential Condition Is (=) Just A Typical Variation On (eb) The Collective Themes Of (e) The Eternal Human Condition.
(4) Our Particular Individuality Emerges From (e) A Collective Mythic Existence.
(5) Individuals May Have (e) An Unconscious Identity With Some Era, Person, Group, Or Object.
(6) Participation Mystique Means (eb) A Mystical Connection In Which One Cannot (e) Clearly Distinguish Self From Other, Bound To It By (e&eb) A Direct Relationship Which Amounts To (=) Partial Identity Or Transference.
(7) Unfortunately, That Is (=) Tantamount To Giving Part Of Yourself Away.
(8) It Is the Mostly Unconscious Instinctive Human Tie To (e) Symbolic Fantasy and (e&eb) Mythic Emanations Leading To (eb) Literalisms and Concretism, The Attempt to Turn the Imaginal Into (e&eb) Events.
(9) Mythological Motifs Project Themselves Into (e&eb) Situations and Objects, Including Other Persons.
(10) Everything Unconscious About Ourselves Is (=) Projected.
(11) When The Boundary between Consciousness and (e&eb) The Unconscious Is Blurred (e) Either Intentionally Or Unintentionally, The Ego Has (e) A Hard Time Differentiating Between What Belongs To Itself And (e&eb) What Belongs To The Other.
(12) The Two Become Identified With (=) Each Other.
(13) When A Projection Is Strongly Fixated, It May Be next To Impossible to Separate One's Projected Material from (e) the Person Who Is Carrying the Projection.
(14) Whether It Is A Person Or An Object, Suddenly, All Those Things Outside Become (=) Carriers, Unconsciously, Of What Resides Within.
(15) Such Illusory States Left Unconscious Turn (e&eb) Toxic and Consuming.
(16) Denial Leaves Room for (e) Serious Errors of Fact.
(17) It Is A Warning Against (e) Extreme Conscious Attitudes Which Inevitably Get Compensated By (e) The Shadow.
(18) In The Process Jung Termed Enantiodromia, Projections Can Break or Revert (e&eb) Into Their Opposites as Psyche seeks to equilibrate or Rebalance Itself. ***This is also questionable in the sense an individual is rarely satisfied, satiated even surfeited. Gratification deprivation complex as we have emphasised several times remains unbalanced. Italics mine***

(19) Rightly or Wrongly, Every Phenomenon Is Subject To (e&eb) Being Named, Labelled and Interpreted, Including (e) The Sense of Self as the Biological Basis of Consciousness.
(20) In The Struggle to Narrate Its Experience, The Mind Tends To Label (e&eb) Phenomena And To Spontaneously Generate (eb) Ideation And Imaginally Fill In (eb) The Gaps In Awareness.
(21) Subcultures Are Dynamic Marketplaces For (e) Knowledge Construction, Authenticating And Authorizing Claims Within (eb) An Overall Belief Structure
(22) Such Authority Structures May Be (=) Benign, Toxic, Exploitive or Malignant.
(23) It's Only A Small Step From Known Psychological Mechanisms That Generate (eb) Illusions To Perceiving An Entity 'Out There' With The Archetypal Characteristics Of A Personality - E.G. The Pater Noster Or Any 'Other', The Not-I, Who Interacts With Us In The Way We Interact Morally With Others.
(24) This Is Where Social Selection Really Favors Mirror Neurons That Can Produce (eb) Neurological Tricks We Interpret (eb) Subjectively, According To Our Beliefs.
(25) Social Order Is Organized around Both (e&eb) Faulty and Plausible Knowledge of Reality.
(26) Ritual Is a Creative Process Used to Construct (eb) Meaning Worlds.
(27) But They Don't Reliably Produce Intuitive Experience With (e&eb) Consensual Meaning.
(28) Most Often Such Experience And Interpretations Are (=) Idiosyncratic, Cannot Be Validated, And Are Rooted Purely In (eb) Beliefs As Assumed Truths.
(29) Most Of This Retrieved Esoteric Knowledge Reflects Superstitious Ideas Arising Prior To The Seventeenth Century. Archaic Consciousness Of The Paleolithic, Mesolithic, And Neolithic Lies Buried Within Us, But To Try To Relive Them Today May Mean Being Less Than Wholly Present Now. Such Beliefs Effectively 'Teleport' One Back In Time Into (eb) Less Sophisticated Modes Of Ideation.
(30) In Archaic Identity There Is (=) No Differentiation Between Object And Subject And No (e) Distinction Between Lived Experience And What The Subject Believes He Or She Perceives (e) About The World.
(31) Perceptual Competence Is Self-Assessed In Grandiose Terms Without (e) Cognitive And Emotional Balance.
(32) Credibility Is Conferred By (e) The Untutored Crowd Based On (e) Otherwise Unworthy Persons, With Charisma And Emotional Appeal.
(33) Orthodoxy Of Interpretation Gives Rise (eb) To Organizations.
(34) The Recognition and Withdrawal of Projections Usually Provokes (eb) A State Of Disenchantment Or, Conversely, Elation And (e&eb) Inflation Of The Ego.
(35) However, These Processes Can Also Open The Way To (eb) A Practice Of The Symbolic Life And Of Human Relations Without Too Much Alienation Or Mystification In The Therapeutic Process.
(36) This Is A 'Knowing' Of The Things Of Life And Their Inherent Mysteries Through (e&eb)The Experience Of The Mundane As Well As The Spiritual.
(37) It Is an Immersion In **The Mysteries Of Nature And The Seeking Of (e&eb) Knowledge Through Mystical Participation. Jung in The 21st Century: Synchronicity And Science By John Ryan Haule Mind Control Countermeasures**

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(1) A number of theoretical and lattice results lead us to believe that quark-gluon plasma not too far from Tc contains not only electrically charged quasiparticles—quarks and gluons—but magnetically charged ones—monopoles and dyons—as well.
(2) Although binary systems such as charge-monopole and charge-dyon were considered in detail before in both classical and quantum settings, this is the first study of coexisting electric and magnetic particles in a many-body context.
(3) Authors perform a molecular dynamics study of strongly coupled plasmas with ∼1000 particles and differing fractions of magnetic charge.
(4) Correlation functions and Kubo formulas lead to transport properties such as the diffusion constant, the shear viscosity, and electric conductivity:
(5) They compare the first two with empirical data from RHIC experiments as well as with results from anti-de-Sitter space/conformal field theory correspondence.
(6) They also study a number of collective excitations in these systems. http://dx.doi.org/10.1103/PhysRevC.75.054907

Nonequilibrium Phase Transitions and a Nonequilibrium Critical Point from Anti–de Sitter Space and Conformal Field Theory Correspondence Shin Nakamura Phys. Rev. Lett. 109, 120602 – Published 18 September 2012

(7) **Shin Nakamura** finds novel phase transitions and critical phenomena that occur only outside the linear-response regime of current-driven nonequilibrium states.
(8) They consider the strongly interacting (3+1)-dimensional N=4 large-Nc SU (Nc) supersymmetric Yang-Mills theory with a single flavor of fundamental N=2 hypermultiplet as a microscopic theory.
(9) They compute its nonlinear nonballistic quark-charge conductivity by using the AdS/CFT correspondence.
(10) Authors find that the system exhibits a novel nonequilibrium first-order phase transition where the conductivity jumps and the sign of the differential conductivity flips at finite current density.
(11) A nonequilibrium critical point is discovered at the end point of the first-order regime.
(12) They propose a nonequilibrium steady-state analogue of thermodynamic potential in terms of the gravity-dual theory in order to define the transition point.
(13) Nonequilibrium analogues of critical exponents are proposed as well.
(14) The critical behavior of the conductivity is numerically confirmed on the basis of these proposals.
(15) The present work provides a new example of nonequilibrium phase transitions and nonequilibrium critical points. © 2012 American Physical Society http://dx.doi.org/10.1103/PhysRevLett.109.120602

From black holes to strange metals Thomas Faulkner, Nabil Iqbal, Hong Liu, John McGreevy, David Vegh

(16) Since the mid-eighties there has been an accumulation of metallic materials whose thermodynamic and transport properties differ significantly from those predicted by Fermi liquid theory.
(17) Examples of these so-called non-Fermi liquids include the strange metal phase of high transition temperature cuprates, and heavy fermion systems near a quantum phase transition.
(18) Authors report on a class of non-Fermi liquids discovered using gauge/gravity duality.
(19) The low energy behavior of these non-Fermi liquids is shown to be governed by a nontrivial infrared (IR) fixed point which exhibits nonanalytic scaling behavior only in the temporal direction
(20) Within this class we find examples whose single-particle spectral function and transport behavior resemble those of strange metals.
(21) In particular, the contribution from the Fermi surface to the conductivity is inversely proportional to the temperature.
(22) In our treatment these properties can be understood as being controlled by the scaling dimension of the fermion operator in the emergent IR fixed point. Report number: MIT-CTP/4105 Cite as:arXiv: 1003.1728 [hep-th]

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T_{15} : Category three of SAS

Module Two

After a long-lasting and intense controversy it was finally clarified that the demon's role does not (e) contradict the second law of thermodynamics, implying that (eb) we can, in principle, convert information to (e&eb) free energy2, 3, 4, 5, 6

G_{16} : Category one of second law of thermodynamics, implying that (eb) we can, in principle, convert information to (e&eb) free energy2, 3, 4, 5, 6

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of demon's role

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

demon's role does not contradict the second law of thermodynamics, implying that (eb) we can, in principle, convert information to (e&eb) free energy2, 3, 4, 5, 6

G_{20} : Category one of demon's role does not contradict the second law of thermodynamics

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of conversion of information to (e&eb) free energy2, 3, 4, 5, 6

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

demon's role does not contradict the second law of thermodynamics, implying that we can, in principle, convert information to (e&eb) free energy2, 3, 4, 5, 6

G_{24} : Category one of demon's role does not contradict the second law of thermodynamics, implying that we can, in principle, convert information; free energy

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of free energy; demon's role does not contradict the second law of thermodynamics, implying that we can, in principle, convert information

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

An experimental demonstration of this information-to-energy conversion, however, has been elusive.

Here authors demonstrate that a

non-equilibrium feedback manipulation (e&eb) of a Brownian particle on the basis of (e) information about its location achieves (eb) a Szilárd-type information-to-energy conversion

G_{28} : Category one of non-equilibrium feedback ; Brownian particle on the basis of (e) information about its location achieves (eb) a Szilárd-type information-to-energy conversion

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of Brownian particle on the basis of (e) information about its location achieves (eb) a Szilárd-type information-to-energy conversion; non-equilibrium feedback

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

non-equilibrium feedback manipulation of a Brownian particle on the basis of (e) information about its location achieves (eb) a Szilárd-type information-to-energy conversion

G_{32} : Category one of information about its location achieves (eb) a Szilárd-type information-to-energy conversion

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of non-equilibrium feedback manipulation of a Brownian particle

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

non-equilibrium feedback manipulation of a Brownian particle on the basis of information about its location achieves (eb) a Szilárd-type information-to-energy conversion

G_{36} : Category one of non-equilibrium feedback manipulation of a Brownian particle on the basis of information about its location

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of Szilárd-type information-to-energy conversion

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

Szilárd-type information-to-energy conversion

G_{40} : Category one of information; energy

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of energy; information

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Using real-time feedback control, the particle is made to climb up a spiral-staircase-like potential exerted by (e) an electric field and gains (e) free energy larger than (e) the amount of work done on it

G_{44} : Category one of electric field and gains (e) free energy larger than (e) the amount of work done on it

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of real-time feedback control, the particle is made to climb up a spiral-staircase-like potential exerted

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$

$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$
$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$
$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$
$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$
$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}),t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}),t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}),t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}),t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}),t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}),t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}),t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$
$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$
$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad (60)$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,

2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{16})^{(2)} & \boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} \quad \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{array}\right] G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{ccc} (a'_{17})^{(2)} & \boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} \quad \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{array}\right] G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{18})^{(2)} & \boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} \quad \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{array}\right] G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[\begin{array}{ccc} (b'_{16})^{(2)} & \boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} \quad \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{array}\right] T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[\begin{array}{ccc} (b'_{17})^{(2)} & \boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} \quad \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{array}\right] T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{18} \quad 66$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)} & +(a''_{20})^{(3)}(T_{21},t) & +(a''_{16})^{(2,2,2)}(T_{17},t) & +(a''_{13})^{(1,1,1,)}(T_{14},t) \\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{36})^{(7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)} & +(a''_{21})^{(3)}(T_{21},t) & +(a''_{17})^{(2,2,2)}(T_{17},t) & +(a''_{14})^{(1,1,1,)}(T_{14},t) \\ +(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{37})^{(7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)} & +(a''_{22})^{(3)}(T_{21},t) & +(a''_{18})^{(2,2,2)}(T_{17},t) & +(a''_{15})^{(1,1,1,)}(T_{14},t) \\ +(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{38})^{(7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{22} \quad 69$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)} & \boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{24} \quad 73$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)} & \boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{25} \quad 74$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)} & \boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
are sixth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
are seventh augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
are eighth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)} & \boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad 76$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)} & \boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* $1, 2$ *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* $1, 2$ *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* $1, 2$ *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
are fourth detrition coefficients for category $1, 2$ *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
are fifth detrition coefficients for category $1, 2$ *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
are sixth detrition coefficients for category $1, 2$ *and* 3

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
are seventh detrition coefficients for category $1, 2$ *and* 3

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category $1, 2$ *and* 3

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{30} \quad 81$$

Where $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* $1, 2$ *and* 3

And $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation coefficient for category* $1, 2$ *and* 3

$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ *are third augmentation coefficient for category* $1, 2$ *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation

coefficients for category 1,2, and 3
$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3
$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3
$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3
$+(a''_{40})^{(8,8\ ,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3
$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3
$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3
$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3
$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3
$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3
$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3
$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3
$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3
$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \quad 85$$

$$- \begin{bmatrix} (a'_{32})^{(6)} & \boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)} & \boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)} & \boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)} & \boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)} & \boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & \boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)} & \boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)} & \boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)} & \boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b_{36}')^{(7)}\boxed{-(b_{36}'')^{(7)}(G_{39},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b_{37}')^{(7)}\boxed{-(b_{37}'')^{(7)}(G_{39},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b_{38}')^{(7)}\boxed{-(b_{38}'')^{(7)}(G_{39},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{matrix} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{matrix}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{matrix} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{matrix}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{matrix} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{matrix}\right] G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\,-(b''_{40})^{(8)}(G_{43},t) & -(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\,-(b''_{41})^{(8)}(G_{43},t) & -(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\,-(b''_{42})^{(8)}(G_{43},t) & -(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq \ (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = \ (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \ , \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\,(a_i)^{(1)} + (a_i')^{(1)} + \ (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}\,(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i')^{(1)} + \ (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \leq (p_i)^{(2)} \leq \left(\hat{A}_{16}\right)^{(2)} \qquad 102$$

$$(b_i'')^{(2)}(G_{19},t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\big((G_{19}),t\big) = \ (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\big((G_{19}),t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\big[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\big]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \le (p_i)^{(3)} \le (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \le (r_i)^{(3)} \le (b_i')^{(3)} \le (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\widehat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\widehat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\widehat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - \ T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\,(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and \ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\,(a_i)^{(6)} + (a_i')^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)}\,(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(A) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(B) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37},t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39},t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

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(C) $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

(D)

$\lim_{G\to\infty} (b_i'')^{(7)}\big((G_{39}),t\big) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}\big((G_{39}),t\big)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$. (T_{37}',t) and (T_{37},t) are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(E) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(F) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$ 138

$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$ 139

$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$ 140

$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t}$ 142

$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t}$ 143

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ 144

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}$, $(r_i)^{(8)}, i = 40,41,42$,
Satisfy the inequalities

$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 145

$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 146

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\hat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)} ||(G_{47}) - (G_{47})'|| e^{-(\hat{M}_{44})^{(9)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}\left[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)} (\hat{k}_{44})^{(9)}\right] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)} t}$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)} t}$, $T_i(0) = T_i^0 > 0$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$, $T_i(0) = T_i^0 > 0$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \ , \ T_i(0) = T_i^0 \ , \ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} , T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\overline{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{40})^{(8)} \text{ , } T_i^0 \le (\hat{Q}_{40})^{(8)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad (169)$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that (170)

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that (171)

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that (172)

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that (173)

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that (174)

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \leq G_{32}^{0}+\int_0^t\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(a) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \leq G_{36}^{0}+\int_0^t\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$
$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \leq G_{40}^{0}+\int_0^t\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}= \quad 180$$
$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t} \leq \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for G_{41} , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \leq G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \leq \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for G_{45} , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \left\{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\right\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)}\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) - (a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right| \; e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} \} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \qquad (186)$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on T_{14} and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: (187)

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. (188)

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying 193
Equations into itself

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$

$sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\hat{M}_{16})^{(2)}t}\}$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $(\widetilde{G_{19}}, \widetilde{T_{19}}) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t (a_{16})^{(2)}|G_{17}^{(1)} - G_{17}^{(2)}|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}ds_{(16)} +$

$\int_0^t\{(a_{16}')^{(2)}|G_{16}^{(1)} - G_{16}^{(2)}|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}\left|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t} \leq$
$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)d\left(\left((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)}\right)\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\}ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0$ for $t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)}G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$:$\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}((G_{23}), (T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$G_{20}^{(2)} \left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ *and* $(\widehat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a'_{22})^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$G_{24}^{(2)} \left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)}\right) d\left(((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on T_{25} and respectively on (G_{27})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (228)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_2 / (a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\tilde{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\tilde{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\tilde{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\tilde{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)}\right), \left((G_{35})^{(2)}, (T_{35})^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})}, \widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})}, \widetilde{(T_{35})}\right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right)\left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)};\ (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ *and* $(\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ (250)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)} \left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)} (T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)}$$ 267

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)}$$ 268

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1,\left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right),\quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{47}}),(\widetilde{T_{47}}):\left((\widetilde{G_{47}}),(\widetilde{T_{47}})\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$

From 99 to 44 it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(9)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$: 281

By $(v_1)^{(1)} > 0$, $(v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{v}_1)^{(1)},, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{v}_1)^{(1)} > 0$, $(\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)}, , (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad (333)$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad (334)$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)} t} - e^{-(a'_{26})^{(4)} t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)} t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad (335)$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad (336)$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28}e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G^0_{28}e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}}G^0_{28}e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}}G^0_{28}e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)}G^0_{28}}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G^0_{30}e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

$$\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_2)^{(5)}((S_1)^{(5)}-(a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t}-e^{-(a'_{30})^{(5)}t}\right]+\ G_{30}^{0}e^{-(a'_{30})^{(5)}t}\Big)$$

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\le T_{30}(t)\le \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$:

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)}\le-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\le-(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)}\le-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\le-(\tau_1)^{(6)}$$

Definition of $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$: 350

By $(v_1)^{(6)}>0$, $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$, $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

Definition of $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)}>0$, $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$, $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

Definition of $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$:-

If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)}=(v_0)^{(6)},(m_1)^{(6)}=(v_1)^{(6)},$ ***if*** $(v_0)^{(6)}<(v_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}$ $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}$ $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}$ $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}$ $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}$ $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})} \left[e^{(S_1)^{(6)} t} - e^{-(a'_{34})^{(6)} t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)} t} \right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(v_1)^{(7)}, (v_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, v^{(7)}, u^{(7)}$: 361

By $(v_1)^{(7)} > 0, (v_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{v}_1)^{(7)}, , (\bar{v}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)} > 0, (\bar{v}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}, \ \boldsymbol{if}\ (v_0)^{(7)} < (v_1)^{(7)}$$

$$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}, \boldsymbol{if}\ (v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)},$$

and $\boxed{(v_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}, \ \boldsymbol{if}\ (\bar{v}_1)^{(7)} < (v_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^{0}e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}}G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}}G_{36}^{0}e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t}-e^{-(S_2)^{(7)}t}\right]+G_{38}^{0}e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left.\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_2)^{(7)}((S_1)^{(7)}-(a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t}-e^{-(a'_{38})^{(7)}t}\right]+\ G_{38}^{0}e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}}T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}}T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right) - (b''_{41})^{(8)}\left((G_{43}), t\right) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(v_1)^{(8)} > 0$, $(v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$, $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{v}_1)^{(8)}$,, $(\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{v}_1)^{(8)} > 0$, $(\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$, $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$:-

If we define $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$, $\boldsymbol{if}$ $(v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$, $\boldsymbol{if}$ $(v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$,
and $\boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}$, $\boldsymbol{if}$ $(\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, $\boldsymbol{if}$ $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, $\boldsymbol{if}$ $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}$ $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t})$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t}\leq T_{42}(t)\leq \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)},(S_2)^{(8)},(R_1)^{(8)},(R_2)^{(8)}$:- 381

Where $(S_1)^{(8)}=(a_{40})^{(8)}(m_2)^{(8)}-(a'_{40})^{(8)}$

$(S_2)^{(8)}=(a_{42})^{(8)}-(p_{42})^{(8)}$

$(R_1)^{(8)}=(b_{40})^{(8)}(\mu_2)^{(8)}-(b'_{40})^{(8)}$

$(R_2)^{(8)}=(b'_{42})^{(8)}-(r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$:

$(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)}\leq -(a'_{44})^{(9)}+(a'_{45})^{(9)}-(a''_{44})^{(9)}(T_{45},t)+(a''_{45})^{(9)}(T_{45},t)\leq -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)}\leq -(b'_{44})^{(9)}+(b'_{45})^{(9)}-(b''_{44})^{(9)}\big((G_{47}),t\big)-(b''_{45})^{(9)}\big((G_{47}),t\big)\leq -(\tau_1)^{(9)}$

Definition of $(v_1)^{(9)},(v_2)^{(9)},(u_1)^{(9)},(u_2)^{(9)},v^{(9)},u^{(9)}$:

By $(v_1)^{(9)}>0$, $(v_2)^{(9)}<0$ and respectively $(u_1)^{(9)}>0$, $(u_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_1)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)}>0$, $(\bar{v}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$, $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

Definition of $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$:-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)},\ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$
and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \le G_{46}(t) \le \right.$$
$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (v_1)^{(1)} \le (\bar{v}_1)^{(1)} \le \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$, we obtain 386

$$(v_1)^{(1)} \le v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $v^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\right) - (a''_{17})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{\mathrm{G}_{16}}{\mathrm{G}_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (\mathrm{C})^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (\mathrm{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)}\right) \le \frac{dv^{(3)}}{dt} \le -\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \ge \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}}$$

it follows $(v_0)^{(3)} \le v^{(3)}(t) \le (v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \le v^{(3)}(t) \le (\bar{v}_1)^{(3)}$

If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)} \le \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}} \le v^{(3)}(t) \le$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (\bar{v}_1)^{(3)}$$

If $0 < (v_1)^{(3)} \le (\bar{v}_1)^{(3)} \le (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$, we obtain 403

$$(v_1)^{(3)} \le v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

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$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{36}'')^{(7)} = (a_{37}'')^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b_{36}'')^{(7)} = (b_{37}'')^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a_{40}')^{(8)} - (a_{41}')^{(8)} + (a_{40}'')^{(8)}(T_{41}, t)\right) - (a_{41}'')^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(8)} and\ (b''_i)^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\,]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\,]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\,]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0$ 479

$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0$ 480

$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0$ 481

$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0$ 482

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(a) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first

equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T^*_{33} for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$$

Definition and uniqueness of T^*_{37} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T^*_{37} for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$$

Definition and uniqueness of T^*_{41} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{25} such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507
$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{29} such that $\varphi((G_{31})^*) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{33} such that $\varphi({G_{35}}^*) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{37} such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{41} such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right]+(b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{45} such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)} G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)} G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)} T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)} T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)} G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)} G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)} T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)} T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)} G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)} G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)} T_{21}^*}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T_{22}^* = \frac{(b_{22})^{(3)} T_{21}^*}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]} \quad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \qquad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \qquad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \qquad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \qquad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \qquad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \qquad 530$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i \qquad 532$$

$$\frac{\partial (a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial (b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \qquad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \qquad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \qquad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \qquad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \qquad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}\mathrm{T}^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}\mathrm{T}^*_{18}\mathbb{G}_j\right) \quad 539$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}\left((G_{27})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\Big)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\Big]$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\Big)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\Big)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\Big]$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\Big)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\Big)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\Big]$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\Big)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\Big)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\Big)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\Big)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

Note: This dovetailing mathematical explanation has to be read with each module, because of typing thousands of superscripts and subscripts, nine module platters is proposed. Each module has different representations and towards the end of felicity of reading the same enucleation is given

SECTION MINUS TWO

Maxwell's Demon At Work?

INTRODUCTION—VARIABLES USED

Information converted to energy Nov 19, 2010 37 comments Edwin Cartlidge is a science writer based in Rome FIGURES AND SOME SENTENCES HAVE BEEN DELETED ATTRIBUTABLE AND ASCRIBABLE TO SPATIAL RESTRICTIONS

(1) Physicists in Japan have shown experimentally that a particle can be made to do work simply by (e)

receiving information, rather than energy.

(2) They say that their demonstration, which uses a feedback system to control (e&eb)the electric potential of tiny polystyrene beads, does not (e) violate the second law of thermodynamics and could in future lead to (eb) new types of microscopic devices. Artist's impression of Maxwell's demon at work

Maxwell's demon hard at work?

(3) The experiment, carried out by Shoichi Toyabe of Chuo University in Tokyo and colleagues, is essentially the practical realization of a thought experiment proposed by James Clerk Maxwell in 1871. Maxwell envisaged a gas initially at uniform temperature contained in (eb) a box separated into two compartments, with a tiny intelligent being, later called "Maxwell's demon", controlling (e&eb) a shutter between the two compartments.

(4) By knowing the velocity of every molecule in the box, the demon can in principle time the opening and closing of (e&eb) the shutter to allow (eb) the build-up of faster molecules in one compartment and slower ones in the other.

(5) In this way, the demon can decrease (e) the entropy inside the box without (e) transferring energy directly to the particles, in apparent contradiction (e) of the second law of thermodynamics.

(6) Among the many responses to this conundrum was that of Leó Szilárd in 1929, who argued that the demon must consume (e) energy in the act of measuring the particle speeds and that this consumption will lead to (eb) a net increase in the system's entropy

NOTATION

Module One

Using real-time feedback control, the particle is made to climb up a spiral-staircase-like potential exerted by an electric field and gains (e) free energy larger than (e) the amount of work done on it

G_{13} : Category one of free energy larger than the amount of work done on it

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of real-time feedback control, the particle is made to climb up a spiral-staircase-like potential exerted by an electric field

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Physicists in Japan have shown experimentally that a

particle can be made to do work simply by (e) receiving information, rather than energy

G_{16} : Category one of receiving information, rather than energy

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of particle can be made to do work

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

They say that their demonstration, which uses a

feedback system to control (e&eb)the electric potential of tiny polystyrene beads, does not (e) violate the second law of thermodynamics and could in future lead to (eb) new types of microscopic devices.

Artist's impression of Maxwell's demon at work (deleted due to spatial constraints. Please excuse me)

G_{20} : Category one of feedback system; electric potential of tiny polystyrene beads, does not (e) violate the second law of thermodynamics and could in future lead to (eb) new types of microscopic devices

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of electric potential of tiny polystyrene beads, does not (e) violate the second law of thermodynamics and could in future lead to (eb) new types of microscopic devices ;feedback system

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

feedback system to control the electric potential of tiny polystyrene beads, does not (e) violate the second law of thermodynamics and could in future lead to new types of microscopic devices

G_{24} : Category one of violate the second law of thermodynamics and could in future lead to (eb) new types of microscopic devices

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of feedback system to control the electric potential of tiny polystyrene beads

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

The experiment, carried out by Shoichi Toyabe of Chuo University in Tokyo and colleagues, is essentially the practical realization of a thought experiment proposed by James Clerk Maxwell in 1871.

Maxwell envisaged a gas initially at uniform temperature contained in (eb) a box separated into two compartments, with a tiny intelligent being, later called "Maxwell's demon", controlling (e&eb) a shutter between the two compartments

G_{28} : Category one of Maxwell envisaged a gas initially at uniform temperature

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of box separated into two compartments, with a tiny intelligent being, later called "Maxwell's demon", controlling (e&eb) a shutter between the two compartments

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Maxwell envisaged a

gas initially at uniform temperature contained in a box separated into two compartments, with a tiny intelligent being, later called "Maxwell's demon", controlling (e&eb) a shutter between the two compartments

G_{32} : Category one of gas initially at uniform temperature contained in a box separated into two compartments, with a tiny intelligent being; shutter between the two compartments

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of shutter between the two compartments; gas initially at uniform temperature contained in a box separated into two compartments, with a tiny intelligent being

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

In this way, the

demon can decrease (e) the entropy inside the box without (e) transferring energy directly to the particles, in apparent contradiction (e) of the second law of thermodynamics

G_{36} : Category one of entropy inside the box without (e) transferring energy directly to the particles, in apparent contradiction (e) of the second law of thermodynamics

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of demon (there can be different demons for different experiments or take all the three categories as equivalent)

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

demon can decrease the entropy inside the box without (e) transferring energy directly to the particles, in apparent contradiction (e) of the second law of thermodynamics

G_{40} : Category one of transferring energy directly to the particles, in apparent contradiction (e) of the second law of thermodynamics

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of demon can decrease the entropy inside the box

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Among the many responses to this conundrum was that of Leó Szilárd in 1929, who argued that the

demon must consume (e) energy in the act of measuring the particle speeds and that this consumption will lead to (eb) a net increase in the system's entropy

G_{44} : Category one of energy in the act of measuring the particle speeds and that this consumption will lead to (eb) a net increase in the system's entropy

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of demon

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}$ $(a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$ $(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}$ $(a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,

$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}), t\big)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}), t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\big((G_{19}), t\big) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$
$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor
$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}),t)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}),t)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}),t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}),t) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth

detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{16})^{(2)} & \boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} \quad \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{array}\right] G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{ccc} (a'_{17})^{(2)} & \boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} \quad \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{array}\right] G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{18})^{(2)} & \boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} \quad \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{array}\right] G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[\begin{array}{ccc} (b'_{16})^{(2)} & \boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} \quad \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{array}\right] T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[\begin{array}{ccc} (b'_{17})^{(2)} & \boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} \quad \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{array}\right] T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)} & \boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{24} \quad 73$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)} & \boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{25} \quad 74$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)} & \boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
are sixth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
are seventh augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
are eighth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)} & \boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad 76$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)} & \boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad (78)$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
are fourth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad (81)$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation

coefficients for category 1,2, and 3
$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3
$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3
$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3
$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3
$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3
$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3
$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3
$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3
$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3
$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3
$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3
$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3
$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)} G_{33} \quad 85$$

$$- \begin{bmatrix} (a'_{32})^{(6)} \boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)} G_{32} - \begin{bmatrix} (a'_{33})^{(6)} \boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)} G_{33} - \begin{bmatrix} (a'_{34})^{(6)} \boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)} T_{33} - \begin{bmatrix} (b'_{32})^{(6)} \boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)} T_{32} - \begin{bmatrix} (b'_{33})^{(6)} \boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}, \boxed{(a''_{37})^{(7)}(T_{37},t)}, \boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b_{36}')^{(7)}\boxed{-(b_{36}'')^{(7)}(G_{39},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b_{37}')^{(7)}\boxed{-(b_{37}'')^{(7)}(G_{39},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b_{38}')^{(7)}\boxed{-(b_{38}'')^{(7)}(G_{39},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$(b_i'')^{(1)}(G,t) \leq \ (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$

$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)}$ 98

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = \ (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$

$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{13})^{(1)}}[\,(a_i)^{(1)} + (a_i')^{(1)} + \ (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}\,(\hat{k}_{13})^{(1)}] < 1$

$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i')^{(1)} + \ (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$(a_i'')^{(2)}(T_{17},t) \leq (p_i)^{(2)} \leq \left(\hat{A}_{16}\right)^{(2)}$ 102

$$(b_i'')^{(2)}(G_{19},t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\big((G_{19}),t\big) = \ (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - \ T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\big((G_{19}),t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\big[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\big]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}} \ , \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\,(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + \ (\hat{P}_{16})^{(2)}\,(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + \ (\hat{Q}_{16})^{(2)}\ \ (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \le (p_i)^{(3)} \le (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \le (r_i)^{(3)} \le (b_i')^{(3)} \le (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\widehat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\widehat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\widehat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - \ T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\,(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\widehat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\widehat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\widehat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\widehat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and \ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{32})^{(6)}}[\,(a_i)^{(6)} + (a_i')^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)}\,(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\widehat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(G) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(H) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37},t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39},t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

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(I) $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

(J)

$\lim_{G\to\infty} (b_i'')^{(7)}((G_{39}),t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}((G_{39}),t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$. (T_{37}',t) and (T_{37},t) are points belonging to the interval $\left[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\right]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(K) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(L) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$ 138

$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$ 139

$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$ 140

$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}$ are positive constants and $i = 40,41,42$

They satisfy Lipschitz condition:

$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t}$ 142

$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t}$ 143

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ 144

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}$, $(r_i)^{(8)}, i = 40,41,42$,
Satisfy the inequalities

$\frac{1}{(\hat{M}_{40})^{(8)}} [(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 145

$\frac{1}{(\hat{M}_{40})^{(8)}} [(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 146

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\hat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)} ||(G_{47}) - (G_{47})'|| e^{-(\hat{M}_{44})^{(9)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}\left[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)} (\hat{k}_{44})^{(9)}\right] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t} \ , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t} \ , \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \ , \ T_i(0) = T_i^0 \ , \ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} , T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\overline{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^{0} + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^{0} + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^{0} + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^{0} + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^{0} + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^{0} + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By

$$\bar{G}_{32}(t) = G_{32}^{0} + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^{0} + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^{0} + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^{0} + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^{0} + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^{0} + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that 174

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \leq G_{32}^{0}+\int_0^t\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)} =$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(b) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \leq G_{36}^{0}+\int_0^t\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)} =$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \leq G_{40}^{0}+\int_0^t\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)} = \quad 180$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t} \leq \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for G_{41} , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \leq G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \leq \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for G_{45} , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \left\{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\right\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $\left(\tilde{G}, \tilde{T}\right) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)}\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) - (a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right| \; e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} \} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \qquad (186)$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on T_{14} and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: (187)

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. (188)

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying 193
Equations into itself

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}} \leq$
$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{\mathrm{A}}_{16})^{(2)} + (\widehat{\mathrm{P}}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)\mathrm{d}\left(\left((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)}\right)\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $\mathrm{G}_i\,(\mathrm{t}) = 0$ and $\mathrm{T}_i\,(\mathrm{t}) = 0$ 200

it results

$\mathrm{G}_i\,(\mathrm{t}) \geq \mathrm{G}_i^0\mathrm{e}^{\left[-\int_0^{\mathrm{t}}\{(a_i')^{(2)} - (a_i'')^{(2)}(\mathrm{T}_{17}(s_{(16)}), s_{(16)})\}\mathrm{d}s_{(16)}\right]} \geq 0$

$\mathrm{T}_i\,(\mathrm{t}) \geq \mathrm{T}_i^0\mathrm{e}^{(-(b_i')^{(2)}\mathrm{t})} > 0 \quad \text{for t} > 0$

Definition of $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1$, $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2$ and $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$\mathrm{G}_{16} < (\widehat{\mathrm{M}}_{16})^{(2)}$ it follows $\frac{\mathrm{dG}_{17}}{\mathrm{dt}} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)}\mathrm{G}_{17}$ and by integrating

$\mathrm{G}_{17} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 = \mathrm{G}_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$

In the same way , one can obtain

$\mathrm{G}_{18} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3 = \mathrm{G}_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{\mathrm{t}\to\infty}((b_i'')^{(2)}\,((G_{19})(\mathrm{t}), \mathrm{t})) = (b_{17}')^{(2)}$ then $\mathrm{T}_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$: $\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0$ *and* $T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$G_{24}^{(2)} \left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)}\right) d\left(((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on T_{25} and respectively on (G_{27})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (228)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_2 / (a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\widehat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{32})^{(6)}\right] \leq (\widehat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}), (\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}), (\widetilde{T_{35}}) \right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right| \; e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} \} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)} (\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)}; \; (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t} \; and \; (\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$ (250)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2 \; and \; \left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33} \; and \; G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)} \left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)} (T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(7)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)}$$ 267

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\widehat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{40})^{(8)}\right] \leq (\widehat{Q}_{40})^{(8)}$$ 268

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1,\left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right),\quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{47}}),(\widetilde{T_{47}}):\left((\widetilde{G_{47}}),(\widetilde{T_{47}})\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$

From 99 to 44 it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(9)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)}, , (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)} t} - e^{-(a'_{26})^{(4)} t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)} t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28}e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G^0_{28}e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}}G^0_{28}e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}}G^0_{28}e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)}G^0_{28}}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G^0_{30}e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

$$\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_2)^{(5)}((S_1)^{(5)}-(a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t}-e^{-(a'_{30})^{(5)}t}\right]+\ G_{30}^{0}e^{-(a'_{30})^{(5)}t}\Big)$$

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\le T_{30}(t)\le \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$:

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$-(\sigma_2)^{(6)}\le-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\le-(\sigma_1)^{(6)}$

$-(\tau_2)^{(6)}\le-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\le-(\tau_1)^{(6)}$

Definition of $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$: 350

By $(v_1)^{(6)}>0$, $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$, $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

Definition of $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)}>0$, $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$, $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

Definition of $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$:-

If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)}=(v_0)^{(6)},(m_1)^{(6)}=(v_1)^{(6)},$ ***if*** $(v_0)^{(6)}<(v_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}$ $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}$ $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}$ $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}$ $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}$ $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})} \left[e^{(S_1)^{(6)} t} - e^{-(a'_{34})^{(6)} t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)} t} \right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}$, $\boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$

$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)}$,
and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}$, $\boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)}$,
and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^{0}e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}}G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}}G_{36}^{0}e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t}-e^{-(S_2)^{(7)}t}\right]+G_{38}^{0}e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left.\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_2)^{(7)}((S_1)^{(7)}-(a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t}-e^{-(a'_{38})^{(7)}t}\right]+\ G_{38}^{0}e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}}T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}}T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right) - (b''_{41})^{(8)}\left((G_{43}), t\right) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(v_1)^{(8)} > 0$, $(v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$, $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{v}_1)^{(8)}$,, $(\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{v}_1)^{(8)} > 0$, $(\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$, $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$:-

If we define $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$, $\boldsymbol{if}$ $(v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$, $\boldsymbol{if}$ $(v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$,
and $\boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}$, $\boldsymbol{if}$ $(\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, $\boldsymbol{if}$ $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, $\boldsymbol{if}$ $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}$ $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t})$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t}\leq T_{42}(t)\leq \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)},(S_2)^{(8)},(R_1)^{(8)},(R_2)^{(8)}$:- 381

Where $(S_1)^{(8)}=(a_{40})^{(8)}(m_2)^{(8)}-(a'_{40})^{(8)}$

$(S_2)^{(8)}=(a_{42})^{(8)}-(p_{42})^{(8)}$

$(R_1)^{(8)}=(b_{40})^{(8)}(\mu_2)^{(8)}-(b'_{40})^{(8)}$

$(R_2)^{(8)}=(b'_{42})^{(8)}-(r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$:

$(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)}\leq -(a'_{44})^{(9)}+(a'_{45})^{(9)}-(a''_{44})^{(9)}(T_{45},t)+(a''_{45})^{(9)}(T_{45},t)\leq -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)}\leq -(b'_{44})^{(9)}+(b'_{45})^{(9)}-(b''_{44})^{(9)}\big((G_{47}),t\big)-(b''_{45})^{(9)}\big((G_{47}),t\big)\leq -(\tau_1)^{(9)}$

Definition of $(v_1)^{(9)},(v_2)^{(9)},(u_1)^{(9)},(u_2)^{(9)},v^{(9)},u^{(9)}$:

By $(v_1)^{(9)}>0$, $(v_2)^{(9)}<0$ and respectively $(u_1)^{(9)}>0$, $(u_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_1)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)}>0$, $(\bar{v}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$, $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

Definition of $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$:-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)},\ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$
and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \le G_{46}(t) \le \right.$$
$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (v_1)^{(1)} \le (\bar{v}_1)^{(1)} \le \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$, we obtain 386

$$(v_1)^{(1)} \le v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $v^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\right) - (a''_{17})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{\mathrm{G}_{16}}{\mathrm{G}_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (\mathrm{C})^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (\mathrm{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)}\right) \le \frac{dv^{(3)}}{dt} \le -\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \ge \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}}$$

it follows $(v_0)^{(3)} \le v^{(3)}(t) \le (v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \le v^{(3)}(t) \le (\bar{v}_1)^{(3)}$

If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)} \le \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}} \le v^{(3)}(t) \le$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (\bar{v}_1)^{(3)}$$

If $0 < (v_1)^{(3)} \le (\bar{v}_1)^{(3)} \le (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$, we obtain 403

$$(v_1)^{(3)} \le v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

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$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(8)} and\ (b''_i)^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\,]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\,]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\,]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0$ 479

$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0$ 480

$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0$ 481

$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0$ 482

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(b) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first

equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T^*_{33} for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$$

Definition and uniqueness of T^*_{37} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T^*_{37} for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$$

Definition and uniqueness of T^*_{41} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{25} such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507
$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{29} such that $\varphi((G_{31})^*) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{33} such that $\varphi({G_{35}}^*) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{37} such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{41} such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right]+(b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{45} such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)} G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)} G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \qquad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \qquad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \qquad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \qquad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \qquad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \qquad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \qquad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i \qquad 532$$

$$\frac{\partial (a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial (b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \qquad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \qquad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \qquad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \qquad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \qquad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}\mathrm{T}^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}\mathrm{T}^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}\left((G_{27})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^{*}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^{*}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^{*}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^{*}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^{*}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^{*}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^{*}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^{*}\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\ B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\Big)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\Big]$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\Big)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\Big)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\Big]$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\Big)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\Big)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\Big]$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\Big)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\Big)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\Big)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\Big)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION MINUS ONE

Spiral Staircase

INTRODUCTION—VARIABLES USED

Information converted to energy Nov 19, 2010 37 comments Edwin Cartlidge is a science writer based in Rome FIGURES AND SOME SENTENCES HAVE BEEN DELETED ATTRIBUTABLE AND ASCRIBABLE TO SPATIAL RESTRICTIONS

(1) In fact, Szilárd formulated equivalence between energy and information, calculating that kTln2 (or about 0.69 kT) is both the minimum amount of work needed to store (e) one bit of binary information and the maximum that is liberated when (e) this bit is erased, where k is Boltzmann's

constant and T is the temperature of the storage medium.

Spiral staircase

(2) Toyabe and colleagues have observed this energy-information equivalence by varying (e&eb) an electric field so that it represents (eb) a kind of spiral staircase.

(3) The difference in electrical potential between successive steps on the staircase is kT, meaning (eb) that a thermally fluctuating particle placed in (eb) the field will occasionally jump up a step but more often than not it will take a step downwards.

(4) What the researchers did was to intervene so that whenever the particle does move upwards they place (eb) the equivalent of a barrier behind it, preventing (e) the particle from falling beyond this point. Repeating the process allows (eb) it to gradually climb the staircase.

(5) The experiment consisted of a 0.3 µm-diameter particle made up of (e) two polystyrene beads that was pinned to (e&eb) a single point on the underside of (e) the top of a glass box containing (e) an aqueous solution.

(6) The shape of an applied electric field forced (eb) the particle to rotate in one direction or, in other words, to fall down (e) the potential-energy staircase.

(7) Buffered by the molecules in (eb) the solution, however, the particle every so often rotated (e&eb) slightly in the opposite direction, allowing (eb) it to take a step upwards

NOTATION

Module One

Model gives the prediction of the relationship between the two parameters

demon must consume energy in the act of measuring the particle speeds and that this consumption will lead to (eb) a net increase in the system's entropy

G_{13} : Category one of demon must consume energy in the act of measuring the particle speeds and that this consumption

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of net increase in the system's entropy

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

In fact, Szilárd formulated equivalence between energy and information, calculating that

kTln2 (or about 0.69 kT) is both the minimum amount of work needed to store (e) one bit of binary information and the maximum that is liberated when (e) this bit is erased, where k is Boltzmann's constant and T is the temperature of the storage medium

G_{16} : Category one of one bit of binary information and the maximum that is liberated when (e) this bit is erased, where k is Boltzmann's constant and T is the temperature of the storage medium

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of kTln2 (or about 0.69 kT) is both the minimum amount of work needed

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

kTln2 (or about 0.69 kT) is both the minimum amount of work needed to store one bit of binary information and the maximum that is liberated when (e) this bit is erased, where k is Boltzmann's constant and T is the temperature of the storage medium

G_{20} : Category one of kTln2 (or about 0.69 kT) is both the minimum amount of work needed to store one bit of binary information and the maximum that is liberated; bit is erased, where k is Boltzmann's constant and T is the temperature of the storage medium

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of bit is erased, where k is Boltzmann's constant and T is the temperature of the storage medium; kTln2 (or about 0.69 kT) is both the minimum amount of work needed to store one bit of binary information and the maximum that is liberated

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

The difference in electrical potential between successive steps on the staircase is kT, meaning (eb) that a thermally fluctuating particle placed in (eb) the field will occasionally jump up a step but more often than not it will take a step downwards

G_{24} : Category one of difference in electrical potential between successive steps on the staircase is kT

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of thermally fluctuating particle placed in (eb) the field will occasionally jump up a step but more often than not it will take a step downwards

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

The difference in electrical potential between successive steps on the staircase is kT, meaning that a thermally fluctuating particle placed in (eb) the field will occasionally jump up a step but more often than not it will take a step downwards

G_{28} : Category one of difference in electrical potential between successive steps on the staircase is kT, meaning that a thermally fluctuating particle

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of field will occasionally jump up a step but more often than not it will take a step downwards

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

What the researchers did was to intervene so that whenever the

particle does move upwards they place (eb) the equivalent of a barrier behind it, preventing (e) the particle from falling beyond this point.

Repeating the process allows (eb) it to gradually climb the staircase

G_{32} : Category one of equivalent of a barrier behind it, preventing (e) the particle from falling beyond this point

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of particle does move upwards

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

particle does move upwards if the equivalent of a barrier is placed behind it, preventing (e) the particle from falling beyond this point

G_{36} : Category one of particle from falling beyond this point

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of particle does move upwards if the equivalent of a barrier is placed behind

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

Repeating the process allows (eb) it to gradually climb the staircase

G_{40} : Category one of Repeating the process

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of climb the staircase gradually (staircases are different for different experiments. Alternately, all the categories could be taken as equivalent).

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

The experiment consisted of a 0.3 µm-diameter particle made up of (e) two polystyrene beads that was pinned to (e&eb) a single point on the underside of (e) the top of a glass box containing (e) an aqueous solution

G_{44} : Category one of experiment consisted of a 0.3 µm-diameter particle; two polystyrene beads that was pinned to (e&eb) a single point on the underside of (e) the top of a glass box containing (e) an aqueous solution

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of two polystyrene beads that was pinned to (e&eb) a single point on the underside of (e) the top of a glass box containing (e) an aqueous solution ;experiment consisted of a 0.3 µm-diameter particle

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION ZERO

Quantification Of The Transformation

INTRODUCTION—VARIABLES USED

Information converted to energy Nov 19, 2010 37 comments Edwin Cartlidge is a science writer based in Rome FIGURES AND SOME SENTENCES HAVE BEEN DELETED ATTRIBUTABLE AND ASCRIBABLE TO SPATIAL RESTRICTIONS

(1) By tracking the particle's motion using a video camera and then using image-analysis software to identify when the particle had rotated against the field, the researchers were able to raise (eb) the metaphorical barrier behind it by inverting (e&eb) the field's phase.
(2) In this way they could gradually raise (eb) the potential of the particle even though they had not (e) imparted any energy to it directly.

Quantifiable breakthrough

(3) In recent years other groups have shown that collections of particles can be rearranged so as to reduce (e) their entropy without providing them with energy directly.

(4) The breakthrough in the latest work is to have quantified (eb) the conversion of information to (e&eb) energy.

(5) By measuring the particle's degree of rotation against the field, Toyabe and colleagues found that they could convert the equivalent of one bit information to (e&eb) 0.28 kTln2 of energy or, in other words, that they could exploit (e) more than a quarter of the information's energy content.

(6) Processes taking place on the nanoscale are completely different to those we are familiar with, and information is part (e&eb) of that picture

Christian Van den Broeck, University of Hasselt

(7) The research is described in Nature Physics, and in an accompanying article Christian Van den Broeck of the University of Hasselt in Belgium describes the result as "a direct verification of information-to-energy conversion" but points out that the conversion factor is an idealized figure. As he explains, it regards just the physics taking place on the microscopic scale and ignores (e) the far larger amount of energy consumed by (e) the macroscopic devices, among them the computers and human operators involved.

(8) He likens the energy gain to that obtained in an experimental fusion facility, which is dwarfed by (e) the energy needed to run the experiment. "They are cheating a little bit," joked Van den Broeck over the telephone. "This is not something you can put on the shelf and sell at this point."

I**nformation converted to energy Nov 19, 2010 37 comments Edwin Cartlidge is a science writer based in Rome FIGURES AND SOME SENTENCES HAVE BEEN DELETED ATTRIBUTABLE AND ASCRIBABLE TO SPATIAL RESTRICTIONS**

However, Van den Broeck does believe that the work could lead to practical applications within perhaps the next 30 or 40 years. He points out that as devices get ever more miniature the energy content of the information used to control them – kT at room temperature being equivalent to about 4 × 10–21 J – will approach that required to operate them. "Nobody thinks of using bits to boil water," he says, "but that would in principle be possible at nanometre scales." And he speculates that molecular processes occurring in nature might already be converting information to (e&eb) energy in some way. "The message is that processes taking place on the nanoscale are completely different from those we are familiar with, and that information is (=) part of that picture."

NOTATION

Module One

experiment consisted of a 0.3 μm-diameter particle made up of two polystyrene beads that was pinned to (e&eb) a single point on the underside of (e) the top of a glass box containing (e) an aqueous solution

G_{13} : Category one of experiment consisted of a 0.3 μm-diameter particle made up of two polystyrene beads ; single point on the underside of (e) the top of a glass box containing (e) an aqueous solution

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of single point on the underside of (e) the top of a glass box containing (e) an aqueous solution ;experiment consisted of a 0.3 μm-diameter particle made up of two polystyrene beads

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

shape of an applied electric field forced (eb) the particle to rotate in one direction or, in other words, to fall down (e) the potential-energy staircase

G_{16} : Category one of shape of an applied electric field

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of particle to rotate in one direction or, in other words, to fall down the potential-energy staircase

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Buffered by the molecules in the solution, however, the particle every so often rotated (e&eb) slightly in the opposite direction, allowing (eb) it to take a step upwards

G_{20} : Category one of particle(s)Buffered by the molecules in the solution; rotated slightly in the opposite direction, allowing (eb) it to take a step upwards

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of rotated slightly in the opposite direction, allowing it to take a step upwards; particle (s) Buffered by the molecules in the solution;

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

In this way they could gradually raise (eb) the

potential of the particle even though they had not (e) imparted any energy to it directly

G_{24} : Category one of potential of the particle; import of energy

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of import of energy; potential of the particle

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

In recent years other groups have shown that

collections of particles can be rearranged so as to reduce (e) their entropy without providing them with

energy directly

G_{28} : Category one of entropy without providing them with energy directly

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of collections of particles can be rearranged

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

By measuring the particle's degree of rotation against the field, Toyabe and colleagues found that they could convert the equivalent of one bit information to (e&eb) 0.28 kTln2 of energy or, in other words, that they could exploit (e) more than a quarter of the information's energy content

G_{32} : Category one of particle's degree of rotation against the field; convert the equivalent of one bit information to (e&eb) 0.28 kTln2 of energy or, in other words, that they could exploit (e) more than a quarter of the information's energy content

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of convert the equivalent of one bit information to (e&eb) 0.28 kTln2 of energy or, in other words, that they could exploit (e) more than a quarter of the information's energy content; particle's degree of rotation against the field

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

more than a quarter of the information's energy content of the particle could be exploited

G_{36} : Category one of more than a quarter of the information's energy content; particle (s) in question whose information's energy content is to be exploited

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of particle (s) in question whose information's energy content is to be exploited; more than a quarter of the information's energy content

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

The research is described in Nature Physics, and in an accompanying article Christian Van den Broeck of the University of Hasselt in Belgium describes the result as "a direct verification of information-to-energy conversion" but points out that the conversion factor is an idealized figure

As he explains, it regards just the physics taking place on the microscopic scale and ignores (e) the far larger amount of energy consumed by (e) the macroscopic devices, among them the computers and human operators involved

G_{40} : Category one of far larger amount of energy consumed by the macroscopic devices, among them the computers and human operators involved

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of experiment(s)

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

He likens the

energy gain to that obtained in an experimental fusion facility, which is dwarfed by (e) the energy needed to run the experiment.

"They are cheating a little bit," joked Van den Broeck over the telephone. "This is not something you can put on the shelf and sell at this point

G_{44} : Category one of energy needed to run the experiment

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of energy gain to that obtained in an experimental fusion facility

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$

$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)} \ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients

for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{c} (a'_{16})^{(2)}\,+(a''_{16})^{(2)}(T_{17},t)\;+(a''_{13})^{(1,1,)}(T_{14},t)\;+(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t)\;+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)\;+(a''_{32})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7)}(T_{37},t)\;+(a''_{40})^{(8,8,8)}(T_{41},t)\;+(a''_{44})^{(9,9)}(T_{45},t) \end{array}\right] G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{c} (a'_{17})^{(2)}\,+(a''_{17})^{(2)}(T_{17},t)\;+(a''_{14})^{(1,1,)}(T_{14},t)\;+(a''_{21})^{(3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t)\;+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)\;+(a''_{33})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{37})^{(7,7,7)}(T_{37},t)\;+(a''_{41})^{(8,8,8)}(T_{41},t)\;+(a''_{45})^{(9,9)}(T_{45},t) \end{array}\right] G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{c} (a'_{18})^{(2)}\,+(a''_{18})^{(2)}(T_{17},t)\;+(a''_{15})^{(1,1,)}(T_{14},t)\;+(a''_{22})^{(3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t)\;+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)\;+(a''_{34})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{38})^{(7,7,7)}(T_{37},t)\;+(a''_{42})^{(8,8,8)}(T_{41},t)\;+(a''_{46})^{(9,9)}(T_{45},t) \end{array}\right] G_{18} \quad (63)$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for

category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)} & \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad 70$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)} & \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition*

coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$ $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t)\\ +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t)\\ +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t)\end{bmatrix}G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)}+(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t)\\ +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t)\\ +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t)\end{bmatrix}G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t)\\ +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t)\\ +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t)\end{bmatrix}G_{26} \quad (75)$$

$(a''_{24})^{(4)}(T_{25},t)$, $(a''_{25})^{(4)}(T_{25},t)$, $(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* $1,2\ 3$

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* $1,2$ *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* $1,2$ *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$
are fourth augmentation coefficients for category $1,2$ *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$
are fifth augmentation coefficients for category $1,2$ *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$
are sixth augmentation coefficients for category $1,2$ *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$
are seventh augmentation coefficients for category $1,2$ *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$
are eighth augmentation coefficients for category $1,2$ *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
are fourth detrition coefficients for category 1, 2 *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
are fifth detrition coefficients for category 1, 2 *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
are sixth detrition coefficients for category 1, 2 *and* 3

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
are seventh detrition coefficients for category 1, 2 *and* 3

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category 1, 2 *and* 3

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{30} \quad (81)$$

$Where\ \boxed{+(a''_{28})^{(5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5)}(T_{29},t)}\ are\ first\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$And\ \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}\ are\ second\ augmentation\ coefficient\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}\ are\ third\ augmentation\ coefficient\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[\begin{array}{ccc} (b'_{28})^{(5)} & \boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} \\ \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} & \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} \\ \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} \\ \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} & & \end{array}\right] T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[\begin{array}{ccc} (b'_{29})^{(5)} & \boxed{-(b''_{29})^{(5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,)}(G_{27},t)} \\ \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} & \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)} \\ \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)} \\ \boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} & & \end{array}\right] T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[\begin{array}{ccc} (b'_{30})^{(5)} & \boxed{-(b''_{30})^{(5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)} \\ \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} & \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)} \\ \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)} \\ \boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} & & \end{array}\right] T_{30} \quad 84$$

$where\ \boxed{-(b''_{28})^{(5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5)}(G_{31},t)}\ are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}\ are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}\ are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients

for category 1,2, and 3

$[-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)]$, $[-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)]$, $[-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)]$ are seventh detrition coefficients for category 1,2, and 3

$[-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)]$, $[-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)]$, $[-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)]$ are eighth detrition coefficients for category 1,2, and 3

$[-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)]$, $[-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)]$, $[-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)]$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)}[+(a''_{32})^{(6)}(T_{33},t)] & [+(a''_{28})^{(5,5,5)}(T_{29},t)] & [+(a''_{24})^{(4,4,4,)}(T_{25},t)] \\ [+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)] & [+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)] & [+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)] \\ [+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)] & [+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)] & [+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)] \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}[+(a''_{33})^{(6)}(T_{33},t)] & [+(a''_{29})^{(5,5,5)}(T_{29},t)] & [+(a''_{25})^{(4,4,4,)}(T_{25},t)] \\ [+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)] & [+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)] & [+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)] \\ [+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)] & [+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)] & [+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)] \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}[+(a''_{34})^{(6)}(T_{33},t)] & [+(a''_{30})^{(5,5,5)}(T_{29},t)] & [+(a''_{26})^{(4,4,4,)}(T_{25},t)] \\ [+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)] & [+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)] & [+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)] \\ [+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)] & [+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)] & [+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)] \end{bmatrix} G_{34} \quad (87)$$

$[+(a''_{32})^{(6)}(T_{33},t)]$, $[+(a''_{33})^{(6)}(T_{33},t)]$, $[+(a''_{34})^{(6)}(T_{33},t)]$ *are first augmentation coefficients for category* 1, 2 *and* 3

$[+(a''_{28})^{(5,5,5)}(T_{29},t)]$, $[+(a''_{29})^{(5,5,5)}(T_{29},t)]$, $[+(a''_{30})^{(5,5,5)}(T_{29},t)]$ *are second augmentation coefficients for category* 1, 2 *and* 3

$[+(a''_{24})^{(4,4,4,)}(T_{25},t)]$, $[+(a''_{25})^{(4,4,4,)}(T_{25},t)]$, $[+(a''_{26})^{(4,4,4,)}(T_{25},t)]$ *are third augmentation coefficients for category* 1, 2 *and* 3

$[+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)]$, $[+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)]$, $[+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)]$ - are fourth augmentation coefficients

$[+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)]$, $[+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)]$, $[+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)]$ - fifth augmentation coefficients

$[+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)]$, $[+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)]$, $[+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)]$ sixth augmentation coefficients

$[+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)]$, $[+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)]$, $[+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)]$ seventh augmentation coefficients

$[+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)]$, $[+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)]$, $[+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)]$ Eighth augmentation coefficients

$[+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)]$, $[+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)]$, $[+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)]$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)}\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)}\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad (90)$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad (91)$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)}\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad (92)$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)}\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$\frac{dG_{38}}{dt}$ *93*

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{36}}{dt} =$ 94

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$\frac{dT_{37}}{dt} =$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$\frac{dT_{38}}{dt} =$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for

category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third

augmentation coefficient for category 1, 2 and 3
$+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3
$+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$,$+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$,$+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3
$+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$,$+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$,$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$
$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\left[-(b''_{40})^{(8)}(G_{43},t)\right] & \left[-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$
$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\left[-(b''_{41})^{(8)}(G_{43},t)\right] & \left[-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$
$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\left[-(b''_{42})^{(8)}(G_{43},t)\right] & \left[-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3
$-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3
$-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3
$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients

for category 1, 2 and 3

$\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{38}'')^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

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$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a_{44}')^{(9)}\boxed{+(a_{44}'')^{(9)}(T_{45},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a_{45}')^{(9)}\boxed{+(a_{45}'')^{(9)}(T_{45},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a_{46}')^{(9)}\boxed{+(a_{46}'')^{(9)}(T_{37},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{44}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{46}'')^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$
$$(b_{44})^{(9)}T_{45} -$$
$$\begin{bmatrix} (b'_{44})^{(9)} & -(b''_{44})^{(9)}(G_{47},t) & -(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{13}$$
$$\frac{dT_{45}}{dt} =$$
$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)} & -(b''_{45})^{(9)}(G_{47},t) & -(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{14}$$
$$\frac{dT_{46}}{dt} =$$
$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)} & -(b''_{46})^{(9)}(G_{47},t) & -(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t) \end{bmatrix} T_{15}$$

Where $-(b''_{44})^{(9)}(G_{47},t)$, $-(b''_{45})^{(9)}(G_{47},t)$, $-(b''_{46})^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15$ *97*

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$(a_i'')^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$

$(b_i'')^{(1)}(G, t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$

$lim_{T_2 \to \infty}(a_i'')^{(1)}(T_{14}, t) = (p_i)^{(1)}$ 98

$\lim_{G \to \infty}(b_i'')^{(1)}(G, t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$|(a_i'')^{(1)}(T_{14}', t) - (a_i'')^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$

$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}', t)$ and $(a_i'')^{(1)}(T_{14}, t)$. (T_{14}', t) and (T_{14}, t) are points belonging to the interval $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$

$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and $(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}} [(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + \ (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq \ (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}\ (T_{21}, t) = (p_i)^{(3)} \quad 113$$

$$\lim_{G \to \infty}(b_i'')^{(3)}\ (G_{23}, t) = \ (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - \ T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)},$ are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\ , \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\ (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}), t\big) \leq (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G \to \infty}(b_i'')^{(4)}\big((G_{27}), t\big) = (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)}(\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \leq (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}$, $(r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G \to \infty}(b_i'')^{(6)}\left((G_{35}), t\right) = (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ (\hat{k}_{32})^{(6)}] < 1$

Where we suppose

(M) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(N) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$

$(b_i'')^{(7)}(G_{39}, t) \leq \ (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$

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(O) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(P)

$\lim_{G \to \infty} (b_i'')^{(7)}\big((G_{39}), t\big) = \ (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, \ (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$

$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}\big((G_{39}), t\big)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(Q) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(R) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants

$(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, \ (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\widehat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\widehat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \tag{136}$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}, \ (r_i)^{(8)}$: (137)

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \tag{138}$$

$$(b_i'')^{(8)}((G_{43}), t) \leq \ (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \tag{139}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)} (T_{41}, t) = (p_i)^{(8)} \tag{140}$$

$$\lim_{G \to \infty} (b_i'')^{(8)} \left((G_{43}), t\right) = \ (r_i)^{(8)} \tag{141}$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\widehat{M}_{40})^{(8)} t} \tag{142}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\left((G_{43}), t\right)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\widehat{M}_{40})^{(8)} t} \tag{143}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\widehat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\widehat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1 \tag{144}$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, \ (r_i)^{(8)}, i = 40,41,42,$

Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \qquad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad 146A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}, \ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq \ (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = \ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, \ (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}, \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$
$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$
$T_i(t) \leq (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , $T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{13})^{(1)}$, $T_i^0 \leq (\hat{Q}_{13})^{(1)}$, 155

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 156

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\tilde{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\tilde{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{20})^{(3)}$, $T_i^0 \leq (\hat{Q}_{20})^{(3)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{24})^{(4)}$, $T_i^0 \leq (\hat{Q}_{24})^{(4)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0,t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)} (T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)} (T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)} (G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)} (G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)} (G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{32})^{(6)}$, $T_i^0 \leq (\hat{Q}_{32})^{(6)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$$

By

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)}) \right) G_{42}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166 A

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)}\left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)}(\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)}\left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)}(\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)}\left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)}\left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(c) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t)\leq G_{36}^{0}+\int_{0}^{t}\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t}\leq\frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\hat{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \tag{180}$$

$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left(e^{(\hat{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that

$$(G_{40}(t) - G_{40}^0) e^{-(\hat{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0\right) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right] \tag{181}$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose $\tag{182}$

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \le (\hat{P}_{13})^{(1)} \tag{183}$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \le (\hat{Q}_{13})^{(1)} \tag{184}$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric $\tag{185}$

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\le \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\,G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)},i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$

From 19 to 24 it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(1)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1,\left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13}<(\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt}\le\left((\widehat{M}_{13})^{(1)}\right)_1-(a'_{14})^{(1)}G_{14}$ and by integrating

$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a'_{14})^{(1)}$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a'_{15})^{(1)}$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t),t)) = (b'_{14})^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t),t) = (b'_{15})^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ *and* $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(((G_{19})^{(1)}, (T_{19})^{(1)}), ((G_{19})^{(2)}, (T_{19})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\le$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t)=0$ and $T_i(t)=0$ 200

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(2)}-(a_i'')^{(2)}(T_{17}(s_{(16)}),s_{(16)})\}ds_{(16)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(2)}t)}>0\quad\text{for } t>0$$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1,\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16}<(\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt}\le\left((\widehat{M}_{16})^{(2)}\right)_1-(a'_{17})^{(2)}G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1/(a'_{17})^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a'_{18})^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t),t)) = (b'_{17})^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t),t\right) = (b'_{18})^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)} + \left((\widehat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\le \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ and\ (\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)},i=20{,}21{,}22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 216

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{\left(-(b_i')^{(3)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1,\left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If $G_{20}\ is$ bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right) = (b'_{21})^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t),t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b''_{22})^{(3)}\left((G_{23})(t),t\right) = (b'_{22})^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)} + \left((\hat{P}_{24})^{(4)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{24})^{(4)}\right] \leq (\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\le \qquad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}\ and\ (\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 227

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$ 228

it results

$$G_i\,(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]}\ge 0$$

$$T_i\,(t)\ge T_i^0e^{\left(-(b_i')^{(4)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1,\left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b'_{25})^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right)$, $t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b'_{26})^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\widehat{P}_{28})^{(5)}$ and $(\widehat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\widehat{P}_{28})^{(5)} + \left((\widehat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[\left((\widehat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{28})^{(5)}\right] \leq (\widehat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})},\widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})},\widetilde{(T_{31})}\right)=\mathcal{A}^{(5)}((G_{31}),(T_{31}))$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\le \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\ (G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}\ and\ (\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i=28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$ 239

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(5)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1,\left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)}G_{29}$ and by integrating

$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a'_{29})^{(5)}$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\ ((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right),\quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\widehat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{32})^{(6)}\right] \leq (\widehat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\le \quad 248$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}\ and\ (\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i=32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$ 250

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(6)}-(a_i'')^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(6)}t)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1,\left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right),\ \ t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i , T_i satisfying

Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})},\widetilde{(T_{39})}:(\widetilde{(G_{39})},\widetilde{(T_{39})})=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$

It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\le\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\le$$ 259
$$\frac{1}{(\widehat{M}_{36})^{(7)}}((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)})d\left(((G_{39})^{(1)},(T_{39})^{(1)};(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i=36,37,38$ depend only on T_{37} and respectively on (G_{39})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 261

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(7)}t)}>0\quad\text{for t}>0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)} \quad 267$$

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$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{40})^{(8)}\right]\leq(\hat{Q}_{40})^{(8)}$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval [0, t] 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40{,}41{,}42$ depend only on T_{41} and respectively on (G_{43})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(8)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also G_{41} *and* G_{42} . indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)} \left((\widehat{M}_{40})^{(8)}\right)_1 / (a_{41}')^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)} \left((\widehat{M}_{40})^{(8)}\right)_2 / (a_{42}')^{(8)}$

If G_{41} *or* G_{42} is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for G_{41} *and* G_{42} . The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b_{42}'')^{(8)}\left((G_{43})(t), t(t), t\right) = (b_{42}')^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{44})^{(9)}\right]\leq(\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\;e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\;(G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t)=0\ and\ T_i\,(t)=0$

From 99 to 44 it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\geq 0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(9)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2$ *and* $\left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also G_{45} *and* G_{46} . indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a'_{45})^{(9)} G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$: 281

By $(v_1)^{(1)} > 0$, $(v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{v}_1)^{(1)},,(\bar{v}_2)^{(1)},(\bar{u}_1)^{(1)},(\bar{u}_2)^{(1)}$: 282

By $(\bar{v}_1)^{(1)} > 0$, $(\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)},(\mu_2)^{(1)},(\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}$, $(m_2)^{(1)}$, $(\mu_1)^{(1)},(\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}\ , if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(\ m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}\ , if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\ \mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined by equation

$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$

$(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \le G_{15}(t) \le$ 286

$\frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}] + G_{15}^0 e^{-(a'_{15})^{(1)}t})$

$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}}$ 287

$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}$ 288

$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \le T_{15}(t) \le$ 289

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- 290

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying
$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(T_{17},t)+(a''_{17})^{(2)}(T_{17},t)\le-(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)},(\nu_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: 295

By $(\nu_1)^{(2)}>0,(\nu_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0,(u_2)^{(2)}<0$ the roots 296

of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2+(\sigma_1)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ 297

and $(b_{14})^{(2)}\left(u^{(2)}\right)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and 298

Definition of $(\bar{\nu}_1)^{(2)},,(\bar{\nu}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)}>0,(\bar{\nu}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0,(\bar{u}_2)^{(2)}<0$ the 300

roots of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2+(\sigma_2)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ 301

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2+(\tau_2)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ 302

Definition of $(m_1)^{(2)},(m_2)^{(2)},(\mu_1)^{(2)},(\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)},(m_2)^{(2)},(\mu_1)^{(2)},(\mu_2)^{(2)}$ by 304

$(m_2)^{(2)}=(\nu_0)^{(2)},(m_1)^{(2)}=(\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)}<(\nu_1)^{(2)}$ 305

$(m_2)^{(2)}=(\nu_1)^{(2)},(m_1)^{(2)}=(\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)}<(\nu_0)^{(2)}<(\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)}=\frac{G_{16}^{0}}{G_{17}^{0}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)},\ \boldsymbol{if}\ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \boldsymbol{if}\ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, ***if*** $(\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, ***if*** $(\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, ***if*** $(\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G^0_{20}e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \le G_{20}(t) \le G^0_{20}e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}}G^0_{20}e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}}G^0_{20}e^{(S_1)^{(3)}t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)}G_{20}^{0}}{(m_1)^{(3)}((S_1)^{(3)}-(p_{20})^{(3)}-(S_2)^{(3)})}\left[e^{((S_1)^{(3)}-(p_{20})^{(3)})t}-e^{-(S_2)^{(3)}t}\right]+G_{22}^{0}e^{-(S_2)^{(3)}t}\le G_{22}(t)\le\right. \quad 324$$

$$\left.\frac{(a_{22})^{(3)}G_{20}^{0}}{(m_2)^{(3)}((S_1)^{(3)}-(a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t}-e^{-(a'_{22})^{(3)}t}\right]+G_{22}^{0}e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^{0}e^{(R_1)^{(3)}t}\le T_{20}(t)\le T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^{0}e^{(R_1)^{(3)}t}\le T_{20}(t)\le\frac{1}{(\mu_2)^{(3)}}T_{20}^{0}e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^{0}}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t}-e^{-(b'_{22})^{(3)}t}\right]+T_{22}^{0}e^{-(b'_{22})^{(3)}t}\le T_{22}(t)\le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^{0}}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t}-e^{-(R_2)^{(3)}t}\right]+T_{22}^{0}e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)},(S_2)^{(3)},(R_1)^{(3)},(R_2)^{(3)}$:- 328

Where $(S_1)^{(3)}=(a_{20})^{(3)}(m_2)^{(3)}-(a'_{20})^{(3)}$

$(S_2)^{(3)}=(a_{22})^{(3)}-(p_{22})^{(3)}$

$(R_1)^{(3)}=(b_{20})^{(3)}(\mu_2)^{(3)}-(b'_{20})^{(3)}$

$(R_2)^{(3)}=(b'_{22})^{(3)}-(r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)},(\sigma_2)^{(4)},(\tau_1)^{(4)},(\tau_2)^{(4)}$:

$(\sigma_1)^{(4)},(\sigma_2)^{(4)},(\tau_1)^{(4)},(\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)}\le-(a'_{24})^{(4)}+(a'_{25})^{(4)}-(a''_{24})^{(4)}(T_{25},t)+(a''_{25})^{(4)}(T_{25},t)\le-(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)}\le-(b'_{24})^{(4)}+(b'_{25})^{(4)}-(b''_{24})^{(4)}\big((G_{27}),t\big)-(b''_{25})^{(4)}\big((G_{27}),t\big)\le-(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)},(v_2)^{(4)},(u_1)^{(4)},(u_2)^{(4)},v^{(4)},u^{(4)}$: 329

By $(v_1)^{(4)}>0$, $(v_2)^{(4)}<0$ and respectively $(u_1)^{(4)}>0$, $(u_2)^{(4)}<0$ the roots of the equations $(a_{25})^{(4)}\big(v^{(4)}\big)^2+(\sigma_1)^{(4)}v^{(4)}-(a_{24})^{(4)}=0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2+(\tau_1)^{(4)}u^{(4)}-(b_{24})^{(4)}=0$ and

Definition of $(\bar{v}_1)^{(4)},,(\bar{v}_2)^{(4)},(\bar{u}_1)^{(4)},(\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)}>0$, $(\bar{v}_2)^{(4)}<0$ and respectively $(\bar{u}_1)^{(4)}>0$, $(\bar{u}_2)^{(4)}<0$ the roots of the equations $(a_{25})^{(4)}\big(v^{(4)}\big)^2+(\sigma_2)^{(4)}v^{(4)}-(a_{24})^{(4)}=0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2+(\tau_2)^{(4)}u^{(4)}-(b_{24})^{(4)}=0$

Definition of $(m_1)^{(4)},(m_2)^{(4)},(\mu_1)^{(4)},(\mu_2)^{(4)},(v_0)^{(4)}$:-

If we define $(m_1)^{(4)},(m_2)^{(4)},(\mu_1)^{(4)},(\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)},\ \boldsymbol{if}\ (\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}\ , \boldsymbol{if}\ (\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)},$

and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(\ m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}\ , \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\ \mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)} t} - e^{-(a'_{26})^{(4)} t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)} t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}((G_{31}), t) - (b''_{29})^{(5)}((G_{31}), t) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)}G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G_{30}^0 e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

$$\left. \frac{(a_{30})^{(5)}G_{28}^0}{(m_2)^{(5)}((S_1)^{(5)}-(a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t} - e^{-(a'_{30})^{(5)}t}\right] + G_{30}^0 e^{-(a'_{30})^{(5)}t}\right)$$

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \le T_{30}(t) \le \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t}\right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \le -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \le -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \le -(\tau_1)^{(6)}$$

Definition of $(v_1)^{(6)}, (v_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, v^{(6)}, u^{(6)}$: 350

By $(v_1)^{(6)} > 0$, $(v_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\big(u^{(6)}\big)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{v}_1)^{(6)}, , (\bar{v}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)} > 0$, $(\bar{v}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the

roots of the equations $(a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (v_0)^{(6)}$:-

If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (v_0)^{(6)}, (m_1)^{(6)} = (v_1)^{(6)}$, ***if*** $(v_0)^{(6)} < (v_1)^{(6)}$

$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (\bar{v}_6)^{(6)}$, ***if*** $(v_1)^{(6)} < (v_0)^{(6)} < (\bar{v}_1)^{(6)}$,
and $\boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)}$, ***if*** $(\bar{v}_1)^{(6)} < (v_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$
$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t} - e^{-(a'_{34})^{(6)}t}\right] + G_{34}^0 e^{-(a'_{34})^{(6)}t}\right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0$, $(\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0$, $(u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)},, (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0$, $(\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0$, $(\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(v_1)^{(8)}, (v_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, v^{(8)}, u^{(8)}$: 372

By $(v_1)^{(8)} > 0$, $(v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$, $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(v^{(8)}\big)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{v}_1)^{(8)}, , (\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{v}_1)^{(8)} > 0$, $(\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$, $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(v^{(8)}\big)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$:-

If we define $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$, $\boldsymbol{if}\ (v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$, $\boldsymbol{if}\ (v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$,
and $\boxed{(v_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}$, $\boldsymbol{if}\ (\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, $\boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, $\boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T^0_{40}}{T^0_{41}}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G^0_{40} e^{\left((S_1)^{(8)} - (p_{40})^{(8)}\right)t} \le G_{40}(t) \le G^0_{40} e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)}-(p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)}-(p_{40})^{(8)}-(S_2)^{(8)})}\left[e^{((S_1)^{(8)}-(p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}\left[e^{(S_1)^{(8)}t} - e^{-(a'_{42})^{(8)}t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)}t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)}t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)}t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t} - e^{-(b'_{42})^{(8)}t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)}t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t} - e^{-(R_2)^{(8)}t}\right] + T_{42}^0 e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}), t\big) - (b''_{45})^{(9)}\big((G_{47}), t\big) \le -(\tau_1)^{(9)}$$

Definition of $(v_1)^{(9)}, (v_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, v^{(9)}, u^{(9)}$:

By $(v_1)^{(9)} > 0$, $(v_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{v}_1)^{(9)}, , (\bar{v}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)} > 0$, $(\bar{v}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)} = 0$ and $(b_{45})^{(9)}\left(u^{(9)}\right)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$, $(v_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}$, $(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (v_0)^{(9)}, (m_1)^{(9)} = (v_1)^{(9)}$, ***if*** $(v_0)^{(9)} < (v_1)^{(9)}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (\bar{v}_1)^{(9)}$, ***if*** $(v_1)^{(9)} < (v_0)^{(9)} < (\bar{v}_1)^{(9)}$,
and $\boxed{(v_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (v_1)^{(9)}, (m_1)^{(9)} = (v_0)^{(9)}$, ***if*** $(\bar{v}_1)^{(9)} < (v_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{\left((S_1)^{(9)} - (p_{44})^{(9)}\right)t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{\left((S_1)^{(9)} - (p_{44})^{(9)}\right)t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)}t}$$

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}\left((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)}\right)}\left[e^{\left((S_1)^{(9)} - (p_{44})^{(9)}\right)t} - e^{-(S_2)^{(9)}t}\right] + G_{46}^0 e^{-(S_2)^{(9)}t} \le G_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}\left((S_1)^{(9)} - (a'_{46})^{(9)}\right)}\left[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}\right] + G_{46}^0 e^{-(a'_{46})^{(9)}t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{\left((R_1)^{(9)} + (r_{44})^{(9)}\right)t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{\left((R_1)^{(9)} + (r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}\left((R_1)^{(9)} - (b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}\left((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)} + (r_{44})^{(9)}\right)t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \leq \frac{dv^{(1)}}{dt} \leq -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \geq \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \leq v^{(1)}(t) \leq (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \leq \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \leq v^{(1)}(t) \leq (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \leq \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \leq v^{(1)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \leq (\bar{v}_1)^{(1)}$$

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If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a_{16}')^{(2)} - (a_{17}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right) - (a_{17}'')^{(2)}(T_{17}, t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{\mathrm{C}})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(\mathrm{C})^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(\mathrm{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{\mathrm{G}_{16}^0}{\mathrm{G}_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{\mathrm{C}})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{\mathrm{C}})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \le \frac{d\nu^{(3)}}{dt} \le -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \ge \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \le \nu^{(3)}(t) \le (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \le \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \le \nu^{(3)}(t) \le (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \le \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}} \le \nu^{(3)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \le (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \le (\bar{\nu}_1)^{(3)} \le (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\quad \boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}}{4+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{4+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

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If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{32}'')^{(6)} = (a_{33}'')^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b_{32}'')^{(6)} = (b_{33}'')^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a_{36}')^{(7)} - (a_{37}')^{(7)} + (a_{36}'')^{(7)}(T_{37}, t)\right) - (a_{37}'')^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\quad \boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get 417

$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have
Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition

$(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(8)}(t)$:-

$$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $v^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then $(u_1)^{(8)} = (\bar{u}_1)^{(8)}$ if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\quad \boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$v^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq v^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)}-(\nu_0)^{(9)}}{(\nu_0)^{(9)}-(\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \le \nu^{(9)}(t) \le (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

with $(p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad 437$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad 438$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\,]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\,]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\,]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

Proof: 486

(c) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$

Definition and uniqueness of T_{14}^{*} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T_{14}^{*} for which $f(T_{14}^{*}) = 0$. With this value , we obtain from the three first

equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T^*_{33} for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$$

Definition and uniqueness of T^*_{37} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that

there exists a unique T_{37}^{*} for which $f(T_{37}^{*})=0$. With this value , we obtain from the three first equations

$$G_{36}=\frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}=\frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

Definition and uniqueness of T_{41}^{*} :- 501

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^{*} for which $f(T_{41}^{*})=0$. With this value , we obtain from the three first equations

$$G_{40}=\frac{(a_{40})^{(8)}G_{41}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}=\frac{(a_{42})^{(8)}G_{41}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

Definition and uniqueness of T_{45}^{*} :- 501 A

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^{*} for which $f(T_{45}^{*})=0$. With this value , we obtain from the three first equations

$$G_{44}=\frac{(a_{44})^{(9)}G_{45}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46}=\frac{(a_{46})^{(9)}G_{45}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G)=(b'_{13})^{(1)}(b'_{14})^{(1)}-(b_{13})^{(1)}(b_{14})^{(1)}-$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G)+(b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right]+(b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G)=0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0)>0$, $\varphi(\infty)<0$ it follows that there exists a unique G_{14}^{*} such that $\varphi(G^{*})=0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19})=(b'_{16})^{(2)}(b'_{17})^{(2)}-(b_{16})^{(2)}(b_{17})^{(2)}-$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19})+(b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19})=0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0)>0$, $\varphi(\infty)<0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*})=0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23})=(b'_{20})^{(3)}(b'_{21})^{(3)}-(b_{20})^{(3)}(b_{21})^{(3)}-$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{21} such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{25} such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{29} such that $\varphi((G_{31})^*) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{33} such that $\varphi({G_{35}}^*) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{37} such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$$

$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{41} such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{45} such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G^*_{14} given by $\varphi(G^*) = 0$, T^*_{14} given by $f(T^*_{14}) = 0$ and

$$G^*_{13} = \frac{(a_{13})^{(1)}G^*_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G^*_{15} = \frac{(a_{15})^{(1)}G^*_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

$$T^*_{13} = \frac{(b_{13})^{(1)}T^*_{14}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^*)]} \quad , \quad T^*_{15} = \frac{(b_{15})^{(1)}T^*_{14}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G^*_{17} given by $\varphi((G_{19})^*) = 0$, T^*_{17} given by $f(T^*_{17}) = 0$ and 512

$$G^*_{16} = \frac{(a_{16})^{(2)}G^*_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G^*_{18} = \frac{(a_{18})^{(2)}G^*_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \quad 513$$

$$T^*_{16} = \frac{(b_{16})^{(2)}T^*_{17}}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T^*_{18} = \frac{(b_{18})^{(2)}T^*_{17}}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G^*_{21} given by $\varphi((G_{23})^*) = 0$, T^*_{21} given by $f(T^*_{21}) = 0$ and

$$G^*_{20} = \frac{(a_{20})^{(3)}G^*_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G^*_{22} = \frac{(a_{22})^{(3)}G^*_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

$$T^*_{20} = \frac{(b_{20})^{(3)}T^*_{21}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T^*_{22} = \frac{(b_{22})^{(3)}T^*_{21}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G^*_{25} given by $\varphi(G_{27}) = 0$, T^*_{25} given by $f(T^*_{25}) = 0$ and

$$G^*_{24} = \frac{(a_{24})^{(4)}G^*_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G^*_{26} = \frac{(a_{26})^{(4)}G^*_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

$T_{24}^{*} = \frac{(b_{24})^{(4)}T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]}$, $T_{26}^{*} = \frac{(b_{26})^{(4)}T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]}$ 517

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$G_{28}^{*} = \frac{(a_{28})^{(5)}G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]}$, $G_{30}^{*} = \frac{(a_{30})^{(5)}G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523

G_{45}^* given by $\varphi((G_{47})^*) = 0$, T_{45}^* given by $f(T_{45}^*) = 0$ and A

$$G_{44}^* = \frac{(a_{44})^{(9)} G_{45}^*}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46}^* = \frac{(a_{46})^{(9)} G_{45}^*}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

$$T_{44}^* = \frac{(b_{44})^{(9)} T_{45}^*}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^*)]} \quad , \quad T_{46}^* = \frac{(b_{46})^{(9)} T_{45}^*}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^*)]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a'_{13})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)} G_{13}^* \mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a'_{14})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)} G_{14}^* \mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a'_{15})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)} G_{15}^* \mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b'_{13})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)} T_{13}^* \mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b'_{14})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)} T_{14}^* \mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b'_{15})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)} T_{15}^* \mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial \mathrm{T}_{17}}(\mathrm{T}_{17}^{*}) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial \mathrm{G}_j}(\,(G_{19})^{*}\,) = s_{ij} \tag{533}$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{\mathrm{d}\mathbb{G}_{16}}{\mathrm{dt}} = -\big((a_{16}')^{(2)} + (p_{16})^{(2)}\big)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}\mathrm{G}_{16}^{*}\mathbb{T}_{17} \tag{534}$$

$$\frac{\mathrm{d}\mathbb{G}_{17}}{\mathrm{dt}} = -\big((a_{17}')^{(2)} + (p_{17})^{(2)}\big)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}\mathrm{G}_{17}^{*}\mathbb{T}_{17} \tag{535}$$

$$\frac{\mathrm{d}\mathbb{G}_{18}}{\mathrm{dt}} = -\big((a_{18}')^{(2)} + (p_{18})^{(2)}\big)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}\mathrm{G}_{18}^{*}\mathbb{T}_{17} \tag{536}$$

$$\frac{\mathrm{d}\mathbb{T}_{16}}{\mathrm{dt}} = -\big((b_{16}')^{(2)} - (r_{16})^{(2)}\big)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \textstyle\sum_{j=16}^{18}\big(s_{(16)(j)}\mathrm{T}_{16}^{*}\mathbb{G}_j\big) \tag{537}$$

$$\frac{\mathrm{d}\mathbb{T}_{17}}{\mathrm{dt}} = -\big((b_{17}')^{(2)} - (r_{17})^{(2)}\big)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \textstyle\sum_{j=16}^{18}\big(s_{(17)(j)}\mathrm{T}_{17}^{*}\mathbb{G}_j\big) \tag{538}$$

$$\frac{\mathrm{d}\mathbb{T}_{18}}{\mathrm{dt}} = -\big((b_{18}')^{(2)} - (r_{18})^{(2)}\big)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \textstyle\sum_{j=16}^{18}\big(s_{(18)(j)}\mathrm{T}_{18}^{*}\mathbb{G}_j\big) \tag{539}$$

ASYMPTOTIC STABILITY ANALYSIS (540)

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}\ and\ (b_i'')^{(3)}$ Belong to $C^{(3)}(\,\mathbb{R}_{+})$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^{*}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}(\,(G_{23})^{*}\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\big((a_{20}')^{(3)} + (p_{20})^{(3)}\big)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^{*}\mathbb{T}_{21} \tag{541}$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\big((a_{21}')^{(3)} + (p_{21})^{(3)}\big)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^{*}\mathbb{T}_{21} \tag{542}$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\big((a_{22}')^{(3)} + (p_{22})^{(3)}\big)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^{*}\mathbb{T}_{21} \tag{543}$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\big((b_{20}')^{(3)} - (r_{20})^{(3)}\big)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \textstyle\sum_{j=20}^{22}\big(s_{(20)(j)}T_{20}^{*}\mathbb{G}_j\big) \tag{544}$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\big((b_{21}')^{(3)} - (r_{21})^{(3)}\big)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \textstyle\sum_{j=20}^{22}\big(s_{(21)(j)}T_{21}^{*}\mathbb{G}_j\big) \tag{545}$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\big((b_{22}')^{(3)} - (r_{22})^{(3)}\big)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \textstyle\sum_{j=20}^{22}\big(s_{(22)(j)}T_{22}^{*}\mathbb{G}_j\big) \tag{546}$$

ASYMPTOTIC STABILITY ANALYSIS (547)

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions

$(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \ , \ \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^* \ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\big((a_{24}')^{(4)} + (p_{24})^{(4)}\big)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\big((a_{25}')^{(4)} + (p_{25})^{(4)}\big)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\big((a_{26}')^{(4)} + (p_{26})^{(4)}\big)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\big((b_{24}')^{(4)} - (r_{24})^{(4)}\big)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(24)(j)}T_{24}^*\mathbb{G}_j\big) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\big((b_{25}')^{(4)} - (r_{25})^{(4)}\big)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\big(s_{(25)(j)}T_{25}^*\mathbb{G}_j\big) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\big((b_{26}')^{(4)} - (r_{26})^{(4)}\big)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\big(s_{(26)(j)}T_{26}^*\mathbb{G}_j\big) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \ , \ \frac{\partial (b_i'')^{(5)}}{\partial G_j}(\ (G_{31})^* \ ) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\big((a_{28}')^{(5)} + (p_{28})^{(5)}\big)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\big((a_{29}')^{(5)} + (p_{29})^{(5)}\big)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\big((a_{30}')^{(5)} + (p_{30})^{(5)}\big)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\big((b_{28}')^{(5)} - (r_{28})^{(5)}\big)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\big(s_{(28)(j)}T_{28}^*\mathbb{G}_j\big) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{37})^{(7)}}{\partial T_{37}}(T^*_{37}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a'_{36})^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G^*_{36}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a'_{37})^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G^*_{37}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a'_{38})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G^*_{38}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b'_{36})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T^*_{36}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b'_{37})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T^*_{37}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b'_{38})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T^*_{38}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{41})^{(8)}}{\partial T_{41}}(T^*_{41}) = (q_{41})^{(8)} \quad , \frac{\partial(b_i'')^{(8)}}{\partial G_j}\left((G_{43})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a'_{40})^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G^*_{40}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a'_{41})^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G^*_{41}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a'_{42})^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G^*_{42}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b'_{40})^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T^*_{40}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \qquad 586\,B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a_{45}')^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G_{45}^*\mathbb{T}_{45} \qquad 586\,C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a_{46}')^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G_{46}^*\mathbb{T}_{45} \qquad 586\,D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b_{44}')^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T_{44}^*\mathbb{G}_j\right) \qquad 586\,E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b_{45}')^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T_{45}^*\mathbb{G}_j\right) \qquad 586\,F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b_{46}')^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T_{46}^*\mathbb{G}_j\right) \qquad 586\,G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b_{15}')^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a_{15}')^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)}G_{13}^*\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T_{14}^* + (b_{14})^{(1)}s_{(13),(14)}T_{14}^*\right)$$

$$+\left(\left((\lambda)^{(1)} + (a_{14}')^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)}G_{14}^*\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T_{14}^* + (b_{14})^{(1)}s_{(13),(13)}T_{13}^*\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b_{13}')^{(1)} + (b_{14}')^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}')^{(1)} + (a_{14}')^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a_{13}')^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G_{13}^*\right)$$

$$\left(\left((\lambda)^{(1)} + (b_{13}')^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T_{14}^* + (b_{14})^{(1)}s_{(13),(15)}T_{13}^*\right)\} = 0$$

$$+$$

$$(\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)})\{((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)})$$

$$\left[\left(((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)})(q_{17})^{(2)}\mathrm{G}^{*}_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)\right]$$

$$\left(((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})s_{(17),(17)}\mathrm{T}^{*}_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^{*}_{17}\right)$$

$$+\left(((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)})(q_{16})^{(2)}\mathrm{G}^{*}_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17}\right)$$

$$\left(((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})s_{(17),(16)}\mathrm{T}^{*}_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^{*}_{16}\right)$$

$$\left((\lambda)^{(2)}\right)^{2} + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$\left((\lambda)^{(2)}\right)^{2} + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$+\left(\left((\lambda)^{(2)}\right)^{2} + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^{*}_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^{*}_{16}\right)$$

$$\left(((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})s_{(17),(18)}\mathrm{T}^{*}_{17} + (b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^{*}_{16}\right)\} = 0$$

$$+$$

$$(\lambda)^{(3)} + (b'_{22})^{(3)} - (r_{22})^{(3)})\{((\lambda)^{(3)} + (a'_{22})^{(3)} + (p_{22})^{(3)})$$

$$\left[\left(((\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)})(q_{21})^{(3)}G^{*}_{21} + (a_{21})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)\right]$$

$$\left(((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)})s_{(21),(21)}T^{*}_{21} + (b_{21})^{(3)}s_{(20),(21)}T^{*}_{21}\right)$$

$$+\left(((\lambda)^{(3)} + (a'_{21})^{(3)} + (p_{21})^{(3)})(q_{20})^{(3)}G^{*}_{20} + (a_{20})^{(3)}(q_{21})^{(1)}G^{*}_{21}\right)$$

$$\left(((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)})s_{(21),(20)}T^{*}_{21} + (b_{21})^{(3)}s_{(20),(20)}T^{*}_{20}\right)$$

$$\left((\lambda)^{(3)}\right)^{2} + \left((a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^{2} + \left((b'_{20})^{(3)} + (b'_{21})^{(3)} - (r_{20})^{(3)} + (r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^{2} + \left((a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^{*}_{21} + (a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^{*}_{20}\right)$$

$$\left(((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)})s_{(21),(22)}T^{*}_{21} + (b_{21})^{(3)}s_{(20),(22)}T^{*}_{20}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(25)}T^{*}_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^{*}_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^{*}_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(24)}T^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^{*}_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^{*}_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^{*}_{25}+(b_{25})^{(4)}s_{(24),(26)}T^{*}_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(29)}T^{*}_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^{*}_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^{*}_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(28)}T^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^{*}_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^{*}_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^{*}_{29}+(b_{29})^{(5)}s_{(28),(30)}T^{*}_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(33)}T^{*}_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^{*}_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^{*}_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(32)}T^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^{*}_{33}+(b_{33})^{(6)}s_{(32),(34)}T^{*}_{32}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(37)}T^{*}_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^{*}_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^{*}_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(36)}T^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^{*}_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^{*}_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^{*}_{37}+(b_{37})^{(7)}s_{(36),(38)}T^{*}_{36}\right)\}=0$$

$$+$$

$$(\lambda)^{(8)} + (b'_{42})^{(8)} - (r_{42})^{(8)})\{((\lambda)^{(8)} + (a'_{42})^{(8)} + (p_{42})^{(8)})$$

$$\left[\left(((\lambda)^{(8)} + (a'_{40})^{(8)} + (p_{40})^{(8)})(q_{41})^{(8)}G^*_{41} + (a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(((\lambda)^{(8)} + (b'_{40})^{(8)} - (r_{40})^{(8)})s_{(41),(41)}T^*_{41} + (b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(((\lambda)^{(8)} + (a'_{41})^{(8)} + (p_{41})^{(8)})(q_{40})^{(8)}G^*_{40} + (a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(((\lambda)^{(8)} + (b'_{40})^{(8)} - (r_{40})^{(8)})s_{(41),(40)}T^*_{41} + (b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(((\lambda)^{(8)})^2 + (\,(a'_{40})^{(8)} + (a'_{41})^{(8)} + (p_{40})^{(8)} + (p_{41})^{(8)})\,(\lambda)^{(8)}\right)$$

$$\left(((\lambda)^{(8)})^2 + (\,(b'_{40})^{(8)} + (b'_{41})^{(8)} - (r_{40})^{(8)} + (r_{41})^{(8)})\,(\lambda)^{(8)}\right)$$

$$+\left(((\lambda)^{(8)})^2 + (\,(a'_{40})^{(8)} + (a'_{41})^{(8)} + (p_{40})^{(8)} + (p_{41})^{(8)})\,(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)} + (a'_{40})^{(8)} + (p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41} + (a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left(((\lambda)^{(8)} + (b'_{40})^{(8)} - (r_{40})^{(8)})s_{(41),(42)}T^*_{41} + (b_{41})^{(8)}s_{(40),(42)}T^*_{40}\right)\} = 0$$

$$+$$

$$(\lambda)^{(9)} + (b'_{46})^{(9)} - (r_{46})^{(9)})\{((\lambda)^{(9)} + (a'_{46})^{(9)} + (p_{46})^{(9)})$$

$$\left[\left(((\lambda)^{(9)} + (a'_{44})^{(9)} + (p_{44})^{(9)})(q_{45})^{(9)}G^*_{45} + (a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(((\lambda)^{(9)} + (b'_{44})^{(9)} - (r_{44})^{(9)})s_{(45),(45)}T^*_{45} + (b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(((\lambda)^{(9)} + (a'_{45})^{(9)} + (p_{45})^{(9)})(q_{44})^{(9)}G^*_{44} + (a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(((\lambda)^{(9)} + (b'_{44})^{(9)} - (r_{44})^{(9)})s_{(45),(44)}T^*_{45} + (b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left(((\lambda)^{(9)})^2 + (\,(a'_{44})^{(9)} + (a'_{45})^{(9)} + (p_{44})^{(9)} + (p_{45})^{(9)})\,(\lambda)^{(9)}\right)$$

$$\left(((\lambda)^{(9)})^2 + (\,(b'_{44})^{(9)} + (b'_{45})^{(9)} - (r_{44})^{(9)} + (r_{45})^{(9)})\,(\lambda)^{(9)}\right)$$

$$+\left(((\lambda)^{(9)})^2 + (\,(a'_{44})^{(9)} + (a'_{45})^{(9)} + (p_{44})^{(9)} + (p_{45})^{(9)})\,(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)} + (a'_{44})^{(9)} + (p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45} + (a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left(((\lambda)^{(9)} + (b'_{44})^{(9)} - (r_{44})^{(9)})s_{(45),(46)}T^*_{45} + (b_{45})^{(9)}s_{(44),(46)}T^*_{44}\right)\} = 0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION ONE

Renormalization Group Flows from Holography

INTRODUCTION—VARIABLES USED

Renormalization Group Flows from Holography--Supersymmetry and a c-Theorem D.Z. Freedman, S.S. Gubser, K. Pilch, N.P. Warner

(1) **D.Z. Freedman, S.S. Gubser, K. Pilch, N.P. Warner** obtain first order equations that determine (eb) a supersymmetric kink solution in (eb) five-dimensional N=8 gauged supergravity.
(2) The kink interpolates between an exterior anti-de Sitter region with maximal supersymmetry and (e&eb) an interior anti-de Sitter region with one quarter of the maximal supersymmetry.
(3) One eighth of supersymmetry is preserved by (e) the kink as a whole. They interpret it as describing the renormalization group flow in (eb) N=4 super-Yang-Mills theory broken to an N=1 theory by (e) the addition of a mass term for one of the three adjoint chiral superfields.
(4) They interpret it as describing the renormalization group flow in (eb) N=4 super-Yang-Mills theory broken to an N=1 theory by (e) the addition of a mass term for one of the three adjoint chiral superfields.
(5) A detailed correspondence is obtained between fields of bulk supergravity in the interior anti-de Sitter region and (e&eb) composite operators of the infrared field theory.
(6) **D.Z. Freedman, S.S. Gubser, K. Pilch, N.P. Warner** also point out that the truncation used to find the reduced symmetry critical point can be extended to obtain (eb) a new N=4 gauged supergravity theory holographically dual to a sector of N=2 gauge theories based on (e) quiver diagrams.
(7) They consider more general kink geometries and construct (eb) a c-function that is positive and monotonic if (e) a weak energy condition holds in the bulk gravity theory.
(8) For even-dimensional boundaries, the c-function coincides with (=) the trace anomaly coefficients of the holographically related field theory in (eb) limits where conformal invariance is recovered. Journal reference: Adv.Theor.Math.Phys.3:363-417, 1999 Report number: CERN-TH/99-86, HUTP-99/A015, USC-99/1, MIT-CTP-2846 Cite as: arXiv: hep-th/9904017

NOTATION

Module One

The Majorana equation is a relativistic wave equation similar to the Dirac equation but includes the charge conjugate ψc of a spinor ψ. It is named after the Italian physicist Ettore Majorana, and it is

$$-i\partial\!\!\!/\psi + m\psi_c = 0 \qquad (1)$$

$$\psi_c := i\psi^*.$$

$$i\partial\!\!\!/\psi_c + m\psi = 0 \qquad (2)$$

$$\psi^L = \frac{1}{2}(1-\gamma_5)\psi$$

$$\psi^R = \frac{1}{2}(1+\gamma_5)\psi$$

Massive boson:

$$Z = \cos\theta_W W_3 - \sin\theta_W B$$

Fourier series: periodic case;

$$F(x) = \beta \sum_{\mathrm{p}} \sum_{r} E_{\mathrm{p}}^{-\frac{1}{2}} \left(a_r(\mathrm{p}) u_r(\mathrm{p}) e^{-\frac{ipx}{\hbar}} + b_r^{\dagger}(\mathrm{p}) v_r(\mathrm{p}) e^{\frac{ipx}{\hbar}} \right)$$

G_{13} : Category one of LHS of the above equations

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of RHS of the above equations (Solution has been exhaustively illustrated many a time. Divide each term in to three categories and apply the model on the same lines. For each equation the model holds and the concatenation can be done at the end as is being done now, extant)

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

D.Z. Freedman, S.S. Gubser, K. Pilch, N.P. Warner obtain

first order equations that determine (eb) a supersymmetric kink solution in (eb) five-dimensional N=8 gauged supergravity

G_{16} : Category one of first order equations

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of supersymmetric kink solution in (eb) five-dimensional N=8 gauged supergravity

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

D.Z. Freedman, S.S. Gubser, K. Pilch, N.P. Warner obtain

first order equations that determine a supersymmetric kink solution in (eb) five-dimensional N=8 gauged supergravity

G_{20} : Category one of supersymmetric kink solution

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of five-dimensional N=8 gauged supergravity

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

The kink interpolates between an exterior anti-de Sitter region with maximal supersymmetry and (e&eb) an interior anti-de Sitter region with one quarter of the maximal supersymmetry

G_{24} : Category one of kink interpolates between an exterior anti-de Sitter region with maximal supersymmetry; interior anti-de Sitter region with one quarter of the maximal supersymmetry

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of interior anti-de Sitter region with one quarter of the maximal supersymmetry; kink interpolates between an exterior anti-de Sitter region with maximal supersymmetry

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

One eighth of supersymmetry is preserved by (e) the kink as a whole.

They interpret it as describing the renormalization group flow in (eb) N=4 super-Yang-Mills theory broken to an N=1 theory by (e) the addition of a mass term for one of the three adjoint chiral superfields

G_{28} : Category one of renormalization group flow

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of N=4 super-Yang-Mills theory broken to an N=1 theory by (e) the addition of a mass term for one of the three adjoint chiral superfields

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

renormalization group flow in N=4 super-Yang-Mills theory broken to an N=1 theory by (e) the addition of a mass term for one of the three adjoint chiral superfields

G_{32} : Category one of addition of a mass term for one of the three adjoint chiral superfields

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of renormalization group flow in N=4 super-Yang-Mills theory broken to an N=1 theory

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

A detailed correspondence is obtained between fields of bulk supergravity in the interior anti-de Sitter region and (e&eb) composite operators of the infrared field theory

G_{36} : Category one of fields of bulk supergravity in the interior anti-de Sitter region; composite operators of the infrared field theory

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of composite operators of the infrared field theory; fields of bulk supergravity in the interior anti-de Sitter region

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

D.Z. Freedman, S.S. Gubser, K. Pilch, N.P. Warner also point out that the truncation used to find the

reduced symmetry critical point can be extended to obtain (eb) a new N=4 gauged supergravity theory holographically dual to a sector of N=2 gauge theories based on (e) quiver diagrams

G_{40} : Category one of reduced symmetry critical point can be extended

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of new N=4 gauged supergravity theory holographically dual to a sector of N=2 gauge theories based on (e) quiver diagrams

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

They consider more general kink geometries and construct (eb) a c-function that is positive and monotonic if (e) a weak energy condition holds in the bulk gravity theory

G_{44} : Category one of weak energy condition holds in the bulk gravity theory

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of general kink geometries and construct (eb) a c-function that is positive and monotonic

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},\ \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}\ ,$
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}),t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}),t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}),t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}),t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}),t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}),t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}),t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}),t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}), t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}), t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}), t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}, t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}, t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}), t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}), t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}), t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45}, t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{13} \quad (55)$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{14} \quad (56)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{15} \quad (57)$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) & \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) & \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) & \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{15} \quad (60)$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) \quad +(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t) \end{array}\right] G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{ccc} (a'_{17})^{(2)} & +(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) \quad +(a''_{21})^{(3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t) \end{array}\right] G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{18})^{(2)} & +(a''_{18})^{(2)}(T_{17},t) & +(a''_{15})^{(1,1,)}(T_{14},t) \quad +(a''_{22})^{(3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{38})^{(7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9)}(T_{45},t) \end{array}\right] G_{18} \quad (63)$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) \end{bmatrix} T_{18} \quad (66)$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)}\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad (72)$$

$\boxed{-(b_{20}'')^{(3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{16}'')^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{13}'')^{(1,1,1,)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,)}(G,t)}$, $\boxed{-(b_{15}'')^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b_{36}'')^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b_{38}'')^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[\begin{array}{ccc} (a_{24}')^{(4)} & \boxed{+(a_{24}'')^{(4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,)}(T_{29},t)} \quad \boxed{+(a_{32}'')^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{16}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{36}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{44}'')^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[\begin{array}{ccc} (a_{25}')^{(4)} & \boxed{+(a_{25}'')^{(4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,)}(T_{29},t)} \quad \boxed{+(a_{33}'')^{(6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{17}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{37}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{45}'')^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[\begin{array}{ccc} (a_{26}')^{(4)} & \boxed{+(a_{26}'')^{(4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,)}(T_{29},t)} \quad \boxed{+(a_{34}'')^{(6,6,)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a_{18}'')^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{38}'')^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{46}'')^{(9,9,9,9)}(T_{45},t)} \end{array}\right] G_{26} \quad (75)$$

$\boxed{(a_{24}'')^{(4)}(T_{25},t)}$, $\boxed{(a_{25}'')^{(4)}(T_{25},t)}$, $\boxed{(a_{26}'')^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a_{28}'')^{(5,5,)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a_{32}'')^{(6,6,)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a_{13}'')^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{14}'')^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a_{16}'')^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a_{20}'')^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3)}(T_{21},t)}$

are sixth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$

are seventh augmentation coefficients for category 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$

are eighth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$

are fourth detrition coefficients for category 1, 2 *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$

are fifth detrition coefficients for category 1, 2 *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$

are sixth detrition coefficients for category 1, 2 *and* 3

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$

are seventh detrition coefficients for category 1, 2 *and* 3

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$

are eighth detrition coefficients for category 1, 2 *and* 3

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad (81)$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8\,,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \quad (84)$$

where $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}$ *are second detrition coefficients*

$for\ category$ 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ $are\ \ third\ \ detrition\ coefficients$ $for\ category$ 1,2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{34} \quad (87)$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ $are\ \ first\ \ augmentation\ coefficients$ $for\ category$ 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ $are\ second\ augmentation$ $coefficients\ for\ category$ 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ $are\ third\ augmentation$ $coefficients\ for\ category$ 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$
seventh augmentation coefficients
$+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)$
Eighth augmentation coefficients
$+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt}=(b_{32})^{(6)}T_{33}-\begin{bmatrix}(b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)}\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt}=(b_{33})^{(6)}T_{32}-\begin{bmatrix}(b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt}=(b_{34})^{(6)}T_{33}-\begin{bmatrix}(b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)}\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}\\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{34} \quad (90)$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients for category* 1, 2 *and* 3
$-(b''_{28})^{(5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3
$-(b''_{24})^{(4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,)}(G_{27},t)$ *are third detrition coefficients for category* 1,2 *and* 3
$-(b''_{13})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3
$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3
$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3
$-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3
$-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category 1, 2, and 3
$-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \qquad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt}$$ *95*

$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt}$$

$$= (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt}$$
$$= (a_{42})^{(8)} G_{41}$$
$$- \left[\begin{array}{ccc} (a'_{42})^{(8)} \boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array} \right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$
$$(b_{40})^{(8)} T_{41} - \left[\begin{array}{ccc} (b'_{40})^{(8)} \boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{13}$$

$$\frac{dT_{41}}{dt} =$$
$$(b_{41})^{(8)} T_{40} - \left[\begin{array}{ccc} (b'_{41})^{(8)} \boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{14}$$

$$\frac{dT_{42}}{dt} =$$
$$(b_{42})^{(8)} T_{41} - \left[\begin{array}{ccc} (b'_{42})^{(8)} \boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{38}'')^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

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$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a_{44}')^{(9)}\boxed{+(a_{44}'')^{(9)}(T_{45},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a_{45}')^{(9)}\boxed{+(a_{45}'')^{(9)}(T_{45},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a_{46}')^{(9)}\boxed{+(a_{46}'')^{(9)}(T_{37},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{44}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{46}'')^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second

augmentation coefficient for category 1, 2 and 3
$+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3
$+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3
$+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3
$+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$
$$(b_{44})^{(9)}T_{45} -$$
$$\begin{bmatrix} (b_{44}')^{(9)}\left[-(b_{44}'')^{(9)}(G_{47},t)\right] & \left[-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{13}$$
$$\frac{dT_{45}}{dt} =$$
$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}\left[-(b_{45}'')^{(9)}(G_{47},t)\right] & \left[-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{14}$$
$$\frac{dT_{46}}{dt} =$$
$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}\left[-(b_{46}'')^{(9)}(G_{47},t)\right] & \left[-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{15}$$

Where $-(b_{44}'')^{(9)}(G_{47},t)$, $-(b_{45}'')^{(9)}(G_{47},t)$, $-(b_{46}'')^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3
$-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}'| e^{-(\hat{M}_{20})^{(3)} t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} ||G_{23} - G_{23}'|| e^{-(\hat{M}_{20})^{(3)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and $(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\ ,\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)},\ (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)},\ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}), t\big) \le \ (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}\,(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\,\big((G_{27}), t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)},\ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - \ T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30$ 122

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)}$$ 123

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}\left((G_{35}), t\right) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(S) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(T) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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(U) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(V)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(W) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(X) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)} \qquad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be

noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\ ,\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}<1 \qquad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+\ (\hat{A}_{40})^{(8)}+(\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \qquad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[\ (b_i)^{(8)}+(b_i')^{(8)}+\ (\hat{B}_{40})^{(8)}+(\hat{Q}_{40})^{(8)}\ (\hat{k}_{40})^{(8)}]<1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \qquad 146A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t)-(a_i'')^{(9)}(T_{45},t)|\le(\hat{k}_{44})^{(9)}|T_{45}-\ T_{45}'|e^{-(\widehat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t)-(b_i'')^{(9)}\big((G_{47}),t\big)|<(\hat{k}_{44})^{(9)}||(G_{47})-(G_{47})'||e^{-(\widehat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$

and$(a_i'')^{(9)}(T_{45},t)$. (T_{45}',t) and (T_{45},t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)},(\widehat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{44})^{(9)}=1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$:

$(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},$ are positive constants

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\ ,\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}<1$$

Definition of $(\hat{P}_{44})^{(9)},(\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},(\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)},(a_i')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(p_i)^{(9)},\ (r_i)^{(9)},i=44,45,46,$
satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)}+(a_i')^{(9)}+\ (\hat{A}_{44})^{(9)}+\ (\hat{P}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)}+(b_i')^{(9)}+\ (\hat{B}_{44})^{(9)}+\ (\hat{Q}_{44})^{(9)}\ \ (\hat{k}_{44})^{(9)}]<1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\ ,T_i(0)$:

$$G_i(t)\leq\ \left(\hat{P}_{13}\right)^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{G_i(0)=G_i^0>0}$$

$$T_i(t)\leq\ (\hat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ \ ,\ \ \boxed{T_i(0)=T_i^0>0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\ ,T_i(0)$

$$G_i(t)\leq\ (\hat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ \ ,\ \ G_i(0)=G_i^0>0$$

$$T_i(t)\leq\ (\hat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ \ ,\ \ T_i(0)=T_i^0>0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t)\leq\ (\hat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ \ ,\ \ G_i(0)=G_i^0>0$$

$$T_i(t)\leq\ (\hat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ \ ,\ \ T_i(0)=T_i^0>0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0)\ ,T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{\left(\hat{M}_{24}\right)^{(4)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{\left(\hat{M}_{24}\right)^{(4)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{\left(\hat{M}_{28}\right)^{(5)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{\left(\hat{M}_{28}\right)^{(5)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{\left(\hat{M}_{32}\right)^{(6)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{\left(\hat{M}_{32}\right)^{(6)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{\left(\hat{M}_{36}\right)^{(7)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{\left(\hat{M}_{36}\right)^{(7)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{\left(\hat{M}_{40}\right)^{(8)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{\left(\hat{M}_{40}\right)^{(8)} t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{\left(\hat{M}_{44}\right)^{(9)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{\left(\hat{M}_{44}\right)^{(9)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{13})^{(1)}, T_i^0 \le (\hat{Q}_{13})^{(1)}$, 155

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} t}$ 156

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}, T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\tilde{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\tilde{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{32})^{(6)}$, $T_i^0 \leq (\hat{Q}_{32})^{(6)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0,t)$

Proof: 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(d) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left(e^{(\tilde{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\tilde{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left(e^{(\tilde{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\tilde{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0\right) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\tilde{M}_{44})^{(9)}} \left(e^{(\tilde{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\tilde{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\tilde{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\tilde{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\tilde{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[(\widehat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on

$G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 19 to 24 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(1)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If $G_{13}\ \ is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}\,(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on

(G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18}. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16}, G_{18} and G_{16}, G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18}. The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2:

Indeed let t_2 be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\,(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on T_{21} and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \quad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 227

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24{,}25{,}26$ depend only on T_{25} and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(4)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25}\ \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t\ > t_4$

$$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\widehat{P}_{28})^{(5)}$ and $(\widehat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\widehat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{31}}),(\widetilde{T_{31}})$: $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\ (G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(5)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} is bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)}+\left((\widehat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\widehat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\widehat{Q}_{32})^{(6)}\right]\leq(\widehat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(6)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose

$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{39}}),(\widetilde{T_{39}}):\left((\widetilde{G_{39}}),(\widetilde{T_{39}})\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$

It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\leq \quad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\ (G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})}, \widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by 274

$(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$. The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right)) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13} e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13} e^{(S_1)^{(1)} t}$

where $(p_i)^{(1)}$ is defined by equation

$\frac{1}{(m_1)^{(1)}} G^0_{13} e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G^0_{13} e^{(S_1)^{(1)} t}$

$$\left(\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t}-e^{-(S_2)^{(1)}t}\right]+G_{15}^{0}e^{-(S_2)^{(1)}t}\le G_{15}(t)\le\right. \quad (286)$$

$$\left.\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}\right]+G_{15}^{0}e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation (291)

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: (292)

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(T_{17},t)+(a''_{17})^{(2)}(T_{17},t)\le-(\sigma_1)^{(2)} \quad (293)$$

$$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)} \quad (294)$$

Definition of $(\nu_1)^{(2)},(\nu_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: (295)

By $(\nu_1)^{(2)}>0$, $(\nu_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0$, $(u_2)^{(2)}<0$ the roots (296)

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_1)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ (297)

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and (298)

Definition of $(\bar{\nu}_1)^{(2)},,(\bar{\nu}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: (299)

By $(\bar{\nu}_1)^{(2)}>0$, $(\bar{\nu}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0$, $(\bar{u}_2)^{(2)}<0$ the (300)

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_2)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ (301)

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t}\right] + T_{18}^0 e^{-(R_2)^{(2)}t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations
$(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$
and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by
$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, ***if*** $(\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, ***if*** $(\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, ***if*** $(\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$$

$$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(v^{(4)}\big)^2 + (\sigma_1)^{(4)} v^{(4)} - (a_{24})^{(4)} = 0$

and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)},,(\bar{v}_2)^{(4)},(\bar{u}_1)^{(4)},(\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$ and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)},(\mu_2)^{(4)},(v_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)},(\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, ***if*** $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)}G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)}G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)}t} - e^{-(a'_{26})^{(4)}t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)}t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^{0}}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t}-e^{-(R_2)^{(4)}t}\right]+T_{26}^{0}e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)},(S_2)^{(4)},(R_1)^{(4)},(R_2)^{(4)}$:- 337

Where $(S_1)^{(4)}=(a_{24})^{(4)}(m_2)^{(4)}-(a'_{24})^{(4)}$

$(S_2)^{(4)}=(a_{26})^{(4)}-(p_{26})^{(4)}$

$(R_1)^{(4)}=(b_{24})^{(4)}(\mu_2)^{(4)}-(b'_{24})^{(4)}$

$(R_2)^{(4)}=(b'_{26})^{(4)}-(r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)},(\sigma_2)^{(5)},(\tau_1)^{(5)},(\tau_2)^{(5)}$:

$(\sigma_1)^{(5)},(\sigma_2)^{(5)},(\tau_1)^{(5)},(\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)}\le-(a'_{28})^{(5)}+(a'_{29})^{(5)}-(a''_{28})^{(5)}(T_{29},t)+(a''_{29})^{(5)}(T_{29},t)\le-(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)}\le-(b'_{28})^{(5)}+(b'_{29})^{(5)}-(b''_{28})^{(5)}\big((G_{31}),t\big)-(b''_{29})^{(5)}\big((G_{31}),t\big)\le-(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)},(\nu_2)^{(5)},(u_1)^{(5)},(u_2)^{(5)},\nu^{(5)},u^{(5)}$: 339

By $(\nu_1)^{(5)}>0$, $(\nu_2)^{(5)}<0$ and respectively $(u_1)^{(5)}>0$, $(u_2)^{(5)}<0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2+(\sigma_1)^{(5)}\nu^{(5)}-(a_{28})^{(5)}=0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2+(\tau_1)^{(5)}u^{(5)}-(b_{28})^{(5)}=0$ and

Definition of $(\bar{\nu}_1)^{(5)},,(\bar{\nu}_2)^{(5)},(\bar{u}_1)^{(5)},(\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)}>0$, $(\bar{\nu}_2)^{(5)}<0$ and respectively $(\bar{u}_1)^{(5)}>0$, $(\bar{u}_2)^{(5)}<0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2+(\sigma_2)^{(5)}\nu^{(5)}-(a_{28})^{(5)}=0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2+(\tau_2)^{(5)}u^{(5)}-(b_{28})^{(5)}=0$

Definition of $(m_1)^{(5)},(m_2)^{(5)},(\mu_1)^{(5)},(\mu_2)^{(5)},(\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)},(m_2)^{(5)},(\mu_1)^{(5)},(\mu_2)^{(5)}$ by

$(m_2)^{(5)}=(\nu_0)^{(5)},(m_1)^{(5)}=(\nu_1)^{(5)}$, $\boldsymbol{if}\ (\nu_0)^{(5)}<(\nu_1)^{(5)}$

$(m_2)^{(5)}=(\nu_1)^{(5)},(m_1)^{(5)}=(\bar{\nu}_1)^{(5)}$, $\boldsymbol{if}\ (\nu_1)^{(5)}<(\nu_0)^{(5)}<(\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)}=\frac{G_{28}^{0}}{G_{29}^{0}}}$

$(m_2)^{(5)}=(\nu_1)^{(5)},(m_1)^{(5)}=(\nu_0)^{(5)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(5)}<(\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)}=(u_0)^{(5)},(\mu_1)^{(5)}=(u_1)^{(5)}$, $\boldsymbol{if}\ (u_0)^{(5)}<(u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, $\boldsymbol{if}$ $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}$ $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{28}(t) \leq G_{28}^0 e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{29}(t) \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \leq G_{30}(t) \leq \right. \quad 344$$

$$\left. \frac{(a_{30})^{(5)} G_{28}^0}{(m_2)^{(5)}((S_1)^{(5)} - (a'_{30})^{(5)})}\left[e^{(S_1)^{(5)} t} - e^{-(a'_{30})^{(5)} t}\right] + G_{30}^0 e^{-(a'_{30})^{(5)} t}\right)$$

$$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \leq T_{28}(t) \leq T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \leq T_{28}(t) \leq \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \leq T_{30}(t) \leq \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \leq -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \leq -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \leq -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \leq -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}$, $(m_2)^{(6)}$, $(\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}$, $(m_2)^{(6)}$, $(\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, $\boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad (354)$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad (355)$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t} - e^{-(a'_{34})^{(6)}t}\right] + G_{34}^0 e^{-(a'_{34})^{(6)}t}\right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad (356)$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}}T_{32}^0 e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(v_1)^{(7)}, (v_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, v^{(7)}, u^{(7)}$: 361

By $(v_1)^{(7)} > 0, (v_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{v}_1)^{(7)}, , (\bar{v}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)} > 0, (\bar{v}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)},\ \boldsymbol{if}\ (\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)},\ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}),t\big) - (b''_{45})^{(9)}\big((G_{47}),t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)},, (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, ***if*** $(\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, ***if*** $(\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, ***if*** $(\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,
and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44}e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44}e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G^0_{44}e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G^0_{44}e^{(S_1)^{(9)}t}$$

(

$$\frac{(a_{46})^{(9)}G^0_{44}}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})}\Big[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t}\Big] + G^0_{46}e^{-(S_2)^{(9)}t} \le G_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}G^0_{44}}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})}[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G^0_{46}e^{-(a'_{46})^{(9)}t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{dv^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)v^{(2)} - (a_{17})^{(2)}v^{(2)}$$

Definition of $v^{(2)}$:- $\boxed{v^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(v^{(2)}\right)^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)}\right) \leq \frac{dv^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(v^{(2)}\right)^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{v}_1)^{(2)}, (v_0)^{(2)}$:-

For $0 < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (v_1)^{(2)} < (\bar{v}_1)^{(2)}$

$$v^{(2)}(t) \geq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(v_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (v_2)^{(2)}}}$$

it follows $(v_0)^{(2)} \leq v^{(2)}(t) \leq (v_1)^{(2)}$

In the same manner , we get 391

$$v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{v}_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (\bar{v}_2)^{(2)}}}$$

From which we deduce $(v_0)^{(2)} \leq v^{(2)}(t) \leq (\bar{v}_1)^{(2)}$ 392

If $0 < (v_1)^{(2)} < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{v}_1)^{(2)}$ we find like in the previous case, 393

$$(v_1)^{(2)} \leq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_2)^{(2)}\right)t\right]}} \leq v^{(2)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \leq (\bar{v}_1)^{(2)}$$

If $0 < (v_1)^{(2)} \leq (\bar{v}_1)^{(2)} \leq (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(v_1)^{(2)} \leq v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \leq (v_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(2)}(t)$:- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

400

From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $v^{(4)}$:- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \leq \frac{dv^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(4)}, (v_0)^{(4)}$:-

For $0 < \boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (v_1)^{(4)} < (\bar{v}_1)^{(4)}$

$$v^{(4)}(t) \geq \frac{(v_1)^{(4)} + (C)^{(4)}(v_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((v_1)^{(4)} - (v_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((v_1)^{(4)} - (v_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(v_1)^{(4)} - (v_0)^{(4)}}{(v_0)^{(4)} - (v_2)^{(4)}}}$$

it follows $(v_0)^{(4)} \leq v^{(4)}(t) \leq (v_1)^{(4)}$

In the same manner , we get 405

$$v^{(4)}(t) \leq \frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)}(\bar{v}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{v}_1)^{(4)} - (v_0)^{(4)}}{(v_0)^{(4)} - (\bar{v}_2)^{(4)}}}$$

From which we deduce $(v_0)^{(4)} \leq v^{(4)}(t) \leq (\bar{v}_1)^{(4)}$

If $0 < (v_1)^{(4)} < (v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{v}_1)^{(4)}$ we find like in the previous case, 406

$$(v_1)^{(4)} \leq \frac{(v_1)^{(4)} + (C)^{(4)}(v_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((v_1)^{(4)} - (v_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((v_1)^{(4)} - (v_2)^{(4)}\right)t\right]}} \leq v^{(4)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)}(\bar{v}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}\right)t\right]}} \leq (\bar{v}_1)^{(4)}$$

If $0 < (v_1)^{(4)} \leq (\bar{v}_1)^{(4)} \leq \boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(v_1)^{(4)} \leq v^{(4)}(t) \leq \frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)}(\bar{v}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}\right)t\right]}} \leq (v_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(4)}(t)$:-

$$(m_2)^{(4)} \leq v^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{v^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then
$(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

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If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\quad \boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$v^{(6)}(t) \le \frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)}=\frac{(\bar{v}_1)^{(6)}-(v_0)^{(6)}}{(v_0)^{(6)}-(\bar{v}_2)^{(6)}}}$$

From which we deduce $(v_0)^{(6)} \le v^{(6)}(t) \le (\bar{v}_1)^{(6)}$

If $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{v}_1)^{(6)}$ we find like in the previous case, 414

$$(v_1)^{(6)} \le \frac{(v_1)^{(6)}+(C)^{(6)}(v_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)}-(v_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)}-(v_2)^{(6)}\right)t\right]}} \le v^{(6)}(t) \le$$

$$\frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \le (\bar{v}_1)^{(6)}$$

If $0 < (v_1)^{(6)} \le (\bar{v}_1)^{(6)} \le \boxed{(v_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(v_1)^{(6)} \le v^{(6)}(t) \le \frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \le (v_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(6)}(t)$:-

$$(m_2)^{(6)} \le v^{(6)}(t) \le (m_1)^{(6)}, \quad \boxed{v^{(6)}(t)=\frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \le u^{(6)}(t) \le (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t)=\frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(v_1)^{(6)} = (\bar{v}_1)^{(6)}$ if in addition $(v_0)^{(6)} = (v_1)^{(6)}$ then $v^{(6)}(t) = (v_0)^{(6)}$ and as a consequence $G_{32}(t) = (v_0)^{(6)}G_{33}(t)$ **this also defines** $(v_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(v_1)^{(6)}$ and $(\bar{v}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)}=\frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}\right)\leq\frac{dv^{(7)}}{dt}\leq-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(7)},(v_0)^{(7)}$:-

For $0<\boxed{(v_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}<(v_1)^{(7)}<(\bar{v}_1)^{(7)}$

$$v^{(7)}(t)\geq\frac{(v_1)^{(7)}+(C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)}-(v_0)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)}-(v_0)^{(7)}\right)t\right]}}\quad,\quad\boxed{(C)^{(7)}=\frac{(v_1)^{(7)}-(v_0)^{(7)}}{(v_0)^{(7)}-(v_2)^{(7)}}}$$

it follows $(v_0)^{(7)}\leq v^{(7)}(t)\leq(v_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t)\leq\frac{(\bar{v}_1)^{(7)}+(\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)}-(\bar{v}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)}-(\bar{v}_2)^{(7)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(7)}=\frac{(\bar{v}_1)^{(7)}-(v_0)^{(7)}}{(v_0)^{(7)}-(\bar{v}_2)^{(7)}}}$$

From which we deduce $(v_0)^{(7)}\leq v^{(7)}(t)\leq(\bar{v}_1)^{(7)}$

If $0<(v_1)^{(7)}<(v_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}<(\bar{v}_1)^{(7)}$ we find like in the previous case, 418

$$(v_1)^{(7)}\leq\frac{(v_1)^{(7)}+(C)^{(7)}(v_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)}-(v_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((v_1)^{(7)}-(v_2)^{(7)}\right)t\right]}}\leq v^{(7)}(t)\leq$$

$$\frac{(\bar{v}_1)^{(7)}+(\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)}-(\bar{v}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)}-(\bar{v}_2)^{(7)}\right)t\right]}}\leq(\bar{v}_1)^{(7)}$$

If $0<(v_1)^{(7)}\leq(\bar{v}_1)^{(7)}\leq\boxed{(v_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(v_1)^{(7)}\leq v^{(7)}(t)\leq\frac{(\bar{v}_1)^{(7)}+(\bar{C})^{(7)}(\bar{v}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)}-(\bar{v}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{v}_1)^{(7)}-(\bar{v}_2)^{(7)}\right)t\right]}}\leq(v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(7)}(t)$:-

$$(m_2)^{(7)}\leq v^{(7)}(t)\leq(m_1)^{(7)},\quad\boxed{v^{(7)}(t)=\frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$ if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\quad \boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \leq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \leq \nu^{(9)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \leq (\bar{\nu}_1)^{(9)} \leq \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \leq \nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \leq \nu^{(9)}(t) \leq (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for**

the special case .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(v_1)^{(9)}$ and $(\bar{v}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a_i'')^{(1)} and\ (b_i'')^{(1)}$ are independent on t , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0$ 447

$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0$ 448

has a unique positive solution , which is an equilibrium solution

$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0$ 449

$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0$ 450

$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0$ 451

$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0$ 452

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(d) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T_{33}^{*} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^{*} for which $f(T_{33}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

Definition and uniqueness of T_{37}^{*} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^{*} for which $f(T_{37}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^{*})]}$$

Definition and uniqueness of T_{41}^{*} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^{*} for which $f(T_{41}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^{*})]}$$

Definition and uniqueness of T_{45}^{*} :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^{*} for which $f(T_{45}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^{*})]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi(G^{*}) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that

there exists a unique G_{37}^{*} such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)}G_{17}^{*}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)}G_{17}^{*}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T_{17}^{*})]} \quad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)}T_{17}^{*}}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)}T_{17}^{*}}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^{*})]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)}G_{21}^{*}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)}G_{21}^{*}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$$T_{28}^* = \frac{(b_{28})^{(5)} T_{29}^*}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^*)]} \quad , \quad T_{30}^* = \frac{(b_{30})^{(5)} T_{29}^*}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^*)]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^* given by $\varphi((G_{35})^*) = 0$, T_{33}^* given by $f(T_{33}^*) = 0$ and

$$G_{32}^* = \frac{(a_{32})^{(6)} G_{33}^*}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34}^* = \frac{(a_{34})^{(6)} G_{33}^*}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

$$T_{32}^* = \frac{(b_{32})^{(6)} T_{33}^*}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^*)]} \quad , \quad T_{34}^* = \frac{(b_{34})^{(6)} T_{33}^*}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^*)]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^* given by $\varphi((G_{39})^*) = 0$, T_{37}^* given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

G_{41}^* given by $\varphi((G_{43})^*) = 0$, T_{41}^* given by $f(T_{41}^*) = 0$ and

$$G_{40}^{*}=\frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}\quad,\quad G_{42}^{*}=\frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

$$T_{40}^{*}=\frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}\quad,\quad T_{42}^{*}=\frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$$

Finally we obtain the unique solution of 89 to 99 523

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G_{45}^{*} given by $\varphi((G_{47})^{*})=0$, T_{45}^{*} given by $f(T_{45}^{*})=0$ and

$$G_{44}^{*}=\frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}\quad,\quad G_{46}^{*}=\frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$$

$$T_{44}^{*}=\frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}\quad,\quad T_{46}^{*}=\frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}\ and\ (b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i=G_i^{*}+\mathbb{G}_i\qquad,T_i=T_i^{*}+\mathbb{T}_i$$

$$\frac{\partial(a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^{*})=(q_{14})^{(1)}\quad,\ \frac{\partial(b_i'')^{(1)}}{\partial G_j}(\,G^{*}\,)=s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt}=-\big((a'_{13})^{(1)}+(p_{13})^{(1)}\big)\mathbb{G}_{13}+(a_{13})^{(1)}\mathbb{G}_{14}-(q_{13})^{(1)}G_{13}^{*}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt}=-\big((a'_{14})^{(1)}+(p_{14})^{(1)}\big)\mathbb{G}_{14}+(a_{14})^{(1)}\mathbb{G}_{13}-(q_{14})^{(1)}G_{14}^{*}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt}=-\big((a'_{15})^{(1)}+(p_{15})^{(1)}\big)\mathbb{G}_{15}+(a_{15})^{(1)}\mathbb{G}_{14}-(q_{15})^{(1)}G_{15}^{*}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt}=-\big((b'_{13})^{(1)}-(r_{13})^{(1)}\big)\mathbb{T}_{13}+(b_{13})^{(1)}\mathbb{T}_{14}+\textstyle\sum_{j=13}^{15}\big(s_{(13)(j)}T_{13}^{*}\mathbb{G}_j\big) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt}=-\big((b'_{14})^{(1)}-(r_{14})^{(1)}\big)\mathbb{T}_{14}+(b_{14})^{(1)}\mathbb{T}_{13}+\textstyle\sum_{j=13}^{15}\big(s_{(14)(j)}T_{14}^{*}\mathbb{G}_j\big) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt}=-\big((b'_{15})^{(1)}-(r_{15})^{(1)}\big)\mathbb{T}_{15}+(b_{15})^{(1)}\mathbb{T}_{14}+\textstyle\sum_{j=13}^{15}\big(s_{(15)(j)}T_{15}^{*}\mathbb{G}_j\big) \quad 530$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions

$(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \tag{532}$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \ , \ \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \tag{533}$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \tag{534}$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \tag{535}$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \tag{536}$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \tag{537}$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}')^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T_{17}^*\mathbb{G}_j\right) \tag{538}$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}')^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T_{18}^*\mathbb{G}_j\right) \tag{539}$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \ , \ \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}')^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \tag{541}$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}')^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \tag{542}$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}')^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \tag{543}$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}')^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T_{20}^*\mathbb{G}_j\right) \tag{544}$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^{*}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T_{22}^{*}\mathbb{G}_j\right) \quad 546$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^{*}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^{*}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^{*}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^{*}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^{*}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^{*}\mathbb{G}_j\right) \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}(\,(G_{31})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^{*}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^{*}) = (q_{41})^{(8)} \quad , \frac{\partial(b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^{*}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^{*}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^{*}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^{*}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \ , \ \frac{\partial(b_i'')^{(9)}}{\partial G_j}(\ (G_{47})^*\ ) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\ B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\ C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\ D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\ E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\ F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\ G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left(\ (a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)\ (\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left(\ (b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)\ (\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14}+(b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\Big]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION TWO

Large-N Collective Fields And Holography

INTRODUCTION—VARIABLES USED

Large-N collective fields and holography Phys. Rev. D 68, 044011 – Published 12 August 2003 Sumit R. Das and Antal Jevicki

(1) Sumit R. Das and Antal Jevicki propose that the Euclidean bilocal collective field theory of (e) critical large-N vector models provides (eb) a complete definition of the proposed dual theory of higher spin fields in anti–de Sitter spaces.
(2) They show how this bilocal field can be decomposed into (eb) an infinite number of even spin fields in one more dimension.
(3) The collective field has (e) a nontrivial classical solution which leads to (eb) an O (N) thermodynamic entropy characteristic of the lower dimensional theory, as required by (e) general considerations of holography.
(4) The collective field has (e) a nontrivial classical solution which leads to (eb) an O (N) thermodynamic entropy characteristic of the lower dimensional theory, as required by (e) general considerations of holography
(5) A subtle cancellation of the entropy coming from (e) the bulk fields in (eb) one higher dimension with O (1) contributions from (e) the classical solution ensures (eb) that the subleading terms in thermodynamic quantities are of the expected form.
(6) A subtle cancellation of the entropy coming from (e) the bulk fields in (eb) one higher dimension with O (1) contributions from (e) the classical solution ensures (eb) that the subleading terms in thermodynamic quantities are of the expected form
(7) While the spin components of the collective field transform properly under (e&eb) dilatational, translational, and rotational isometries of AdS, special conformal transformations mix fields of (e) different spins indicating (eb) a need for a nonlocal map between the two sets of fields.
(8) They discuss the nature of the propagating degrees of freedom through (e&eb) a Hamiltonian form of collective field theory and argue that nonsinglet states which are present in (eb) a Euclidean version are related to (e&eb) nontrivial backgrounds. DOI: http://dx.doi.org/10.1103/PhysRevD.68.044011
(9) They discuss the nature of the propagating degrees of freedom through (e&eb) a Hamiltonian form of collective field theory and argue that nonsinglet states which are present in (eb) a Euclidean version are related to (e&eb) nontrivial backgrounds. DOI: http://dx.doi.org/10.1103/PhysRevD.68.044011

NOTATION

Module One

c-function For even-dimensional boundaries coincides with (=) the trace anomaly coefficients of the holographically related field theory in (eb) limits where conformal invariance is recovered.

Journal reference: Adv.Theor.Math.Phys.3:363-417, 1999 Report number: CERN-TH/99-86, HUTP-99/A015, USC-99/1, MIT-CTP-2846 Cite as: arXiv: hep-th/9904017

G_{13} : Category one of c-function For even-dimensional boundaries

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of trace anomaly coefficients of the holographically related field theory in (eb) limits where conformal invariance is recovered.

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

c-function For even-dimensional boundaries coincides with the trace anomaly coefficients of the holographically related field theory in (eb) limits where conformal invariance is recovered

G_{16} : Category one of c-function For even-dimensional boundaries coincides with the trace anomaly coefficients of the holographically related field theory

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of limits where conformal invariance is recovered

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Sumit R. Das and Antal Jevicki propose that the

Euclidean bilocal collective field theory of (e) critical large-N vector models provides (eb) a complete definition of the proposed dual theory of higher spin fields in anti–de Sitter spaces

G_{20} : Category one of critical large-N vector models provides (eb) a complete definition of the proposed dual theory of higher spin fields in anti–de Sitter spaces

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of Euclidean bilocal collective field theory

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Euclidean bilocal collective field theory of critical large-N vector models provides (eb) a complete definition of the proposed dual theory of higher spin fields in anti–de Sitter spaces

G_{24} : Category one of Euclidean bilocal collective field theory of critical large-N vector models

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of complete definition of the proposed dual theory of higher spin fields in anti–de Sitter spaces

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

They show how this

bilocal field can be decomposed into (eb) an infinite number of even spin fields in one more dimension

G_{28} : Category one of bilocal field

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of infinite number of even spin fields in one more dimension

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

The collective field has (e) a nontrivial classical solution which leads to (eb) an O (N) thermodynamic entropy characteristic of the lower dimensional theory, as required by (e) general considerations of holography

G_{32} : Category one of nontrivial classical solution which leads to (eb) an O (N) thermodynamic entropy characteristic of the lower dimensional theory, as required by (e) general considerations of holography

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of collective field

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

collective field has a nontrivial classical solution which leads to (eb) an O (N) thermodynamic entropy characteristic of the lower dimensional theory, as required by general considerations of holography

G_{36} : Category one of collective field has a nontrivial classical solution

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of O (N) thermodynamic entropy characteristic of the lower dimensional theory, as required by (e) general considerations of holography

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

A subtle cancellation of the entropy coming from (e) the bulk fields in (eb) one higher dimension with O (1) contributions from (e) the classical solution ensures (eb) that the subleading terms in thermodynamic quantities are of the expected form

G_{40} : Category one of bulk fields in (eb) one higher dimension with O (1) contributions from (e) the classical solution ensures (eb) that the subleading terms in thermodynamic quantities are of the expected form

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of subtle cancellation of the entropy

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

subtle cancellation of the entropy coming from the bulk fields in (eb) one higher dimension with O (1) contributions from (e) the classical solution ensures (eb) that the subleading terms in thermodynamic quantities are of the expected form

G_{44} : Category one of subtle cancellation of the entropy coming from the bulk fields

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of one higher dimension with O (1) contributions from (e) the classical solution ensures (eb) that the subleading terms in thermodynamic quantities are of the expected form

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},\ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**
$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)}\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad (60)$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t) \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & +(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) & +(a''_{21})^{(3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t) \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & +(a''_{18})^{(2)}(T_{17},t) & +(a''_{15})^{(1,1,)}(T_{14},t) & +(a''_{22})^{(3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{38})^{(7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9)}(T_{45},t) \end{bmatrix} G_{18} \quad (63)$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{20})^{(3)}\,+(a''_{20})^{(3)}(T_{21},t)\,+(a''_{16})^{(2,2,2)}(T_{17},t)\,+(a''_{13})^{(1,1,1,)}(T_{14},t) \\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)\,+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)\,+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7,7)}(T_{37},t)\,+(a''_{40})^{(8,8,8,8)}(T_{41},t)\,+(a''_{44})^{(9,9,9)}(T_{45},t) \end{array}\right] G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c} (a'_{21})^{(3)}\,+(a''_{21})^{(3)}(T_{21},t)\,+(a''_{17})^{(2,2,2)}(T_{17},t)\,+(a''_{14})^{(1,1,1,)}(T_{14},t) \\ +(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)\,+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)\,+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{37})^{(7,7,7,7)}(T_{37},t)\,+(a''_{41})^{(8,8,8,8)}(T_{41},t)\,+(a''_{45})^{(9,9,9)}(T_{45},t) \end{array}\right] G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{22})^{(3)}\,+(a''_{22})^{(3)}(T_{21},t)\,+(a''_{18})^{(2,2,2)}(T_{17},t)\,+(a''_{15})^{(1,1,1,)}(T_{14},t) \\ +(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)\,+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)\,+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{38})^{(7,7,7,7)}(T_{37},t)\,+(a''_{42})^{(8,8,8,8)}(T_{41},t)\,+(a''_{46})^{(9,9,9)}(T_{45},t) \end{array}\right] G_{22} \quad (69)$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[\begin{array}{c} (b'_{20})^{(3)}\,-(b''_{20})^{(3)}(G_{23},t)\,-(b''_{16})^{(2,2,2)}(G_{19},t)\,-(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)\,-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)\,-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t)\,-(b''_{40})^{(8,8,8,8)}(G_{43},t)\,-(b''_{44})^{(9,9,9)}(G_{47},t) \end{array}\right] T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[\begin{array}{c} (b'_{21})^{(3)}\,-(b''_{21})^{(3)}(G_{23},t)\,-(b''_{17})^{(2,2,2)}(G_{19},t)\,-(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)\,-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)\,-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t)\,-(b''_{41})^{(8,8,8,8)}(G_{43},t)\,-(b''_{45})^{(9,9,9)}(G_{47},t) \end{array}\right] T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$

$are\ fourth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$
$are\ fifth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$
$are\ sixth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$
$are\ seventh\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$
$are\ eighth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

$Where\ -(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ $are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ $are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ $are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
$are\ eighth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{30} \quad (81)$$

Where $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1)}(G,t) & & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1)}(G,t) & & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,)}(G,t) & & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

$where\ \boxed{-(b''_{28})^{(5)}(G_{31},t)}\ ,\boxed{-(b''_{29})^{(5)}(G_{31},t)}\ ,\boxed{-(b''_{30})^{(5)}(G_{31},t)}\ \ are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}\ ,\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}\ ,\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}\ are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}\ ,\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}\ ,\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}\ are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}\ ,\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}\ ,\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}\ ,\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}\ ,\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}\ ,\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}\ ,\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}\ ,\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}\ ,\boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}\ ,\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}\ ,\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}\ ,\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}\ ,\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt}=(a_{32})^{(6)}G_{33} \qquad 85$$

$$-\left[\begin{matrix}(a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)}\ \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}\end{matrix}\right]G_{32}$$

$$\frac{dG_{33}}{dt}=(a_{33})^{(6)}G_{32}-\left[\begin{matrix}(a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)}\ \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}\end{matrix}\right]G_{33} \qquad 86$$

$$\frac{dG_{34}}{dt}=(a_{34})^{(6)}G_{33}-\left[\begin{matrix}(a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)}\ \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}\end{matrix}\right]G_{34} \qquad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}\ ,\boxed{+(a''_{33})^{(6)}(T_{33},t)}\ ,\boxed{+(a''_{34})^{(6)}(T_{33},t)}\ \ are\ first\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\ ,\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\ ,\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\ are\ second\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}\ ,\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}\ ,\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}\ \ are\ third\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\ ,\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\ ,\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\ ,\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\ ,\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation

coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$ Eighth augmentation coefficients

$+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)} & -(b''_{32})^{(6)}(G_{35},t) & -(b''_{28})^{(5,5,5)}(G_{31},t) & -(b''_{24})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{13})^{(1,1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)} & -(b''_{33})^{(6)}(G_{35},t) & -(b''_{29})^{(5,5,5)}(G_{31},t) & -(b''_{25})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{14})^{(1,1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & -(b''_{34})^{(6)}(G_{35},t) & -(b''_{30})^{(5,5,5)}(G_{31},t) & -(b''_{26})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{15})^{(1,1,1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{34} \quad (90)$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,)}(G_{27},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3

$-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3

$-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2, and 3

$-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \qquad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt}$$ *95*

$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt}$$

$$= (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt}$$
$$= (a_{42})^{(8)} G_{41}$$
$$- \left[\begin{array}{c} (a'_{42})^{(8)} \boxed{+(a''_{42})^{(8)}(T_{41},t)} \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array} \right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$
$$(b_{40})^{(8)} T_{41} - \left[\begin{array}{c} (b'_{40})^{(8)} \boxed{-(b''_{40})^{(8)}(G_{43},t)} \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{13}$$

$$\frac{dT_{41}}{dt} =$$
$$(b_{41})^{(8)} T_{40} - \left[\begin{array}{c} (b'_{41})^{(8)} \boxed{-(b''_{41})^{(8)}(G_{43},t)} \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{14}$$

$$\frac{dT_{42}}{dt} =$$
$$(b_{42})^{(8)} T_{41} - \left[\begin{array}{c} (b'_{42})^{(8)} \boxed{-(b''_{42})^{(8)}(G_{43},t)} \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{38}'')^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

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$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a_{44}')^{(9)}\boxed{+(a_{44}'')^{(9)}(T_{45},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a_{45}')^{(9)}\boxed{+(a_{45}'')^{(9)}(T_{45},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a_{46}')^{(9)}\boxed{+(a_{46}'')^{(9)}(T_{37},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{44}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{46}'')^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second

augmentation coefficient for category 1, 2 and 3
$+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3
$+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3
$+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3
$+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$
$$(b_{44})^{(9)}T_{45} -$$
$$\begin{bmatrix} (b_{44}')^{(9)}\left[-(b_{44}'')^{(9)}(G_{47},t)\right] & \left[-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{13}$$
$$\frac{dT_{45}}{dt} =$$
$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}\left[-(b_{45}'')^{(9)}(G_{47},t)\right] & \left[-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{14}$$
$$\frac{dT_{46}}{dt} =$$
$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}\left[-(b_{46}'')^{(9)}(G_{47},t)\right] & \left[-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{15}$$

Where $-(b_{44}'')^{(9)}(G_{47},t)$, $-(b_{45}'')^{(9)}(G_{47},t)$, $-(b_{46}'')^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3
$-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}'| e^{-(\hat{M}_{20})^{(3)} t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} ||G_{23} - G_{23}'|| e^{-(\hat{M}_{20})^{(3)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and $(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\ ,\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)},\ (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\ (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)},\ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}), t\big) \le \ (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}\,(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\,\big((G_{27}), t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)},\ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - \ T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30$$ 122

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \leq (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)}$$ 123

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}\left((G_{35}), t\right) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(Y) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(Z) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

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(AA) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(BB)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(CC) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(DD) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)} \qquad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be

noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\ ,\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}<1 \qquad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+\ (\hat{A}_{40})^{(8)}+(\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \qquad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[\ (b_i)^{(8)}+(b_i')^{(8)}+\ (\hat{B}_{40})^{(8)}+(\hat{Q}_{40})^{(8)}\ (\hat{k}_{40})^{(8)}]<1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \qquad 146A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t)-(a_i'')^{(9)}(T_{45},t)|\le(\hat{k}_{44})^{(9)}|T_{45}-\ T_{45}'|e^{-(\widehat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t)-(b_i'')^{(9)}\big((G_{47}),t\big)|<(\hat{k}_{44})^{(9)}||(G_{47})-(G_{47})'||e^{-(\widehat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$

and$(a_i'')^{(9)}(T_{45},t)$. (T_{45}',t) and (T_{45},t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)},(\widehat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{44})^{(9)}=1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$:

$(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},$ are positive constants

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\ ,\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}<1$$

Definition of $(\hat{P}_{44})^{(9)},(\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},(\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)},(a_i')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(p_i)^{(9)},\ (r_i)^{(9)},i=44,45,46,$
satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)}+(a_i')^{(9)}+\ (\hat{A}_{44})^{(9)}+\ (\hat{P}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)}+(b_i')^{(9)}+\ (\hat{B}_{44})^{(9)}+\ (\hat{Q}_{44})^{(9)}\ \ (\hat{k}_{44})^{(9)}]<1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\ ,T_i(0)$:

$$G_i(t)\leq\ \left(\hat{P}_{13}\right)^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{G_i(0)=G_i^0>0}$$

$$T_i(t)\leq\ (\hat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\quad ,\quad \boxed{T_i(0)=T_i^0>0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\ ,T_i(0)$

$$G_i(t)\leq\ (\hat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\quad ,\quad G_i(0)=G_i^0>0$$

$$T_i(t)\leq\ (\hat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\quad ,\quad T_i(0)=T_i^0>0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t)\leq\ (\hat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\quad ,\quad G_i(0)=G_i^0>0$$

$$T_i(t)\leq\ (\hat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\quad ,\quad T_i(0)=T_i^0>0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0)\ ,T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{\left(\hat{M}_{24}\right)^{(4)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{\left(\hat{M}_{24}\right)^{(4)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{\left(\hat{M}_{28}\right)^{(5)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{\left(\hat{M}_{28}\right)^{(5)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{\left(\hat{M}_{32}\right)^{(6)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{\left(\hat{M}_{32}\right)^{(6)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{\left(\hat{M}_{36}\right)^{(7)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{\left(\hat{M}_{36}\right)^{(7)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{\left(\hat{M}_{40}\right)^{(8)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{\left(\hat{M}_{40}\right)^{(8)} t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{\left(\hat{M}_{44}\right)^{(9)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{\left(\hat{M}_{44}\right)^{(9)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{13})^{(1)}, T_i^0 \le (\hat{Q}_{13})^{(1)}$, 155

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} t}$ 156

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}, T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\tilde{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\tilde{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{20})^{(3)}, T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{32})^{(6)}$, $T_i^0 \leq (\hat{Q}_{32})^{(6)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0,t)$

Proof: 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(e) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left(e^{(\tilde{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\tilde{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left(e^{(\tilde{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\tilde{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0\right) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\tilde{M}_{44})^{(9)}} \left(e^{(\tilde{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\tilde{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\tilde{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\tilde{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\tilde{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[(\widehat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on

$G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 19 to 24 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(1)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If $G_{13}\ \ is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}\,(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on

(G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)} t)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18}. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16}, G_{18} and G_{16}, G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18}. The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\,(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on T_{21} and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \quad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}\ and\ (\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 227

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24{,}25{,}26$ depend only on T_{25} and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(4)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25}\ \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t\ > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25}\ \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25}\ \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}\ , G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\ , \frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\widehat{P}_{28})^{(5)}$ and $(\widehat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\widehat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \tag{234}$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \tag{235}$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric (236)

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{31}}),(\widetilde{T_{31}})$: $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\leq \tag{237}$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\ (G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(5)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)}+\left((\widehat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\widehat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\widehat{Q}_{32})^{(6)}\right]\leq(\widehat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(6)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose

$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{39}}),(\widetilde{T_{39}}):\left((\widetilde{G_{39}}),(\widetilde{T_{39}})\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$

It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\leq \quad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\ (G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})}, \widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by 274

$(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$ and to choose $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[(\hat{P}_{44})^{(9)} + \left((\hat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\left[\left((\hat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{44})^{(9)}\right] \leq (\hat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$. The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right)) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13} e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13} e^{(S_1)^{(1)} t}$

where $(p_i)^{(1)}$ is defined by equation

$\frac{1}{(m_1)^{(1)}} G^0_{13} e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G^0_{13} e^{(S_1)^{(1)} t}$

$$\left(\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t}-e^{-(S_2)^{(1)}t}\right]+G_{15}^{0}e^{-(S_2)^{(1)}t}\le G_{15}(t)\le\right. \quad (286)$$

$$\left.\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}\right]+G_{15}^{0}e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation (291)

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: (292)

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(\mathrm{T}_{17},t)+(a''_{17})^{(2)}(\mathrm{T}_{17},t)\le-(\sigma_1)^{(2)}$ (293)

$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)}$ (294)

Definition of $(\nu_1)^{(2)},(\nu_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: (295)

By $(\nu_1)^{(2)}>0$, $(\nu_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0$, $(u_2)^{(2)}<0$ the roots (296)

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_1)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ (297)

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and (298)

Definition of $(\bar{\nu}_1)^{(2)},,(\bar{\nu}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: (299)

By $(\bar{\nu}_1)^{(2)}>0$, $(\bar{\nu}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0$, $(\bar{u}_2)^{(2)}<0$ the (300)

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_2)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ (301)

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t}\right] + T_{18}^0 e^{-(R_2)^{(2)}t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations

$(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, ***if*** $(\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, ***if*** $(\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, ***if*** $(\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}\left((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)}\right)}\left[e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}\left((S_1)^{(3)} - (a'_{22})^{(3)}\right)}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{\left((R_1)^{(3)} + (r_{20})^{(3)}\right)t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{\left((R_1)^{(3)} + (r_{20})^{(3)}\right)t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}\left((R_1)^{(3)} - (b'_{22})^{(3)}\right)}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}\left((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)}\right)}\left[e^{\left((R_1)^{(3)} + (r_{20})^{(3)}\right)t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$$

$$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right) - (b''_{25})^{(4)}\left((G_{27}), t\right) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$

and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)},,(\bar{v}_2)^{(4)},(\bar{u}_1)^{(4)},(\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$ and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}$, $(\mu_2)^{(4)}$, $(v_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}$, $(\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, ***if*** $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)}G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)}G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)}t} - e^{-(a'_{26})^{(4)}t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)}t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^{0}}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t}-e^{-(R_2)^{(4)}t}\right]+T_{26}^{0}e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)},(S_2)^{(4)},(R_1)^{(4)},(R_2)^{(4)}$:- 337

Where $(S_1)^{(4)}=(a_{24})^{(4)}(m_2)^{(4)}-(a'_{24})^{(4)}$

$(S_2)^{(4)}=(a_{26})^{(4)}-(p_{26})^{(4)}$

$(R_1)^{(4)}=(b_{24})^{(4)}(\mu_2)^{(4)}-(b'_{24})^{(4)}$

$(R_2)^{(4)}=(b'_{26})^{(4)}-(r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)},(\sigma_2)^{(5)},(\tau_1)^{(5)},(\tau_2)^{(5)}$:

$(\sigma_1)^{(5)},(\sigma_2)^{(5)},(\tau_1)^{(5)},(\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)}\le-(a'_{28})^{(5)}+(a'_{29})^{(5)}-(a''_{28})^{(5)}(T_{29},t)+(a''_{29})^{(5)}(T_{29},t)\le-(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)}\le-(b'_{28})^{(5)}+(b'_{29})^{(5)}-(b''_{28})^{(5)}\big((G_{31}),t\big)-(b''_{29})^{(5)}\big((G_{31}),t\big)\le-(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)},(\nu_2)^{(5)},(u_1)^{(5)},(u_2)^{(5)},\nu^{(5)},u^{(5)}$: 339

By $(\nu_1)^{(5)}>0$, $(\nu_2)^{(5)}<0$ and respectively $(u_1)^{(5)}>0$, $(u_2)^{(5)}<0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2+(\sigma_1)^{(5)}\nu^{(5)}-(a_{28})^{(5)}=0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2+(\tau_1)^{(5)}u^{(5)}-(b_{28})^{(5)}=0$ and

Definition of $(\bar{\nu}_1)^{(5)},,(\bar{\nu}_2)^{(5)},(\bar{u}_1)^{(5)},(\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)}>0$, $(\bar{\nu}_2)^{(5)}<0$ and respectively $(\bar{u}_1)^{(5)}>0$, $(\bar{u}_2)^{(5)}<0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2+(\sigma_2)^{(5)}\nu^{(5)}-(a_{28})^{(5)}=0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2+(\tau_2)^{(5)}u^{(5)}-(b_{28})^{(5)}=0$

Definition of $(m_1)^{(5)},(m_2)^{(5)},(\mu_1)^{(5)},(\mu_2)^{(5)},(\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)},(m_2)^{(5)},(\mu_1)^{(5)},(\mu_2)^{(5)}$ by

$(m_2)^{(5)}=(\nu_0)^{(5)},(m_1)^{(5)}=(\nu_1)^{(5)}$, $\boldsymbol{if}\ (\nu_0)^{(5)}<(\nu_1)^{(5)}$

$(m_2)^{(5)}=(\nu_1)^{(5)},(m_1)^{(5)}=(\bar{\nu}_1)^{(5)}$, $\boldsymbol{if}\ (\nu_1)^{(5)}<(\nu_0)^{(5)}<(\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)}=\frac{G_{28}^{0}}{G_{29}^{0}}}$

$(m_2)^{(5)}=(\nu_1)^{(5)},(m_1)^{(5)}=(\nu_0)^{(5)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(5)}<(\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)}=(u_0)^{(5)},(\mu_1)^{(5)}=(u_1)^{(5)}$, $\boldsymbol{if}\ (u_0)^{(5)}<(u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, $\boldsymbol{if}$ $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}$ $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{28}(t) \leq G_{28}^0 e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{29}(t) \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \leq G_{30}(t) \leq \right. \quad 344$$

$$\left. \frac{(a_{30})^{(5)} G_{28}^0}{(m_2)^{(5)}((S_1)^{(5)} - (a'_{30})^{(5)})}\left[e^{(S_1)^{(5)} t} - e^{-(a'_{30})^{(5)} t}\right] + G_{30}^0 e^{-(a'_{30})^{(5)} t}\right)$$

$$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \leq T_{28}(t) \leq T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \leq T_{28}(t) \leq \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \leq T_{30}(t) \leq \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \leq -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \leq -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \leq -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}((G_{35}), t) - (b''_{33})^{(6)}((G_{35}), t) \leq -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}$, $(m_2)^{(6)}$, $(\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}$, $(m_2)^{(6)}$, $(\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad (354)$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad (355)$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t} - e^{-(a'_{34})^{(6)}t}\right] + G_{34}^0 e^{-(a'_{34})^{(6)}t}\right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad (356)$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}}T_{32}^0 e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(v_1)^{(7)}, (v_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, v^{(7)}, u^{(7)}$: 361

By $(v_1)^{(7)} > 0, (v_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{v}_1)^{(7)}, , (\bar{v}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)} > 0, (\bar{v}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)},\ \boldsymbol{if}\ (\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)},\ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}),t\big) - (b''_{45})^{(9)}\big((G_{47}),t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)},, (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, ***if*** $(\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, ***if*** $(\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, ***if*** $(\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,
and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44}e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44}e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G^0_{44}e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G^0_{44}e^{(S_1)^{(9)}t}$$

(

$$\frac{(a_{46})^{(9)}G^0_{44}}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})}\Big[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t}\Big] + G^0_{46}e^{-(S_2)^{(9)}t} \le G_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}G^0_{44}}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})}[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G^0_{46}e^{-(a'_{46})^{(9)}t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \le \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \le \nu^{(1)}(t) \le (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \le \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \le \nu^{(1)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \le (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \le (\bar{\nu}_1)^{(1)} \le \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \le \nu^{(1)}(t) \le \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \le (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \le \nu^{(1)}(t) \le (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \le u^{(1)}(t) \le (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{dv^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)v^{(2)} - (a_{17})^{(2)}v^{(2)}$$

Definition of $v^{(2)}$:- $\boxed{v^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(v^{(2)}\right)^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)}\right) \leq \frac{dv^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(v^{(2)}\right)^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{v}_1)^{(2)}, (v_0)^{(2)}$:-

For $0 < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (v_1)^{(2)} < (\bar{v}_1)^{(2)}$

$$v^{(2)}(t) \geq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(v_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (v_2)^{(2)}}}$$

it follows $(v_0)^{(2)} \leq v^{(2)}(t) \leq (v_1)^{(2)}$

In the same manner , we get 391

$$v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{v}_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (\bar{v}_2)^{(2)}}}$$

From which we deduce $(v_0)^{(2)} \leq v^{(2)}(t) \leq (\bar{v}_1)^{(2)}$ 392

If $0 < (v_1)^{(2)} < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{v}_1)^{(2)}$ we find like in the previous case, 393

$$(v_1)^{(2)} \leq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_2)^{(2)}\right)t\right]}} \leq v^{(2)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \leq (\bar{v}_1)^{(2)}$$

If $0 < (v_1)^{(2)} \leq (\bar{v}_1)^{(2)} \leq (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(v_1)^{(2)} \leq v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \leq (v_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(2)}(t)$:- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then
$(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

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If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\quad \boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$v^{(6)}(t) \le \frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)}=\frac{(\bar{v}_1)^{(6)}-(v_0)^{(6)}}{(v_0)^{(6)}-(\bar{v}_2)^{(6)}}}$$

From which we deduce $(v_0)^{(6)} \le v^{(6)}(t) \le (\bar{v}_1)^{(6)}$

If $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{v}_1)^{(6)}$ we find like in the previous case, 414

$$(v_1)^{(6)} \le \frac{(v_1)^{(6)}+(C)^{(6)}(v_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)}-(v_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)}-(v_2)^{(6)}\right)t\right]}} \le v^{(6)}(t) \le$$

$$\frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \le (\bar{v}_1)^{(6)}$$

If $0 < (v_1)^{(6)} \le (\bar{v}_1)^{(6)} \le \boxed{(v_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(v_1)^{(6)} \le v^{(6)}(t) \le \frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \le (v_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(6)}(t)$:-

$$(m_2)^{(6)} \le v^{(6)}(t) \le (m_1)^{(6)}, \quad \boxed{v^{(6)}(t)=\frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \le u^{(6)}(t) \le (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t)=\frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(v_1)^{(6)} = (\bar{v}_1)^{(6)}$ if in addition $(v_0)^{(6)} = (v_1)^{(6)}$ then $v^{(6)}(t) = (v_0)^{(6)}$ and as a consequence $G_{32}(t) = (v_0)^{(6)}G_{33}(t)$ **this also defines** $(v_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(v_1)^{(6)}$ and $(\bar{v}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)}=\frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}\right)\leq\frac{dv^{(7)}}{dt}\leq-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0<\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}<(\nu_1)^{(7)}<(\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t)\geq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}\quad,\quad\boxed{(C)^{(7)}=\frac{(\nu_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)}\leq\nu^{(7)}(t)\leq(\nu_1)^{(7)}$

In the same manner , we get 417

$$\nu^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(7)}=\frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)}\leq\nu^{(7)}(t)\leq(\bar{\nu}_1)^{(7)}$

If $0<(\nu_1)^{(7)}<(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}<(\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)}\leq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}\leq\nu^{(7)}(t)\leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\bar{\nu}_1)^{(7)}$$

If $0<(\nu_1)^{(7)}\leq(\bar{\nu}_1)^{(7)}\leq\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)}\leq\nu^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)}\leq\nu^{(7)}(t)\leq(m_1)^{(7)},\quad\boxed{\nu^{(7)}(t)=\frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

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If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case,

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

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If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$ if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\quad \boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \leq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \leq \nu^{(9)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \leq (\bar{\nu}_1)^{(9)} \leq \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \leq \nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \leq \nu^{(9)}(t) \leq (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for**

the special case .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(v_1)^{(9)}$ and $(\bar{v}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a_i'')^{(1)} and\ (b_i'')^{(1)}$ are independent on t , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0$ 447

$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0$ 448

has a unique positive solution , which is an equilibrium solution

$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0$ 449

$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0$ 450

$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0$ 451

$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0$ 452

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(e) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T_{33}^* :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that

there exists a unique G_{37}^{*} such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)}G_{17}^{*}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)}G_{17}^{*}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T_{17}^{*})]} \quad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)}T_{17}^{*}}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)}T_{17}^{*}}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^{*})]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)}G_{21}^{*}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)}G_{21}^{*}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$$G_{36}^{*} = \frac{(a_{36})^{(7)} G_{37}^{*}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}^{*} = \frac{(a_{38})^{(7)} G_{37}^{*}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*} = \frac{(b_{36})^{(7)} T_{37}^{*}}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^{*})]} \quad , \quad T_{38}^{*} = \frac{(b_{38})^{(7)} T_{37}^{*}}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$$G_{40}^{*}=\frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}\quad,\quad G_{42}^{*}=\frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

$$T_{40}^{*}=\frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}\quad,\quad T_{42}^{*}=\frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$$

Finally we obtain the unique solution of 89 to 99 523
A

G_{45}^{*} given by $\varphi((G_{47})^{*})=0$, T_{45}^{*} given by $f(T_{45}^{*})=0$ and

$$G_{44}^{*}=\frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}\quad,\quad G_{46}^{*}=\frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$$

$$T_{44}^{*}=\frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}\quad,\quad T_{46}^{*}=\frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}\ and\ (b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i=G_i^{*}+\mathbb{G}_i\qquad,T_i=T_i^{*}+\mathbb{T}_i$$

$$\frac{\partial(a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^{*})=(q_{14})^{(1)}\quad,\ \frac{\partial(b_i'')^{(1)}}{\partial G_j}(\,G^{*}\,)=s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt}=-\big((a'_{13})^{(1)}+(p_{13})^{(1)}\big)\mathbb{G}_{13}+(a_{13})^{(1)}\mathbb{G}_{14}-(q_{13})^{(1)}G_{13}^{*}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt}=-\big((a'_{14})^{(1)}+(p_{14})^{(1)}\big)\mathbb{G}_{14}+(a_{14})^{(1)}\mathbb{G}_{13}-(q_{14})^{(1)}G_{14}^{*}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt}=-\big((a'_{15})^{(1)}+(p_{15})^{(1)}\big)\mathbb{G}_{15}+(a_{15})^{(1)}\mathbb{G}_{14}-(q_{15})^{(1)}G_{15}^{*}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt}=-\big((b'_{13})^{(1)}-(r_{13})^{(1)}\big)\mathbb{T}_{13}+(b_{13})^{(1)}\mathbb{T}_{14}+\textstyle\sum_{j=13}^{15}\big(s_{(13)(j)}T_{13}^{*}\mathbb{G}_j\big) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt}=-\big((b'_{14})^{(1)}-(r_{14})^{(1)}\big)\mathbb{T}_{14}+(b_{14})^{(1)}\mathbb{T}_{13}+\textstyle\sum_{j=13}^{15}\big(s_{(14)(j)}T_{14}^{*}\mathbb{G}_j\big) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt}=-\big((b'_{15})^{(1)}-(r_{15})^{(1)}\big)\mathbb{T}_{15}+(b_{15})^{(1)}\mathbb{T}_{14}+\textstyle\sum_{j=13}^{15}\big(s_{(15)(j)}T_{15}^{*}\mathbb{G}_j\big) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions

$(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \tag{532}$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \ , \ \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \tag{533}$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \tag{534}$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \tag{535}$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \tag{536}$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \tag{537}$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}')^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T_{17}^*\mathbb{G}_j\right) \tag{538}$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}')^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T_{18}^*\mathbb{G}_j\right) \tag{539}$$

ASYMPTOTIC STABILITY ANALYSIS (540)

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \ , \ \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}')^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \tag{541}$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}')^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \tag{542}$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}')^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \tag{543}$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}')^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T_{20}^*\mathbb{G}_j\right) \tag{544}$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^{*}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T_{22}^{*}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^{*}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^{*}) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^{*}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^{*}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^{*}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^{*}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^{*}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^{*}\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^{*}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}((G_{31})^{*}) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^{*}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^{*}) = (q_{41})^{(8)} \quad , \frac{\partial(b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^{*}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^{*}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^{*}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^{*}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \ , \ \frac{\partial(b_i'')^{(9)}}{\partial G_j}(\ (G_{47})^*\ ) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\ B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\ C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\ D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\ E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\ F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\ G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14}+(b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)})\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)})\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\Big]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION TWO A

Dilatational, Translational, And Rotational Isometries Of Ads

INTRODUCTION—VARIABLES USED

Large-N collective fields and holography Phys. Rev. D 68, 044011 – Published 12 August 2003 Sumit R. Das and Antal Jevicki

(1) While the spin components of the collective field transform properly under (e&eb) dilatational, translational, and rotational isometries of AdS, special conformal transformations mix fields of (e) different spins indicating (eb) a need for a nonlocal map between the two sets of fields.
(2) They discuss the nature of the propagating degrees of freedom through (e&eb) a Hamiltonian form of collective field theory and argue that nonsinglet states which are present in (eb) a Euclidean version are related to (e&eb) nontrivial backgrounds. DOI: http://dx.doi.org/10.1103/PhysRevD.68.044011
(3) They discuss the nature of the propagating degrees of freedom through (e&eb) a Hamiltonian form of collective field theory and argue that nonsinglet states which are present in (eb) a Euclidean version are related to (e&eb) nontrivial backgrounds. DOI: http://dx.doi.org/10.1103/PhysRevD.68.044011

NOTATION

Module One

subtle cancellation of the entropy coming from the bulk fields in one higher dimension with O (1) contributions from (e) the classical solution ensures (eb) that the subleading terms in thermodynamic quantities are of the expected form

G_{13} : Category one of classical solution ensures (eb) that the subleading terms in thermodynamic quantities are of the expected form

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of subtle cancellation of the entropy coming from the bulk fields in one higher dimension with O (1) contributions

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

subtle cancellation of the entropy coming from the bulk fields in one higher dimension with O (1) contributions from the classical solution ensures (eb) that the subleading terms in thermodynamic quantities are of the expected form

G_{16} : Category one of subtle cancellation of the entropy coming from the bulk fields in one higher dimension with O (1) contributions from the classical solution

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of subleading terms in thermodynamic quantities are of the expected form

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

While the spin components of the collective field transform properly under (e&eb) dilatational, translational, and rotational isometries of AdS, special conformal transformations mix fields of (e) different spins indicating (eb) a need for a nonlocal map between the two sets of fields

G_{20} : Category one of spin components of the collective field; dilatational, translational, and rotational isometries of AdS, special conformal transformations mix fields of (e) different spins indicating (eb) a need for a nonlocal map between the two sets of fields

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of dilatational, translational, and rotational isometries of AdS, special conformal transformations mix fields of (e) different spins indicating (eb) a need for a nonlocal map between the two sets of fields; spin components of the collective field

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

While the spin components of the collective field transform properly under dilatational, translational, and rotational isometries of AdS, special conformal transformations mix fields of (e) different spins indicating (eb) a need for a nonlocal map between the two sets of fields

G_{24} : Category one of spin components of the collective field transform properly under dilatational, translational, and rotational isometries of AdS, special conformal transformations; fields of (e) different spins indicating (eb) a need for a nonlocal map between the two sets of fields

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of fields of (e) different spins indicating (eb) a need for a nonlocal map between the two sets of fields; spin components of the collective field transform properly under dilatational, translational,

and rotational isometries of AdS, special conformal transformations

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

spin components of the collective field transform properly under dilatational, translational, and rotational isometries of AdS, special conformal transformations mix fields of (e) different spins indicating (eb) a need for a nonlocal map between the two sets of fields

G_{28} : Category one of different spins indicating (eb) a need for a nonlocal map between the two sets of fields

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of spin components of the collective field transform properly under dilatational, translational, and rotational isometries of AdS, special conformal transformations mix fields

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

spin components of the collective field transform properly under dilatational, translational, and rotational isometries of AdS, special conformal transformations mix fields of different spins indicating (eb) a need for a nonlocal map between the two sets of fields

G_{32} : Category one of spin components of the collective field transform properly under dilatational, translational, and rotational isometries of AdS, special conformal transformations mix fields of different spins; need for a nonlocal map between the two sets of fields

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of need for a nonlocal map between the two sets of fields; spin components of the collective field transform properly under dilatational, translational, and rotational isometries of AdS, special conformal transformations mix fields of different spins

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

spin components of the collective field transform properly under dilatational, translational, and rotational isometries of AdS, special conformal transformations mix fields of different spins indicating a need for a nonlocal map between the two sets of fields

G_{36} : Category one of spin components of the collective field transform properly under dilatational, translational, and rotational isometries of AdS, special conformal transformations mix fields of different spins indicating a need for a nonlocal map; two sets of fields

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of two sets of fields ;spin components of the collective field transform properly under dilatational, translational, and rotational isometries of AdS, special conformal transformations mix fields of different spins indicating a need for a nonlocal map

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

They discuss the

nature of the propagating degrees of freedom through (e&eb) a Hamiltonian form of collective field theory and argue that nonsinglet states which are present in (eb) a Euclidean version are related to (e&eb) nontrivial backgrounds.

DOI: http://dx.doi.org/10.1103/PhysRevD.68.044011

G_{40} : Category one of nature of the propagating degrees of freedom; Hamiltonian form of collective field theory and argue that nonsinglet states which are present in (eb) a Euclidean version are related to (e&eb) nontrivial backgrounds

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of Hamiltonian form of collective field theory and argue that nonsinglet states which are present in (eb) a Euclidean version are related to (e&eb) nontrivial backgrounds; nature of the propagating degrees of freedom

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

nature of the propagating degrees of freedom through a Hamiltonian form of collective field theory and argue that nonsinglet states which are present in (eb) a Euclidean version are related to (e&eb) nontrivial backgrounds

G_{44} : Category one of nature of the propagating degrees of freedom through a Hamiltonian form of collective field theory and argue that nonsinglet states which are present in a Euclidean version; nontrivial backgrounds

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of nontrivial backgrounds; nature of the propagating degrees of freedom through a Hamiltonian form of collective field theory and argue that nonsinglet states which are present in a Euclidean version

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION THREE

Strongly Correlated Flat-Band Systems

INTRODUCTION—VARIABLES USED

Strongly correlated flat-band systems: The route from Heisenberg spins to Hubbard electrons Oleg Derzhko, Johannes Richter, Mykola Maksymenko

(1) In this review authors recapitulate the basic features of the flat-band spin systems and briefly summarize earlier studies in the field. Main emphasis is made on recent developments which include (e) results for both spin and electron flat-band models.

(2) In particular, for flat-band spin systems they highlight field-driven phase transitions for (e) frustrated quantum Heisenberg antiferromagnets at (eb) low temperatures, chiral flat-band states, as well as the effect (e&eb) of a slight dispersion of a previously strictly flat band due to (e) nonideal lattice geometry.

(3) In particular, for flat-band spin systems they highlight field-driven phase transitions for (e) frustrated quantum Heisenberg antiferromagnets at (eb) low temperatures, chiral flat-band states, as well as the effect (e&eb) of a slight dispersion of a previously strictly flat band due to (e) nonideal lattice geometry.

(4) For electronic systems, they discuss the universal low-temperature behavior of (e) several flat-band Hubbard models, the emergence of ground-state ferromagnetism in (eb) the square-lattice Tasaki-Hubbard model and the related (e&eb) Pauli-correlated percolation problem, as well as the dispersion-driven (eb) ground-state ferromagnetism in (eb) flat-band Hubbard systems.

(5) For electronic systems, they discuss the universal low-temperature behavior of (e) several flat-band Hubbard models, the emergence of ground-state ferromagnetism in (eb) the square-lattice Tasaki-Hubbard model and the related (e&eb) Pauli-correlated percolation problem, as well as the dispersion-driven (eb) ground-state ferromagnetism in (eb) flat-band Hubbard systems

(6) For electronic systems, they discuss the universal low-temperature behavior of (e) several flat-band Hubbard models, the emergence of ground-state ferromagnetism in (eb) the square-lattice Tasaki-Hubbard model and the related (e&eb) Pauli-correlated percolation problem, as well as the dispersion-driven (eb) ground-state ferromagnetism in (eb) flat-band Hubbard systems. Closely related studies and possible experimental realizations of the flat-band physics are also described briefly. Subjects: Strongly Correlated Electrons (cond-mat.str-el) Cite as: arXiv: 1502.02729 [cond-mat.str-el]

NOTATION

Module One

$$\frac{\partial \rho E}{\partial t} + \left(\rho u_\alpha E\right)_{,\alpha} = \left(u_\alpha \tau_{\alpha\beta}\right)_{,\beta} + \left(kT_{,\alpha}\right)_{,\alpha} + \rho u_\alpha f_\alpha^b + \rho q$$

$$\sigma = k\mathbf{n}\cdot\mathbf{grad}T$$

$$W = \int_{V(t)} \left\{\rho u_\alpha f_\alpha^b + \left(u_\alpha \tau_{\alpha\beta}\right)_{,\beta}\right\} dV$$

$$\rho\frac{Du_i}{Dt} = -\nabla p + \rho\mathbf{g}$$

$$\left.\begin{aligned}
\rho\frac{du}{dt} &= \rho F_x - \frac{\partial p}{\partial x} + \left(\frac{\partial \sigma'_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right) \\
\rho\frac{dv}{dt} &= \rho F_y - \frac{\partial p}{\partial y} + \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma'_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}\right) \\
\rho\frac{dw}{dt} &= \rho F_z - \frac{\partial p}{\partial z} + \left(\frac{\partial \tau'_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma'_z}{\partial z}\right)
\end{aligned}\right\}$$

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}\cdot(\rho\boldsymbol{u})$$

Source: Wikipedia

N = 4 supersymmetric Yang–Mills theory is a mathematical and physical model created to study particles through a simple system, similar to string theory, with conformal symmetry. It is a simplified toy theory based on Yang–Mills theory that does not describe the real world, but is useful because it can act as a proving ground for approaches for attacking problems in more complex theories.[1] It describes a universe containing boson fields and fermion fields which are related by 4 supersymmetries (this means that swapping boson, fermion and scalar fields in a certain way leaves the predictions of the theory invariant). It is one of the simplest (because it has no free parameters except for the gauge group) and one of the few finite quantum field theories in 4 dimensions. It can be thought of as the most symmetric field theory that does not involve gravity.

$$L = \mathrm{tr}\left\{-\frac{1}{2g^2}F_{\mu\nu}F^{\mu\nu} + \frac{\theta_I}{8\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu} - i\lambda^a\sigma^\mu D_\mu\lambda_a - D_\mu X^i D^\mu X^i + gC_i^{ab}\lambda_a[X^i,\lambda_b] + C_{iab}\lambda^a[X^i,\lambda^b] + \frac{g^2}{2}[X^i,X^j]^2\right\}$$

Ten-dimensional Lagrangian

The above Lagrangian can be found by beginning with the simpler ten-dimensional Lagrangian

$$L = \mathrm{tr}\left\{\frac{1}{g^2}F_{IJ}F^{IJ} - i\bar{\lambda}\Gamma^I D_I\lambda\right\}$$

S-duality:

$$\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2}.$$

D-brane From Wikipedia, the free encyclopedia

In string theory, D-branes are a class of extended objects upon which open strings can end with Dirichlet

boundary conditions, after which they are named. D-branes were discovered by Dai, Leigh and Polchinski, and independently by Hořava in 1989. In 1995, Polchinski identified D-branes with black p-brane solutions of supergravity, a discovery that triggered the Second Superstring Revolution and led to both holographic and M-theory dualities. D-branes are typically classified by their spatial dimension, which is indicated by a number written after the D. A D0-brane is a single point, a D1-brane is a line (sometimes called a "D-string"), a D2-brane is a plane, and a D25-brane fills the highest-dimensional space considered in bosonic string theory. There are also instantonic D(-1)-branes, which are localized in both space and time.

S-duality in quantum field theory:

$$\nabla \cdot \mathbf{E} = 0,$$
$$\nabla \cdot \mathbf{B} = 0,$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$
$$\nabla \times \mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}.$$

$$\mathbf{E} \to \mathbf{B}$$
$$\mathbf{B} \to -\frac{1}{c^2}\mathbf{E}.$$

Montonen–Olive duality:

In quantum field theory, the electric and magnetic fields are unified into a single entity called the electromagnetic field, and this field is described by a special type of quantum field theory called a gauge theory or Yang–Mills theory. In a gauge theory, the physical fields have a high degree of symmetry which can be understood mathematically using the notion of a Lie group. This Lie group is known as the gauge group. The electromagnetic field is described by a very simple gauge theory corresponding to the abelian gauge group U(1), but there are other gauge theories with more complicated non-abelian gauge groups.[5] It is natural to ask whether there is an analog in gauge theory of the symmetry interchanging the electric and magnetic fields in Maxwell's equations. The answer was given in the late 1970s by Claus Montonen and David Olive,[6] building on earlier work of Peter Goddard, Jean Nuyts, and Olive.[7] Their work provides an example of S-duality now known as Montonen–Olive duality. Montonen–Olive duality applies to a very special type of gauge theory called N = 4 supersymmetric Yang–Mills theory, and it says that two such theories may be equivalent in a certain precise sense.[1] If one of the theories has a gauge group G, then the dual theory has gauge group {^L}G where {^L}G denotes the Langlands dual group which is in general different from G.[8] An important quantity in quantum field theory is complexified coupling constant. This is a complex number defined by the formula [9]

$$\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2}$$

Change in entropy:

$$\mathrm{d}S = \frac{\mathrm{d}M}{T}.$$

The supersymmetry algebra

Traditional symmetries of physics are generated by objects that transform by the tensor representations of the Poincaré group and internal symmetries. Supersymmetries, however, are generated by objects that transform by the spinor representations. According to the spin-statistics theorem, bosonic fields commute while fermionic fields anticommute. Combining the two kinds of fields into a single algebra requires the introduction of a Z2-grading under which the bosons are the even elements and the fermions are the odd elements. Such algebra is called Lie superalgebra. The simplest supersymmetric extension of the Poincaré algebra is the Super-Poincaré algebra. Expressed in terms of two Weyl spinors has the following anti-commutation relation:

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2(\sigma^\mu)_{\alpha\dot{\beta}} P_\mu$$

Solution via extra dimensions:

$$\frac{1}{M_{\mathrm{Pl}}^2 r^2} = \frac{1}{M_{\mathrm{Pl}_{3+1+\delta}}^{2+\delta} r^2 n^\delta} \Rightarrow -m\frac{\mathbf{e_r}}{M_{\mathrm{Pl}}^2 r^2} = -m\frac{\mathbf{e_r}}{M_{\mathrm{Pl}_{3+1+\delta}}^{2+\delta} r^2 n^\delta}$$

Supersymmetric solution:

$$\mathcal{L}_{\mathbf{Yukawa}} = -\lambda_f \bar{\psi} H \psi$$

Symplectic Groups and Quaternion Representations:

$$SU(4,H)_L \times H_R = Sp(8) \times SU(2) \supset SU(4) \times SU(2) \supset SU(3) \times SU(2) \times U(1)$$

If \psi is a quaternion valued spinor, A^{ab}_\mu is quaternion hermitian 4 × 4 matrix coming from Sp(8) and B_\mu is a pure imaginary quaternion (both of which are 4-vector bosons) then the interaction term is:

$$\overline{\psi^a}\gamma_\mu \left(A_\mu^{ab}\psi^b + \psi^a B_\mu\right)$$

In 1973 Jackiw and Johnson[6] and Cornwall and Norton[7] studied the possibility that a (non-vectorial) gauge interaction of fermions can break itself; i.e., is strong enough to form a Goldstone boson coupled to the gauge current. Using Abelian gauge models, they showed that, if such a Goldstone boson is formed, it is "eaten" by the Higgs mechanism, becoming the longitudinal component of the now massive gauge boson. Technically, the polarization function Π(p2) appearing in the gauge boson propagator, Δμν = (pμ pν/p2 - gμν)/[p2(1 - g2 Π(p2))] develops a pole at p2 = 0 with residue F2, the square of the Goldstone boson's decay constant, and the gauge boson acquires mass M ≅ g F. In 1973, Weinstein [8] showed that composite Goldstone bosons whose constituent fermions transform in the "standard" way under SU (2) ⊗ U(1) generate the weak boson masses

$$(1) \qquad M_{W^\pm} = \frac{1}{2} g F_{EW} \quad \text{and} \quad M_Z = \frac{1}{2}\sqrt{g^2 + g'^2} F_{EW} \equiv \frac{M_W}{\cos\theta_W}.$$

$$(2) \qquad m_{q,\ell}(M_{ETC}) \cong \frac{g_{ETC}^2 \langle \bar{T}T \rangle_{ETC}}{M_{ETC}^2} \cong \frac{4\pi F_{EW}^3}{\Lambda_{ETC}^2} .$$

$$(3) \qquad \langle \bar{T}T \rangle_{ETC} = \exp\left(\int_{\Lambda_{TC}}^{M_{ETC}} \frac{d\mu}{\mu} \gamma_m(\mu) \right) \langle \bar{T}T \rangle_{TC} ,$$

$$(4) \qquad F_{EW}^2 M_{\pi T}^2 \cong \frac{g_{ETC}^2 \langle \bar{T}T\bar{T}T \rangle_{ETC}}{M_{ETC}^2} \cong \frac{16\pi^2 F_{EW}^6}{\Lambda_{ETC}^2} .$$

$$(5) \qquad S = 16\pi \frac{d}{dq^2} \left[\Pi_{33}^{\text{new}}(q^2) - \Pi_{3Q}^{\text{new}}(q^2) \right]_{q^2=0}$$
$$= 4\pi \int \frac{dm^2}{m^4} \left[\sigma_V^3(m^2) - \sigma_A^3(m^2) \right]^{\text{new}} ;$$

$$(6) \qquad T = \frac{16\pi}{M_Z^2 \sin^2 2\theta_W} \left[\Pi_{11}^{\text{new}}(0) - \Pi_{33}^{\text{new}}(0) \right]$$
$$= \frac{4\pi}{M_Z^2 \sin^2 2\theta_W} \int_0^\infty \frac{dm^2}{m^2} \left[\sigma_V^1(m^2) + \sigma_A^1(m^2) - \sigma_V^3(m^2) - \sigma_A^3(m^2) \right]^{\text{new}} ,$$

Functional renormalization group:

$$\partial_k \Gamma_k = \frac{1}{2} \mathrm{STr}\, \partial_k R_k \left(\Gamma_k^{(2)} + R_k \right)^{-1},$$

Functional renormalization-group for Wick-ordered effective interaction:

$$\mathcal{V}[\eta, \eta^+] = -\ln Z[G_0^{-1}\eta, G_0^{-1}\eta^+] - \eta G_0^{-1} \eta^+$$

$$\frac{\partial}{\partial \Lambda} V_\Lambda(\psi) = -\dot{\Delta}_{G_{0,\Lambda}} V_\Lambda(\psi) + \Delta^{12}_{\dot{G}_{0,\Lambda}} \mathcal{V}_\Lambda^{(1)} \mathcal{V}_\Lambda^{(2)}$$

$$\mathcal{W}[\eta, \eta^+] = \exp(-\Delta_D) \mathcal{V}[\eta, \eta^+]$$

$$\partial_\Lambda \mathcal{W}_\Lambda = -\Delta_{\dot{D}_\Lambda + \dot{G}_{0,\Lambda}} \mathcal{W}_\Lambda + e^{-\Delta^{12}_{D_\Lambda}} \Delta^{12}_{\dot{G}_{0,\Lambda}} \mathcal{W}_\Lambda^{(1)} \mathcal{W}_\Lambda^{(2)}$$

Ising Model:

$$H(\sigma) = -\sum_{<i\ j>} J_{ij} \sigma_i \sigma_j - \mu \sum_j h_j \sigma_j$$

$$P_\beta(\sigma) = \frac{e^{-\beta H(\sigma)}}{Z_\beta},$$

Classical Heisenberg model:

$$\mathcal{H} = -\sum_{i,j} \mathcal{J}_{ij} \vec{s}_i \cdot \vec{s}_j \quad (2)$$

$$\mathcal{J}_{ij} = \begin{cases} J & \text{if } i,j \text{ are neighbors} \\ 0 & \text{else.} \end{cases}$$

Quantum Heisenberg Model:

$$\hat{H} = -J \sum_{j=1}^{N} \sigma_j \sigma_{j+1} - h \sum_{j=1}^{N} \sigma_j$$

$$\hat{H} = -\frac{1}{2} \sum_{j=1}^{N} \left(J_x \sigma_j^x \sigma_{j+1}^x + J_y \sigma_j^y \sigma_{j+1}^y + J_z \sigma_j^z \sigma_{j+1}^z - h \sigma_j^z \right)$$

DMRG form of Heisenberg Model:

$$\mathbf{H}_{SB} = \mathbf{H}_B + \mathbf{H}_U + \sum_{<i,j>} \mathbf{S}_{x_i} \mathbf{S}_{x_j} + \mathbf{S}_{y_i} \mathbf{S}_{y_j} + \mathbf{S}_{z_i} \mathbf{S}_{z_j}$$

Quantum Rotor Model:

$$H_R = \frac{J\bar{g}}{2} \sum_i \mathbf{L}_i^2 - J \sum_{\langle ij \rangle} \mathbf{n}_i \cdot \mathbf{n}_j$$

Kuramoto Model:

$$\frac{d\theta_i}{dt} = \omega_i + \zeta_i + \frac{K}{N} \sum_{j=1}^{N} \sin(\theta_j - \theta_i)$$

$$\langle \zeta_i(t) \zeta_j(t') \rangle = 2D\delta_{ij}\delta(t - t')$$

Potts model:

$$H_p = -J_p \sum_{(i,j)} \delta(s_i, s_j)$$

Classical XY model:

$$H(\mathbf{s}) = -\sum_{i\neq j} J_{ij}\,\mathbf{s}_i \cdot \mathbf{s}_j - \sum_j \mathbf{h}_j \cdot \mathbf{s}_j = -\sum_{i\neq j} J_{ij}\,\cos(\theta_i - \theta_j) - \sum_j h_j \cos\theta_j$$

Configuration Probability:

$$P(\mathbf{s}) = \frac{e^{-\beta H(\mathbf{s})}}{Z} \qquad Z = \int_{[-\pi,\pi]^\Lambda} \prod_{j\in\Lambda} d\theta_j\; e^{-\beta H(\mathbf{s})}.$$

Hubbard's Model:

$$H = -t \sum_{\langle i,j\rangle,\sigma} (c^\dagger_{i,\sigma} c_{j,\sigma} + c^\dagger_{j,\sigma} c_{i,\sigma}) + U \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow},$$

t-J model:

$$\hat{H} = -t \sum_{<ij>\sigma} \left(\hat{a}^\dagger_{i\sigma}\hat{a}_{j\sigma} + \hat{a}^\dagger_{j\sigma}\hat{a}_{i\sigma}\right) + J \sum_{<ij>} (\vec{S}_i \cdot \vec{S}_j - n_i n_j/4)$$

J(1)-J(2) model:

$$\hat{H} = J_1 \sum_{\langle ij\rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle ij\rangle\rangle} \vec{S}_i \cdot \vec{S}_j$$

Parameter dependent Yang Baxter equation:

$$R_{12}(u)\; R_{13}(u+v)\; R_{23}(v) = R_{23}(v)\; R_{13}(u+v)\; R_{12}(u),$$

$$(R\otimes \mathbf{1})(\mathbf{1}\otimes R)(R\otimes \mathbf{1}) = (\mathbf{1}\otimes R)(R\otimes \mathbf{1})(\mathbf{1}\otimes R)$$

Majumdar Ghosh Model;

$$\hat{H} = J \sum_{j=1}^{N} \vec{S}_j \cdot \vec{S}_{j+1} + \frac{J}{2} \sum_{j=1}^{N} \vec{S}_j \cdot \vec{S}_{j+2}$$

G_{13} : Category one of LHS of the above equations

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of RHs of the above equations

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Molecular dynamics From Wikipedia, the free encyclopedia (for references please see Wikipedia)

Molecular dynamics (MD) is a computer simulation of physical movements of atoms and molecules in the context of N-body simulation. The atoms and molecules are allowed to interact for a period of time, giving a view of the motion of the atoms. In the most common version, the trajectories of atoms and molecules are determined by numerically solving the Newton's equations of motion for a system of interacting particles, where forces between the particles and potential energy are defined by interatomic potentials or molecular mechanics force fields. The method was originally conceived within theoretical physics in the late 1950s [1][2] but is applied today mostly in chemical physics, materials science and the modeling of biomolecules. Because molecular systems consist of a vast number of particles, it is impossible to find the properties of such complex systems analytically; MD simulation circumvents this problem by using numerical methods. However, long MD simulations are mathematically ill-conditioned, generating cumulative errors in numerical integration that can be minimized with proper selection of algorithms and parameters, but not eliminated entirely. For systems which obey the ergodic hypothesis, the evolution of a single molecular dynamics simulation may be used to determine macroscopic thermodynamic properties of the system: the time averages of an ergodic system correspond to microcanonical ensemble averages. MD has also been termed "statistical mechanics by numbers" and "Laplace's vision of Newtonian mechanics" of predicting the future by animating nature's forces [3] [4] and allowing insight into molecular motion on an atomic scale.

Ensemble average of molecular dynamics: (Wikipedia)

$$\langle A \rangle_{ensemble} = \iint dp^N dr^N A\left(p^N, r^N\right)\rho\left(p^N, r^N\right)$$

This algorithm yields positions, velocities and accelerations at time t.

There is no compromise on precision

$$r(t+\delta t) = r(t) + v(t)\delta t + \frac{1}{2}a(t)\delta t^2$$

$$v(t+\delta t) = v(t) + \frac{1}{2}\left[a(t) + a(t+\delta t)\right]\delta t$$

Bee man's algorithm:

$$r(t+\delta t) = r(t) + v(t)\delta t + \frac{2}{3}a(t)\delta t^2 - \frac{1}{6}a(t-\delta t)\delta t^2$$

$$v(t+\delta t) = v(t) + v(t)\delta t + \frac{1}{3}a(t)\delta t + \frac{5}{6}a(t)\delta t - \frac{1}{6}a(t-\delta t)\delta t$$

Velocity Verlet algorithm:

$$r(t+\delta t) = r(t) + v(t)\delta t + \frac{1}{2}a(t)\delta t^2$$

$$v(t+\delta t) = v(t) + \frac{1}{2}\left[a(t) + a(t+\delta t)\right]\delta t$$

Leap frog algorithm:

$$r(t+\delta t) = r(t) + v\left(t + \frac{1}{2}\delta t\right)\delta t$$

$$v\left(t + \frac{1}{2}\delta t\right) = v\left(t - \frac{1}{2}\delta t\right) + a(t)\delta t$$

The velocities, vi, are often chosen randomly from a Maxwell-Boltzmann or Gaussian distribution at a given temperature, which gives the probability that an atom i has a velocity vx in the x direction at a temperature T.

$$p(v_{ix}) = \left(\frac{m_i}{2\pi k_B T}\right)^{1/2} \exp\left[-\frac{1}{2}\frac{m_i v_{ix}^2}{k_B T}\right]$$

temperature can be calculated from the velocities using the relation

$$T = \frac{1}{(3N)}\sum_{i=1}^{N}\frac{|p_i|}{2m_i}$$

Average Kinetic energy:

$$K = \langle K \rangle = \frac{1}{M}\sum_{j=1}^{M}\left\{\sum_{i=1}^{N}\frac{m_i}{2}v_i \cdot v_i\right\}_j$$

Average potential energy:

$$V = \langle V \rangle = \frac{1}{M}\sum_{i=1}^{M}V_i$$

G_{16} : Category one of LHS of the above equations

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of RHS of the above equations

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

In this review authors recapitulate the basic features of the flat-band spin systems and briefly summarize earlier studies in the field. Main emphasis is made on recent developments which include (e) results for both

Spin and electron flat-band models.

G_{20} : Category one of Spin models; flat-band models

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of flat-band models ;Spin models;

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

In particular, for flat-band spin systems they highlight field-driven phase transitions for (e) frustrated quantum Heisenberg antiferromagnets at (eb) low temperatures, chiral flat-band states, as well as the effect (e&eb) of a slight dispersion of a previously strictly flat band due to (e) nonideal lattice geometry

G_{24} : Category one of frustrated quantum Heisenberg antiferromagnets at (eb) low temperatures, chiral flat-band states, as well as the effect (e&eb) of a slight dispersion of a previously strictly flat band due to (e) nonideal lattice geometry

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of flat-band spin systems field-driven phase transitions

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

flat-band spin systems field-driven phase transitions for frustrated quantum Heisenberg antiferromagnets at (eb) low temperatures, chiral flat-band states, as well as the effect (e&eb) of a slight dispersion of a previously strictly flat band due to (e) nonideal lattice geometry

G_{28} : Category one of flat-band spin systems field-driven phase transitions for frustrated quantum Heisenberg antiferromagnets

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of low temperatures, chiral flat-band states, as well as the effect (e&eb) of a slight dispersion of a previously strictly flat band due to (e) nonideal lattice geometry

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

flat-band spin systems field-driven phase transitions for frustrated quantum Heisenberg antiferromagnets at low temperatures, chiral flat-band states, as well as the effect (e&eb) of a slight dispersion of a previously strictly flat band due to (e) nonideal lattice geometry

G_{32} : Category one of nonideal lattice geometry

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of flat-band spin systems field-driven phase transitions for frustrated quantum Heisenberg antiferromagnets at low temperatures, chiral flat-band states, as well as the effect of a slight dispersion of a previously strictly flat band

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

For electronic systems, they discuss the

universal low-temperature behavior of (e) several flat-band Hubbard models, the emergence of ground-state ferromagnetism in (eb) the square-lattice Tasaki-Hubbard model and the related (e&eb) Pauli-correlated percolation problem, as well as the dispersion-driven (eb) ground-state ferromagnetism in (eb) flat-band Hubbard systems

G_{36} : Category one of several flat-band Hubbard models

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of universal low-temperature behavior

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

emergence of ground-state ferromagnetism in (eb) the square-lattice Tasaki-Hubbard model and the related (e&eb) Pauli-correlated percolation problem, as well as the dispersion-driven (eb) ground-state ferromagnetism in (eb) flat-band Hubbard systems

G_{40} : Category one of emergence of ground-state ferromagnetism

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of square-lattice Tasaki-Hubbard model and the related (e&eb) Pauli-correlated percolation problem, as well as the dispersion-driven (eb) ground-state ferromagnetism in (eb) flat-band Hubbard systems

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

emergence of ground-state ferromagnetism in the square-lattice Tasaki-Hubbard model and the related (e&eb) Pauli-correlated percolation problem, as well as the dispersion-driven (eb) ground-state ferromagnetism in (eb) flat-band Hubbard systems

G_{44} : Category one of emergence of ground-state ferromagnetism in the square-lattice Tasaki-Hubbard model; Pauli-correlated percolation problem, as well as the dispersion-driven (eb) ground-state ferromagnetism in (eb) flat-band Hubbard systems

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of Pauli-correlated percolation problem, as well as the dispersion-driven (eb) ground-state ferromagnetism in (eb) flat-band Hubbard systems ;emergence of ground-state ferromagnetism in the square-lattice Tasaki-Hubbard model

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},\ \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}\ ,$
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}), t\big)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}), t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\big((G_{19}), t\big) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}),t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}),t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}),t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}),t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}),t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\begin{array}{ccc} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) \quad +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{array}\right]G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad (56)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad (57)$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)}\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)}\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$-(b_{20}'')^{(3,3,)}(G_{23},t)$, $-(b_{21}'')^{(3,3,)}(G_{23},t)$, $-(b_{22}'')^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b_{24}'')^{(4,4,4,4,)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b_{28}'')^{(5,5,5,5,)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b_{32}'')^{(6,6,6,6,)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b_{37}'')^{(7,7,)}(G_{39},t)$, $-(b_{36}'')^{(7,7,)}(G_{39},t)$, $-(b_{38}'')^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b_{40}'')^{(8,8)}(G_{43},t)$ $-(b_{41}'')^{(8,8)}(G_{43},t)$ $-(b_{42}'')^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{c} (a_{16}')^{(2)} + (a_{16}'')^{(2)}(T_{17},t) + (a_{13}'')^{(1,1,)}(T_{14},t) + (a_{20}'')^{(3,3,3)}(T_{21},t) \\ + (a_{24}'')^{(4,4,4,4,4)}(T_{25},t) + (a_{28}'')^{(5,5,5,5,5)}(T_{29},t) + (a_{32}'')^{(6,6,6,6,6)}(T_{33},t) \\ + (a_{36}'')^{(7,7,7)}(T_{37},t) + (a_{40}'')^{(8,8,8)}(T_{41},t) + (a_{44}'')^{(9,9)}(T_{45},t) \end{array}\right] G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{c} (a_{17}')^{(2)} + (a_{17}'')^{(2)}(T_{17},t) + (a_{14}'')^{(1,1,)}(T_{14},t) + (a_{21}'')^{(3,3,3)}(T_{21},t) \\ + (a_{25}'')^{(4,4,4,4,4)}(T_{25},t) + (a_{29}'')^{(5,5,5,5,5)}(T_{29},t) + (a_{33}'')^{(6,6,6,6,6)}(T_{33},t) \\ + (a_{37}'')^{(7,7,7)}(T_{37},t) + (a_{41}'')^{(8,8,8)}(T_{41},t) + (a_{45}'')^{(9,9)}(T_{45},t) \end{array}\right] G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{c} (a_{18}')^{(2)} + (a_{18}'')^{(2)}(T_{17},t) + (a_{15}'')^{(1,1,)}(T_{14},t) + (a_{22}'')^{(3,3,3)}(T_{21},t) \\ + (a_{26}'')^{(4,4,4,4,4)}(T_{25},t) + (a_{30}'')^{(5,5,5,5,5)}(T_{29},t) + (a_{34}'')^{(6,6,6,6,6)}(T_{33},t) \\ + (a_{38}'')^{(7,7,7)}(T_{37},t) + (a_{42}'')^{(8,8,8)}(T_{41},t) + (a_{46}'')^{(9,9)}(T_{45},t) \end{array}\right] G_{18} \quad 63$$

Where $+(a_{16}'')^{(2)}(T_{17},t)$, $+(a_{17}'')^{(2)}(T_{17},t)$, $+(a_{18}'')^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{13}'')^{(1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) \end{bmatrix} T_{18} \quad (66)$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)} & +(a''_{20})^{(3)}(T_{21},t) & +(a''_{16})^{(2,2,2)}(T_{17},t) & +(a''_{13})^{(1,1,1,)}(T_{14},t) \\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9)}(T_{45},t) \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)} & +(a''_{21})^{(3)}(T_{21},t) & +(a''_{17})^{(2,2,2)}(T_{17},t) & +(a''_{14})^{(1,1,1,)}(T_{14},t) \\ +(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{37})^{(7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)} & +(a''_{22})^{(3)}(T_{21},t) & +(a''_{18})^{(2,2,2)}(T_{17},t) & +(a''_{15})^{(1,1,1,)}(T_{14},t) \\ +(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{38})^{(7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{22} \quad (69)$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for*

category 1, 2 and 3

$-(b_{13}'')^{(1,1,1,)}(G,t)$, $-(b_{14}'')^{(1,1,1,)}(G,t)$, $-(b_{15}'')^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b_{36}'')^{(7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7)}(G_{39},t)$ $-(b_{38}'')^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b_{40}'')^{(8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b_{46}'')^{(9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a_{24}')^{(4)} & +(a_{24}'')^{(4)}(T_{25},t) & +(a_{28}'')^{(5,5,)}(T_{29},t) & +(a_{32}'')^{(6,6,)}(T_{33},t) \\ & +(a_{13}'')^{(1,1,1,1)}(T_{14},t) & +(a_{16}'')^{(2,2,2,2)}(T_{17},t) & +(a_{20}'')^{(3,3,3,3)}(T_{21},t) \\ & +(a_{36}'')^{(7,7,7,7,7)}(T_{37},t) & +(a_{40}'')^{(8,8,8,8,8)}(T_{41},t) & +(a_{44}'')^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a_{25}')^{(4)} & +(a_{25}'')^{(4)}(T_{25},t) & +(a_{29}'')^{(5,5,)}(T_{29},t) & +(a_{33}'')^{(6,6)}(T_{33},t) \\ & +(a_{14}'')^{(1,1,1,1)}(T_{14},t) & +(a_{17}'')^{(2,2,2,2)}(T_{17},t) & +(a_{21}'')^{(3,3,3,3)}(T_{21},t) \\ & +(a_{37}'')^{(7,7,7,7,7)}(T_{37},t) & +(a_{41}'')^{(8,8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a_{26}')^{(4)} & +(a_{26}'')^{(4)}(T_{25},t) & +(a_{30}'')^{(5,5,)}(T_{29},t) & +(a_{34}'')^{(6,6,)}(T_{33},t) \\ & +(a_{15}'')^{(1,1,1,1)}(T_{14},t) & +(a_{18}'')^{(2,2,2,2)}(T_{17},t) & +(a_{22}'')^{(3,3,3,3)}(T_{21},t) \\ & +(a_{38}'')^{(7,7,7,7,7)}(T_{37},t) & +(a_{42}'')^{(8,8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad 75$$

$(a_{24}'')^{(4)}(T_{25},t)$, $(a_{25}'')^{(4)}(T_{25},t)$, $(a_{26}'')^{(4)}(T_{25},t)$ *are first augmentation coefficients category* $1,2\ 3$

$+(a_{28}'')^{(5,5,)}(T_{29},t)$, $+(a_{29}'')^{(5,5,)}(T_{29},t)$, $+(a_{30}'')^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* $1,2$ *and* 3

$+(a_{32}'')^{(6,6,)}(T_{33},t)$, $+(a_{33}'')^{(6,6,)}(T_{33},t)$, $+(a_{34}'')^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* $1,2$ *and* 3

$+(a_{13}'')^{(1,1,1,1)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1)}(T_{14},t)$
are fourth augmentation coefficients for category $1,2$ *and* 3

$+(a_{16}'')^{(2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2)}(T_{17},t)$
are fifth augmentation coefficients for category $1,2$ *and* 3

$+(a_{20}'')^{(3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3)}(T_{21},t)$
are sixth augmentation coefficients for category $1,2$ *and* 3

$+(a_{36}'')^{(7,7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7,7)}(T_{37},t)$
are seventh augmentation coefficients for category $1,2$ *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$

are eighth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$

are fourth detrition coefficients for category 1, 2 *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$

are fifth detrition coefficients for category 1, 2 *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$

are sixth detrition coefficients for category 1, 2 *and* 3

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$

are seventh detrition coefficients for category 1, 2 *and* 3

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$

are eighth detrition coefficients for category 1, 2 *and* 3

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & \boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} & \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} & \end{bmatrix} G_{30} \quad (81)$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & \boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} & \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} & \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & \boxed{-(b''_{29})^{(5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} & \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} & \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & \boxed{-(b''_{30})^{(5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} & \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} & \end{bmatrix} T_{30} \quad (84)$$

where $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}$ *are second detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for

category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)} & +(a''_{32})^{(6)}(T_{33},t) & +(a''_{28})^{(5,5,5)}(T_{29},t) & +(a''_{24})^{(4,4,4,)}(T_{25},t) \\ +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t) & \\ +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)} & +(a''_{33})^{(6)}(T_{33},t) & +(a''_{29})^{(5,5,5)}(T_{29},t) & +(a''_{25})^{(4,4,4,)}(T_{25},t) \\ +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) & \\ +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)} & +(a''_{34})^{(6)}(T_{33},t) & +(a''_{30})^{(5,5,5)}(T_{29},t) & +(a''_{26})^{(4,4,4,)}(T_{25},t) \\ +(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t) & \\ +(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{34} \quad (87)$$

$+(a''_{32})^{(6)}(T_{33},t)$, $+(a''_{33})^{(6)}(T_{33},t)$, $+(a''_{34})^{(6)}(T_{33},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{28})^{(5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5)}(T_{29},t)$ *are second augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{24})^{(4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,)}(T_{25},t)$ *are third augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)$ - are fourth augmentation coefficients

$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)$ - fifth augmentation coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \qquad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[\begin{array}{ccc} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{13}$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[\begin{array}{ccc} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{14}$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[\begin{array}{ccc} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{38}'')^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

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$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a_{44}')^{(9)}\boxed{+(a_{44}'')^{(9)}(T_{45},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a_{45}')^{(9)}\boxed{+(a_{45}'')^{(9)}(T_{45},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a_{46}')^{(9)}\boxed{+(a_{46}'')^{(9)}(T_{37},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{44}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{46}'')^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second

augmentation coefficient for category 1, 2 and 3
$+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3
$+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3
$+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3
$+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$
$$(b_{44})^{(9)}T_{45} -$$
$$\begin{bmatrix} (b_{44}')^{(9)}\left[-(b_{44}'')^{(9)}(G_{47},t)\right] & \left[-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{13}$$
$$\frac{dT_{45}}{dt} =$$
$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}\left[-(b_{45}'')^{(9)}(G_{47},t)\right] & \left[-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{14}$$
$$\frac{dT_{46}}{dt} =$$
$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}\left[-(b_{46}'')^{(9)}(G_{47},t)\right] & \left[-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{15}$$

Where $-(b_{44}'')^{(9)}(G_{47},t)$, $-(b_{45}'')^{(9)}(G_{47},t)$, $-(b_{46}'')^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3
$-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\ ,\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)},\ (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)},\ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}), t\big) \le \ (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}\,(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\,\big((G_{27}), t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)},\ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - \ T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \leq (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}\left((G_{35}), t\right) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(EE) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(FF) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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(GG) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(HH)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(II) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(JJ) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)} \qquad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be

noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\ ,\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}<1 \qquad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+\ (\hat{A}_{40})^{(8)}+\ (\hat{P}_{40})^{(8)}\ (\hat{k}_{40})^{(8)}]<1 \qquad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[\ (b_i)^{(8)}+(b_i')^{(8)}+\ (\hat{B}_{40})^{(8)}+\ (\hat{Q}_{40})^{(8)}\ \ (\hat{k}_{40})^{(8)}]<1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \qquad 146A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t)-(a_i'')^{(9)}(T_{45},t)|\le(\hat{k}_{44})^{(9)}|T_{45}-\ T_{45}'|e^{-(\widehat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t)-(b_i'')^{(9)}\big((G_{47}),t\big)|<(\hat{k}_{44})^{(9)}||(G_{47})-(G_{47})'||e^{-(\widehat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$

and$(a_i'')^{(9)}(T_{45},t)$. (T_{45}',t) and (T_{45},t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)},(\widehat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{44})^{(9)}=1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$:

$(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},$ are positive constants

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\ ,\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}<1$$

Definition of $(\hat{P}_{44})^{(9)},(\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},(\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)},(a_i')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(p_i)^{(9)},\ (r_i)^{(9)},i=44,45,46,$
satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)}+(a_i')^{(9)}+\ (\hat{A}_{44})^{(9)}+\ (\hat{P}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)}+(b_i')^{(9)}+\ (\hat{B}_{44})^{(9)}+\ (\hat{Q}_{44})^{(9)}\ \ (\hat{k}_{44})^{(9)}]<1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\ ,T_i(0)$:

$$G_i(t)\leq\ \left(\hat{P}_{13}\right)^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{G_i(0)=G_i^0>0}$$

$$T_i(t)\leq\ (\hat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ \ ,\ \ \boxed{T_i(0)=T_i^0>0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\ ,T_i(0)$

$$G_i(t)\leq\ (\hat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ \ ,\ \ G_i(0)=G_i^0>0$$

$$T_i(t)\leq\ (\hat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ \ ,\ \ T_i(0)=T_i^0>0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t)\leq\ (\hat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ \ ,\ \ G_i(0)=G_i^0>0$$

$$T_i(t)\leq\ (\hat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ \ ,\ \ T_i(0)=T_i^0>0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0)\ ,T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{\left(\hat{M}_{24}\right)^{(4)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{\left(\hat{M}_{24}\right)^{(4)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{\left(\hat{M}_{28}\right)^{(5)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{\left(\hat{M}_{28}\right)^{(5)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{\left(\hat{M}_{32}\right)^{(6)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{\left(\hat{M}_{32}\right)^{(6)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{\left(\hat{M}_{36}\right)^{(7)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{\left(\hat{M}_{36}\right)^{(7)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{\left(\hat{M}_{40}\right)^{(8)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{\left(\hat{M}_{40}\right)^{(8)} t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{\left(\hat{M}_{44}\right)^{(9)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{\left(\hat{M}_{44}\right)^{(9)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{13})^{(1)}$, $T_i^0 \le (\hat{Q}_{13})^{(1)}$, 155

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 156

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{28})^{(5)}$, $T_i^0 \le (\hat{Q}_{28})^{(5)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0,t)$

Proof: 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(f) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left(e^{(\tilde{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\tilde{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left(e^{(\tilde{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\tilde{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0\right) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\tilde{M}_{44})^{(9)}} \left(e^{(\tilde{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\tilde{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\tilde{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\tilde{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\tilde{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[(\widehat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on

$G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 19 to 24 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(1)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If $G_{13}\ \ is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}\,(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on

(G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\,(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on T_{21} and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \quad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 227

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24{,}25{,}26$ depend only on T_{25} and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(4)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25}\ \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t\ > t_4$

$$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\widehat{P}_{28})^{(5)}$ and $(\widehat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\widehat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \tag{234}$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \tag{235}$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric (236)

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{31}}),(\widetilde{T_{31}})$: $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\leq \tag{237}$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\ (G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(5)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)}+\left((\widehat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\widehat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\widehat{Q}_{32})^{(6)}\right]\leq(\widehat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(6)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose

$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \tag{256}$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \tag{257}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric (258)

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})},\widetilde{(T_{39})}:\left(\widetilde{(G_{39})},\widetilde{(T_{39})}\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$

It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\leq \tag{259}$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. (260)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})}, \widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by 274

$(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right| \; e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$. The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right)) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}, \ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}, \ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13} e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13} e^{(S_1)^{(1)} t}$

where $(p_i)^{(1)}$ is defined by equation

$\frac{1}{(m_1)^{(1)}} G^0_{13} e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G^0_{13} e^{(S_1)^{(1)} t}$

$$\left(\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t}-e^{-(S_2)^{(1)}t}\right]+G_{15}^{0}e^{-(S_2)^{(1)}t}\le G_{15}(t)\le\right. \quad (286)$$

$$\left.\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}\right]+G_{15}^{0}e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation (291)

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: (292)

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(T_{17},t)+(a''_{17})^{(2)}(T_{17},t)\le-(\sigma_1)^{(2)} \quad (293)$$

$$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)} \quad (294)$$

Definition of $(\nu_1)^{(2)},(\nu_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: (295)

By $(\nu_1)^{(2)}>0$, $(\nu_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0$, $(u_2)^{(2)}<0$ the roots (296)

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_1)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ (297)

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and (298)

Definition of $(\bar{\nu}_1)^{(2)},,(\bar{\nu}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: (299)

By $(\bar{\nu}_1)^{(2)}>0$, $(\bar{\nu}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0$, $(\bar{u}_2)^{(2)}<0$ the (300)

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_2)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ (301)

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t}\right] + T_{18}^0 e^{-(R_2)^{(2)}t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, ***if*** $(\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, ***if*** $(\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, ***if*** $(\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$$

$$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)} v^{(4)} - (a_{24})^{(4)} = 0$

and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)},,(\bar{v}_2)^{(4)},(\bar{u}_1)^{(4)},(\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)},(\mu_2)^{(4)},(v_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)},(\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, ***if*** $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)}G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)}G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)}t} - e^{-(a'_{26})^{(4)}t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)}t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^{0}}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t}-e^{-(R_2)^{(4)}t}\right]+T_{26}^{0}e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)},(S_2)^{(4)},(R_1)^{(4)},(R_2)^{(4)}$:- 337

Where $(S_1)^{(4)}=(a_{24})^{(4)}(m_2)^{(4)}-(a'_{24})^{(4)}$

$(S_2)^{(4)}=(a_{26})^{(4)}-(p_{26})^{(4)}$

$(R_1)^{(4)}=(b_{24})^{(4)}(\mu_2)^{(4)}-(b'_{24})^{(4)}$

$(R_2)^{(4)}=(b'_{26})^{(4)}-(r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)},(\sigma_2)^{(5)},(\tau_1)^{(5)},(\tau_2)^{(5)}$:

$(\sigma_1)^{(5)},(\sigma_2)^{(5)},(\tau_1)^{(5)},(\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)}\le-(a'_{28})^{(5)}+(a'_{29})^{(5)}-(a''_{28})^{(5)}(T_{29},t)+(a''_{29})^{(5)}(T_{29},t)\le-(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)}\le-(b'_{28})^{(5)}+(b'_{29})^{(5)}-(b''_{28})^{(5)}\big((G_{31}),t\big)-(b''_{29})^{(5)}\big((G_{31}),t\big)\le-(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)},(\nu_2)^{(5)},(u_1)^{(5)},(u_2)^{(5)},\nu^{(5)},u^{(5)}$: 339

By $(\nu_1)^{(5)}>0$, $(\nu_2)^{(5)}<0$ and respectively $(u_1)^{(5)}>0$, $(u_2)^{(5)}<0$ the roots of the equations $(a_{29})^{(5)}\left(\nu^{(5)}\right)^2+(\sigma_1)^{(5)}\nu^{(5)}-(a_{28})^{(5)}=0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2+(\tau_1)^{(5)}u^{(5)}-(b_{28})^{(5)}=0$ and

Definition of $(\bar{\nu}_1)^{(5)},,(\bar{\nu}_2)^{(5)},(\bar{u}_1)^{(5)},(\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)}>0$, $(\bar{\nu}_2)^{(5)}<0$ and respectively $(\bar{u}_1)^{(5)}>0$, $(\bar{u}_2)^{(5)}<0$ the roots of the equations $(a_{29})^{(5)}\left(\nu^{(5)}\right)^2+(\sigma_2)^{(5)}\nu^{(5)}-(a_{28})^{(5)}=0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2+(\tau_2)^{(5)}u^{(5)}-(b_{28})^{(5)}=0$

Definition of $(m_1)^{(5)},(m_2)^{(5)},(\mu_1)^{(5)},(\mu_2)^{(5)},(\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)},(m_2)^{(5)},(\mu_1)^{(5)},(\mu_2)^{(5)}$ by

$(m_2)^{(5)}=(\nu_0)^{(5)},(m_1)^{(5)}=(\nu_1)^{(5)}$, $\boldsymbol{if}\ (\nu_0)^{(5)}<(\nu_1)^{(5)}$

$(m_2)^{(5)}=(\nu_1)^{(5)},(m_1)^{(5)}=(\bar{\nu}_1)^{(5)}$, $\boldsymbol{if}\ (\nu_1)^{(5)}<(\nu_0)^{(5)}<(\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)}=\frac{G_{28}^{0}}{G_{29}^{0}}}$

$(m_2)^{(5)}=(\nu_1)^{(5)},(m_1)^{(5)}=(\nu_0)^{(5)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(5)}<(\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)}=(u_0)^{(5)},(\mu_1)^{(5)}=(u_1)^{(5)}$, $\boldsymbol{if}\ (u_0)^{(5)}<(u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, $\boldsymbol{if}$ $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}$ $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{28}(t) \leq G_{28}^0 e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{29}(t) \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \leq G_{30}(t) \leq \right. \quad 344$$

$$\left. \frac{(a_{30})^{(5)} G_{28}^0}{(m_2)^{(5)}((S_1)^{(5)} - (a'_{30})^{(5)})}\left[e^{(S_1)^{(5)} t} - e^{-(a'_{30})^{(5)} t}\right] + G_{30}^0 e^{-(a'_{30})^{(5)} t}\right)$$

$$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \leq T_{28}(t) \leq T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \leq T_{28}(t) \leq \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \leq T_{30}(t) \leq \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \leq -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \leq -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \leq -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}((G_{35}), t) - (b''_{33})^{(6)}((G_{35}), t) \leq -(\tau_1)^{(6)}$$

Definition of $(v_1)^{(6)}, (v_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, v^{(6)}, u^{(6)}$: 350

By $(v_1)^{(6)} > 0$, $(v_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{v}_1)^{(6)},, (\bar{v}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)} > 0$, $(\bar{v}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}$, $(m_2)^{(6)}$, $(\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}$, $(m_2)^{(6)}$, $(\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t} - e^{-(a'_{34})^{(6)}t}\right] + G_{34}^0 e^{-(a'_{34})^{(6)}t}\right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(v_1)^{(7)}, (v_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, v^{(7)}, u^{(7)}$: 361

By $(v_1)^{(7)} > 0, (v_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{v}_1)^{(7)}, , (\bar{v}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)} > 0, (\bar{v}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)},\ \boldsymbol{if}\ (\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)},\ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}),t\big) - (b''_{45})^{(9)}\big((G_{47}),t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)},, (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, ***if*** $(\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, ***if*** $(\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, ***if*** $(\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,
and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44}e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44}e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G^0_{44}e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G^0_{44}e^{(S_1)^{(9)}t}$$

(

$$\frac{(a_{46})^{(9)}G^0_{44}}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})}\Big[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t}\Big] + G^0_{46}e^{-(S_2)^{(9)}t} \le G_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}G^0_{44}}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})}[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G^0_{46}e^{-(a'_{46})^{(9)}t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \le \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \le \nu^{(1)}(t) \le (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \le \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \le \nu^{(1)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \le (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \le (\bar{\nu}_1)^{(1)} \le \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \le \nu^{(1)}(t) \le \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \le (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \le \nu^{(1)}(t) \le (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \le u^{(1)}(t) \le (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{dv^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)v^{(2)} - (a_{17})^{(2)}v^{(2)}$$

Definition of $v^{(2)}$:- $\boxed{v^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(v^{(2)}\right)^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)}\right) \leq \frac{dv^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(v^{(2)}\right)^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{v}_1)^{(2)}, (v_0)^{(2)}$:-

For $0 < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (v_1)^{(2)} < (\bar{v}_1)^{(2)}$

$$v^{(2)}(t) \geq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(v_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (v_2)^{(2)}}}$$

it follows $(v_0)^{(2)} \leq v^{(2)}(t) \leq (v_1)^{(2)}$

In the same manner , we get 391

$$v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{v}_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (\bar{v}_2)^{(2)}}}$$

From which we deduce $(v_0)^{(2)} \leq v^{(2)}(t) \leq (\bar{v}_1)^{(2)}$ 392

If $0 < (v_1)^{(2)} < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{v}_1)^{(2)}$ we find like in the previous case, 393

$$(v_1)^{(2)} \leq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_2)^{(2)}\right)t\right]}} \leq v^{(2)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \leq (\bar{v}_1)^{(2)}$$

If $0 < (v_1)^{(2)} \leq (\bar{v}_1)^{(2)} \leq (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(v_1)^{(2)} \leq v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \leq (v_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(2)}(t)$:- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then
$(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

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If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\quad \boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$v^{(6)}(t) \le \frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)}=\frac{(\bar{v}_1)^{(6)}-(v_0)^{(6)}}{(v_0)^{(6)}-(\bar{v}_2)^{(6)}}}$$

From which we deduce $(v_0)^{(6)} \le v^{(6)}(t) \le (\bar{v}_1)^{(6)}$

If $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{v}_1)^{(6)}$ we find like in the previous case, 414

$$(v_1)^{(6)} \le \frac{(v_1)^{(6)}+(C)^{(6)}(v_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)}-(v_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)}-(v_2)^{(6)}\right)t\right]}} \le v^{(6)}(t) \le$$

$$\frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \le (\bar{v}_1)^{(6)}$$

If $0 < (v_1)^{(6)} \le (\bar{v}_1)^{(6)} \le \boxed{(v_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(v_1)^{(6)} \le v^{(6)}(t) \le \frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \le (v_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(6)}(t)$:-

$$(m_2)^{(6)} \le v^{(6)}(t) \le (m_1)^{(6)}, \quad \boxed{v^{(6)}(t)=\frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \le u^{(6)}(t) \le (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t)=\frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(v_1)^{(6)} = (\bar{v}_1)^{(6)}$ if in addition $(v_0)^{(6)} = (v_1)^{(6)}$ then $v^{(6)}(t) = (v_0)^{(6)}$ and as a consequence $G_{32}(t) = (v_0)^{(6)}G_{33}(t)$ **this also defines** $(v_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(v_1)^{(6)}$ and $(\bar{v}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)}=\frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}\right)\leq\frac{dv^{(7)}}{dt}\leq-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0<\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}<(\nu_1)^{(7)}<(\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t)\geq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}\quad,\quad\boxed{(C)^{(7)}=\frac{(\nu_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)}\leq\nu^{(7)}(t)\leq(\nu_1)^{(7)}$

In the same manner , we get 417

$$\nu^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(7)}=\frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)}\leq\nu^{(7)}(t)\leq(\bar{\nu}_1)^{(7)}$

If $0<(\nu_1)^{(7)}<(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}<(\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)}\leq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}\leq\nu^{(7)}(t)\leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\bar{\nu}_1)^{(7)}$$

If $0<(\nu_1)^{(7)}\leq(\bar{\nu}_1)^{(7)}\leq\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)}\leq\nu^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)}\leq\nu^{(7)}(t)\leq(m_1)^{(7)},\quad\boxed{\nu^{(7)}(t)=\frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$ if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\quad \boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \leq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \leq \nu^{(9)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \leq (\bar{\nu}_1)^{(9)} \leq \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \leq \nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \leq \nu^{(9)}(t) \leq (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for**

the special case .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(v_1)^{(9)}$ and $(\bar{v}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a_i'')^{(1)} and\ (b_i'')^{(1)}$ are independent on t , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0$ 447

$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0$ 448

has a unique positive solution , which is an equilibrium solution

$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0$ 449

$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0$ 450

$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0$ 451

$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0$ 452

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(f) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T_{33}^* :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that

there exists a unique G_{37}^{*} such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)}G_{17}^{*}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)}G_{17}^{*}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T_{17}^{*})]} \quad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)}T_{17}^{*}}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)}T_{17}^{*}}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^{*})]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)}G_{21}^{*}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)}G_{21}^{*}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$$T_{28}^* = \frac{(b_{28})^{(5)} T_{29}^*}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^*)]} \quad , \quad T_{30}^* = \frac{(b_{30})^{(5)} T_{29}^*}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^*)]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^* given by $\varphi((G_{35})^*) = 0$, T_{33}^* given by $f(T_{33}^*) = 0$ and

$$G_{32}^* = \frac{(a_{32})^{(6)} G_{33}^*}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34}^* = \frac{(a_{34})^{(6)} G_{33}^*}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

$$T_{32}^* = \frac{(b_{32})^{(6)} T_{33}^*}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^*)]} \quad , \quad T_{34}^* = \frac{(b_{34})^{(6)} T_{33}^*}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^*)]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^* given by $\varphi((G_{39})^*) = 0$, T_{37}^* given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

G_{41}^* given by $\varphi((G_{43})^*) = 0$, T_{41}^* given by $f(T_{41}^*) = 0$ and

$$G_{40}^{*}=\frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}\quad,\quad G_{42}^{*}=\frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

$$T_{40}^{*}=\frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}\quad,\quad T_{42}^{*}=\frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$$

Finally we obtain the unique solution of 89 to 99 523

A

G_{45}^{*} given by $\varphi((G_{47})^{*})=0$, T_{45}^{*} given by $f(T_{45}^{*})=0$ and

$$G_{44}^{*}=\frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}\quad,\quad G_{46}^{*}=\frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$$

$$T_{44}^{*}=\frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}\quad,\quad T_{46}^{*}=\frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}\ and\ (b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i=G_i^{*}+\mathbb{G}_i\qquad,T_i=T_i^{*}+\mathbb{T}_i$$

$$\frac{\partial(a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^{*})=(q_{14})^{(1)}\quad,\ \frac{\partial(b_i'')^{(1)}}{\partial G_j}(\,G^{*}\,)=s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt}=-\big((a'_{13})^{(1)}+(p_{13})^{(1)}\big)\mathbb{G}_{13}+(a_{13})^{(1)}\mathbb{G}_{14}-(q_{13})^{(1)}G_{13}^{*}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt}=-\big((a'_{14})^{(1)}+(p_{14})^{(1)}\big)\mathbb{G}_{14}+(a_{14})^{(1)}\mathbb{G}_{13}-(q_{14})^{(1)}G_{14}^{*}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt}=-\big((a'_{15})^{(1)}+(p_{15})^{(1)}\big)\mathbb{G}_{15}+(a_{15})^{(1)}\mathbb{G}_{14}-(q_{15})^{(1)}G_{15}^{*}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt}=-\big((b'_{13})^{(1)}-(r_{13})^{(1)}\big)\mathbb{T}_{13}+(b_{13})^{(1)}\mathbb{T}_{14}+\textstyle\sum_{j=13}^{15}\big(s_{(13)(j)}T_{13}^{*}\mathbb{G}_j\big) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt}=-\big((b'_{14})^{(1)}-(r_{14})^{(1)}\big)\mathbb{T}_{14}+(b_{14})^{(1)}\mathbb{T}_{13}+\textstyle\sum_{j=13}^{15}\big(s_{(14)(j)}T_{14}^{*}\mathbb{G}_j\big) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt}=-\big((b'_{15})^{(1)}-(r_{15})^{(1)}\big)\mathbb{T}_{15}+(b_{15})^{(1)}\mathbb{T}_{14}+\textstyle\sum_{j=13}^{15}\big(s_{(15)(j)}T_{15}^{*}\mathbb{G}_j\big) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions

$(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \tag{532}$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \ , \ \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \tag{533}$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \tag{534}$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \tag{535}$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \tag{536}$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \tag{537}$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}')^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T_{17}^*\mathbb{G}_j\right) \tag{538}$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}')^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T_{18}^*\mathbb{G}_j\right) \tag{539}$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \ , \ \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}')^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \tag{541}$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}')^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \tag{542}$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}')^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \tag{543}$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}')^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T_{20}^*\mathbb{G}_j\right) \tag{544}$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^{*}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T_{22}^{*}\mathbb{G}_j\right) \quad 546$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^{*}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^{*}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^{*}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^{*}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^{*}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^{*}\mathbb{G}_j\right) \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}(\,(G_{31})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^{*}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^{*}) = (q_{41})^{(8)} \quad , \frac{\partial(b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^{*}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^{*}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^{*}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^{*}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \ , \ \frac{\partial(b_i'')^{(9)}}{\partial G_j}(\ (G_{47})^*\ ) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\ B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\ C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\ D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\ E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\ F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\ G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14}+(b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\Big]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION FOUR

Permeable Conformal Walls And Holography

INTRODUCTION—VARIABLES USED

Constantin Bachas et al JHEP06 (2002)027 doi:10.1088/1126-6708/2002/06/027 Permeable conformal walls and holography Constantin Bachas1, 5, Jan de Boer1, 5, Robbert Dijkgraaf1, 5 and Hirosi Ooguri1,5

(1) Constantin Bachas1, 5, Jan de Boer1, 5, Robbert Dijkgraaf1, 5 and Hirosi Ooguri1,5 investigate conformal field theories in (eb) two dimensions separated by domain walls, which preserve (eb&eb) at least one Virasoro algebra.
(2) They develop tools to study such domain walls, extending and clarifying the concept of `folding' discussed in the condensed-matter literature. They analyze the conditions for (e) unbroken supersymmetry, and discuss the holographic duals in (eb) AdS3 when they exist.
(3) One of the interesting observables is (=) the Casimir energy between a wall and an anti-wall.
(4) When these separate (e&eb) free scalar field theories with different target-space radii, the Casimir energy is given by (eb) the dilogarithm function of the reflection probability.
(5) The walls with holographic duals in (eb) AdS3 separate two sigma models, whose target spaces are (=) moduli spaces of (e) Yang-Mills instantons on (eb) T4 or K3.
(6) In the supergravity limit, the Casimir energy is computable as (=) classical energy of a brane that connects the walls through (e&eb) AdS3.
(7) They compare this result with (e&eb) expectations from (e) the sigma-model point of view

NOTATION

Module One

emergence of ground-state ferromagnetism in the square-lattice Tasaki-Hubbard model and the related Pauli-correlated percolation problem, as well as the dispersion-driven (eb) ground-state ferromagnetism in (eb) flat-band Hubbard systems

G_{13} : Category one of emergence of ground-state ferromagnetism in the square-lattice Tasaki-Hubbard model and the related Pauli-correlated percolation problem, as well as the dispersion-driven

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of ground-state ferromagnetism in flat-band Hubbard systems

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

For electronic systems, they discuss the

universal low-temperature behavior of (e) several flat-band Hubbard models, the emergence of ground-state ferromagnetism in (eb) the square-lattice Tasaki-Hubbard model and the related (e&eb) Pauli-correlated percolation problem, as well as the dispersion-driven (eb) ground-state ferromagnetism in (eb) flat-band Hubbard systems. Closely related studies and possible experimental realizations of the flat-band physics are also described briefly. Subjects: Strongly Correlated Electrons (cond-mat.str-el) Cite as: arXiv: 1502.02729 [cond-mat.str-el]

G_{16} : Category one of several flat-band Hubbard models, the emergence of ground-state ferromagnetism in (eb) the square-lattice Tasaki-Hubbard model and the related (e&eb) Pauli-correlated percolation problem, as well as the dispersion-driven (eb) ground-state ferromagnetism in (eb) flat-band Hubbard systems. Closely related studies and possible experimental realizations of the flat-band physics are also described briefly. Subjects: Strongly Correlated Electrons (cond-mat.str-el) Cite as: arXiv: 1502.02729 [cond-mat.str-el]

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of universal low-temperature behavior

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

universal low-temperature behavior of several flat-band Hubbard models, the emergence of ground-state ferromagnetism in (eb) the square-lattice Tasaki-Hubbard model and the related (e&eb) Pauli-correlated percolation problem, as well as the dispersion-driven (eb) ground-state ferromagnetism in (eb) flat-band Hubbard systems. Closely related studies and possible experimental realizations of the flat-band physics are also described briefly. Subjects: Strongly Correlated Electrons (cond-mat.str-el) Cite as: arXiv: 1502.02729 [cond-mat.str-el]

G_{20} : Category one of universal low-temperature behavior of several flat-band Hubbard models, the emergence of ground-state ferromagnetism

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of square-lattice Tasaki-Hubbard model and the related (e&eb) Pauli-correlated percolation problem, as well as the dispersion-driven (eb) ground-state ferromagnetism in (eb) flat-band Hubbard systems. Closely related studies and possible experimental realizations of the flat-band physics are also described briefly. Subjects: Strongly Correlated Electrons (cond-mat.str-el) Cite as: arXiv: 1502.02729 [cond-mat.str-el]

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Universal low-temperature behavior of several flat-band Hubbard models, the emergence of ground-state ferromagnetism in the square-lattice Tasaki-Hubbard model and the related (e&eb) Pauli-correlated percolation problem, as well as the dispersion-driven (eb) ground-state ferromagnetism in (eb) flat-band Hubbard systems. Closely related studies and possible experimental realizations of the flat-band physics are also described briefly. Subjects: Strongly Correlated Electrons (cond-mat.str-el) Cite as: arXiv: 1502.02729 [cond-mat.str-el]

G_{24} : Category one of universal low-temperature behavior of several flat-band Hubbard models, the emergence of ground-state ferromagnetism in the square-lattice Tasaki-Hubbard model; Pauli-correlated percolation problem, as well as the dispersion-driven (eb) ground-state ferromagnetism in (eb) flat-band Hubbard systems. Closely related studies and possible experimental realizations of the flat-band physics are also described briefly. Subjects: Strongly Correlated Electrons (cond-mat.str-el) Cite as: arXiv: 1502.02729 [cond-mat.str-el]

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of Pauli-correlated percolation problem, as well as the dispersion-driven (eb) ground-state ferromagnetism in (eb) flat-band Hubbard systems. Closely related studies and possible experimental realizations of the flat-band physics are also described briefly. Subjects: Strongly Correlated Electrons (cond-mat.str-el) Cite as: arXiv: 1502.02729 [cond-mat.str-el]; universal low-temperature behavior of several flat-band Hubbard models, the emergence of ground-state ferromagnetism in the square-lattice Tasaki-Hubbard model

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Universal low-temperature behavior of several flat-band Hubbard models, the emergence of ground-state ferromagnetism in the square-lattice Tasaki-Hubbard model and the related Pauli-correlated percolation problem, as well as the dispersion-driven (eb) ground-state ferromagnetism in (eb) flat-band Hubbard systems.

Closely related studies and possible experimental realizations of the flat-band physics are also described briefly.

G_{28} : Category one of Universal low-temperature behavior of several flat-band Hubbard models, the emergence of ground-state ferromagnetism in the square-lattice Tasaki-Hubbard model and the related Pauli-correlated percolation problem, as well as the dispersion-driven

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of ground-state ferromagnetism in (eb) flat-band Hubbard systems.

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Universal low-temperature behavior of several flat-band Hubbard models, the emergence of ground-state ferromagnetism in the square-lattice Tasaki-Hubbard model and the related Pauli-correlated percolation problem, as well as the dispersion-driven ground-state ferromagnetism in (eb) flat-band Hubbard systems

G_{32} : Category one of Universal low-temperature behavior of several flat-band Hubbard models, the emergence of ground-state ferromagnetism in the square-lattice Tasaki-Hubbard model and the related Pauli-correlated percolation problem, as well as the dispersion-driven ground-state ferromagnetism

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of flat-band Hubbard systems

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Constantin Bachas1, 5, Jan de Boer1, 5, Robbert Dijkgraaf1, 5 and Hirosi Ooguri1,5 investigate

conformal field theories in (eb) two dimensions separated by domain walls, which preserve (eb&eb) at least one Virasoro algebra

G_{36} : Category one of conformal field theories

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of two dimensions separated by domain walls, which preserve (eb&eb) at least one Virasoro algebra

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

conformal field theories in two dimensions separated by domain walls, which preserve (eb&eb) at least one Virasoro algebra

G_{40} : Category one of conformal field theories in two dimensions separated by domain walls

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of Virasoro algebras

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

They develop tools to study such domain walls, extending and clarifying the concept of `folding' discussed in the condensed-matter literature. They analyze the

conditions for (e) unbroken supersymmetry, and discuss the holographic duals in (eb) AdS3 when they exist

G_{44} : Category one of unbroken supersymmetry

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of conditions

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \qquad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**
$-(b''_{24})^{(4)}((G_{27}),t) =$ **First detritions factor**
Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$
$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$
$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$
$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28} \quad 28$$
$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$
$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**
$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**
Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**
Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**
Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}),t\right)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}),t\right)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}),t\right)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}),t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}),t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t) \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & +(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) & +(a''_{21})^{(3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t) \end{bmatrix} G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition

coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)}\ \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\ \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\ \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{22} \quad (69)$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for

category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)} & \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)} & \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad (72)$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}, \boxed{(a''_{25})^{(4)}(T_{25},t)}, \boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
are sixth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
are seventh augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
are eighth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad 78$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$

are fourth detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad (81)$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3
And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3
$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3
$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3
$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3
$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3
$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3
$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3
$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation

coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)} & +(a''_{32})^{(6)}(T_{33},t) & +(a''_{28})^{(5,5,5)}(T_{29},t) & +(a''_{24})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)} & +(a''_{33})^{(6)}(T_{33},t) & +(a''_{29})^{(5,5,5)}(T_{29},t) & +(a''_{25})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[\begin{array}{ccc} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* $1,2$ *and* 3

$-(b_{13}'')^{(1,1,1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1,1,1)}(G,t)$, $-(b_{15}'')^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b_{16}'')^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b_{20}'')^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3

$-(b_{36}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3

$-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2, and 3

$-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a_{36}')^{(7)} & +(a_{36}'')^{(7)}(T_{37},t) & +(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t) & +(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a_{37}')^{(7)} & +(a_{37}'')^{(7)}(T_{37},t) & +(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a_{38}')^{(7)} & +(a_{38}'')^{(7)}(T_{37},t) & +(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15}$$

Where $(a_{36}'')^{(7)}(T_{37},t)$, $(a_{37}'')^{(7)}(T_{37},t)$, $(a_{38}'')^{(7)}(T_{37},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{40}}{dt} =$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt}$$

$$= (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt}$$
$$= (a_{45})^{(9)} G_{44}$$
$$- \left[\begin{array}{ccc} (a'_{45})^{(9)} \boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array} \right] G_{14}$$

$$\frac{dG_{46}}{dt}$$
$$= (a_{46})^{(9)} G_{45}$$
$$- \left[\begin{array}{ccc} (a'_{46})^{(9)} \boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array} \right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$
$$(b_{44})^{(9)} T_{45} -$$
$$\left[\begin{array}{ccc} (b'_{44})^{(9)} \boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{array} \right] T_{13}$$
$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b''_i)^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}', t) - (a_i'')^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}', t)$ and $(a_i'')^{(1)}(T_{14}, t)$. (T_{14}', t) and (T_{14}, t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}\big((G_{19}), t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$\lim_{G\to\infty}(b_i'')^{(3)}\,(G_{23},t) = \ (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$

$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)},$ are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$

$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$

$(b_i'')^{(4)}\big((G_{27}),t\big) \le \ (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$

$lim_{T_2\to\infty}(a_i'')^{(4)}\,(T_{25},t) = (p_i)^{(4)}$ 118

$\lim_{G\to\infty}(b_i'')^{(4)}\,\big((G_{27}),t\big) = \ (r_i)^{(4)}$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24,25,26,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30$$ 122

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}\,(T_{29}, t) = (p_i)^{(5)}$$ 123

$$\lim_{G \to \infty}(b_i'')^{(5)}\,(G_{31}, t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\,(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ \ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}\,(T_{33}, t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G \to \infty}(b_i'')^{(6)}\,\big((G_{35}), t\big) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\big((G_{35}), t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\,(a_i)^{(6)} + (a_i')^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)}\,(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ \ (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(KK) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(LL) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq \ (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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(MM) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(NN)

$\lim_{G\to\infty}(b_i'')^{(7)}\left((G_{39}),t\right) = \ (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, \ (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}\left((G_{39}),t\right)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $\left[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\right]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(OO) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(PP) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, \ (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\,(a_i)^{(7)} + (a_i')^{(7)} + \ (\hat{A}_{36})^{(7)} + \ (\hat{P}_{36})^{(7)}\,(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\ (b_i)^{(7)} + (b_i')^{(7)} + \ (\hat{B}_{36})^{(7)} + \ (\hat{Q}_{36})^{(7)}\ (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}, \ (r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41},t) \le (p_i)^{(8)} \le (\hat{A}_{40})^{(8)} \quad 138$$

$$(b_i'')^{(8)}((G_{43}),t) \le (r_i)^{(8)} \le (b_i')^{(8)} \le (\hat{B}_{40})^{(8)} \quad 139$$

$$\lim_{T_2\to\infty}(a_i'')^{(8)}(T_{41},t) = (p_i)^{(8)} \quad 140$$

$$\lim_{G\to\infty}(b_i'')^{(8)}((G_{43}),t) = (r_i)^{(8)} \quad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}',t) - (a_i'')^{(8)}(T_{41},t)| \le (\hat{k}_{40})^{(8)}|T_{41} - T_{41}'|e^{-(\hat{M}_{40})^{(8)}t} \quad 142$$

$$|(b_i'')^{(8)}((G_{43})',t) - (b_i'')^{(8)}((G_{43}),t)| < (\hat{k}_{40})^{(8)}||(G_{43}) - (G_{43})'||e^{-(\hat{M}_{40})^{(8)}t} \quad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}',t)$ and $(a_i'')^{(8)}(T_{41},t)$. (T_{41}',t) and (T_{41},t) are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \quad 144$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad 146A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t) \le (p_i)^{(9)} \le (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t) \le (r_i)^{(9)} \le (b_i')^{(9)} \le (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty} (a_i'')^{(9)}(T_{45},t) = (p_i)^{(9)}$$

$$\lim_{G\to\infty} (b_i'')^{(9)}(G_{47},t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t) - (a_i'')^{(9)}(T_{45},t)| \le (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t) - (b_i'')^{(9)}\big((G_{47}),t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$ and$(a_i'')^{(9)}(T_{45},t)$. (T_{45}',t) and (T_{45},t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}$, $(r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\,, T_i(0)$

$$G_i(t) \le \left(\hat{P}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t} \;,\quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le \left(\hat{Q}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t} \;,\quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le \left(\hat{P}_{20}\right)^{(3)} e^{(\hat{M}_{20})^{(3)}t} \;,\quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le \left(\hat{Q}_{20}\right)^{(3)} e^{(\hat{M}_{20})^{(3)}t} \;,\quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq \left(\hat{P}_{13}\right)^{(1)}$, $T_i^0 \leq \left(\hat{Q}_{13}\right)^{(1)}$, 155

$0 \leq G_i(t) - G_i^0 \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 156

$0 \leq T_i(t) - T_i^0 \leq \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{16})^{(2)}$, $T_i^0 \leq (\hat{Q}_{16})^{(2)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''^{(2)}_{16}\right)\left(T_{17}(s_{(16)}), s_{(16)}\right)\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}\left(T_{17}(s_{(16)}), s_{(17)}\right)\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}\left(T_{17}(s_{(16)}), s_{(16)}\right)\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{20})^{(3)}$, $T_i^0 \leq (\hat{Q}_{20})^{(3)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''^{(3)}_{20}\right)\left(T_{21}(s_{(20)}), s_{(20)}\right)\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}\left(T_{21}(s_{(20)}), s_{(20)}\right)\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}\left(T_{21}(s_{(20)}), s_{(20)}\right)\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)},\ T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^{0} + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^{0} + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^{0} + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^{0} + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^{0} + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^{0} + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)},\ T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^{0} + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{32})^{(6)},\ T_i^0 \le (\hat{Q}_{32})^{(6)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{36})^{(7)},\ T_i^0 \le (\hat{Q}_{36})^{(7)},$$

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that 174

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \leq G_{32}^{0}+\int_0^t\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(g) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \leq G_{36}^{0}+\int_0^t\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$
$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \leq G_{40}^{0}+\int_0^t\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}= \quad 180$$
$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t} \leq \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for G_{41} , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \leq G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \leq \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for G_{45} , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \left\{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\right\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T} : (\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)} \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} +$$

$$(a''_{13})^{(1)} \left(T_{14}^{(1)}, s_{(13)}\right) \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}} +$$

$$G_{13}^{(2)} \left|(a''_{13})^{(1)} \left(T_{14}^{(1)}, s_{(13)}\right) - (a''_{13})^{(1)} \left(T_{14}^{(2)}, s_{(13)}\right)\right| \; e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}} \} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} t} \leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}} \left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)} (\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13{,}14{,}15$ depend only on T_{14} and respectively on G(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)} (T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(1)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}} \leq$
$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{\mathrm{A}}_{16})^{(2)} + (\widehat{\mathrm{P}}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)\mathrm{d}\left(\left((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)}\right)\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $\mathrm{G}_i\,(\mathrm{t}) = 0$ and $\mathrm{T}_i\,(\mathrm{t}) = 0$ 200

it results

$\mathrm{G}_i\,(\mathrm{t}) \geq \mathrm{G}_i^0\mathrm{e}^{\left[-\int_0^{\mathrm{t}}\{(a_i')^{(2)} - (a_i'')^{(2)}(\mathrm{T}_{17}(s_{(16)}), s_{(16)})\}\mathrm{d}s_{(16)}\right]} \geq 0$

$\mathrm{T}_i\,(\mathrm{t}) \geq \mathrm{T}_i^0\mathrm{e}^{(-(b_i')^{(2)}\mathrm{t})} > 0 \quad \text{for t} > 0$

Definition of $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1$, $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2$ and $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$\mathrm{G}_{16} < (\widehat{\mathrm{M}}_{16})^{(2)}$ it follows $\frac{\mathrm{dG}_{17}}{\mathrm{dt}} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)}\mathrm{G}_{17}$ and by integrating

$\mathrm{G}_{17} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 = \mathrm{G}_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$

In the same way , one can obtain

$\mathrm{G}_{18} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3 = \mathrm{G}_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{\mathrm{t}\to\infty}((b_i'')^{(2)}\,((G_{19})(\mathrm{t}), \mathrm{t})) = (b_{17}')^{(2)}$ then $\mathrm{T}_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$: $\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0$ *and* $T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$G_{24}^{(2)} \left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)}\right) d\left(((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on T_{25} and respectively on (G_{27})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (228)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_2 / (a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\widehat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{32})^{(6)}\right] \leq (\widehat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}), (\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}), (\widetilde{T_{35}}) \right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right| \; e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} \} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)} (\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)}; \; (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t} \; and \; (\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$ (250)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2 \; and \; \left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33} \; and \; G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)} \left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on (G_{39})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)} (T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(7)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}),(\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}),(\widetilde{T_{43}})\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1,\left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right),\quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{47}}),(\widetilde{T_{47}}):\left((\widetilde{G_{47}}),(\widetilde{T_{47}})\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$

From 99 to 44 it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(9)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$: 281

By $(v_1)^{(1)} > 0$, $(v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{v}_1)^{(1)},, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{v}_1)^{(1)} > 0$, $(\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)},, (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)}, , (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad (333)$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad (334)$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)} t} - e^{-(a'_{26})^{(4)} t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)} t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad (335)$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad (336)$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G^0_{28} e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G^0_{28}}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G^0_{30} e^{-(S_2)^{(5)} t} \le G_{30}(t) \le \right. \quad 344$$

$$\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_2)^{(5)}((S_1)^{(5)}-(a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t}-e^{-(a'_{30})^{(5)}t}\right]+\ G_{30}^{0}e^{-(a'_{30})^{(5)}t}\Big)$$

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\le T_{30}(t)\le \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$:

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)}\le-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\le-(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)}\le-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\le-(\tau_1)^{(6)}$$

Definition of $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$: 350

By $(v_1)^{(6)}>0$, $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$, $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

Definition of $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)}>0$, $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$, $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

Definition of $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$:-

If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)}=(v_0)^{(6)},(m_1)^{(6)}=(v_1)^{(6)},$ ***if*** $(v_0)^{(6)}<(v_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}$ $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}$ $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}$ $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}$ $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}$ $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})} \left[e^{(S_1)^{(6)} t} - e^{-(a'_{34})^{(6)} t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)} t} \right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}$, $\boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$

$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)}$,
and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}$, $\boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)}$,
and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^{0}e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}}G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}}G_{36}^{0}e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t}-e^{-(S_2)^{(7)}t}\right]+G_{38}^{0}e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left.\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_2)^{(7)}((S_1)^{(7)}-(a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t}-e^{-(a'_{38})^{(7)}t}\right]+\ G_{38}^{0}e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}}T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}}T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right) - (b''_{41})^{(8)}\left((G_{43}), t\right) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(v_1)^{(8)} > 0$, $(v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$, $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{v}_1)^{(8)}$,, $(\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{v}_1)^{(8)} > 0$, $(\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$, $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$:-

If we define $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$, $\boldsymbol{if}$ $(v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$, $\boldsymbol{if}$ $(v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$,
and $\boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}$, $\boldsymbol{if}$ $(\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, $\boldsymbol{if}$ $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, $\boldsymbol{if}$ $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}$ $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t})$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t}\leq T_{42}(t)\leq \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)},(S_2)^{(8)},(R_1)^{(8)},(R_2)^{(8)}$:- 381

Where $(S_1)^{(8)}=(a_{40})^{(8)}(m_2)^{(8)}-(a'_{40})^{(8)}$

$(S_2)^{(8)}=(a_{42})^{(8)}-(p_{42})^{(8)}$

$(R_1)^{(8)}=(b_{40})^{(8)}(\mu_2)^{(8)}-(b'_{40})^{(8)}$

$(R_2)^{(8)}=(b'_{42})^{(8)}-(r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$:

$(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)}\leq -(a'_{44})^{(9)}+(a'_{45})^{(9)}-(a''_{44})^{(9)}(T_{45},t)+(a''_{45})^{(9)}(T_{45},t)\leq -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)}\leq -(b'_{44})^{(9)}+(b'_{45})^{(9)}-(b''_{44})^{(9)}\big((G_{47}),t\big)-(b''_{45})^{(9)}\big((G_{47}),t\big)\leq -(\tau_1)^{(9)}$

Definition of $(v_1)^{(9)},(v_2)^{(9)},(u_1)^{(9)},(u_2)^{(9)},v^{(9)},u^{(9)}$:

By $(v_1)^{(9)}>0$, $(v_2)^{(9)}<0$ and respectively $(u_1)^{(9)}>0$, $(u_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_1)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)}>0$, $(\bar{v}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$, $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

Definition of $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$:-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)},\ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$
and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \le G_{46}(t) \le \right.$$
$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (v_1)^{(1)} \le (\bar{v}_1)^{(1)} \le \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$, we obtain 386

$$(v_1)^{(1)} \le v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $v^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\right) - (a''_{17})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{\mathrm{G}_{16}}{\mathrm{G}_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (\mathrm{C})^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (\mathrm{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)}\right) \le \frac{dv^{(3)}}{dt} \le -\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \ge \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}}$$

it follows $(v_0)^{(3)} \le v^{(3)}(t) \le (v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \le v^{(3)}(t) \le (\bar{v}_1)^{(3)}$

If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)} \le \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}} \le v^{(3)}(t) \le$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (\bar{v}_1)^{(3)}$$

If $0 < (v_1)^{(3)} \le (\bar{v}_1)^{(3)} \le (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$, we obtain 403

$$(v_1)^{(3)} \le v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

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If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case,

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

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If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

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$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{36}'')^{(7)} = (a_{37}'')^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b_{36}'')^{(7)} = (b_{37}'')^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a_{40}')^{(8)} - (a_{41}')^{(8)} + (a_{40}'')^{(8)}(T_{41}, t)\right) - (a_{41}'')^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(8)} and\ (b''_i)^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\,]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\,]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\,]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0$ 479

$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0$ 480

$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0$ 481

$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0$ 482

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(g) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first

equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T^*_{33} for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$$

Definition and uniqueness of T^*_{37} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T^*_{37} for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$$

Definition and uniqueness of T^*_{41} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{25} such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507
$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{29} such that $\varphi((G_{31})^*) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{33} such that $\varphi({G_{35}}^*) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{37} such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{41} such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right]+(b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{45} such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)} G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)} G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)} T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)} T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)} G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)} G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)} T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)} T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)} G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)} G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)} T_{21}^*}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T_{22}^* = \frac{(b_{22})^{(3)} T_{21}^*}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]} \quad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \qquad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \qquad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \qquad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \qquad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \qquad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \qquad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i \qquad 532$$

$$\frac{\partial (a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial (b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \qquad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \qquad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \qquad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \qquad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \qquad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}\mathrm{T}^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}\mathrm{T}^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}\left((G_{27})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^{*}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^{*}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^{*}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^{*}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^{*}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^{*}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^{*}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^{*}\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\ B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\Big)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\Big]$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\Big)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\Big)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\Big]$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\Big)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\Big)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\Big]$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\Big)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\Big)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\Big)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\Big)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION FOUR A

Holographic Duals In AdS3

INTRODUCTION—VARIABLES USED

Constantin Bachas et al JHEP06 (2002)027 doi:10.1088/1126-6708/2002/06/027 Permeable conformal walls and holography Constantin Bachas1, 5, Jan de Boer1, 5, Robbert Dijkgraaf1, 5 and Hirosi Ooguri1,5

(1) One of the interesting observables is (=) the Casimir energy between a wall and an anti-wall.
(2) When these separate (e&eb) free scalar field theories with different target-space radii, the Casimir energy is given by (eb) the dilogarithm function of the reflection probability.

(3) The walls with holographic duals in (eb) AdS3 separate two sigma models, whose target spaces are (=) moduli spaces of (e) Yang-Mills instantons on (eb) T4 or K3.

(4) In the supergravity limit, the Casimir energy is computable as (=) classical energy of a brane that connects the walls through (e&eb) AdS3.

They compare this result with (e&eb) expectations from (e) the sigma-model point of view

NOTATION

Module One

holographic duals in (eb) AdS3

G_{13} : Category one of holographic duals

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of AdS3

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

One of the interesting observables is (=) the Casimir energy between a wall and an anti-wall.

G_{16} : Category one of observables

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of Casimir energy between a wall and an anti-wall

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

interesting observables is the Casimir energy between a wall and an anti-wall

G_{20} : Category one of observables is the Casimir energy; wall and an anti-wall

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of wall and an anti-wall; observables is the Casimir energy

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

When observables separate free scalar field theories with different target-space radii, the Casimir energy is

given by (eb) the dilogarithm function of the reflection probability

G_{24} : Category one of observables separate free scalar field theories; different target-space radii, the Casimir energy is given by (eb) the dilogarithm function of the reflection probability

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of different target-space radii, the Casimir energy is given by (eb) the dilogarithm function of the reflection probability; observables separate free scalar field theories

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

observables separate free scalar field theories with different target-space radii, the Casimir energy is given by (eb) the dilogarithm function of the reflection probability

G_{28} : Category one of observables separate free scalar field theories with different target-space radii, the Casimir energy

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of dilogarithm function of the reflection probability

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

observables separate free scalar field theories with different target-space radii, the Casimir energy is given by the dilogarithm function of the reflection probability

G_{32} : Category one of reflection probability

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of observables separate free scalar field theories with different target-space radii, the Casimir energy is given by the dilogarithm function

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

The walls with holographic duals in (eb) AdS3 separate two sigma models, whose target spaces are (=)

moduli spaces of (e) Yang-Mills instantons on (eb) T4 or K3

G_{36} : Category one of walls with holographic duals

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of AdS3 separate two sigma models, whose target spaces are (=) moduli spaces of (e) Yang-Mills instantons on (eb) T4 or K3

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

walls with holographic duals in AdS3 separate two sigma models, whose target spaces are (=) moduli spaces of (e) Yang-Mills instantons on (eb) T4 or K3

G_{40} : Category one of walls with holographic duals in AdS3 separate two sigma models, whose target spaces

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of moduli spaces of (e) Yang-Mills instantons on (eb) T4 or K3

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

walls with holographic duals in AdS3 separate two sigma models, whose target spaces are moduli spaces of (e) Yang-Mills instantons on (eb) T4 or K3

G_{44} : Category one of Yang-Mills instantons on T4 or K3

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of walls with holographic duals in AdS3 separate two sigma models, whose target spaces are moduli spaces

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}), t\big)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}), t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\big((G_{19}), t\big) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}),t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}),t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}),t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}),t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}),t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\begin{array}{ccc} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) \quad +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{array}\right]G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) & \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) & \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) & \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b_{20}'')^{(3,3,)}(G_{23},t)$, $-(b_{21}'')^{(3,3,)}(G_{23},t)$, $-(b_{22}'')^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b_{24}'')^{(4,4,4,4,)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b_{28}'')^{(5,5,5,5,)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b_{32}'')^{(6,6,6,6,)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b_{37}'')^{(7,7,)}(G_{39},t)$, $-(b_{36}'')^{(7,7,)}(G_{39},t)$, $-(b_{38}'')^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b_{40}'')^{(8,8)}(G_{43},t)$ $-(b_{41}'')^{(8,8)}(G_{43},t)$ $-(b_{42}'')^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{c} (a_{16}')^{(2)}+(a_{16}'')^{(2)}(T_{17},t)\ +(a_{13}'')^{(1,1,)}(T_{14},t)\ +(a_{20}'')^{(3,3,3)}(T_{21},t) \\ +(a_{24}'')^{(4,4,4,4,4)}(T_{25},t)\ +(a_{28}'')^{(5,5,5,5,5)}(T_{29},t)\ +(a_{32}'')^{(6,6,6,6,6)}(T_{33},t) \\ +(a_{36}'')^{(7,7,7)}(T_{37},t)\ +(a_{40}'')^{(8,8,8)}(T_{41},t)\ +(a_{44}'')^{(9,9)}(T_{45},t) \end{array}\right] G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{c} (a_{17}')^{(2)}+(a_{17}'')^{(2)}(T_{17},t)\ +(a_{14}'')^{(1,1,)}(T_{14},t)\ +(a_{21}'')^{(3,3,3)}(T_{21},t) \\ +(a_{25}'')^{(4,4,4,4,4)}(T_{25},t)\ +(a_{29}'')^{(5,5,5,5,5)}(T_{29},t)\ +(a_{33}'')^{(6,6,6,6,6)}(T_{33},t) \\ +(a_{37}'')^{(7,7,7)}(T_{37},t)\ +(a_{41}'')^{(8,8,8)}(T_{41},t)\ +(a_{45}'')^{(9,9)}(T_{45},t) \end{array}\right] G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{c} (a_{18}')^{(2)}+(a_{18}'')^{(2)}(T_{17},t)\ +(a_{15}'')^{(1,1,)}(T_{14},t)\ +(a_{22}'')^{(3,3,3)}(T_{21},t) \\ +(a_{26}'')^{(4,4,4,4,4)}(T_{25},t)\ +(a_{30}'')^{(5,5,5,5,5)}(T_{29},t)\ +(a_{34}'')^{(6,6,6,6,6)}(T_{33},t) \\ +(a_{38}'')^{(7,7,7)}(T_{37},t)\ +(a_{42}'')^{(8,8,8)}(T_{41},t)\ +(a_{46}'')^{(9,9)}(T_{45},t) \end{array}\right] G_{18} \quad 63$$

Where $+(a_{16}'')^{(2)}(T_{17},t)$, $+(a_{17}'')^{(2)}(T_{17},t)$, $+(a_{18}'')^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{13}'')^{(1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) \end{bmatrix} T_{18} \quad (66)$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)} & +(a''_{20})^{(3)}(T_{21},t) & +(a''_{16})^{(2,2,2)}(T_{17},t) & +(a''_{13})^{(1,1,1,)}(T_{14},t) \\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9)}(T_{45},t) \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)} & +(a''_{21})^{(3)}(T_{21},t) & +(a''_{17})^{(2,2,2)}(T_{17},t) & +(a''_{14})^{(1,1,1,)}(T_{14},t) \\ +(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{37})^{(7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)} & +(a''_{22})^{(3)}(T_{21},t) & +(a''_{18})^{(2,2,2)}(T_{17},t) & +(a''_{15})^{(1,1,1,)}(T_{14},t) \\ +(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a''_{38})^{(7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{22} \quad 69$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad 70$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{22} \quad 72$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for*

category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & \boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & \boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & \boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* $1, 2\ 3$

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category $1, 2$ *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category $1, 2$ *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
are sixth augmentation coefficients for category $1, 2$ *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
are seventh augmentation coefficients for category $1, 2$ *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$

are eighth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$

are fourth detrition coefficients for category 1, 2 *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$

are fifth detrition coefficients for category 1, 2 *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$

are sixth detrition coefficients for category 1, 2 *and* 3

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$

are seventh detrition coefficients for category 1, 2 *and* 3

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$

are eighth detrition coefficients for category 1, 2 *and* 3

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) & \\ +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{30} \quad 81$$

Where $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) & \\ -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) & \\ -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) & \\ -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{30} \quad 84$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for

category 1,2, and 3

$-(b_{16}'')^{(2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b_{20}'')^{(3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b_{36}'')^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b_{42}'')^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b_{40}'')^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b_{46}'')^{(9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a_{32}')^{(6)} & +(a_{32}'')^{(6)}(T_{33},t) & +(a_{28}'')^{(5,5,5)}(T_{29},t) & +(a_{24}'')^{(4,4,4,)}(T_{25},t) \\ +(a_{13}'')^{(1,1,1,1,1,1)}(T_{14},t) & & +(a_{16}'')^{(2,2,2,2,2,2)}(T_{17},t) & +(a_{20}'')^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a_{36}'')^{(7,7,7,7,7,7,7)}(T_{37},t) & & +(a_{40}'')^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a_{44}'')^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a_{33}')^{(6)} & +(a_{33}'')^{(6)}(T_{33},t) & +(a_{29}'')^{(5,5,5)}(T_{29},t) & +(a_{25}'')^{(4,4,4,)}(T_{25},t) \\ +(a_{14}'')^{(1,1,1,1,1,1)}(T_{14},t) & & +(a_{17}'')^{(2,2,2,2,2,2)}(T_{17},t) & +(a_{21}'')^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a_{37}'')^{(7,7,7,7,7,7,7)}(T_{37},t) & & +(a_{41}'')^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a_{34}')^{(6)} & +(a_{34}'')^{(6)}(T_{33},t) & +(a_{30}'')^{(5,5,5)}(T_{29},t) & +(a_{26}'')^{(4,4,4,)}(T_{25},t) \\ +(a_{15}'')^{(1,1,1,1,1,1)}(T_{14},t) & & +(a_{18}'')^{(2,2,2,2,2,2)}(T_{17},t) & +(a_{22}'')^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a_{38}'')^{(7,7,7,7,7,7,7)}(T_{37},t) & & +(a_{42}'')^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{34} \quad (87)$$

$+(a_{32}'')^{(6)}(T_{33},t)$, $+(a_{33}'')^{(6)}(T_{33},t)$, $+(a_{34}'')^{(6)}(T_{33},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

$+(a_{28}'')^{(5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5)}(T_{29},t)$ *are second augmentation coefficients for category* 1, 2 *and* 3

$+(a_{24}'')^{(4,4,4,)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,)}(T_{25},t)$ *are third augmentation coefficients for category* 1, 2 *and* 3

$+(a_{13}'')^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1,1)}(T_{14},t)$ - are fourth augmentation coefficients

$+(a_{16}'')^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2,2,2)}(T_{17},t)$ - fifth augmentation coefficients

$+(a_{20}'')^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a_{36}'')^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$+(a_{40}'')^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8,8,8)}(T_{41},t)$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \qquad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt}$$
$$= (a_{42})^{(8)} G_{41}$$
$$- \left[\begin{array}{c} (a'_{42})^{(8)} \boxed{+(a''_{42})^{(8)}(T_{41},t)} \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array} \right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$
$$(b_{40})^{(8)} T_{41} - \left[\begin{array}{c} (b'_{40})^{(8)} \boxed{-(b''_{40})^{(8)}(G_{43},t)} \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{13}$$

$$\frac{dT_{41}}{dt} =$$
$$(b_{41})^{(8)} T_{40} - \left[\begin{array}{c} (b'_{41})^{(8)} \boxed{-(b''_{41})^{(8)}(G_{43},t)} \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{14}$$

$$\frac{dT_{42}}{dt} =$$
$$(b_{42})^{(8)} T_{41} - \left[\begin{array}{c} (b'_{42})^{(8)} \boxed{-(b''_{42})^{(8)}(G_{43},t)} \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{38}'')^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

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$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a_{44}')^{(9)}\boxed{+(a_{44}'')^{(9)}(T_{45},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a_{45}')^{(9)}\boxed{+(a_{45}'')^{(9)}(T_{45},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a_{46}')^{(9)}\boxed{+(a_{46}'')^{(9)}(T_{37},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{44}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{46}'')^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second

augmentation coefficient for category 1, 2 and 3
$+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3
$+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3
$+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3
$+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$
$$(b_{44})^{(9)}T_{45} -$$
$$\begin{bmatrix} (b_{44}')^{(9)}\left[-(b_{44}'')^{(9)}(G_{47},t)\right] & \left[-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{13}$$
$$\frac{dT_{45}}{dt} =$$
$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}\left[-(b_{45}'')^{(9)}(G_{47},t)\right] & \left[-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{14}$$
$$\frac{dT_{46}}{dt} =$$
$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}\left[-(b_{46}'')^{(9)}(G_{47},t)\right] & \left[-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{15}$$

Where $-(b_{44}'')^{(9)}(G_{47},t)$, $-(b_{45}'')^{(9)}(G_{47},t)$, $-(b_{46}'')^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3
$-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}'| e^{-(\hat{M}_{20})^{(3)} t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} ||G_{23} - G_{23}'|| e^{-(\hat{M}_{20})^{(3)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and $(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\ ,\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)},\ (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)},\ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}), t\big) \le \ (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}\,(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\,\big((G_{27}), t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)},\ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - \ T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30$$ 122

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)}$$ 123

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}\left((G_{35}), t\right) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(QQ) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(RR) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

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(SS) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(TT)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(UU) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(VV) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)} \qquad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be

noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\ ,\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}<1 \qquad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+\ (\hat{A}_{40})^{(8)}+(\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \qquad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[\ (b_i)^{(8)}+(b_i')^{(8)}+\ (\hat{B}_{40})^{(8)}+(\hat{Q}_{40})^{(8)}\ (\hat{k}_{40})^{(8)}]<1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \qquad 146A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t)-(a_i'')^{(9)}(T_{45},t)|\le(\hat{k}_{44})^{(9)}|T_{45}-\ T_{45}'|e^{-(\widehat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t)-(b_i'')^{(9)}\big((G_{47}),t\big)|<(\hat{k}_{44})^{(9)}||(G_{47})-(G_{47})'||e^{-(\widehat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$

and $(a_i'')^{(9)}(T_{45},t)$. (T_{45}',t) and (T_{45},t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)},(\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)}=1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},$ are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\ ,\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}<1$$

Definition of $(\hat{P}_{44})^{(9)},(\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},(\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)},(a_i')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(p_i)^{(9)},\ (r_i)^{(9)},i=44,45,46,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)}+(a_i')^{(9)}+\ (\hat{A}_{44})^{(9)}+\ (\hat{P}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)}+(b_i')^{(9)}+\ (\hat{B}_{44})^{(9)}+\ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\ ,T_i(0)$:

$$G_i(t)\le\ (\hat{P}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}\ ,\ \boxed{G_i(0)=G_i^0>0}$$

$$T_i(t)\le\ (\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}\ ,\ \boxed{T_i(0)=T_i^0>0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\ ,T_i(0)$

$$G_i(t)\le\ (\hat{P}_{16})^{(2)}e^{(\hat{M}_{16})^{(2)}t}\ ,\quad G_i(0)=G_i^0>0$$

$$T_i(t)\le\ (\hat{Q}_{16})^{(2)}e^{(\hat{M}_{16})^{(2)}t}\ ,\quad T_i(0)=T_i^0>0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t)\le\ (\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}t}\ ,\quad G_i(0)=G_i^0>0$$

$$T_i(t)\le\ (\hat{Q}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}t}\ ,\quad T_i(0)=T_i^0>0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0)\ ,T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{\left(\hat{M}_{24}\right)^{(4)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{\left(\hat{M}_{24}\right)^{(4)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{\left(\hat{M}_{28}\right)^{(5)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{\left(\hat{M}_{28}\right)^{(5)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{\left(\hat{M}_{32}\right)^{(6)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{\left(\hat{M}_{32}\right)^{(6)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{\left(\hat{M}_{36}\right)^{(7)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{\left(\hat{M}_{36}\right)^{(7)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{\left(\hat{M}_{40}\right)^{(8)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{\left(\hat{M}_{40}\right)^{(8)} t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{\left(\hat{M}_{44}\right)^{(9)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{\left(\hat{M}_{44}\right)^{(9)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{13})^{(1)}, T_i^0 \le (\hat{Q}_{13})^{(1)}$, 155

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} t}$ 156

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}, T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\tilde{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\tilde{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{32})^{(6)}$, $T_i^0 \leq (\hat{Q}_{32})^{(6)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0,t)$

Proof: 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(h) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left(e^{(\tilde{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\tilde{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left(e^{(\tilde{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\tilde{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0\right) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\tilde{M}_{44})^{(9)}} \left(e^{(\tilde{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\tilde{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\tilde{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\tilde{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\tilde{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[(\widehat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on

$G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 19 to 24 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(1)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If $G_{13}\ \ is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}\,(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on

(G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)} t)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18}. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16}, G_{18} and G_{16}, G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18}. The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad (208)$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad (209)$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself (210)

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric (211)

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{20})^{(3)}t}\}$$

Indeed if we denote (212)

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results (213)

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad (214)$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\,(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. (215)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on T_{21} and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \quad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 227

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on T_{25} and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(4)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\,((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\widehat{P}_{28})^{(5)}$ and $(\widehat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\widehat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{31}}),(\widetilde{T_{31}})$: $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\leq \quad 237$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\ (G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 238

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(5)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)}+\left((\widehat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\widehat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\widehat{Q}_{32})^{(6)}\right]\leq(\widehat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(6)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose

$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{39}}),(\widetilde{T_{39}}):\left((\widetilde{G_{39}}),(\widetilde{T_{39}})\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$

It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\leq \quad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\ (G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})}, \widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by 274

$(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$. The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right)) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0, (\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0, (u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0, (\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0, (\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}, \ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}, \ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13} e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13} e^{(S_1)^{(1)} t}$

where $(p_i)^{(1)}$ is defined by equation

$\frac{1}{(m_1)^{(1)}} G^0_{13} e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G^0_{13} e^{(S_1)^{(1)} t}$

$$\left(\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t}-e^{-(S_2)^{(1)}t}\right]+G_{15}^{0}e^{-(S_2)^{(1)}t}\le G_{15}(t)\le\right. \quad (286)$$

$$\left.\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}\right]+G_{15}^{0}e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation (291)

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: (292)

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(T_{17},t)+(a''_{17})^{(2)}(T_{17},t)\le-(\sigma_1)^{(2)} \quad (293)$$

$$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)} \quad (294)$$

Definition of $(\nu_1)^{(2)},(\nu_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: (295)

By $(\nu_1)^{(2)}>0$, $(\nu_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0$, $(u_2)^{(2)}<0$ the roots (296)

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_1)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ (297)

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and (298)

Definition of $(\bar{\nu}_1)^{(2)},,(\bar{\nu}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: (299)

By $(\bar{\nu}_1)^{(2)}>0$, $(\bar{\nu}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0$, $(\bar{u}_2)^{(2)}<0$ the (300)

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_2)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ (301)

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t}\right] + T_{18}^0 e^{-(R_2)^{(2)}t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$

$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations
$(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$
and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by
$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$$

$$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(v^{(4)}\big)^2 + (\sigma_1)^{(4)} v^{(4)} - (a_{24})^{(4)} = 0$

and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)},,(\bar{v}_2)^{(4)},(\bar{u}_1)^{(4)},(\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$ and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)},(\mu_2)^{(4)},(v_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)},(\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, ***if*** $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)}t} - e^{-(a'_{26})^{(4)}t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)}t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^{0}}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t}-e^{-(R_2)^{(4)}t}\right]+T_{26}^{0}e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)},(S_2)^{(4)},(R_1)^{(4)},(R_2)^{(4)}$:- 337

Where $(S_1)^{(4)}=(a_{24})^{(4)}(m_2)^{(4)}-(a'_{24})^{(4)}$

$(S_2)^{(4)}=(a_{26})^{(4)}-(p_{26})^{(4)}$

$(R_1)^{(4)}=(b_{24})^{(4)}(\mu_2)^{(4)}-(b'_{24})^{(4)}$

$(R_2)^{(4)}=(b'_{26})^{(4)}-(r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)},(\sigma_2)^{(5)},(\tau_1)^{(5)},(\tau_2)^{(5)}$:

$(\sigma_1)^{(5)},(\sigma_2)^{(5)},(\tau_1)^{(5)},(\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)}\le-(a'_{28})^{(5)}+(a'_{29})^{(5)}-(a''_{28})^{(5)}(T_{29},t)+(a''_{29})^{(5)}(T_{29},t)\le-(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)}\le-(b'_{28})^{(5)}+(b'_{29})^{(5)}-(b''_{28})^{(5)}\big((G_{31}),t\big)-(b''_{29})^{(5)}\big((G_{31}),t\big)\le-(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)},(\nu_2)^{(5)},(u_1)^{(5)},(u_2)^{(5)},\nu^{(5)},u^{(5)}$: 339

By $(\nu_1)^{(5)}>0$, $(\nu_2)^{(5)}<0$ and respectively $(u_1)^{(5)}>0$, $(u_2)^{(5)}<0$ the roots of the equations $(a_{29})^{(5)}\left(\nu^{(5)}\right)^2+(\sigma_1)^{(5)}\nu^{(5)}-(a_{28})^{(5)}=0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2+(\tau_1)^{(5)}u^{(5)}-(b_{28})^{(5)}=0$ and

Definition of $(\bar{\nu}_1)^{(5)},,(\bar{\nu}_2)^{(5)},(\bar{u}_1)^{(5)},(\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)}>0$, $(\bar{\nu}_2)^{(5)}<0$ and respectively $(\bar{u}_1)^{(5)}>0$, $(\bar{u}_2)^{(5)}<0$ the roots of the equations $(a_{29})^{(5)}\left(\nu^{(5)}\right)^2+(\sigma_2)^{(5)}\nu^{(5)}-(a_{28})^{(5)}=0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2+(\tau_2)^{(5)}u^{(5)}-(b_{28})^{(5)}=0$

Definition of $(m_1)^{(5)},(m_2)^{(5)},(\mu_1)^{(5)},(\mu_2)^{(5)},(\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)},(m_2)^{(5)},(\mu_1)^{(5)},(\mu_2)^{(5)}$ by

$(m_2)^{(5)}=(\nu_0)^{(5)},(m_1)^{(5)}=(\nu_1)^{(5)}$, $\boldsymbol{if}\ (\nu_0)^{(5)}<(\nu_1)^{(5)}$

$(m_2)^{(5)}=(\nu_1)^{(5)},(m_1)^{(5)}=(\bar{\nu}_1)^{(5)}$, $\boldsymbol{if}\ (\nu_1)^{(5)}<(\nu_0)^{(5)}<(\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)}=\frac{G_{28}^{0}}{G_{29}^{0}}}$

$(m_2)^{(5)}=(\nu_1)^{(5)},(m_1)^{(5)}=(\nu_0)^{(5)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(5)}<(\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)}=(u_0)^{(5)},(\mu_1)^{(5)}=(u_1)^{(5)}$, $\boldsymbol{if}\ (u_0)^{(5)}<(u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, $\boldsymbol{if}$ $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}$ $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{28}(t) \leq G_{28}^0 e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{29}(t) \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})} \left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t} \right] + G_{30}^0 e^{-(S_2)^{(5)} t} \leq G_{30}(t) \leq \right. \quad 344$$

$$\left. \frac{(a_{30})^{(5)} G_{28}^0}{(m_2)^{(5)}((S_1)^{(5)} - (a'_{30})^{(5)})} \left[e^{(S_1)^{(5)} t} - e^{-(a'_{30})^{(5)} t} \right] + G_{30}^0 e^{-(a'_{30})^{(5)} t} \right)$$

$$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \leq T_{28}(t) \leq T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \leq T_{28}(t) \leq \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})} \left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t} \right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \leq T_{30}(t) \leq \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})} \left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t} \right] + T_{30}^0 e^{-(R_2)^{(5)} t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \leq -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \leq -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \leq -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}((G_{35}), t) - (b''_{33})^{(6)}((G_{35}), t) \leq -(\tau_1)^{(6)}$$

Definition of $(v_1)^{(6)}, (v_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, v^{(6)}, u^{(6)}$: 350

By $(v_1)^{(6)} > 0$, $(v_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{v}_1)^{(6)},, (\bar{v}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)} > 0$, $(\bar{v}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}$, $(m_2)^{(6)}$, $(\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}$, $(m_2)^{(6)}$, $(\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t} - e^{-(a'_{34})^{(6)}t}\right] + G_{34}^0 e^{-(a'_{34})^{(6)}t}\right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}}T_{32}^0 e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(v_1)^{(7)}, (v_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, v^{(7)}, u^{(7)}$: 361

By $(v_1)^{(7)} > 0, (v_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{v}_1)^{(7)}, , (\bar{v}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)} > 0, (\bar{v}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)},\ \boldsymbol{if}\ (\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)},\ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}),t\big) - (b''_{45})^{(9)}\big((G_{47}),t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)},, (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, ***if*** $(\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, ***if*** $(\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, ***if*** $(\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,
and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44}e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44}e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G^0_{44}e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G^0_{44}e^{(S_1)^{(9)}t}$$

(

$$\frac{(a_{46})^{(9)}G^0_{44}}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})}\Big[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t}\Big] + G^0_{46}e^{-(S_2)^{(9)}t} \le G_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}G^0_{44}}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})}[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G^0_{46}e^{-(a'_{46})^{(9)}t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \le \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \le \nu^{(1)}(t) \le (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \le \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \le \nu^{(1)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \le (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \le (\bar{\nu}_1)^{(1)} \le \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \le \nu^{(1)}(t) \le \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \le (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \le \nu^{(1)}(t) \le (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \le u^{(1)}(t) \le (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{dv^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)v^{(2)} - (a_{17})^{(2)}v^{(2)}$$

Definition of $v^{(2)}$:- $\boxed{v^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(v^{(2)}\right)^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)}\right) \leq \frac{dv^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(v^{(2)}\right)^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{v}_1)^{(2)}, (v_0)^{(2)}$:-

For $0 < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (v_1)^{(2)} < (\bar{v}_1)^{(2)}$

$$v^{(2)}(t) \geq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(v_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (v_2)^{(2)}}}$$

it follows $(v_0)^{(2)} \leq v^{(2)}(t) \leq (v_1)^{(2)}$

In the same manner , we get 391

$$v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{v}_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (\bar{v}_2)^{(2)}}}$$

From which we deduce $(v_0)^{(2)} \leq v^{(2)}(t) \leq (\bar{v}_1)^{(2)}$ 392

If $0 < (v_1)^{(2)} < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{v}_1)^{(2)}$ we find like in the previous case, 393

$$(v_1)^{(2)} \leq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_2)^{(2)}\right)t\right]}} \leq v^{(2)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \leq (\bar{v}_1)^{(2)}$$

If $0 < (v_1)^{(2)} \leq (\bar{v}_1)^{(2)} \leq (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(v_1)^{(2)} \leq v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \leq (v_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(2)}(t)$:- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t)\right) - (a_{21}'')^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $v^{(4)}$:- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \leq \frac{dv^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(4)}, (v_0)^{(4)}$:-

For $0 < \boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (v_1)^{(4)} < (\bar{v}_1)^{(4)}$

$$v^{(4)}(t) \geq \frac{(v_1)^{(4)} + (C)^{(4)}(v_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((v_1)^{(4)} - (v_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((v_1)^{(4)} - (v_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(v_1)^{(4)} - (v_0)^{(4)}}{(v_0)^{(4)} - (v_2)^{(4)}}}$$

it follows $(v_0)^{(4)} \leq v^{(4)}(t) \leq (v_1)^{(4)}$

In the same manner , we get 405

$$v^{(4)}(t) \leq \frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)}(\bar{v}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{v}_1)^{(4)} - (v_0)^{(4)}}{(v_0)^{(4)} - (\bar{v}_2)^{(4)}}}$$

From which we deduce $(v_0)^{(4)} \leq v^{(4)}(t) \leq (\bar{v}_1)^{(4)}$

If $0 < (v_1)^{(4)} < (v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{v}_1)^{(4)}$ we find like in the previous case, 406

$$(v_1)^{(4)} \leq \frac{(v_1)^{(4)} + (C)^{(4)}(v_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((v_1)^{(4)} - (v_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((v_1)^{(4)} - (v_2)^{(4)}\right)t\right]}} \leq v^{(4)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)}(\bar{v}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}\right)t\right]}} \leq (\bar{v}_1)^{(4)}$$

If $0 < (v_1)^{(4)} \leq (\bar{v}_1)^{(4)} \leq \boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(v_1)^{(4)} \leq v^{(4)}(t) \leq \frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)}(\bar{v}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}\right)t\right]}} \leq (v_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(4)}(t)$:-

$$(m_2)^{(4)} \leq v^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{v^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then
$(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

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If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\quad \boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$v^{(6)}(t) \le \frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)}=\frac{(\bar{v}_1)^{(6)}-(v_0)^{(6)}}{(v_0)^{(6)}-(\bar{v}_2)^{(6)}}}$$

From which we deduce $(v_0)^{(6)} \le v^{(6)}(t) \le (\bar{v}_1)^{(6)}$

If $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{v}_1)^{(6)}$ we find like in the previous case, 414

$$(v_1)^{(6)} \le \frac{(v_1)^{(6)}+(C)^{(6)}(v_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)}-(v_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)}-(v_2)^{(6)}\right)t\right]}} \le v^{(6)}(t) \le$$

$$\frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \le (\bar{v}_1)^{(6)}$$

If $0 < (v_1)^{(6)} \le (\bar{v}_1)^{(6)} \le \boxed{(v_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(v_1)^{(6)} \le v^{(6)}(t) \le \frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \le (v_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(6)}(t)$:-

$$(m_2)^{(6)} \le v^{(6)}(t) \le (m_1)^{(6)}, \quad \boxed{v^{(6)}(t)=\frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \le u^{(6)}(t) \le (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t)=\frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(v_1)^{(6)} = (\bar{v}_1)^{(6)}$ if in addition $(v_0)^{(6)} = (v_1)^{(6)}$ then $v^{(6)}(t) = (v_0)^{(6)}$ and as a consequence $G_{32}(t) = (v_0)^{(6)}G_{33}(t)$ **this also defines** $(v_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(v_1)^{(6)}$ and $(\bar{v}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)}=\frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}\right)\leq\frac{dv^{(7)}}{dt}\leq-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0<\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}<(\nu_1)^{(7)}<(\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t)\geq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}\quad,\quad\boxed{(C)^{(7)}=\frac{(\nu_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)}\leq\nu^{(7)}(t)\leq(\nu_1)^{(7)}$

In the same manner , we get 417

$$\nu^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(7)}=\frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)}\leq\nu^{(7)}(t)\leq(\bar{\nu}_1)^{(7)}$

If $0<(\nu_1)^{(7)}<(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}<(\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)}\leq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}\leq\nu^{(7)}(t)\leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\bar{\nu}_1)^{(7)}$$

If $0<(\nu_1)^{(7)}\leq(\bar{\nu}_1)^{(7)}\leq\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)}\leq\nu^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)}\leq\nu^{(7)}(t)\leq(m_1)^{(7)},\quad\boxed{\nu^{(7)}(t)=\frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{40}'')^{(8)} = (a_{41}'')^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b_{40}'')^{(8)} = (b_{41}'')^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$ if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a_{44}')^{(9)} - (a_{45}')^{(9)} + (a_{44}'')^{(9)}(T_{45}, t)\right) - (a_{45}'')^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\quad \boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \leq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \leq \nu^{(9)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \leq (\bar{\nu}_1)^{(9)} \leq \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \leq \nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \leq \nu^{(9)}(t) \leq (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for**

the special case .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(v_1)^{(9)}$ and $(\bar{v}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a_i'')^{(1)} and\ (b_i'')^{(1)}$ are independent on t , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0$ 447

$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0$ 448

has a unique positive solution , which is an equilibrium solution

$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0$ 449

$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0$ 450

$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0$ 451

$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0$ 452

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(h) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T_{33}^* :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that

there exists a unique G_{37}^{*} such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)}G_{17}^{*}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)}G_{17}^{*}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T_{17}^{*})]} \quad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)}T_{17}^{*}}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)}T_{17}^{*}}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^{*})]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)}G_{21}^{*}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)}G_{21}^{*}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$$T_{28}^* = \frac{(b_{28})^{(5)} T_{29}^*}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^*)]} \quad , \quad T_{30}^* = \frac{(b_{30})^{(5)} T_{29}^*}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^*)]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^* given by $\varphi((G_{35})^*) = 0$, T_{33}^* given by $f(T_{33}^*) = 0$ and

$$G_{32}^* = \frac{(a_{32})^{(6)} G_{33}^*}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34}^* = \frac{(a_{34})^{(6)} G_{33}^*}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

$$T_{32}^* = \frac{(b_{32})^{(6)} T_{33}^*}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^*)]} \quad , \quad T_{34}^* = \frac{(b_{34})^{(6)} T_{33}^*}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^*)]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^* given by $\varphi((G_{39})^*) = 0$, T_{37}^* given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

G_{41}^* given by $\varphi((G_{43})^*) = 0$, T_{41}^* given by $f(T_{41}^*) = 0$ and

$$G_{40}^{*}=\frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}\quad,\quad G_{42}^{*}=\frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

$$T_{40}^{*}=\frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}\quad,\quad T_{42}^{*}=\frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$$

Finally we obtain the unique solution of 89 to 99 523
A

G_{45}^{*} given by $\varphi((G_{47})^{*})=0$, T_{45}^{*} given by $f(T_{45}^{*})=0$ and

$$G_{44}^{*}=\frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}\quad,\quad G_{46}^{*}=\frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$$

$$T_{44}^{*}=\frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}\quad,\quad T_{46}^{*}=\frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}\ and\ (b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i=G_i^{*}+\mathbb{G}_i\qquad,T_i=T_i^{*}+\mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^{*})=(q_{14})^{(1)}\quad,\ \frac{\partial(b_i'')^{(1)}}{\partial G_j}(\,G^{*}\,)=s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt}=-\big((a'_{13})^{(1)}+(p_{13})^{(1)}\big)\mathbb{G}_{13}+(a_{13})^{(1)}\mathbb{G}_{14}-(q_{13})^{(1)}G_{13}^{*}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt}=-\big((a'_{14})^{(1)}+(p_{14})^{(1)}\big)\mathbb{G}_{14}+(a_{14})^{(1)}\mathbb{G}_{13}-(q_{14})^{(1)}G_{14}^{*}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt}=-\big((a'_{15})^{(1)}+(p_{15})^{(1)}\big)\mathbb{G}_{15}+(a_{15})^{(1)}\mathbb{G}_{14}-(q_{15})^{(1)}G_{15}^{*}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt}=-\big((b'_{13})^{(1)}-(r_{13})^{(1)}\big)\mathbb{T}_{13}+(b_{13})^{(1)}\mathbb{T}_{14}+\textstyle\sum_{j=13}^{15}\big(s_{(13)(j)}T_{13}^{*}\mathbb{G}_j\big) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt}=-\big((b'_{14})^{(1)}-(r_{14})^{(1)}\big)\mathbb{T}_{14}+(b_{14})^{(1)}\mathbb{T}_{13}+\textstyle\sum_{j=13}^{15}\big(s_{(14)(j)}T_{14}^{*}\mathbb{G}_j\big) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt}=-\big((b'_{15})^{(1)}-(r_{15})^{(1)}\big)\mathbb{T}_{15}+(b_{15})^{(1)}\mathbb{T}_{14}+\textstyle\sum_{j=13}^{15}\big(s_{(15)(j)}T_{15}^{*}\mathbb{G}_j\big) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions

$(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \tag{532}$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \ , \ \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \tag{533}$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \tag{534}$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \tag{535}$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \tag{536}$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \tag{537}$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}')^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T_{17}^*\mathbb{G}_j\right) \tag{538}$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}')^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T_{18}^*\mathbb{G}_j\right) \tag{539}$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \ , \ \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}')^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \tag{541}$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}')^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \tag{542}$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}')^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \tag{543}$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}')^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T_{20}^*\mathbb{G}_j\right) \tag{544}$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^{*}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T_{22}^{*}\mathbb{G}_j\right) \quad 546$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^{*}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^{*}) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^{*}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^{*}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^{*}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^{*}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^{*}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^{*}\mathbb{G}_j\right) \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^{*}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}((G_{31})^{*}) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^{*}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^{*}) = (q_{41})^{(8)} \quad , \frac{\partial(b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^{*}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^{*}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^{*}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^{*}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \ , \ \frac{\partial(b''_i)^{(9)}}{\partial G_j}(\ (G_{47})^*\ ) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\ B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\ C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\ D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\ E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\ F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\ G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14}+(b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\Big]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION FIVE

Ads/CFT Correspondence

INTRODUCTION—VARIABLES USED

David Berenstein JHEP07 (2004)018 doi:10.1088/1126-6708/2004/07/018 A toy model for the AdS/CFT correspondence

(1) David Berenstein studies the large-N gauged quantum mechanics for (e) a single Hermitian matrix in (eb) the Harmonic oscillator potential well as (=) a toy model for (e) the AdS/CFT correspondence.
(2) His argument is that the dual geometry should be (=) a string in two dimensions with a curvature of stringy size.
(3) Even though the dual geometry is not weakly curved, one can still gain (e) knowledge of the system from a detailed study of the open-closed string duality.
(4) David Berenstein gives a mapping between the basis of states made of traces (closed strings) and (e&eb) the eigenvalues of the matrix (D-brane picture) in terms of (e&eb) Schur polynomials.
(5) This is interpreted as (=) an exact open-closed duality.
(6) They connect this model with (e&eb) a decoupling limit of Script N = 4 SYM and the (e&eb) study of giant gravitons in AdS5 × S5.
(7) They show that the two giant gravitons that expand along (e&eb) AdS5 and S5 can be interpreted in (eb)the matrix model as taking an eigenvalue from (e) the Fermi Sea and exciting (eb) it very much, or as making a hole in (eb) the Fermi sea respectively.
(8) This is similar to recent studies of the c = 1 string. This connection (see proposition 14) gives (eb) new insight on how to perform calculations for giant gravitons.

NOTATION

Module One

David Berenstein studies the
large-N gauged quantum mechanics for (e) a single Hermitian matrix in (eb) the Harmonic oscillator potential well as (=) a toy model for (e) the AdS/CFT correspondence

G_{13} : Category one of single Hermitian matrix in (eb) the Harmonic oscillator potential well as (=) a toy model for (e) the AdS/CFT correspondence

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of large-N gauged quantum mechanics

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

large-N gauged quantum mechanics for a single Hermitian matrix in (eb) the Harmonic oscillator potential well as (=) a toy model for (e) the AdS/CFT correspondence

G_{16} : Category one of large-N gauged quantum mechanics for a single Hermitian matrix

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of Harmonic oscillator potential well as (=) a toy model for (e) the AdS/CFT correspondence

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

large-N gauged quantum mechanics for a single Hermitian matrix in the Harmonic oscillator potential well as (=) a toy model for the AdS/CFT correspondence

G_{20} : Category one of large-N gauged quantum mechanics for a single Hermitian matrix in the Harmonic oscillator potential well

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of toy model for the AdS/CFT correspondence

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

His argument is that the dual geometry should be (=) a string in two dimensions with a curvature of stringy size

G_{24} : Category one of dual geometry

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of string in two dimensions with a curvature of stringy size

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Even though the dual geometry is not weakly curved, one can still gain (e) knowledge of the system from a detailed study of the open-closed string duality

G_{28} : Category one of open-closed string duality ; dual geometry is not weakly curved

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of dual geometry is not weakly curved; open-closed string duality

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

David Berenstein gives a

mapping between the basis of states made of traces (closed strings) and (e&eb) the eigenvalues of the matrix (D-brane picture) in terms of (e&eb) Schur polynomials

G_{32} : Category one of basis of states made of traces (closed strings); eigenvalues of the matrix (D-brane picture) in terms of (e&eb) Schur polynomials

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of eigenvalues of the matrix (D-brane picture) in terms of (e&eb) Schur polynomials; basis of states made of traces (closed strings)

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

mapping between the basis of states made of traces (closed strings) and the eigenvalues of the matrix (D-brane picture) in terms of (e&eb) Schur polynomials

G_{36} : Category one of mapping between the basis of states made of traces (closed strings) and the eigenvalues of the matrix (D-brane picture); Schur polynomials

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of Schur polynomials ;mapping between the basis of states made of traces (closed strings) and the eigenvalues of the matrix (D-brane picture)

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

They connect this model with a decoupling limit of Script N = 4 SYM and the (e&eb) study of giant gravitons in AdS5 × S5

G_{40} : Category one of decoupling limit of Script N = 4 SYM; study of giant gravitons in AdS5 × S5

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of study of giant gravitons in AdS5 × S5; decoupling limit of Script N = 4 SYM

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

They show that the

two giant gravitons that expand along (e&eb) AdS5 and S5 can be interpreted in (eb)the matrix model as taking an eigenvalue from (e) the Fermi Sea and exciting (eb) it very much, or as making a hole in (eb) the Fermi sea respectively

G_{44} : Category one of two giant gravitons that expand along AdS5 and S5 can be interpreted

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of matrix model as taking an eigenvalue from (e) the Fermi Sea and exciting (eb) it very much, or as making a hole in (eb) the Fermi sea respectively

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION SIX

Chaotic Fast Scrambling At Black Holes

INTRODUCTION—VARIABLES USED

LUIS J. GARAY, Int. J. Mod. Phys. A, 10, 145 (1995). DOI: 10.1142/S0217751X95000085 QUANTUM GRAVITY AND MINIMUM LENGTH

(1) The existence of a fundamental scale, a lower bound to (e) any output of a position measurement, seems to be (=) a model-independent feature of quantum gravity.

(2) In fact, different approaches to this theory lead to this result. The key ingredients for the appearance

of this minimum length are quantum mechanics, special relativity and general relativity. As a consequence, classical notions such as causality or distance between events cannot be expected to be applicable at this scale. They must be replaced by some other, yet unknown, structure.

Chaotic fast scrambling at black holes Phys. Rev. D 84, 106012 – Published 29 November 2011 José L. F. Barbón and Javier M. Magán

(3) Fast scramblers process (e&eb) information in (eb) characteristic times scaling logarithmically with the entropy, a behavior which has been conjectured (eb) for black hole horizons.

(4) In this paper authors use the AdS/CFT context to argue that causality bounds on (eb) information flow only depend on (=) the properties of a single thermal cell, and admit (e) a geometrical interpretation in terms of (e&eb) the optical depth, i.e. the thickness of the Rindler region in (eb) the so-called optical metric.

(5) The spatial sections of the optical metric are well approximated by (e) constant-curvature hyperboloids.

(6) Authors use this fact to propose (eb) an effective kinetic model of scrambling which behaves (e&eb) like a compact hyperbolic billiard, furnishing a classic example of hard chaos.

(7) It is suggested that classical chaos at large N is (=) a crucial ingredient in reconciling the notion of fast scrambling with (e&eb) the required saturation of causality. DOI: http://dx.doi.org/10.1103/PhysRevD.84.106012

NOTATION

Module One

two giant gravitons that expand along AdS5 and S5 can be interpreted in the matrix model as taking an eigenvalue from (e) the Fermi Sea and exciting (eb) it very much, or as making a hole in (eb) the Fermi sea respectively

G_{13} : Category one of Fermi Sea and exciting (eb) it very much, or as making a hole in (eb) the Fermi sea respectively

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of two giant gravitons that expand along AdS5 and S5 can be interpreted in the matrix model as taking an eigenvalue

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

two giant gravitons that expand along AdS5 and S5 can be interpreted in the matrix model as taking an eigenvalue from the Fermi Sea and exciting (eb) it very much, or as making a hole in (eb) the Fermi sea respectively

G_{16} : Category one of two giant gravitons that expand along AdS5 and S5 can be interpreted in the matrix model as taking an eigenvalue from the Fermi Sea and exciting

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of it very much, or as making a hole in the Fermi sea respectively

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

The existence of a fundamental scale, a lower bound to (e) any output of a position measurement, seems to be (=) a model-independent feature of quantum gravity

G_{20} : Category one of any output of a position measurement, seems to be (=) a model-independent feature of quantum gravity

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of existence of a fundamental scale, a lower bound

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

existence of a fundamental scale, a lower bound to any output of a position measurement, seems to be (=) a model-independent feature of quantum gravity

G_{24} : Category one of existence of a fundamental scale, a lower bound to any output of a position measurement

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of model-independent feature of quantum gravity

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Fast scramblers process (e&eb) information in (eb) characteristic times scaling logarithmically with the entropy, a behavior which has been conjectured (eb) for black hole horizons

G_{28} : Category one of Fast scramblers; information in (eb) characteristic times scaling logarithmically with the entropy, a behavior which has been conjectured (eb) for black hole horizons

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of information in (eb) characteristic times scaling logarithmically with the entropy, a behavior which has been conjectured (eb) for black hole horizons; Fast scramblers

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Fast scramblers process information in (eb) characteristic times scaling logarithmically with the entropy, a behavior which has been conjectured (eb) for black hole horizons

G_{32} : Category one of Fast scramblers process information

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of characteristic times scaling logarithmically with the entropy, a behavior which has been conjectured (eb) for black hole horizons

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Fast scramblers process information in characteristic times scaling logarithmically with the entropy, a behavior which has been conjectured (eb) for black hole horizons

G_{36} : Category one of black hole horizons

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of Fast scramblers process information in characteristic times scaling logarithmically with the entropy, a behavior

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

In this paper authors use the
AdS/CFT context to argue that causality bounds on (eb) information flow only depend on (=) the properties of a single thermal cell, and admit (e) a geometrical interpretation in terms of (e&eb) the optical depth, i.e. the thickness of the Rindler region in (eb) the so-called optical metric

G_{40} : Category one of AdS/CFT context with causality bounds

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of information flow only depend on (=) the properties of a single thermal cell, and admit (e) a geometrical interpretation in terms of (e&eb) the optical depth, i.e. the thickness of the Rindler region in (eb) the so-called optical metric

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

AdS/CFT context to argue that causality bounds on information flow only depend on the properties of a single thermal cell, and admit (e) a geometrical interpretation in terms of (e&eb) the optical depth, i.e. the thickness of the Rindler region in (eb) the so-called optical metric

G_{44} : Category one of geometrical interpretation in terms of (e&eb) the optical depth, i.e. the thickness of the Rindler region in (eb) the so-called optical metric

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of AdS/CFT context to argue that causality bounds on information flow only depend on the properties of a single thermal cell

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}), t\big)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}), t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}\big((G_{19}), t\big) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}), t\right)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}), t\right)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\left((G_{39}), t\right)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\begin{array}{ccc} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) \quad +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{array}\right]G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[\begin{array}{ccc} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) \quad +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{array}\right]G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\begin{array}{ccc} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) \quad +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{array}\right]G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\,-(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\,-(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\,-(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad (60)$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21},t)\ +(a''_{16})^{(2,2,2)}(T_{17},t)\ +(a''_{13})^{(1,1,1,)}(T_{14},t)\\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)\ +(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)\ +(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a''_{36})^{(7,7,7,7)}(T_{37},t)\ +(a''_{40})^{(8,8,8,8)}(T_{41},t)\ +(a''_{44})^{(9,9,9)}(T_{45},t)\end{array}\right]G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c}(a'_{21})^{(3)}+(a''_{21})^{(3)}(T_{21},t)\ +(a''_{17})^{(2,2,2)}(T_{17},t)\ +(a''_{14})^{(1,1,1,)}(T_{14},t)\\ +(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)\ +(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)\ +(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a''_{37})^{(7,7,7,7)}(T_{37},t)\ +(a''_{41})^{(8,8,8,8)}(T_{41},t)\ +(a''_{45})^{(9,9,9)}(T_{45},t)\end{array}\right]G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21},t)\ +(a''_{18})^{(2,2,2)}(T_{17},t)\ +(a''_{15})^{(1,1,1,)}(T_{14},t)\\ +(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)\ +(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)\ +(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a''_{38})^{(7,7,7,7)}(T_{37},t)\ +(a''_{42})^{(8,8,8,8)}(T_{41},t)\ +(a''_{46})^{(9,9,9)}(T_{45},t)\end{array}\right]G_{22} \quad (69)$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\left[-(b''_{20})^{(3)}(G_{23},t)\right] & \left[-(b''_{16})^{(2,2,2)}(G_{19},t)\right] & \left[-(b''_{13})^{(1,1,1,)}(G,t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{36})^{(7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{40})^{(8,8,8,8)}(G_{43},t)\right] & \left[-(b''_{44})^{(9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\left[-(b''_{21})^{(3)}(G_{23},t)\right] & \left[-(b''_{17})^{(2,2,2)}(G_{19},t)\right] & \left[-(b''_{14})^{(1,1,1,)}(G,t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{37})^{(7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{41})^{(8,8,8,8)}(G_{43},t)\right] & \left[-(b''_{45})^{(9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\left[-(b''_{22})^{(3)}(G_{23},t)\right] & \left[-(b''_{18})^{(2,2,2)}(G_{19},t)\right] & \left[-(b''_{15})^{(1,1,1,)}(G,t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{38})^{(7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{42})^{(8,8,8,8)}(G_{43},t)\right] & \left[-(b''_{46})^{(9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$ $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad 75$$

$(a''_{24})^{(4)}(T_{25},t)$, $(a''_{25})^{(4)}(T_{25},t)$, $(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$
are fourth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$
are fifth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$
are sixth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$
are seventh augmentation coefficients for category 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$
are eighth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

$Where\ \boxed{-(b_{24}'')^{(4)}(G_{27},t)}, \boxed{-(b_{25}'')^{(4)}(G_{27},t)}, \boxed{-(b_{26}'')^{(4)}(G_{27},t)}\ are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{28}'')^{(5,5,)}(G_{31},t)}, \boxed{-(b_{29}'')^{(5,5,)}(G_{31},t)}, \boxed{-(b_{30}'')^{(5,5,)}(G_{31},t)}\ are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{32}'')^{(6,6,)}(G_{35},t)}, \boxed{-(b_{33}'')^{(6,6,)}(G_{35},t)}, \boxed{-(b_{34}'')^{(6,6,)}(G_{35},t)}\ are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{13}'')^{(1,1,1,1)}(G,t)}, \boxed{-(b_{14}'')^{(1,1,1,1)}(G,t)}, \boxed{-(b_{15}'')^{(1,1,1,1)}(G,t)}$

$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{16}'')^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b_{17}'')^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b_{18}'')^{(2,2,2,2)}(G_{19},t)}$

$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{20}'')^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b_{21}'')^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b_{22}'')^{(3,3,3,3)}(G_{23},t)}$

$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{36}'')^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b_{37}'')^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b_{38}'')^{(7,7,7,7,7)}(G_{39},t)}$

$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{40}'')^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b_{41}'')^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b_{42}'')^{(8,8,8,8,8)}(G_{43},t)}$

$are\ eighth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{46}'')^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b_{45}'')^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b_{44}'')^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{c} (a_{28}')^{(5)}\boxed{+(a_{28}'')^{(5)}(T_{29},t)}\ \boxed{+(a_{24}'')^{(4,4,)}(T_{25},t)}\ \boxed{+(a_{32}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{36}'')^{(7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a_{40}'')^{(8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a_{44}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{c} (a_{29}')^{(5)}\boxed{+(a_{29}'')^{(5)}(T_{29},t)}\ \boxed{+(a_{25}'')^{(4,4,)}(T_{25},t)}\ \boxed{+(a_{33}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a_{45}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{c} (a_{30}')^{(5)}\boxed{+(a_{30}'')^{(5)}(T_{29},t)}\ \boxed{+(a_{26}'')^{(4,4,)}(T_{25},t)}\ \boxed{+(a_{34}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a_{46}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{30} \quad (81)$$

$Where\ \boxed{+(a_{28}'')^{(5)}(T_{29},t)}, \boxed{+(a_{29}'')^{(5)}(T_{29},t)}, \boxed{+(a_{30}'')^{(5)}(T_{29},t)}\ are\ first\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$And\ \boxed{+(a_{24}'')^{(4,4,)}(T_{25},t)}, \boxed{+(a_{25}'')^{(4,4,)}(T_{25},t)}, \boxed{+(a_{26}'')^{(4,4,)}(T_{25},t)}\ are\ second\ augmentation\ coefficient\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a_{32}'')^{(6,6,6)}(T_{33},t)}, \boxed{+(a_{33}'')^{(6,6,6)}(T_{33},t)}, \boxed{+(a_{34}'')^{(6,6,6)}(T_{33},t)}\ are\ third\ \ augmentation\ coefficient\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$+(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a_{36}'')^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a_{40}'')^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a_{46}'')^{(9,9,9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9,9,9)}(T_{45},t)$, $+(a_{44}'')^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b_{28}')^{(5)} & -(b_{28}'')^{(5)}(G_{31},t) & -(b_{24}'')^{(4,4,)}(G_{27},t) & -(b_{32}'')^{(6,6,6)}(G_{35},t) \\ & -(b_{13}'')^{(1,1,1,1,1)}(G,t) & -(b_{16}'')^{(2,2,2,2,2)}(G_{19},t) & -(b_{20}'')^{(3,3,3,3,3)}(G_{23},t) \\ & -(b_{36}'')^{(7,7,7,7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8,8,8,8)}(G_{43},t) & -(b_{44}'')^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b_{29}')^{(5)} & -(b_{29}'')^{(5)}(G_{31},t) & -(b_{25}'')^{(4,4,)}(G_{27},t) & -(b_{33}'')^{(6,6,6)}(G_{35},t) \\ & -(b_{14}'')^{(1,1,1,1,1)}(G,t) & -(b_{17}'')^{(2,2,2,2,2)}(G_{19},t) & -(b_{21}'')^{(3,3,3,3,3)}(G_{23},t) \\ & -(b_{37}'')^{(7,7,7,7,7,7)}(G_{39},t) & -(b_{41}'')^{(8,8,8,8,8,8)}(G_{43},t) & -(b_{45}'')^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b_{30}')^{(5)} & -(b_{30}'')^{(5)}(G_{31},t) & -(b_{26}'')^{(4,4,)}(G_{27},t) & -(b_{34}'')^{(6,6,6)}(G_{35},t) \\ & -(b_{15}'')^{(1,1,1,1,1,)}(G,t) & -(b_{18}'')^{(2,2,2,2,2)}(G_{19},t) & -(b_{22}'')^{(3,3,3,3,3)}(G_{23},t) \\ & -(b_{38}'')^{(7,7,7,7,7,7)}(G_{39},t) & -(b_{42}'')^{(8,8,8,8,8,8)}(G_{43},t) & -(b_{46}'')^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

where $-(b_{28}'')^{(5)}(G_{31},t)$, $-(b_{29}'')^{(5)}(G_{31},t)$, $-(b_{30}'')^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b_{24}'')^{(4,4,)}(G_{27},t)$, $-(b_{25}'')^{(4,4,)}(G_{27},t)$, $-(b_{26}'')^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b_{32}'')^{(6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b_{13}'')^{(1,1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1,1)}(G,t)$, $-(b_{15}'')^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b_{16}'')^{(2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b_{20}'')^{(3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b_{36}'')^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b_{42}'')^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b_{40}'')^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b_{46}'')^{(9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \quad 85$$

$$- \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}, \boxed{(a''_{37})^{(7)}(T_{37},t)}, \boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\left[-(b''_{40})^{(8)}(G_{43},t)\right] & -(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\left[-(b''_{41})^{(8)}(G_{43},t)\right] & -(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\left[-(b''_{42})^{(8)}(G_{43},t)\right] & -(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq \ (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = \ (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \ , \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\,(a_i)^{(1)} + (a_i')^{(1)} + \ (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}\,(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i')^{(1)} + \ (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \leq (p_i)^{(2)} \leq \left(\hat{A}_{16}\right)^{(2)} \qquad 102$$

$$(b_i'')^{(2)}(G_{19},t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\big((G_{19}),t\big) = \ (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\big((G_{19}),t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\big[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\big]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \le (p_i)^{(3)} \le (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \le (r_i)^{(3)} \le (b_i')^{(3)} \le (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - \ T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\,(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and \ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\,(a_i)^{(6)} + (a_i')^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)}\,(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(WW) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(XX) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$(a_i'')^{(7)}(T_{37},t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$

$(b_i'')^{(7)}(G_{39},t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$

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(YY) $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

(ZZ)

$\lim_{G\to\infty}(b_i'')^{(7)}((G_{39}),t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$

$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}((G_{39}),t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$. (T_{37}',t) and (T_{37},t) are points belonging to the interval $\left[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\right]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(AAA) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(BBB) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$

$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$

Where we suppose

$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$ 138

$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$ 139

$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$ 140

$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t}$ 142

$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t}$ 143

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ 144

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}$, $(r_i)^{(8)}, i = 40,41,42$,
Satisfy the inequalities

$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 145

$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 146

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\hat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)} ||(G_{47}) - (G_{47})'|| e^{-(\hat{M}_{44})^{(9)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)} (\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t} \ , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t} \ , \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \ , \ T_i(0) = T_i^0 \ , \ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} , T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\overline{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{16})^{(2)}, T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{20})^{(3)}, T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^{0} + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^{0} + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^{0} + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^{0} + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^{0} + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^{0} + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By

$$\bar{G}_{32}(t) = G_{32}^{0} + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^{0} + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^{0} + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^{0} + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^{0} + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^{0} + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that 174

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \leq G_{32}^{0}+\int_0^t\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)} =$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(i) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \leq G_{36}^{0}+\int_0^t\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)} =$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \leq G_{40}^{0}+\int_0^t\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)} = \quad 180$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t} \leq \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for G_{41} , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \leq G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \leq \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for G_{45} , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \left\{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\right\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)}\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) - (a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right| \; e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \qquad (186)$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: (187)

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. (188)

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}\left|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t} \leq$
$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)d\left(\left((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)}\right)\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\}ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0$ for $t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)}G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$: $\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})($and\ not\ on\ \ t$) and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$G_{24}^{(2)} \left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)}\right) d\left(((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on T_{25} and respectively on (G_{27})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (228)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_2 / (a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\widehat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{32})^{(6)}\right] \leq (\widehat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)}, (T_{35})^{(1)}), ((G_{35})^{(2)}, (T_{35})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}), (\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}), (\widetilde{T_{35}}) \right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right| \; e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} \} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)} (\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)}; \; (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t} \; and \; (\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$ (250)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2 \; and \; \left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33} \; and \; G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$$
$$sup_i\{max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}, max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)} \left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)} (T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b'_i)^{(7)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)}$$ 267

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)}$$ 268

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1,\left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right),\quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$

From 99 to 44 it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(9)}t)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$: 281

By $(v_1)^{(1)} > 0$, $(v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{v}_1)^{(1)},, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{v}_1)^{(1)} > 0$, $(\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)}, , (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)} t} - e^{-(a'_{26})^{(4)} t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)} t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G^0_{28} e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G^0_{28}}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G^0_{30} e^{-(S_2)^{(5)} t} \le G_{30}(t) \le \right. \quad 344$$

$$\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_2)^{(5)}((S_1)^{(5)}-(a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t}-e^{-(a'_{30})^{(5)}t}\right]+\ G_{30}^{0}e^{-(a'_{30})^{(5)}t}\Big)$$

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \qquad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le \frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \qquad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\le T_{30}(t)\le \qquad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$:

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)}\le -(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)}\le -(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\le -(\tau_1)^{(6)}$$

Definition of $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$: 350

By $(v_1)^{(6)}>0$, $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$, $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

Definition of $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)}>0$, $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$, $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

Definition of $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$:-

If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)}=(v_0)^{(6)},(m_1)^{(6)}=(v_1)^{(6)},$ ***if*** $(v_0)^{(6)}<(v_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}$ $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}$ $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}$ $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}$ $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}$ $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})} \left[e^{(S_1)^{(6)} t} - e^{-(a'_{34})^{(6)} t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)} t} \right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^{0}e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}}G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}}G_{36}^{0}e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t}-e^{-(S_2)^{(7)}t}\right]+G_{38}^{0}e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left.\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_2)^{(7)}((S_1)^{(7)}-(a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t}-e^{-(a'_{38})^{(7)}t}\right]+\ G_{38}^{0}e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}}T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}}T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right) - (b''_{41})^{(8)}\left((G_{43}), t\right) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(v_1)^{(8)} > 0$, $(v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$, $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{v}_1)^{(8)}$,, $(\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{v}_1)^{(8)} > 0$, $(\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$, $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$:-

If we define $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$, $\boldsymbol{if}$ $(v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$, $\boldsymbol{if}$ $(v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$,
and $\boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}$, $\boldsymbol{if}$ $(\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, $\boldsymbol{if}$ $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, $\boldsymbol{if}$ $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}$ $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t})$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t}\leq T_{42}(t)\leq \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)},(S_2)^{(8)},(R_1)^{(8)},(R_2)^{(8)}$:- 381

Where $(S_1)^{(8)}=(a_{40})^{(8)}(m_2)^{(8)}-(a'_{40})^{(8)}$

$(S_2)^{(8)}=(a_{42})^{(8)}-(p_{42})^{(8)}$

$(R_1)^{(8)}=(b_{40})^{(8)}(\mu_2)^{(8)}-(b'_{40})^{(8)}$

$(R_2)^{(8)}=(b'_{42})^{(8)}-(r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$:

$(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)}\leq -(a'_{44})^{(9)}+(a'_{45})^{(9)}-(a''_{44})^{(9)}(T_{45},t)+(a''_{45})^{(9)}(T_{45},t)\leq -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)}\leq -(b'_{44})^{(9)}+(b'_{45})^{(9)}-(b''_{44})^{(9)}\big((G_{47}),t\big)-(b''_{45})^{(9)}\big((G_{47}),t\big)\leq -(\tau_1)^{(9)}$

Definition of $(v_1)^{(9)},(v_2)^{(9)},(u_1)^{(9)},(u_2)^{(9)},v^{(9)},u^{(9)}$:

By $(v_1)^{(9)}>0$, $(v_2)^{(9)}<0$ and respectively $(u_1)^{(9)}>0$, $(u_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_1)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)}>0$, $(\bar{v}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$, $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

Definition of $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$:-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)},\ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$
and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \le G_{46}(t) \le \right.$$
$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)} (m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)} (\mu_2)^{(9)} - (b'_{44})^{(9)}$

$$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \Big((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\Big) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\Big((a_{14})^{(1)}\big(v^{(1)}\big)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\Big) \le \frac{dv^{(1)}}{dt} \le -\Big((a_{14})^{(1)}\big(v^{(1)}\big)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\Big)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (v_1)^{(1)} \le (\bar{v}_1)^{(1)} \le \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$, we obtain 386

$$(v_1)^{(1)} \le v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $v^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\right) - (a''_{17})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{\mathrm{G}_{16}}{\mathrm{G}_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (\mathrm{C})^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (\mathrm{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)}\right) \le \frac{dv^{(3)}}{dt} \le -\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \ge \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}}$$

it follows $(v_0)^{(3)} \le v^{(3)}(t) \le (v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \le v^{(3)}(t) \le (\bar{v}_1)^{(3)}$

If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)} \le \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}} \le v^{(3)}(t) \le$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (\bar{v}_1)^{(3)}$$

If $0 < (v_1)^{(3)} \le (\bar{v}_1)^{(3)} \le (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$, we obtain 403

$$(v_1)^{(3)} \le v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

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$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(8)} and\ (b''_i)^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\,]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\,]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\,]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0$ 479

$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0$ 480

$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0$ 481

$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0$ 482

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(i) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first

equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T^*_{33} for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$$

Definition and uniqueness of T^*_{37} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T^*_{37} for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$$

Definition and uniqueness of T^*_{41} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{25} such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507
$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{29} such that $\varphi((G_{31})^*) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{33} such that $\varphi({G_{35}}^*) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{37} such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{41} such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right]+(b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{45} such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)} G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)} G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)} T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)} T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)} G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)} G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)} T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)} T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)} G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)} G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)} T_{21}^*}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T_{22}^* = \frac{(b_{22})^{(3)} T_{21}^*}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]} \quad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}\mathrm{T}^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}\mathrm{T}^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}\left((G_{27})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^{*}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^{*}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^{*}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^{*}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^{*}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^{*}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^{*}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^{*}\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\Big)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\Big]$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\Big)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\Big)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\Big]$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\Big)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\Big)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\Big]$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\Big)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\Big)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\Big)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\Big)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\Big)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\Big)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION SEVEN

Higgs Bosons In A Nonminimal Supersymmetric Model

INTRODUCTION—VARIABLES USED

Higgs bosons in a nonminimal supersymmetric model Phys. Rev. D 39, 844 – Published 1 February 1989 J. Ellis, J. F. Gunion, H. E. Haber, L. Roszkowski, and F. Zwirner

(1) The minimal supersymmetric standard model contains (e) two Higgs doublets which must mix via (e&eb) a mass parameter whose magnitude remains to be explained.
(2) Authors explore an extension of the minimal model to include (e) a singlet Higgs field whose vacuum expectation value determines (eb) the mixing.

(3) Authors study the spectrum and couplings of Higgs bosons in this extended model and compare (e&eb) them with those in the minimal model.

(4) They examine a number of limiting cases analytically and also make numerical studies of the extended model both with and without (e&eb) constraints from the renormalization-group analysis of (e) a parent superstring-inspired grand-unified-theory model.

(5) They establish the conditions for there to be a charged Higgs boson lighter than (eb) the W± and the circumstances under which there is (=) no light neutral Higgs boson.

(6) With a particularly simple set of boundary conditions at the unification scale, the renormalization-group equations imply (eb) that one or more Higgs bosons are light enough to be found at (eb) the CERN LEP or SLAC Linear Collider and that many supersymmetric particles should be (=) accessible to these accelerators and the Fermilab Tevatron; relatively few would require (e) the Superconducting Super Collider, Large Hadron Collider, or a TeV-scale e+e− collider for discovery.

(7) Finally, authors analyze the possible production mechanisms and phenomenological signatures of (e) the different Higgs bosons at these machines. DOI: http://dx.doi.org/10.1103/PhysRevD.39.844

NOTATION

Module One

AdS/CFT context to argue that causality bounds on information flow only depend on the properties of a single thermal cell, and admit a geometrical interpretation in terms of (e&eb) the optical depth, i.e. the thickness of the Rindler region in (eb) the so-called optical metric

G_{13} : Category one of AdS/CFT context to argue that causality bounds on information flow only depend on the properties of a single thermal cell, and admit a geometrical interpretation; optical depth, i.e. the thickness of the Rindler region in (eb) the so-called optical metric

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of optical depth, i.e. the thickness of the Rindler region in (eb) the so-called optical metric; AdS/CFT context to argue that causality bounds on information flow only depend on the properties of a single thermal cell, and admit a geometrical interpretation

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

AdS/CFT context to argue that causality bounds on information flow only depend on the properties of a single thermal cell, and admit a geometrical interpretation in terms of the optical depth, i.e. the thickness of the Rindler region in (eb) the so-called optical metric

G_{16} : Category one of AdS/CFT context to argue that causality bounds on information flow only depend on the properties of a single thermal cell, and admit a geometrical interpretation in terms of the optical depth, i.e. the thickness of the Rindler region

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of optical metric

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

(1) The spatial sections of the optical metric are well approximated by (e) constant-curvature hyperboloids.
(2) Authors use this fact to propose (eb) an effective kinetic model of scrambling which behaves (e&eb) like a compact hyperbolic billiard, furnishing a classic example of hard chaos.
(3) It is suggested that classical chaos at large N is (=) a crucial ingredient in reconciling the notion of fast scrambling with (e&eb) the required saturation of causality. DOI: http://dx.doi.org/10.1103/PhysRevD.84.106012

G_{20} : Category one of constant-curvature hyperboloids

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of spatial sections of the optical metric are well approximated

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Authors use this fact to propose (eb) an

effective kinetic model of scrambling which behaves (e&eb) like a compact hyperbolic billiard, furnishing a classic example of hard chaos

G_{24} : Category one of effective kinetic model of scrambling; compact hyperbolic billiard, furnishing a classic example of hard chaos

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of compact hyperbolic billiard, furnishing a classic example of hard chaos; effective kinetic model of scrambling

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Classical chaos at large N is (=) a crucial ingredient in reconciling the notion of fast scrambling with (e&eb) the required saturation of causality.

DOI: http://dx.doi.org/10.1103/PhysRevD.84.106012

G_{28} : Category one of Classical chaos at large N

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of crucial ingredient in reconciling the notion of fast scrambling with (e&eb) the required saturation of causality

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Classical chaos at large N is a crucial ingredient in reconciling the notion of fast scrambling with (e&eb) the required saturation of causality

G_{32} : Category one of Classical chaos at large N is a crucial ingredient in reconciling the notion of fast scrambling; required saturation of causality

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of required saturation of causality; Classical chaos at large N is a crucial ingredient in reconciling the notion of fast scrambling

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

minimal supersymmetric standard model contains (e) two Higgs doublets which must mix via (e&eb) a mass parameter whose magnitude remains to be explained

G_{36} : Category one of two Higgs doublets which must mix via (e&eb) a mass parameter whose magnitude remains to be explained

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of minimal supersymmetric standard model

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

Predictions of the MSSM regarding hadron colliders

The Tevatron and LHC have active experimental programs searching for supersymmetric particles. Since both of these machines are hadron colliders — proton antiproton for the Tevatron and proton proton for the

LHC — they search best for strongly interacting particles. Therefore most experimental signature involves production of squarks or gluinos. Since the MSSM has R-parity, the lightest supersymmetric particle is stable and after the squarks and gluinos decay each decay chain will contain one LSP that will leave the detector unseen. This leads to the generic prediction that the MSSM will produce a 'missing energy' signal from these particles leaving the detector.

Neutralinos

There are four neutralinos that are fermions and are electrically neutral, the lightest of which is typically stable. They are typically labeled Ν0
1, Ν0
2, Ν0
3, Ν0
4 (although sometimes $\tilde{\chi}_1^0, \cdots, \tilde{\chi}_4^0$ is used instead). These four states are mixtures of the Bino and the neutral Wino (which are the neutral electroweak Gauginos), and the neutral Higgsinos. As the neutralinos are Majorana fermions, each of them is identical with its antiparticle. Because these particles only interact with the weak vector bosons, they are not directly produced at hadron colliders in copious numbers. They primarily appear as particles in cascade decays of heavier particles usually originating from colored supersymmetric particles such as squarks or gluinos.

In R-parity conserving models, the lightest neutralino is stable and all supersymmetric cascades decays end up decaying into this particle which leaves the detector unseen and its existence can only be inferred by looking for unbalanced momentum in a detector.

The heavier neutralinos typically decay through a Z0 to a lighter neutralino or through a W± to chargino. Thus a typical decay is

$$\text{Ν0}_2 \rightarrow \text{Ν0}_1 + \text{Z0} \rightarrow \text{Missing energy} + \underline{\text{l+}} + \underline{\text{l−}}$$

$$\text{Ν0}_2 \rightarrow \text{C}\tilde{\chi}_1^{\pm} + \text{W}\mp \rightarrow \text{Ν0}_1 + \text{W}\pm + \text{W}\mp \rightarrow \text{Missing energy} + \text{l+} + \text{l−}$$

The mass splittings between the different Neutralinos will dictate which patterns of decays are allowed.

minimal supersymmetric standard model contains two Higgs doublets which must mix via (e&eb) a mass parameter whose magnitude remains to be explained

G_{40} : Category one of minimal supersymmetric standard model contains two Higgs doublets; mass parameter

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of mass parameter; minimal supersymmetric standard model contains two Higgs doublets

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Authors explore an extension of the minimal model to include (e) a singlet Higgs field whose vacuum expectation value determines (eb) the mixing

G_{44} : Category one of singlet Higgs field whose vacuum expectation value determines (eb) the mixing

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of extension of the minimal model

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION SEVEN A

Spectrum And Couplings Of Higgs Bosons

INTRODUCTION—VARIABLES USED

Higgs bosons in a nonminimal supersymmetric model Phys. Rev. D 39, 844 – Published 1 February 1989 J. Ellis, J. F. Gunion, H. E. Haber, L. Roszkowski, and F. Zwirner

(1) Authors explore an extension of the minimal model to include (e) a singlet Higgs field whose vacuum expectation value determines (eb) the mixing.
(2) Authors study the spectrum and couplings of Higgs bosons in this extended model and compare (e&eb) them with those in the minimal model.
(3) They examine a number of limiting cases analytically and also make numerical studies of the extended model both with and without (e&eb) constraints from the renormalization-group analysis of (e) a parent superstring-inspired grand-unified-theory model.
(4) They establish the conditions for there to be a charged Higgs boson lighter than (eb) the W± and the circumstances under which there is (=) no light neutral Higgs boson.
(5) With a particularly simple set of boundary conditions at the unification scale, the renormalization-group equations imply (eb) that one or more Higgs bosons are light enough to be found at (eb) the CERN LEP or SLAC Linear Collider and that many supersymmetric particles should be (=) accessible to these accelerators and the Fermilab Tevatron; relatively few would require (e) the Superconducting Super Collider, Large Hadron Collider, or a TeV-scale e+e− collider for discovery.
(6) Finally, authors analyze the possible production mechanisms and phenomenological signatures of (e) the different Higgs bosons at these machines. DOI: http://dx.doi.org/10.1103/PhysRevD.39.844

NOTATION

Module One

Authors explore an

extension of the minimal model to include a singlet Higgs field whose vacuum expectation value determines

(eb) the mixing

G_{13} : Category one of extension of the minimal model to include a singlet Higgs field whose vacuum expectation value

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of mixing

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Authors study the spectrum and couplings of Higgs bosons in this extended model and compare (e&eb) them with those in the minimal model

G_{16} : Category one of spectrum and couplings of Higgs bosons in this extended model; those in the minimal model

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of those in the minimal model; spectrum and couplings of Higgs bosons in this extended model

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

They examine a number of limiting cases analytically and also make

numerical studies of the extended model both with and without (e&eb) constraints from the renormalization-group analysis of (e) a parent superstring-inspired grand-unified-theory model

G_{20} : Category one of numerical studies of the extended model; constraints from the renormalization-group analysis of (e) a parent superstring-inspired grand-unified-theory model

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of constraints from the renormalization-group analysis of (e) a parent superstring-inspired grand-unified-theory model ;numerical studies of the extended model

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

numerical studies of the extended model both with and without constraints from the renormalization-group analysis of (e) a parent superstring-inspired grand-unified-theory model

G_{24} : Category one of parent superstring-inspired grand-unified-theory model

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of numerical studies of the extended model both with and without constraints from the renormalization-group analysis

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

They establish the conditions for there to be a charged Higgs boson lighter than (eb) the W± and the circumstances under which there is (=) no light neutral Higgs boson

G_{28} : Category one of conditions for there to be a charged Higgs boson

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of W± and the circumstances under which there is (=) no light neutral Higgs boson

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

conditions for there to be a charged Higgs boson lighter than the W± and the circumstances under which there is (=) no light neutral Higgs boson

G_{32} : Category one of conditions for there to be a charged Higgs boson lighter than the W± and the circumstance

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of no light neutral Higgs boson

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Finally, authors analyze the

Possible production mechanisms and phenomenological signatures of (e) the different Higgs bosons at these machines.

DOI: http://dx.doi.org/10.1103/PhysRevD.39.844

G_{36} : Category one of different Higgs bosons at these machines

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of possible production mechanisms and phenomenological signatures

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

Thus a typical decay is

$$\tilde{N}^0_2 \rightarrow \tilde{N}^0_1 + Z^0 \rightarrow \text{Missing energy} + l^+ + l^-$$

$$\tilde{N}^0_2 \rightarrow C^{\pm}_1 + W^{\mp} \rightarrow \tilde{N}^0_1 + W^{\pm} + W^{\mp} \rightarrow \text{Missing energy} + l^+ + l^-$$

G_{40} : Category one of LHS of the above decay

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of RHS of the above decay

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

As the neutralinos are Majorana fermions, each of them is identical with its antiparticle. Because these particles only interact with the weak vector bosons, they are not directly produced at hadron colliders in copious numbers.

They primarily appear as particles in cascade decays of heavier particles usually originating from colored supersymmetric particles such as squarks or gluinos.

G_{44} : Category one of neutralinos

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of Majorana fermions

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}),t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}),t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}),t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}),t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}),t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}),t\right)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}),t\right)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\left((G_{39}),t\right)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}),t\right)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}),t\right)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}),t\right)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\begin{array}{ccc} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) \quad +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{array}\right]G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[\begin{array}{ccc} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) \quad +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{array}\right]G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\begin{array}{ccc} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) \quad +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{array}\right]G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[\begin{array}{ccc} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) \quad -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{array}\right]T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[\begin{array}{ccc} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) \quad -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{array}\right]T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) & \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{15} \quad (60)$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t) & \\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t) & \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & +(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) & +(a''_{21})^{(3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t) & \\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t) & \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & +(a''_{18})^{(2)}(T_{17},t) & +(a''_{15})^{(1,1,)}(T_{14},t) & +(a''_{22})^{(3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33},t) & \\ +(a''_{38})^{(7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9)}(T_{45},t) & \end{bmatrix} G_{18} \quad (63)$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$-(b_{44}'')^{(9,9)}(G_{47},t)$, $-(b_{46}'')^{(9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c} (a_{20}')^{(3)}\,+(a_{20}'')^{(3)}(T_{21},t)\,+(a_{16}'')^{(2,2,2)}(T_{17},t)\,+(a_{13}'')^{(1,1,1,)}(T_{14},t) \\ +(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)\,+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)\,+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t) \\ +(a_{36}'')^{(7,7,7,7)}(T_{37},t)\,+(a_{40}'')^{(8,8,8,8)}(T_{41},t)\,+(a_{44}'')^{(9,9,9)}(T_{45},t) \end{array}\right] G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c} (a_{21}')^{(3)}\,+(a_{21}'')^{(3)}(T_{21},t)\,+(a_{17}'')^{(2,2,2)}(T_{17},t)\,+(a_{14}'')^{(1,1,1,)}(T_{14},t) \\ +(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)\,+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)\,+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t) \\ +(a_{37}'')^{(7,7,7,7)}(T_{37},t)\,+(a_{41}'')^{(8,8,8,8)}(T_{41},t)\,+(a_{45}'')^{(9,9,9)}(T_{45},t) \end{array}\right] G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c} (a_{22}')^{(3)}\,+(a_{22}'')^{(3)}(T_{21},t)\,+(a_{18}'')^{(2,2,2)}(T_{17},t)\,+(a_{15}'')^{(1,1,1,)}(T_{14},t) \\ +(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)\,+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)\,+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t) \\ +(a_{38}'')^{(7,7,7,7)}(T_{37},t)\,+(a_{42}'')^{(8,8,8,8)}(T_{41},t)\,+(a_{46}'')^{(9,9,9)}(T_{45},t) \end{array}\right] G_{22} \quad (69)$$

$+(a_{20}'')^{(3)}(T_{21},t)$, $+(a_{21}'')^{(3)}(T_{21},t)$, $+(a_{22}'')^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a_{40}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a_{44}'')^{(9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[\begin{array}{c} (b_{20}')^{(3)}\,-(b_{20}'')^{(3)}(G_{23},t)\,-(b_{16}'')^{(2,2,2)}(G_{19},t)\,-(b_{13}'')^{(1,1,1,)}(G,t) \\ -(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t)\,-(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t)\,-(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b_{36}'')^{(7,7,7,7)}(G_{39},t)\,-(b_{40}'')^{(8,8,8,8)}(G_{43},t)\,-(b_{44}'')^{(9,9,9)}(G_{47},t) \end{array}\right] T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[\begin{array}{c} (b_{21}')^{(3)}\,-(b_{21}'')^{(3)}(G_{23},t)\,-(b_{17}'')^{(2,2,2)}(G_{19},t)\,-(b_{14}'')^{(1,1,1,)}(G,t) \\ -(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t)\,-(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t)\,-(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b_{37}'')^{(7,7,7,7)}(G_{39},t)\,-(b_{41}'')^{(8,8,8,8)}(G_{43},t)\,-(b_{45}'')^{(9,9,9)}(G_{47},t) \end{array}\right] T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$

$are\ fourth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$
$are\ fifth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$
$are\ sixth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$
$are\ seventh\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$
$are\ eighth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

$Where\ -(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ $are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ $are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ $are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
$are\ eighth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad (81)$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \quad (84)$$

$where\ \boxed{-(b''_{28})^{(5)}(G_{31},t)}\ ,\boxed{-(b''_{29})^{(5)}(G_{31},t)}\ ,\boxed{-(b''_{30})^{(5)}(G_{31},t)}\ \ are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}\ ,\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}\ ,\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}\ are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}\ ,\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}\ ,\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}\ are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}\ ,\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}\ ,\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}\ ,\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}\ ,\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}\ ,\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}\ ,\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}\ ,\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}\ ,\boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}\ ,\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}\ ,\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}\ ,\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}\ ,\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad (87)$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}\ ,\boxed{+(a''_{33})^{(6)}(T_{33},t)}\ ,\boxed{+(a''_{34})^{(6)}(T_{33},t)}\ are\ first\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\ ,\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\ ,\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\ are\ second\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}\ ,\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}\ ,\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}\ \ are\ third\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\ ,\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\ ,\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\ ,\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\ ,\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation

coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$ Eighth augmentation coefficients

$+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)} & -(b''_{32})^{(6)}(G_{35},t) & -(b''_{28})^{(5,5,5)}(G_{31},t) & -(b''_{24})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{13})^{(1,1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)} & -(b''_{33})^{(6)}(G_{35},t) & -(b''_{29})^{(5,5,5)}(G_{31},t) & -(b''_{25})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{14})^{(1,1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & -(b''_{34})^{(6)}(G_{35},t) & -(b''_{30})^{(5,5,5)}(G_{31},t) & -(b''_{26})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{15})^{(1,1,1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{34} \quad (90)$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,)}(G_{27},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3

$-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3

$-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2, and 3

$-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \qquad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt}$$

$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt}$$

$$= (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt}$$
$$= (a_{42})^{(8)} G_{41}$$
$$- \left[\begin{array}{c} (a'_{42})^{(8)} \boxed{+(a''_{42})^{(8)}(T_{41},t)} \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array} \right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$
$$(b_{40})^{(8)} T_{41} - \left[\begin{array}{c} (b'_{40})^{(8)} \boxed{-(b''_{40})^{(8)}(G_{43},t)} \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{13}$$

$$\frac{dT_{41}}{dt} =$$
$$(b_{41})^{(8)} T_{40} - \left[\begin{array}{c} (b'_{41})^{(8)} \boxed{-(b''_{41})^{(8)}(G_{43},t)} \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{14}$$

$$\frac{dT_{42}}{dt} =$$
$$(b_{42})^{(8)} T_{41} - \left[\begin{array}{c} (b'_{42})^{(8)} \boxed{-(b''_{42})^{(8)}(G_{43},t)} \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{38}'')^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

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$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a_{44}')^{(9)}\boxed{+(a_{44}'')^{(9)}(T_{45},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a_{45}')^{(9)}\boxed{+(a_{45}'')^{(9)}(T_{45},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a_{46}')^{(9)}\boxed{+(a_{46}'')^{(9)}(T_{37},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{44}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{46}'')^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second

augmentation coefficient for category 1, 2 and 3
$+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3
$+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3
$+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3
$+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$
$$(b_{44})^{(9)}T_{45} -$$
$$\begin{bmatrix} (b_{44}')^{(9)}\left[-(b_{44}'')^{(9)}(G_{47},t)\right] & \left[-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{13}$$
$$\frac{dT_{45}}{dt} =$$
$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}\left[-(b_{45}'')^{(9)}(G_{47},t)\right] & \left[-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{14}$$
$$\frac{dT_{46}}{dt} =$$
$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}\left[-(b_{46}'')^{(9)}(G_{47},t)\right] & \left[-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{15}$$

Where $-(b_{44}'')^{(9)}(G_{47},t)$, $-(b_{45}'')^{(9)}(G_{47},t)$, $-(b_{46}'')^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3
$-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}'| e^{-(\hat{M}_{20})^{(3)} t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} ||G_{23} - G_{23}'|| e^{-(\hat{M}_{20})^{(3)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\ ,\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)},\ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)},\ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}), t\big) \le \ (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}\,(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\,\big((G_{27}), t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)},\ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - \ T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \leq (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\ ,\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)},\ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ \ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)},\ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \le\ \ (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}\,(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G\to\infty}(b_i'')^{(6)}\left((G_{35}), t\right) = \ \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)},\ \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \le (\hat{k}_{32})^{(6)}|T_{33} - \ T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(CCC) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(DDD) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

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(EEE) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(FFF)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(GGG) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(HHH) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)} \qquad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be

noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\ ,\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}<1 \qquad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+\ (\hat{A}_{40})^{(8)}+(\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \qquad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[\ (b_i)^{(8)}+(b_i')^{(8)}+\ (\hat{B}_{40})^{(8)}+(\hat{Q}_{40})^{(8)}\ (\hat{k}_{40})^{(8)}]<1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \qquad 146A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t)-(a_i'')^{(9)}(T_{45},t)|\le(\hat{k}_{44})^{(9)}|T_{45}-\ T_{45}'|e^{-(\widehat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t)-(b_i'')^{(9)}\big((G_{47}),t\big)|<(\hat{k}_{44})^{(9)}||(G_{47})-(G_{47})'||e^{-(\widehat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$

and $(a_i'')^{(9)}(T_{45},t)$. (T_{45}',t) and (T_{45},t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)},(\widehat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{44})^{(9)}=1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$:

$(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},$ are positive constants

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\ ,\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}<1$$

Definition of $(\hat{P}_{44})^{(9)},(\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},(\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)},(a_i')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(p_i)^{(9)},\ (r_i)^{(9)},i=44,45,46,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)}+(a_i')^{(9)}+\ (\hat{A}_{44})^{(9)}+\ (\hat{P}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)}+(b_i')^{(9)}+\ (\hat{B}_{44})^{(9)}+\ (\hat{Q}_{44})^{(9)}\ \ (\hat{k}_{44})^{(9)}]<1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\ ,T_i(0)$:

$$G_i(t)\leq\ \left(\hat{P}_{13}\right)^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{G_i(0)=G_i^0>0}$$

$$T_i(t)\leq\ (\hat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{T_i(0)=T_i^0>0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\ ,T_i(0)$

$$G_i(t)\leq\ (\hat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ ,\ \ G_i(0)=G_i^0>0$$

$$T_i(t)\leq\ (\hat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ ,\ \ T_i(0)=T_i^0>0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t)\leq\ (\hat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ ,\ \ G_i(0)=G_i^0>0$$

$$T_i(t)\leq\ (\hat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ ,\ \ T_i(0)=T_i^0>0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0)\ ,T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{24}\right)^{(4)} e^{\left(\hat{M}_{24}\right)^{(4)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{24}\right)^{(4)} e^{\left(\hat{M}_{24}\right)^{(4)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{28}\right)^{(5)} e^{\left(\hat{M}_{28}\right)^{(5)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{28}\right)^{(5)} e^{\left(\hat{M}_{28}\right)^{(5)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{32}\right)^{(6)} e^{\left(\hat{M}_{32}\right)^{(6)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{32}\right)^{(6)} e^{\left(\hat{M}_{32}\right)^{(6)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{36}\right)^{(7)} e^{\left(\hat{M}_{36}\right)^{(7)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{36}\right)^{(7)} e^{\left(\hat{M}_{36}\right)^{(7)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{40}\right)^{(8)} e^{\left(\hat{M}_{40}\right)^{(8)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{40}\right)^{(8)} e^{\left(\hat{M}_{40}\right)^{(8)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{44}\right)^{(9)} e^{\left(\hat{M}_{44}\right)^{(9)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{44}\right)^{(9)} e^{\left(\hat{M}_{44}\right)^{(9)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{13})^{(1)}, T_i^0 \le (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{16})^{(2)}, T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\tilde{M}_{16})^{(2)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\tilde{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{32})^{(6)}, T_i^0 \le (\hat{Q}_{32})^{(6)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0,t)$

Proof: 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(j) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left(e^{(\tilde{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\tilde{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left(e^{(\tilde{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\tilde{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0\right) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\tilde{M}_{44})^{(9)}} \left(e^{(\tilde{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\tilde{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\tilde{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\tilde{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\tilde{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[(\widehat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on

$G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 19 to 24 it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(1)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If $G_{13}\ \ is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}\,(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on

(G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\,(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on T_{21} and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \quad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 227

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on T_{25} and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(4)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25}\ \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t\ > t_4$

$$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\widehat{P}_{28})^{(5)}$ and $(\widehat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\widehat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \tag{234}$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \tag{235}$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric (236)

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{31}}),(\widetilde{T_{31}})$: $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\leq \tag{237}$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\ (G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(5)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)}+\left((\widehat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\widehat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\widehat{Q}_{32})^{(6)}\right]\leq(\widehat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(6)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose

$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \tag{256}$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \tag{257}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric (258)

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})},\widetilde{(T_{39})}:\left(\widetilde{(G_{39})},\widetilde{(T_{39})}\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$

It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\leq \tag{259}$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. (260)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\widehat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{40})^{(8)}\right] \leq (\widehat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})}, \widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by 274

$(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$. The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right)) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)}, , (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13} e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13} e^{(S_1)^{(1)} t}$

where $(p_i)^{(1)}$ is defined by equation

$\frac{1}{(m_1)^{(1)}} G^0_{13} e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G^0_{13} e^{(S_1)^{(1)} t}$

$$\left(\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t}-e^{-(S_2)^{(1)}t}\right]+G_{15}^{0}e^{-(S_2)^{(1)}t}\le G_{15}(t)\le\right. \quad (286)$$

$$\left.\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}\right]+G_{15}^{0}e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation (291)

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: (292)

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(T_{17},t)+(a''_{17})^{(2)}(T_{17},t)\le-(\sigma_1)^{(2)} \quad (293)$$

$$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)} \quad (294)$$

Definition of $(\nu_1)^{(2)},(\nu_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: (295)

By $(\nu_1)^{(2)}>0$, $(\nu_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0$, $(u_2)^{(2)}<0$ the roots (296)

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_1)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ (297)

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and (298)

Definition of $(\bar{\nu}_1)^{(2)},,(\bar{\nu}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: (299)

By $(\bar{\nu}_1)^{(2)}>0$, $(\bar{\nu}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0$, $(\bar{u}_2)^{(2)}<0$ the (300)

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_2)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ (301)

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t}\right] + T_{18}^0 e^{-(R_2)^{(2)}t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$

$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations
$(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$
and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by
$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$$

$$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(v^{(4)}\big)^2 + (\sigma_1)^{(4)} v^{(4)} - (a_{24})^{(4)} = 0$

and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)},,(\bar{v}_2)^{(4)},(\bar{u}_1)^{(4)},(\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$ and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}$, $(\mu_2)^{(4)}$, $(v_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}$, $(\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, ***if*** $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)}t} - e^{-(a'_{26})^{(4)}t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)}t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^{0}}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t}-e^{-(R_2)^{(4)}t}\right]+T_{26}^{0}e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)},(S_2)^{(4)},(R_1)^{(4)},(R_2)^{(4)}$:- 337

Where $(S_1)^{(4)}=(a_{24})^{(4)}(m_2)^{(4)}-(a'_{24})^{(4)}$

$(S_2)^{(4)}=(a_{26})^{(4)}-(p_{26})^{(4)}$

$(R_1)^{(4)}=(b_{24})^{(4)}(\mu_2)^{(4)}-(b'_{24})^{(4)}$

$(R_2)^{(4)}=(b'_{26})^{(4)}-(r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)},(\sigma_2)^{(5)},(\tau_1)^{(5)},(\tau_2)^{(5)}$:

$(\sigma_1)^{(5)},(\sigma_2)^{(5)},(\tau_1)^{(5)},(\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)}\le-(a'_{28})^{(5)}+(a'_{29})^{(5)}-(a''_{28})^{(5)}(T_{29},t)+(a''_{29})^{(5)}(T_{29},t)\le-(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)}\le-(b'_{28})^{(5)}+(b'_{29})^{(5)}-(b''_{28})^{(5)}\big((G_{31}),t\big)-(b''_{29})^{(5)}\big((G_{31}),t\big)\le-(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)},(\nu_2)^{(5)},(u_1)^{(5)},(u_2)^{(5)},\nu^{(5)},u^{(5)}$: 339

By $(\nu_1)^{(5)}>0$, $(\nu_2)^{(5)}<0$ and respectively $(u_1)^{(5)}>0$, $(u_2)^{(5)}<0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2+(\sigma_1)^{(5)}\nu^{(5)}-(a_{28})^{(5)}=0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2+(\tau_1)^{(5)}u^{(5)}-(b_{28})^{(5)}=0$ and

Definition of $(\bar{\nu}_1)^{(5)},,(\bar{\nu}_2)^{(5)},(\bar{u}_1)^{(5)},(\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)}>0$, $(\bar{\nu}_2)^{(5)}<0$ and respectively $(\bar{u}_1)^{(5)}>0$, $(\bar{u}_2)^{(5)}<0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2+(\sigma_2)^{(5)}\nu^{(5)}-(a_{28})^{(5)}=0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2+(\tau_2)^{(5)}u^{(5)}-(b_{28})^{(5)}=0$

Definition of $(m_1)^{(5)},(m_2)^{(5)},(\mu_1)^{(5)},(\mu_2)^{(5)},(\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)},(m_2)^{(5)},(\mu_1)^{(5)},(\mu_2)^{(5)}$ by

$(m_2)^{(5)}=(\nu_0)^{(5)},(m_1)^{(5)}=(\nu_1)^{(5)}$, $\boldsymbol{if}\ (\nu_0)^{(5)}<(\nu_1)^{(5)}$

$(m_2)^{(5)}=(\nu_1)^{(5)},(m_1)^{(5)}=(\bar{\nu}_1)^{(5)}$, $\boldsymbol{if}\ (\nu_1)^{(5)}<(\nu_0)^{(5)}<(\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)}=\frac{G_{28}^{0}}{G_{29}^{0}}}$

$(m_2)^{(5)}=(\nu_1)^{(5)},(m_1)^{(5)}=(\nu_0)^{(5)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(5)}<(\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)}=(u_0)^{(5)},(\mu_1)^{(5)}=(u_1)^{(5)}$, $\boldsymbol{if}\ (u_0)^{(5)}<(u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, $\boldsymbol{if}$ $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}$ $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{28}(t) \leq G_{28}^0 e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{29}(t) \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})} \left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t} \right] + G_{30}^0 e^{-(S_2)^{(5)} t} \leq G_{30}(t) \leq \right. \quad 344$$

$$\left. \frac{(a_{30})^{(5)} G_{28}^0}{(m_2)^{(5)}((S_1)^{(5)} - (a'_{30})^{(5)})} \left[e^{(S_1)^{(5)} t} - e^{-(a'_{30})^{(5)} t} \right] + G_{30}^0 e^{-(a'_{30})^{(5)} t} \right)$$

$$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \leq T_{28}(t) \leq T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \leq T_{28}(t) \leq \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})} \left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t} \right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \leq T_{30}(t) \leq \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})} \left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t} \right] + T_{30}^0 e^{-(R_2)^{(5)} t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \leq -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \leq -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \leq -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}((G_{35}), t) - (b''_{33})^{(6)}((G_{35}), t) \leq -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}$, $(m_2)^{(6)}$, $(\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}$, $(m_2)^{(6)}$, $(\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t} - e^{-(a'_{34})^{(6)}t}\right] + G_{34}^0 e^{-(a'_{34})^{(6)}t}\right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(v_1)^{(7)}, (v_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, v^{(7)}, u^{(7)}$: 361

By $(v_1)^{(7)} > 0, (v_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{v}_1)^{(7)}, , (\bar{v}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)} > 0, (\bar{v}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)},\ \boldsymbol{if}\ (\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)},\ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}),t\big) - (b''_{45})^{(9)}\big((G_{47}),t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)},, (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, ***if*** $(\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, ***if*** $(\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, ***if*** $(\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,
and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44}e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44}e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G^0_{44}e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G^0_{44}e^{(S_1)^{(9)}t}$$

(

$$\frac{(a_{46})^{(9)}G^0_{44}}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})}\Big[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t}\Big] + G^0_{46}e^{-(S_2)^{(9)}t} \le G_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}G^0_{44}}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})}[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G^0_{46}e^{-(a'_{46})^{(9)}t})$$

$$\boxed{T_{44}^{0}e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^{0}e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^{0}e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^{0}e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^{0}}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t}-e^{-(b'_{46})^{(9)}t}\right]+T_{46}^{0}e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^{0}}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t}-e^{-(R_2)^{(9)}t}\right]+T_{46}^{0}e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^{0}}{G_{14}^{0}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{dv^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)v^{(2)} - (a_{17})^{(2)}v^{(2)}$$

Definition of $v^{(2)}$:- $\boxed{v^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(v^{(2)}\right)^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)}\right) \leq \frac{dv^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(v^{(2)}\right)^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{v}_1)^{(2)}, (v_0)^{(2)}$:-

For $0 < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (v_1)^{(2)} < (\bar{v}_1)^{(2)}$

$$v^{(2)}(t) \geq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(v_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (v_2)^{(2)}}}$$

it follows $(v_0)^{(2)} \leq v^{(2)}(t) \leq (v_1)^{(2)}$

In the same manner , we get 391

$$v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{v}_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (\bar{v}_2)^{(2)}}}$$

From which we deduce $(v_0)^{(2)} \leq v^{(2)}(t) \leq (\bar{v}_1)^{(2)}$ 392

If $0 < (v_1)^{(2)} < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{v}_1)^{(2)}$ we find like in the previous case, 393

$$(v_1)^{(2)} \leq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_2)^{(2)}\right)t\right]}} \leq v^{(2)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \leq (\bar{v}_1)^{(2)}$$

If $0 < (v_1)^{(2)} \leq (\bar{v}_1)^{(2)} \leq (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(v_1)^{(2)} \leq v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \leq (v_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(2)}(t)$:- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then
$(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

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If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\quad \boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$v^{(6)}(t) \le \frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)}=\frac{(\bar{v}_1)^{(6)}-(v_0)^{(6)}}{(v_0)^{(6)}-(\bar{v}_2)^{(6)}}}$$

From which we deduce $(v_0)^{(6)} \le v^{(6)}(t) \le (\bar{v}_1)^{(6)}$

If $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{v}_1)^{(6)}$ we find like in the previous case, 414

$$(v_1)^{(6)} \le \frac{(v_1)^{(6)}+(C)^{(6)}(v_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)}-(v_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)}-(v_2)^{(6)}\right)t\right]}} \le v^{(6)}(t) \le$$

$$\frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \le (\bar{v}_1)^{(6)}$$

If $0 < (v_1)^{(6)} \le (\bar{v}_1)^{(6)} \le \boxed{(v_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(v_1)^{(6)} \le v^{(6)}(t) \le \frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \le (v_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(6)}(t)$:-

$$(m_2)^{(6)} \le v^{(6)}(t) \le (m_1)^{(6)}, \quad \boxed{v^{(6)}(t)=\frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \le u^{(6)}(t) \le (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t)=\frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(v_1)^{(6)} = (\bar{v}_1)^{(6)}$ if in addition $(v_0)^{(6)} = (v_1)^{(6)}$ then $v^{(6)}(t) = (v_0)^{(6)}$ and as a consequence $G_{32}(t) = (v_0)^{(6)}G_{33}(t)$ **this also defines** $(v_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(v_1)^{(6)}$ and $(\bar{v}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)}=\frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}\right)\leq\frac{dv^{(7)}}{dt}\leq-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)},(\nu_0)^{(7)}$:-

$$\text{For } 0<\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}<(\nu_1)^{(7)}<(\bar{\nu}_1)^{(7)}$$

$$v^{(7)}(t)\geq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}\quad,\quad\boxed{(C)^{(7)}=\frac{(\nu_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)}\leq v^{(7)}(t)\leq(\nu_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(7)}=\frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)}\leq v^{(7)}(t)\leq(\bar{\nu}_1)^{(7)}$

If $0<(\nu_1)^{(7)}<(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}<(\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)}\leq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}\leq v^{(7)}(t)\leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\bar{\nu}_1)^{(7)}$$

If $0<(\nu_1)^{(7)}\leq(\bar{\nu}_1)^{(7)}\leq\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)}\leq v^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(7)}(t)$:-

$$(m_2)^{(7)}\leq v^{(7)}(t)\leq(m_1)^{(7)},\quad\boxed{v^{(7)}(t)=\frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

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If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case,

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

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If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{40}'')^{(8)} = (a_{41}'')^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b_{40}'')^{(8)} = (b_{41}'')^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a_{44}')^{(9)} - (a_{45}')^{(9)} + (a_{44}'')^{(9)}(T_{45}, t)\right) - (a_{45}'')^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\quad \boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \leq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \leq \nu^{(9)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \leq (\bar{\nu}_1)^{(9)} \leq \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \leq \nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \leq \nu^{(9)}(t) \leq (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for**

the special case .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a_i'')^{(1)} and\ (b_i'')^{(1)}$ are independent on t , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0$ 447

$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0$ 448

has a unique positive solution , which is an equilibrium solution

$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0$ 449

$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0$ 450

$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0$ 451

$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0$ 452

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(j) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T_{33}^* :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that

there exists a unique G_{37}^{*} such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)}G_{17}^{*}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)}G_{17}^{*}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T_{17}^{*})]} \quad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)}T_{17}^{*}}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)}T_{17}^{*}}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^{*})]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)}G_{21}^{*}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)}G_{21}^{*}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$$T_{28}^* = \frac{(b_{28})^{(5)} T_{29}^*}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^*)]} \quad , \quad T_{30}^* = \frac{(b_{30})^{(5)} T_{29}^*}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^*)]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^* given by $\varphi((G_{35})^*) = 0$, T_{33}^* given by $f(T_{33}^*) = 0$ and

$$G_{32}^* = \frac{(a_{32})^{(6)} G_{33}^*}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34}^* = \frac{(a_{34})^{(6)} G_{33}^*}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

$$T_{32}^* = \frac{(b_{32})^{(6)} T_{33}^*}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^*)]} \quad , \quad T_{34}^* = \frac{(b_{34})^{(6)} T_{33}^*}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^*)]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^* given by $\varphi((G_{39})^*) = 0$, T_{37}^* given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

G_{41}^* given by $\varphi((G_{43})^*) = 0$, T_{41}^* given by $f(T_{41}^*) = 0$ and

$$G_{40}^{*}=\frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}\quad,\quad G_{42}^{*}=\frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

$$T_{40}^{*}=\frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}\quad,\quad T_{42}^{*}=\frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$$

Finally we obtain the unique solution of 89 to 99 523
A

G_{45}^{*} given by $\varphi((G_{47})^{*})=0$, T_{45}^{*} given by $f(T_{45}^{*})=0$ and

$$G_{44}^{*}=\frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}\quad,\quad G_{46}^{*}=\frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$$

$$T_{44}^{*}=\frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}\quad,\quad T_{46}^{*}=\frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}\ and\ (b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i=G_i^{*}+\mathbb{G}_i\qquad,T_i=T_i^{*}+\mathbb{T}_i$$

$$\frac{\partial(a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^{*})=(q_{14})^{(1)}\quad,\ \frac{\partial(b_i'')^{(1)}}{\partial G_j}(\,G^{*}\,)=s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt}=-\big((a'_{13})^{(1)}+(p_{13})^{(1)}\big)\mathbb{G}_{13}+(a_{13})^{(1)}\mathbb{G}_{14}-(q_{13})^{(1)}G_{13}^{*}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt}=-\big((a'_{14})^{(1)}+(p_{14})^{(1)}\big)\mathbb{G}_{14}+(a_{14})^{(1)}\mathbb{G}_{13}-(q_{14})^{(1)}G_{14}^{*}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt}=-\big((a'_{15})^{(1)}+(p_{15})^{(1)}\big)\mathbb{G}_{15}+(a_{15})^{(1)}\mathbb{G}_{14}-(q_{15})^{(1)}G_{15}^{*}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt}=-\big((b'_{13})^{(1)}-(r_{13})^{(1)}\big)\mathbb{T}_{13}+(b_{13})^{(1)}\mathbb{T}_{14}+\textstyle\sum_{j=13}^{15}\big(s_{(13)(j)}T_{13}^{*}\mathbb{G}_j\big) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt}=-\big((b'_{14})^{(1)}-(r_{14})^{(1)}\big)\mathbb{T}_{14}+(b_{14})^{(1)}\mathbb{T}_{13}+\textstyle\sum_{j=13}^{15}\big(s_{(14)(j)}T_{14}^{*}\mathbb{G}_j\big) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt}=-\big((b'_{15})^{(1)}-(r_{15})^{(1)}\big)\mathbb{T}_{15}+(b_{15})^{(1)}\mathbb{T}_{14}+\textstyle\sum_{j=13}^{15}\big(s_{(15)(j)}T_{15}^{*}\mathbb{G}_j\big) \quad 530$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions

$(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \tag{532}$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \ , \ \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \tag{533}$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \tag{534}$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \tag{535}$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \tag{536}$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \tag{537}$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}')^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T_{17}^*\mathbb{G}_j\right) \tag{538}$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}')^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T_{18}^*\mathbb{G}_j\right) \tag{539}$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \ , \ \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}')^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \tag{541}$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}')^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \tag{542}$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}')^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \tag{543}$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}')^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T_{20}^*\mathbb{G}_j\right) \tag{544}$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^{*}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T_{22}^{*}\mathbb{G}_j\right) \quad 546$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^{*}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^{*}) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^{*}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^{*}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^{*}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^{*}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^{*}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^{*}\mathbb{G}_j\right) \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^{*}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}((G_{31})^{*}) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^{*}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}(\,(G_{35})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^{*}) = (q_{41})^{(8)} \quad , \frac{\partial(b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^{*}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^{*}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^{*}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^{*}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

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ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \ , \ \frac{\partial(b''_i)^{(9)}}{\partial G_j}(\ (G_{47})^*\ ) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\ B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\ C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\ D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\ E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\ F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\ G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left(\ (a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)\ (\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left(\ (b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)\ (\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14}+(b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\Big]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION EIGHT

Higgs Scalar-Pseudoscalar Mixing

INTRODUCTION—VARIABLES USED

Physics Letters B Volume 435, Issues 1–2, 3 September 1998, Pages 88–100 Higgs scalar-pseudoscalar mixing in the minimal supersymmetric Standard Model Apostolos Pilaftsis doi:10.1016/S0370-2693(98)00771-0

(1) In the minimal supersymmetric Standard Model, the heaviest CP-even Higgs boson H and (e&eb) the CP-odd Higgs scalar A are predicted to be (=) almost degenerate in mass at (eb) the tree level, for (e) the wide kinematic range MA>2MZ and tanβ≥2.

(2) In the minimal supersymmetric Standard Model, the heaviest CP-even Higgs boson H and (e&eb) the CP-odd Higgs scalar A are predicted to be (=) almost degenerate in mass at (eb) the tree level, for (e) the wide kinematic range MA>2MZ and tanβ≥2.

(3) However, if large soft-CP-violating Yukawa interactions involving (e&eb) scalar quarks of the third generation are present in the theory, then (eb) the CP invariance of the Higgs potential can be maximally broken beyond (e) the Born approximation, and the high degree of (e) mass degeneracy between H and A may be lifted (e) through a sizeable HA mixing.

(4) After taking the CP-odd tadpole renormalization of (e) the A boson into account, they find that the small mass difference MH−MA, which is (=) about 1% of the A-boson mass at the tree level, can be substantially enhanced to (eb) the 25% level at the one-loop order.

(5) After taking the CP-odd tadpole renormalization of (e) the A boson into account, they find that the small mass difference MH−MA, which is (=) about 1% of the A-boson mass at the tree level, can be substantially enhanced to (eb) the 25% level at the one-loop order.

(6) They also find that the loop-induced (eb) mixing between the lightest CP-even Higgs boson h and (e&eb) the A boson may be of (=) comparable size to Mh.

(7) They briefly discuss the main phenomenological implications of the predicted hA and HA mixings for (e) the general Higgs-boson mass spectrum and for (e) CP-violating observables at (eb) collider and lower energies. Copyright © 1998 Elsevier Science B.V. All rights reserved.

(8) They briefly discuss the main phenomenological implications of the predicted hA and HA mixings for (e) the general Higgs-boson mass spectrum and for (e) CP-violating observables at (eb) collider and lower energies. Copyright © 1998 Elsevier Science B.V. All rights reserved.

NOTATION

Module One

In the minimal supersymmetric Standard Model, the

heaviest CP-even Higgs boson H and (e&eb) the CP-odd Higgs scalar A are predicted to be (=) almost degenerate in mass at (eb) the tree level, for (e) the wide kinematic range MA>2MZ and tanβ≥2

G_{13} : Category one of heaviest CP-even Higgs boson H; CP-odd Higgs scalar A are predicted to be (=) almost degenerate in mass at (eb) the tree level, for (e) the wide kinematic range MA>2MZ and tanβ≥2

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of CP-odd Higgs scalar A are predicted to be (=) almost degenerate in mass at (eb) the tree level, for (e) the wide kinematic range MA>2MZ and tanβ≥2;heaviest CP-even Higgs boson H

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

heaviest CP-even Higgs boson H and the CP-odd Higgs scalar A are predicted to be (=) almost degenerate in mass at (eb) the tree level, for (e) the wide kinematic range MA>2MZ and tanβ≥2

G_{16} : Category one of heaviest CP-even Higgs boson H and the CP-odd Higgs scalar A

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of degenerate in mass at (eb) the tree level, for (e) the wide kinematic range MA>2MZ and tanβ≥2

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

heaviest CP-even Higgs boson H and the CP-odd Higgs scalar A are predicted to be almost degenerate in mass at the tree level, for (e) the wide kinematic range MA>2MZ and tanβ≥2

G_{20} : Category one of wide kinematic range MA>2MZ and tanβ≥2

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of heaviest CP-even Higgs boson H and the CP-odd Higgs scalar A are predicted to be almost degenerate in mass at the tree level

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

However, if large soft-CP-violating Yukawa interactions involving (e&eb) scalar quarks of the third generation are present in the theory, then (eb) the CP invariance of the Higgs potential can be maximally broken beyond (e) the Born approximation, and the high degree of (e) mass degeneracy between H and A

may be lifted (e) through a sizeable HA mixing

G_{24} : Category one of large soft-CP-violating Yukawa interactions involving (e&eb) scalar quarks of the third generation are present in the theory,

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one ofCP invariance of the Higgs potential can be maximally broken beyond (e) the Born approximation, and the high degree of (e) mass degeneracy between H and A may be lifted (e) through a sizeable HA mixing

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

large soft-CP-violating Yukawa interactions involving scalar quarks of the third generation are present in the theory, then the CP invariance of the Higgs potential can be maximally broken beyond (e) the Born approximation, and the high degree of (e) mass degeneracy between H and A may be lifted (e) through a sizeable HA mixing

G_{28} : Category one of Born approximation, and the high degree of (e) mass degeneracy between H and A may be lifted (e) through a sizeable HA mixing

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of large soft-CP-violating Yukawa interactions involving scalar quarks of the third generation are present in the theory, then the CP invariance of the Higgs potential can be maximally broken

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

large soft-CP-violating Yukawa interactions involving scalar quarks of the third generation are present in the theory, then the CP invariance of the Higgs potential can be maximally broken beyond the Born approximation, and the high degree of mass degeneracy between H and A may be lifted (e) through a sizeable HA mixing

G_{32} : Category one of sizeable HA mixing

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of large soft-CP-violating Yukawa interactions involving scalar quarks of the third generation are present in the theory, then the CP invariance of the Higgs potential can be maximally broken beyond the Born approximation, and the high degree of mass degeneracy between H and A

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

After taking the CP-odd tadpole renormalization of the A boson into account, they find that the small mass difference MH−MA, which is (=) about 1% of the A-boson mass at the tree level, can be substantially enhanced to (eb) the 25% level at the one-loop order

G_{36} : Category one of A boson into account, they find that the small mass difference MH−MA, which is (=) about 1% of the A-boson mass at the tree level, can be substantially enhanced to (eb) the 25% level at the one-loop order

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of CP-odd tadpole renormalization

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

taking the CP-odd tadpole renormalization of the A boson into account, they find that the small mass difference MH−MA, which is (=) about 1% of the A-boson mass at the tree level, can be substantially enhanced to (eb) the 25% level at the one-loop order

G_{40} : Category one of taking the CP-odd tadpole renormalization of the A boson into account, they find that the small mass difference MH−MA, which is (=) about 1% of the A-boson mass at the tree level

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of25% level at the one-loop order (different for different experiments; alternately take all the three categories as equivalent)

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

They also find that the loop-induced mixing between the lightest CP-even Higgs boson h and (e&eb) the A boson may be of (=) comparable size to Mh

G_{44} : Category one of loop-induced mixing between the lightest CP-even Higgs boson h; A boson may be of comparable size to Mh

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of A boson may be of (=) comparable size to Mh; loop-induced mixing between the lightest CP-even Higgs boson h

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION NINE

Renormalization-Group-Improved Higgs Sector

INTRODUCTION—VARIABLES USED

Renormalization-group-improved Higgs sector of the minimal supersymmetric model Phys. Rev. D 48, 4280 – Published 1 November 1993 Howard E. Haber and Ralf Hempfling

(1) In the minimal supersymmetric model all Higgs self-coupling parameters are related to (e&eb) gauge couplings at the tree level.
(2) Leading-logarithmic radiative corrections to (e) these quantities can be summed using (e) renormalization-group techniques.
(3) By this procedure authors obtain complete leading-logarithmic radiative corrections to (e) the Higgs boson masses, the CP-even Higgs boson mixing angle, and trilinear Higgs boson couplings.
(4) Additional corrections due to (e) squark mixing can be explicitly incorporated into (e) this formalism.
(5) These results incorporate nearly all potentially large corrections. Mass shifts to (e) the neutral CP-even Higgs bosons grow with (e&eb) the fourth power of the top-quark mass and can be significant. The phenomenological consequences of these results are examined. DOI: http://dx.doi.org/10.1103/PhysRevD.48.4280

NOTATION

Module One

all Higgs self-coupling parameters In the minimal supersymmetric model are related to (e&eb) gauge couplings at the tree level

G_{13} : Category one of all Higgs self-coupling parameters In the minimal supersymmetric model; gauge couplings at the tree level

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of gauge couplings at the tree level ;all Higgs self-coupling parameters In the minimal supersymmetric model

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Leading-logarithmic radiative corrections to (e) these quantities can be summed using (e) renormalization-group techniques

G_{16} : Category one of these quantities can be summed using (e) renormalization-group techniques

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of Leading-logarithmic radiative corrections

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Leading-logarithmic radiative corrections to these quantities can be summed using (e) renormalization-group techniques

G_{20} : Category one of renormalization-group techniques

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of Leading-logarithmic radiative corrections to these quantities can be summed

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

By this procedure authors obtain

complete leading-logarithmic radiative corrections to (e) the Higgs boson masses, the CP-even Higgs boson mixing angle, and trilinear Higgs boson couplings

G_{24} : Category one of Higgs boson masses, the CP-even Higgs boson mixing angle, and trilinear Higgs boson couplings

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of complete leading-logarithmic radiative corrections

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Higgs boson masses, the CP-even Higgs boson mixing angle, and trilinear Higgs boson couplings

G_{28} : Category one of Higgs boson masses; CP-even Higgs boson mixing angle

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of CP-even Higgs boson mixing angle; Higgs boson masses

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

G_{32} : Category one of CP-even Higgs boson mixing angle; trilinear Higgs boson couplings

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of trilinear Higgs boson couplings; CP-even Higgs boson mixing angle

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Additional corrections due to (e) squark mixing can be explicitly incorporated into (e) this formalism.

G_{36} : Category one of squark mixing can be explicitly incorporated into (e) this formalism

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of Additional corrections

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

Additional corrections due to squark mixing can be explicitly incorporated into (e) this formalism

G_{40} : Category one of formalism

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of Additional corrections due to squark mixing can be explicitly incorporated

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

These results incorporate nearly all potentially large corrections.

Mass shifts to the neutral CP-even Higgs bosons grow with (e&eb) the fourth power of the top-quark mass and can be significant.

The phenomenological consequences of these results are examined. DOI: http://dx.doi.org/10.1103/PhysRevD.48.4280

G_{44} : Category one of Mass shifts to the neutral CP-even Higgs bosons; fourth power of the top-quark mass

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of fourth power of the top-quark mass; Mass shifts to the neutral CP-even Higgs bosons

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},\ \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}\ ,$
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G,t)\right]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t)\right]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}),t\big)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}),t\big)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}),t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\big((G_{19}),t\big) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27}),t\big)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27}),t\big)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27}),t\big)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\big((G_{27}),t\big) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}),t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}),t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}),t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}),t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}),t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}),t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}),t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}),t\right)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}),t\right)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\left((G_{39}),t\right)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}),t\right)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}),t\right)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}),t\right)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}),t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}),t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad 63$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition

coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)}\ \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\ \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\ \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{22} \quad (69)$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for

category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad 70$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad 72$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$ $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}, \boxed{(a''_{25})^{(4)}(T_{25},t)}, \boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
are sixth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
are seventh augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
are eighth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad 78$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$

are fourth detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{ccc} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{ccc} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{ccc} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{30} \quad (81)$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3
And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3
$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3
$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3
$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3
$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3
$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3
$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3
$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation

coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)} & +(a''_{32})^{(6)}(T_{33},t) & +(a''_{28})^{(5,5,5)}(T_{29},t) & +(a''_{24})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)} & +(a''_{33})^{(6)}(T_{33},t) & +(a''_{29})^{(5,5,5)}(T_{29},t) & +(a''_{25})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)}\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)}\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)}\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* $1,2$ *and* 3

$-(b_{13}'')^{(1,1,1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1,1,1)}(G,t)$, $-(b_{15}'')^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b_{16}'')^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b_{20}'')^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3

$-(b_{36}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3

$-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2, and 3

$-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a_{36}')^{(7)} & +(a_{36}'')^{(7)}(T_{37},t) & +(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t) & +(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a_{37}')^{(7)} & +(a_{37}'')^{(7)}(T_{37},t) & +(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a_{38}')^{(7)} & +(a_{38}'')^{(7)}(T_{37},t) & +(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15}$$

Where $(a_{36}'')^{(7)}(T_{37},t)$, $(a_{37}'')^{(7)}(T_{37},t)$, $(a_{38}'')^{(7)}(T_{37},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{40}}{dt} =$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt}$$

$$= (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{ccc} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[\begin{array}{ccc} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{array}\right] T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b''_i)^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}', t) - (a_i'')^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}', t)$ and $(a_i'')^{(1)}(T_{14}, t)$. (T_{14}', t) and (T_{14}, t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}, \ (r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq \left(\hat{A}_{16}\right)^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}\left((G_{19}), t\right) = \ (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}\big((G_{19}), t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, \ (\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq \ (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$\lim_{G\to\infty}(b_i'')^{(3)}\,(G_{23},t) = \ (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$

$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)},$ are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$

$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$

$(b_i'')^{(4)}\big((G_{27}),t\big) \le \ (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$

$lim_{T_2\to\infty}(a_i'')^{(4)}\,(T_{25},t) = (p_i)^{(4)}$ 118

$\lim_{G\to\infty}(b_i'')^{(4)}\,\big((G_{27}),t\big) = \ (r_i)^{(4)}$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}\,(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}\,(G_{31}, t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\,(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}\,(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}\,\big((G_{35}), t\big) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\big((G_{35}), t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\,(a_i)^{(6)} + (a_i')^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)}\,(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ \ (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(III) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(JJJ) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq \ (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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(KKK) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(LLL)

$\lim_{G\to\infty}(b_i'')^{(7)}\left((G_{39}),t\right) = \ (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, \ (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}\left((G_{39}),t\right)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$. (T_{37}',t) and (T_{37},t) are points belonging to the interval $\left[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\right]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(MMM) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(NNN) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, \ (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\,(a_i)^{(7)} + (a_i')^{(7)} + \ (\hat{A}_{36})^{(7)} + \ (\hat{P}_{36})^{(7)}\,(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\ (b_i)^{(7)} + (b_i')^{(7)} + \ (\hat{B}_{36})^{(7)} + \ (\hat{Q}_{36})^{(7)}\ (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}, \ (r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41},t) \le (p_i)^{(8)} \le (\hat{A}_{40})^{(8)} \quad 138$$

$$(b_i'')^{(8)}((G_{43}),t) \le (r_i)^{(8)} \le (b_i')^{(8)} \le (\hat{B}_{40})^{(8)} \quad 139$$

$$\lim_{T_2\to\infty}(a_i'')^{(8)}(T_{41},t) = (p_i)^{(8)} \quad 140$$

$$\lim_{G\to\infty}(b_i'')^{(8)}((G_{43}),t) = (r_i)^{(8)} \quad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}',t) - (a_i'')^{(8)}(T_{41},t)| \le (\hat{k}_{40})^{(8)}|T_{41} - T_{41}'|e^{-(\hat{M}_{40})^{(8)}t} \quad 142$$

$$|(b_i'')^{(8)}((G_{43})',t) - (b_i'')^{(8)}((G_{43}),t)| < (\hat{k}_{40})^{(8)}||(G_{43}) - (G_{43})'||e^{-(\hat{M}_{40})^{(8)}t} \quad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}',t)$ and $(a_i'')^{(8)}(T_{41},t)$. (T_{41}',t) and (T_{41},t) are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \quad 144$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad 146A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t) \le (p_i)^{(9)} \le (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t) \le (r_i)^{(9)} \le (b_i')^{(9)} \le (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty} (a_i'')^{(9)}(T_{45},t) = (p_i)^{(9)}$$

$$\lim_{G\to\infty} (b_i'')^{(9)}(G_{47},t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t) - (a_i'')^{(9)}(T_{45},t)| \le (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t) - (b_i'')^{(9)}\big((G_{47}),t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$ and$(a_i'')^{(9)}(T_{45},t)$. (T_{45}',t) and (T_{45},t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}$, $(r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\,, T_i(0)$

$$G_i(t) \le \left(\hat{P}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t} \;,\quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le \left(\hat{Q}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t} \;,\quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le \left(\hat{P}_{20}\right)^{(3)} e^{(\hat{M}_{20})^{(3)}t} \;,\quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le \left(\hat{Q}_{20}\right)^{(3)} e^{(\hat{M}_{20})^{(3)}t} \;,\quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}$, $T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}$, 155

$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 156

$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{16})^{(2)}$, $T_i^0 \leq (\hat{Q}_{16})^{(2)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''^{(2)}_{16}\right)\left(T_{17}(s_{(16)}), s_{(16)}\right)\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}\left(T_{17}(s_{(16)}), s_{(17)}\right)\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}\left(T_{17}(s_{(16)}), s_{(16)}\right)\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{20})^{(3)}$, $T_i^0 \leq (\hat{Q}_{20})^{(3)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''^{(3)}_{20}\right)\left(T_{21}(s_{(20)}), s_{(20)}\right)\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}\left(T_{21}(s_{(20)}), s_{(20)}\right)\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}\left(T_{21}(s_{(20)}), s_{(20)}\right)\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)},\ T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)},\ T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad (169)$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that (170)

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that (171)

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that (172)

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that (173)

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that (174)

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \leq G_{32}^{0}+\int_0^t\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)} =$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(k) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \leq G_{36}^{0}+\int_0^t\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)} =$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \leq G_{40}^{0}+\int_0^t\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)} = \quad 180$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t} \leq \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \leq G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \leq \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \left\{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\right\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $\left(\tilde{G}, \tilde{T}\right) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+\ (a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\ (t)=0\ and\ T_i\ (t)=0$

From 19 to 24 it results

$$G_i\ (t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\geq 0$$

$$T_i\ (t)\geq T_i^0e^{\left(-(b_i')^{(1)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1,\left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13}<(\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt}\leq\left((\widehat{M}_{13})^{(1)}\right)_1-(a'_{14})^{(1)}G_{14}$ and by integrating

$$G_{14}\leq\left((\widehat{M}_{13})^{(1)}\right)_2=G_{14}^0+2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15}\leq\left((\widehat{M}_{13})^{(1)}\right)_3=G_{15}^0+2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a'_{15})^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}} \leq$
$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{\mathrm{A}}_{16})^{(2)} + (\widehat{\mathrm{P}}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)}, (T_{19})^{(1)};\ (G_{19})^{(2)}, (T_{19})^{(2)})\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $\mathrm{G}_i\,(\mathrm{t}) = 0$ and $\mathrm{T}_i\,(\mathrm{t}) = 0$ 200

it results

$\mathrm{G}_i\,(\mathrm{t}) \geq \mathrm{G}_i^0\mathrm{e}^{\left[-\int_0^{\mathrm{t}}\{(a_i')^{(2)} - (a_i'')^{(2)}(\mathrm{T}_{17}(s_{(16)}), s_{(16)})\}\mathrm{d}s_{(16)}\right]} \geq 0$

$\mathrm{T}_i\,(\mathrm{t}) \geq \mathrm{T}_i^0\mathrm{e}^{(-(b_i')^{(2)}\mathrm{t})} > 0 \quad \text{for t} > 0$

Definition of $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1$, $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2$ and $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$\mathrm{G}_{16} < (\widehat{\mathrm{M}}_{16})^{(2)}$ it follows $\frac{\mathrm{dG}_{17}}{\mathrm{dt}} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)}\mathrm{G}_{17}$ and by integrating

$\mathrm{G}_{17} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 = \mathrm{G}_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$

In the same way , one can obtain

$\mathrm{G}_{18} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3 = \mathrm{G}_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{\mathrm{t}\to\infty}((b_i'')^{(2)}\,((G_{19})(\mathrm{t}), \mathrm{t})) = (b_{17}')^{(2)}$ then $\mathrm{T}_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$: $\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+\ (a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right) d\left(((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ (228)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)}G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})}, \widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})}, \widetilde{(T_{35})}\right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right)\left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)};\ (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ *and* $(\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ (250)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)} \left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)}; (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)} (T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b'_i)^{(7)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1,\left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right),\quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{47}}),(\widetilde{T_{47}}):\left((\widetilde{G_{47}}),(\widetilde{T_{47}})\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$

From 99 to 44 it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(9)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$: 281

By $(v_1)^{(1)} > 0$, $(v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{v}_1)^{(1)},, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{v}_1)^{(1)} > 0$, $(\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)}, , (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad (333)$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad (334)$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)} t} - e^{-(a'_{26})^{(4)} t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)} t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad (335)$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad (336)$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G^0_{28} e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G^0_{28}}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G^0_{30} e^{-(S_2)^{(5)} t} \le G_{30}(t) \le \right. \quad 344$$

$$\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_2)^{(5)}((S_1)^{(5)}-(a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t}-e^{-(a'_{30})^{(5)}t}\right]+\ G_{30}^{0}e^{-(a'_{30})^{(5)}t}\Big)$$

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \qquad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \qquad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\le T_{30}(t)\le \qquad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$:

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$-(\sigma_2)^{(6)}\le-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\le-(\sigma_1)^{(6)}$

$-(\tau_2)^{(6)}\le-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\le-(\tau_1)^{(6)}$

Definition of $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$: 350

By $(v_1)^{(6)}>0$, $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$, $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

Definition of $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)}>0$, $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$, $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

Definition of $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$:-

If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)}=(v_0)^{(6)},(m_1)^{(6)}=(v_1)^{(6)},$ ***if*** $(v_0)^{(6)}<(v_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}$ $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}$ $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}$ $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}$ $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}$ $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})} \left[e^{(S_1)^{(6)} t} - e^{-(a'_{34})^{(6)} t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)} t} \right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^{0}e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}}G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}}G_{36}^{0}e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t}-e^{-(S_2)^{(7)}t}\right]+G_{38}^{0}e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left.\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_2)^{(7)}((S_1)^{(7)}-(a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t}-e^{-(a'_{38})^{(7)}t}\right]+\ G_{38}^{0}e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}}T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}}T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right) - (b''_{41})^{(8)}\left((G_{43}), t\right) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(v_1)^{(8)} > 0$, $(v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$, $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{v}_1)^{(8)}$,, $(\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{v}_1)^{(8)} > 0$, $(\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$, $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$:-

If we define $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$, $\boldsymbol{if}$ $(v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$, $\boldsymbol{if}$ $(v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$,
and $\boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}$, $\boldsymbol{if}$ $(\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, $\boldsymbol{if}$ $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, $\boldsymbol{if}$ $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}$ $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t})$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t}\leq T_{42}(t)\leq \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)},(S_2)^{(8)},(R_1)^{(8)},(R_2)^{(8)}$:- 381

Where $(S_1)^{(8)}=(a_{40})^{(8)}(m_2)^{(8)}-(a'_{40})^{(8)}$

$(S_2)^{(8)}=(a_{42})^{(8)}-(p_{42})^{(8)}$

$(R_1)^{(8)}=(b_{40})^{(8)}(\mu_2)^{(8)}-(b'_{40})^{(8)}$

$(R_2)^{(8)}=(b'_{42})^{(8)}-(r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$:

$(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)}\leq -(a'_{44})^{(9)}+(a'_{45})^{(9)}-(a''_{44})^{(9)}(T_{45},t)+(a''_{45})^{(9)}(T_{45},t)\leq -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)}\leq -(b'_{44})^{(9)}+(b'_{45})^{(9)}-(b''_{44})^{(9)}\big((G_{47}),t\big)-(b''_{45})^{(9)}\big((G_{47}),t\big)\leq -(\tau_1)^{(9)}$

Definition of $(v_1)^{(9)},(v_2)^{(9)},(u_1)^{(9)},(u_2)^{(9)},v^{(9)},u^{(9)}$:

By $(v_1)^{(9)}>0$, $(v_2)^{(9)}<0$ and respectively $(u_1)^{(9)}>0$, $(u_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_1)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$ and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)}>0$, $(\bar{v}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$, $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$ and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

Definition of $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$:-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)},\ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$
and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \le G_{46}(t) \le \right.$$
$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (v_1)^{(1)} \le (\bar{v}_1)^{(1)} \le \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$, we obtain 386

$$(v_1)^{(1)} \le v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $v^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\right) - (a''_{17})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{\mathrm{G}_{16}}{\mathrm{G}_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (\mathrm{C})^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (\mathrm{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)}\right) \le \frac{dv^{(3)}}{dt} \le -\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \ge \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}}$$

it follows $(v_0)^{(3)} \le v^{(3)}(t) \le (v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \le v^{(3)}(t) \le (\bar{v}_1)^{(3)}$

If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)} \le \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}} \le v^{(3)}(t) \le$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (\bar{v}_1)^{(3)}$$

If $0 < (v_1)^{(3)} \le (\bar{v}_1)^{(3)} \le (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$, we obtain 403

$$(v_1)^{(3)} \le v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

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$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{36}'')^{(7)} = (a_{37}'')^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b_{36}'')^{(7)} = (b_{37}'')^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a_{40}')^{(8)} - (a_{41}')^{(8)} + (a_{40}'')^{(8)}(T_{41}, t)\right) - (a_{41}'')^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(8)} and\ (b''_i)^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\,]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\,]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\,]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0$ 479

$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0$ 480

$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0$ 481

$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0$ 482

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(k) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first

equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T^*_{33} for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$$

Definition and uniqueness of T^*_{37} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T^*_{37} for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$$

Definition and uniqueness of T^*_{41} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{25} such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507
$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{29} such that $\varphi((G_{31})^*) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{33} such that $\varphi({G_{35}}^*) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{37} such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{41} such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right]+(b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{45} such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)} G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)} G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \qquad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad (525)$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad (526)$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad (527)$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \quad (528)$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \quad (529)$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \quad (530)$$

ASYMPTOTIC STABILITY ANALYSIS (531)

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i \quad (532)$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad (533)$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad (534)$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad (535)$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \quad (536)$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \quad (537)$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}\mathrm{T}^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}\mathrm{T}^*_{18}\mathbb{G}_j\right) \quad 539$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}\left((G_{27})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^{*}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^{*}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^{*}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^{*}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^{*}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^{*}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^{*}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^{*}\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

ASYMPTOTIC STABILITY ANALYSIS (586 A)

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\Big)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\Big]$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\Big)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\Big)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\Big]$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\Big)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\Big)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\Big]$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\Big)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\Big)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\Big)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\Big)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\Big)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\Big)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION TEN

Molecular Dynamics

INTRODUCTION—VARIABLES USED

Computer Physics Communications Volume 190, May 2015, Pages 1–14 Molecular dynamics-based triangulation algorithm of free-form parametric surfaces for computer-aided engineering A.L. Zheleznyakova doi:10.1016/j.cpc.2014.12.018

(1) A new computational approach for (e) automated triangulation of Computer-Aided Design (CAD) surface models; applicable to (e&eb) various CFD (Computational Fluid Dynamics) problems of (e) practical interest are proposed.

(2) The complex shaped product configurations are represented by (eb) a set of Non-Uniform Rational B-Splines (NURBS) surface patches.
(3) The suggested technique is based on (e) the **molecular dynamics** method.

Computer Physics Communications Volume 190, May 2015, Pages 1–14 Molecular dynamics-based triangulation algorithm of free-form parametric surfaces for computer-aided engineering A.L. Zheleznyakova doi:10.1016/j.cpc.2014.12.018

(1) The main idea of the approach is that the mesh nodes are (=) considered as similarly charged interacting particles which move (e&eb) within the region to be meshed under the influence of (e&eb) internal (such as particle–particle interaction forces) and external forces as well as optional additional forces.
(2) Moreover, the particles experience (e) a medium resistance due to which the system comes to (=) equilibrium within (eb) a relatively short period of time.
(3) The proposed 3D surface mesh generation algorithm uses (e) a **parametric NURBS representation** as (=) initial definition of the domain boundary.
(4) This method first distributes the interacting nodes into (e&eb) optimal locations in (eb) the parametric domain of (e) the NURBS surface patch using (e) molecular dynamics simulation.
(5) Then, the well-shaped triangles can be created (eb) after connecting the nodes by Delaunay triangulation.
(6) Finally, the mapping from parametric space to (e&eb) 3D physical space is performed

NOTATION

Module One

main idea of the approach is that the
mesh nodes are (=) considered as similarly charged interacting particles which move (e&eb) within the region to be meshed under the influence of (e&eb) internal (such as particle–particle interaction forces) and external forces as well as optional additional forces

G_{13} : Category one of mesh nodes

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of as similarly charged interacting particles which move (e&eb) within the region to be meshed under the influence of (e&eb) internal (such as particle–particle interaction forces) and external forces as well as optional additional forces

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

mesh nodes are considered as similarly charged interacting particles which move (e&eb) within the region to be meshed under the influence of (e&eb) internal (such as particle–particle interaction forces) and external forces as well as optional additional forces

G_{16} : Category one of mesh nodes are considered as similarly charged interacting particles; region to be meshed under the influence of (e&eb) internal (such as particle–particle interaction forces) and external forces as well as optional additional forces

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of region to be meshed under the influence of (e&eb) internal (such as particle–particle interaction forces) and external forces as well as optional additional forces; mesh nodes are considered as similarly charged interacting particles

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

mesh nodes are considered as similarly charged interacting particles which move within the region to be meshed under the influence of (e&eb) internal (such as particle–particle interaction forces) and external forces as well as optional additional forces

G_{20} : Category one of mesh nodes are considered as similarly charged interacting particles which move within the region to be meshed; internal (such as particle–particle interaction forces) and external forces as well as optional additional forces

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of internal (such as particle–particle interaction forces) and external forces as well as optional additional forces; mesh nodes are considered as similarly charged interacting particles which move within the region to be meshed

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

mesh nodes are considered as similarly charged interacting particles which move within the region to be meshed under the influence of internal (such as particle–particle interaction forces) and external forces as well as optional additional forces

G_{24} : Category one of mesh nodes are considered as similarly charged interacting particles which move within the region to be meshed under the influence of internal (such as particle–particle interaction forces); external forces as well as optional additional forces

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of external forces as well as optional additional forces; mesh nodes are considered as similarly charged interacting particles which move within the region to be meshed under the influence of internal (such as particle–particle interaction forces)

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

G_{28} : Category one of external forces; optional additional forces

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of optional additional forces; external forces

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Moreover, the particles experience (e) a medium resistance due to which the system comes to (=) equilibrium within (eb) a relatively short period of time

G_{32} : Category one of particles; medium resistance due to which the system comes to (=) equilibrium within (eb) a relatively short period of time

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of medium resistance due to which the system comes to (=) equilibrium within (eb) a relatively short period of time; particles

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

particles experience a medium resistance due to which the system comes to (=) equilibrium within (eb) a relatively short period of time

G_{36} : Category one of particles experience a medium resistance due to which the system

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of equilibrium within a relatively short period of time

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

The proposed 3D surface mesh generation algorithm uses (e) a **parametric NURBS representation** as (=) initial definition of the domain boundary

G_{40} : Category one of proposed 3D surface mesh generation algorithm

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of **parametric NURBS representation** as (=) initial definition of the domain boundary

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

proposed 3D surface mesh generation algorithm uses a **parametric NURBS representation** as (=) initial definition of the domain boundary

G_{44} : Category one of proposed 3D surface mesh generation algorithm uses a **parametric NURBS representation**

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of initial definition of the domain boundary

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}), t\big)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}), t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}\big((G_{19}), t\big) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}), t\right)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}), t\right)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\left((G_{39}), t\right)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\,-(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\,-(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\,-(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad (60)$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad 63$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}\ \boxed{-(b''_{13})^{(1,1,)}(G,t)}\ \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}\ \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)}\ \boxed{-(b''_{14})^{(1,1,)}(G,t)}\ \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}\ \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}\ \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)}\ \boxed{-(b''_{15})^{(1,1,)}(G,t)}\ \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}\ \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}\ \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)}\ \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\ \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\ \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\left[-(b''_{20})^{(3)}(G_{23},t)\right] & \left[-(b''_{16})^{(2,2,2)}(G_{19},t)\right] & \left[-(b''_{13})^{(1,1,1,)}(G,t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{36})^{(7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{40})^{(8,8,8,8)}(G_{43},t)\right] & \left[-(b''_{44})^{(9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\left[-(b''_{21})^{(3)}(G_{23},t)\right] & \left[-(b''_{17})^{(2,2,2)}(G_{19},t)\right] & \left[-(b''_{14})^{(1,1,1,)}(G,t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{37})^{(7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{41})^{(8,8,8,8)}(G_{43},t)\right] & \left[-(b''_{45})^{(9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\left[-(b''_{22})^{(3)}(G_{23},t)\right] & \left[-(b''_{18})^{(2,2,2)}(G_{19},t)\right] & \left[-(b''_{15})^{(1,1,1,)}(G,t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{38})^{(7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{42})^{(8,8,8,8)}(G_{43},t)\right] & \left[-(b''_{46})^{(9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$ $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad 75$$

$(a''_{24})^{(4)}(T_{25},t)$, $(a''_{25})^{(4)}(T_{25},t)$, $(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$
are fourth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$
are fifth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$
are sixth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$
are seventh augmentation coefficients for category 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$
are eighth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

$Where\ \boxed{-(b_{24}'')^{(4)}(G_{27},t)}, \boxed{-(b_{25}'')^{(4)}(G_{27},t)}, \boxed{-(b_{26}'')^{(4)}(G_{27},t)}\ are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{28}'')^{(5,5,)}(G_{31},t)}, \boxed{-(b_{29}'')^{(5,5,)}(G_{31},t)}, \boxed{-(b_{30}'')^{(5,5,)}(G_{31},t)}\ are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{32}'')^{(6,6,)}(G_{35},t)}, \boxed{-(b_{33}'')^{(6,6,)}(G_{35},t)}, \boxed{-(b_{34}'')^{(6,6,)}(G_{35},t)}\ are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{13}'')^{(1,1,1,1)}(G,t)}, \boxed{-(b_{14}'')^{(1,1,1,1)}(G,t)}, \boxed{-(b_{15}'')^{(1,1,1,1)}(G,t)}$

$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{16}'')^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b_{17}'')^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b_{18}'')^{(2,2,2,2)}(G_{19},t)}$

$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{20}'')^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b_{21}'')^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b_{22}'')^{(3,3,3,3)}(G_{23},t)}$

$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{36}'')^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b_{37}'')^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b_{38}'')^{(7,7,7,7,7)}(G_{39},t)}$

$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{40}'')^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b_{41}'')^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b_{42}'')^{(8,8,8,8,8)}(G_{43},t)}$

$are\ eighth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{46}'')^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b_{45}'')^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b_{44}'')^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{c} (a_{28}')^{(5)}\boxed{+(a_{28}'')^{(5)}(T_{29},t)}\ \boxed{+(a_{24}'')^{(4,4,)}(T_{25},t)}\ \boxed{+(a_{32}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{36}'')^{(7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a_{40}'')^{(8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a_{44}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{c} (a_{29}')^{(5)}\boxed{+(a_{29}'')^{(5)}(T_{29},t)}\ \boxed{+(a_{25}'')^{(4,4,)}(T_{25},t)}\ \boxed{+(a_{33}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a_{45}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{c} (a_{30}')^{(5)}\boxed{+(a_{30}'')^{(5)}(T_{29},t)}\ \boxed{+(a_{26}'')^{(4,4,)}(T_{25},t)}\ \boxed{+(a_{34}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a_{46}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{30} \quad (81)$$

$Where\ \boxed{+(a_{28}'')^{(5)}(T_{29},t)}, \boxed{+(a_{29}'')^{(5)}(T_{29},t)}, \boxed{+(a_{30}'')^{(5)}(T_{29},t)}\ are\ first\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$And\ \boxed{+(a_{24}'')^{(4,4,)}(T_{25},t)}, \boxed{+(a_{25}'')^{(4,4,)}(T_{25},t)}, \boxed{+(a_{26}'')^{(4,4,)}(T_{25},t)}\ are\ second\ augmentation\ coefficient\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a_{32}'')^{(6,6,6)}(T_{33},t)}, \boxed{+(a_{33}'')^{(6,6,6)}(T_{33},t)}, \boxed{+(a_{34}'')^{(6,6,6)}(T_{33},t)}\ are\ third\ \ augmentation\ coefficient\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8\ ,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \quad 85$$

$$- \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}, \boxed{(a''_{37})^{(7)}(T_{37},t)}, \boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \qquad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b_{36}')^{(7)}\boxed{-(b_{36}'')^{(7)}(G_{39},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b_{37}')^{(7)}\boxed{-(b_{37}'')^{(7)}(G_{39},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b_{38}')^{(7)}\boxed{-(b_{38}'')^{(7)}(G_{39},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{matrix} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)}\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{matrix}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{matrix} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)}\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{matrix}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{matrix} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)}\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{matrix}\right] G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\left[-(b''_{40})^{(8)}(G_{43},t)\right] & -(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\left[-(b''_{41})^{(8)}(G_{43},t)\right] & -(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\left[-(b''_{42})^{(8)}(G_{43},t)\right] & -(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq \ (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = \ (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \ , \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (a_i)^{(1)} + (a_i')^{(1)} + \ (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i')^{(1)} + \ (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \leq (p_i)^{(2)} \leq \left(\hat{A}_{16}\right)^{(2)} \qquad 102$$

$$(b_i'')^{(2)}(G_{19},t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\big((G_{19}),t\big) = \ (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - \ T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\big((G_{19}),t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\big[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\big]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}} \ , \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\,(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + \ (\hat{P}_{16})^{(2)}\,(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + \ (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \le (p_i)^{(3)} \le (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \le (r_i)^{(3)} \le (b_i')^{(3)} \le (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - \ T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\,(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and \ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\,(a_i)^{(6)} + (a_i')^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)}\,(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(OOO) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(PPP) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$(a_i'')^{(7)}(T_{37},t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$

$(b_i'')^{(7)}(G_{39},t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$

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(QQQ) $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

(RRR)

$\lim_{G\to\infty}(b_i'')^{(7)}\big((G_{39}),t\big) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$

$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}\big((G_{39}),t\big)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$. (T_{37}',t) and (T_{37},t) are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(SSS) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(TTT) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$

$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$

Where we suppose

$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$ 138

$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$ 139

$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$ 140

$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t}$ 142

$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t}$ 143

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ 144

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}$, $(r_i)^{(8)}, i = 40,41,42$,
Satisfy the inequalities

$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 145

$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 146

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\hat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)} ||(G_{47}) - (G_{47})'|| e^{-(\hat{M}_{44})^{(9)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}\left[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)} (\hat{k}_{44})^{(9)}\right] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t} \ , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t} \ , \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \ , \ T_i(0) = T_i^0 \ , \ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} , T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\overline{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^{0} + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^{0} + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^{0} + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^{0} + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^{0} + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^{0} + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By

$$\bar{G}_{32}(t) = G_{32}^{0} + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^{0} + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^{0} + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^{0} + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^{0} + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^{0} + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that 174

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \leq G_{32}^{0}+\int_0^t\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)} =$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(l) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \leq G_{36}^{0}+\int_0^t\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)} =$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \leq G_{40}^{0}+\int_0^t\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)} = \quad 180$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t} \leq \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for G_{41} , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \leq G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \leq \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for G_{45} , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \left\{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\right\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)}\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) - (a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right| \; e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \qquad (186)$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: (187)

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. (188)

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}\left|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t} \leq$
$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)})\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\}ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0$ for $t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)}G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$: $\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0$ *and* $T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$G_{24}^{(2)} \left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)}\right) d\left(((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on T_{25} and respectively on (G_{27})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (228)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_2 / (a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right)$, $t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\widehat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{32})^{(6)}\right] \leq (\widehat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}), (\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}), (\widetilde{T_{35}}) \right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right| \; e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} \} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)} (\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)}; \; (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t} \; and \; (\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$ (250)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2 \; and \; \left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33} \; and \; G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)} \left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)} (T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(7)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}),(\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}),(\widetilde{T_{43}})\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1,\left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right),\quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$

From 99 to 44 it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(9)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)}, , (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)} t} - e^{-(a'_{26})^{(4)} t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)} t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G^0_{28} e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G^0_{28}}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G^0_{30} e^{-(S_2)^{(5)} t} \le G_{30}(t) \le \right. \quad 344$$

$$\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_2)^{(5)}((S_1)^{(5)}-(a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t}-e^{-(a'_{30})^{(5)}t}\right]+\ G_{30}^{0}e^{-(a'_{30})^{(5)}t}\Big)$$

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\le T_{30}(t)\le \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$:

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$-(\sigma_2)^{(6)}\le-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\le-(\sigma_1)^{(6)}$

$-(\tau_2)^{(6)}\le-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\le-(\tau_1)^{(6)}$

Definition of $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$: 350

By $(v_1)^{(6)}>0$, $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$, $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

Definition of $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)}>0$, $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$, $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

Definition of $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$:-

If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)}=(v_0)^{(6)},(m_1)^{(6)}=(v_1)^{(6)},$ ***if*** $(v_0)^{(6)}<(v_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}$ $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}$ $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}$ $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}$ $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}$ $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})} \left[e^{(S_1)^{(6)} t} - e^{-(a'_{34})^{(6)} t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)} t} \right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^{0}e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}}G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}}G_{36}^{0}e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t}-e^{-(S_2)^{(7)}t}\right]+G_{38}^{0}e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left.\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_2)^{(7)}((S_1)^{(7)}-(a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t}-e^{-(a'_{38})^{(7)}t}\right]+\ G_{38}^{0}e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}}T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}}T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right) - (b''_{41})^{(8)}\left((G_{43}), t\right) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(v_1)^{(8)} > 0$, $(v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$, $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{v}_1)^{(8)}$,, $(\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{v}_1)^{(8)} > 0$, $(\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$, $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$:-

If we define $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$, $\boldsymbol{if}$ $(v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$, $\boldsymbol{if}$ $(v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$,
and $\boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}$, $\boldsymbol{if}$ $(\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, $\boldsymbol{if}$ $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, $\boldsymbol{if}$ $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}$ $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t})$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t}\leq T_{42}(t)\leq \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)},(S_2)^{(8)},(R_1)^{(8)},(R_2)^{(8)}$:- 381

Where $(S_1)^{(8)}=(a_{40})^{(8)}(m_2)^{(8)}-(a'_{40})^{(8)}$

$(S_2)^{(8)}=(a_{42})^{(8)}-(p_{42})^{(8)}$

$(R_1)^{(8)}=(b_{40})^{(8)}(\mu_2)^{(8)}-(b'_{40})^{(8)}$

$(R_2)^{(8)}=(b'_{42})^{(8)}-(r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$:

$(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)}\leq -(a'_{44})^{(9)}+(a'_{45})^{(9)}-(a''_{44})^{(9)}(T_{45},t)+(a''_{45})^{(9)}(T_{45},t)\leq -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)}\leq -(b'_{44})^{(9)}+(b'_{45})^{(9)}-(b''_{44})^{(9)}\big((G_{47}),t\big)-(b''_{45})^{(9)}\big((G_{47}),t\big)\leq -(\tau_1)^{(9)}$

Definition of $(v_1)^{(9)},(v_2)^{(9)},(u_1)^{(9)},(u_2)^{(9)},v^{(9)},u^{(9)}$:

By $(v_1)^{(9)}>0$, $(v_2)^{(9)}<0$ and respectively $(u_1)^{(9)}>0$, $(u_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_1)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)}>0$, $(\bar{v}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$, $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

Definition of $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$:-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)},\ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$
and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \le G_{46}(t) \le \right.$$
$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (v_1)^{(1)} \le (\bar{v}_1)^{(1)} \le \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$, we obtain 386

$$(v_1)^{(1)} \le v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $v^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\right) - (a''_{17})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{\mathrm{G}_{16}}{\mathrm{G}_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (\mathrm{C})^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (\mathrm{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)}\right) \le \frac{dv^{(3)}}{dt} \le -\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \ge \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}}$$

it follows $(v_0)^{(3)} \le v^{(3)}(t) \le (v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \le v^{(3)}(t) \le (\bar{v}_1)^{(3)}$

If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)} \le \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}} \le v^{(3)}(t) \le$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (\bar{v}_1)^{(3)}$$

If $0 < (v_1)^{(3)} \le (\bar{v}_1)^{(3)} \le (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$, we obtain 403

$$(v_1)^{(3)} \le v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

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$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{36}'')^{(7)} = (a_{37}'')^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b_{36}'')^{(7)} = (b_{37}'')^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a_{40}')^{(8)} - (a_{41}')^{(8)} + (a_{40}'')^{(8)}(T_{41}, t)\right) - (a_{41}'')^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(8)} and\ (b''_i)^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\,]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\,]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\,]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0$ 479

$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0$ 480

$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0$ 481

$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0$ 482

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(l) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first

equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T^*_{33} for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$$

Definition and uniqueness of T^*_{37} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T^*_{37} for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$$

Definition and uniqueness of T^*_{41} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{25} such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507
$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{29} such that $\varphi((G_{31})^*) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{33} such that $\varphi({G_{35}}^*) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{37} such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{41} such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right]+(b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{45} such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)} G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)} G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)} T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)} T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)} G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)} G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)} T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)} T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)} G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)} G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)} T_{21}^*}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T_{22}^* = \frac{(b_{22})^{(3)} T_{21}^*}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]} \quad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \qquad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \qquad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \qquad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \qquad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \qquad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \qquad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i \qquad 532$$

$$\frac{\partial (a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial (b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \qquad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \qquad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \qquad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \qquad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \qquad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}\mathrm{T}^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}\mathrm{T}^*_{18}\mathbb{G}_j\right) \quad 539$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}\left((G_{27})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^{*}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^{*}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^{*}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^{*}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^{*}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^{*}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^{*}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^{*}\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\Big)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\Big]$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\Big)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\Big)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\Big]$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\Big)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\Big)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\Big]$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\Big)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\Big)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\Big)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\Big)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION ELEVEN

Ads/CFT Correspondence And The Renormalization Group Flow And Molecular Dynamics-Based Triangulation Algorithm

INTRODUCTION—VARIABLES USED

Computer Physics Communications Volume 190, May 2015, Pages 1–14 Molecular dynamics-based triangulation algorithm of free-form parametric surfaces for computer-aided engineering A.L. Zheleznyakova doi:10.1016/j.cpc.2014.12.018

(1) Since the presented interactive algorithm allows to control (e&eb) the distance between (e&eb) a pair of nodes depending on (=) the curvature of the NURBS surface, the method generates (eb) high

quality triangular mesh.

(2) The algorithm enables to produce (eb) uniform mesh, as well as anisotropic adaptive mesh with refinement (e&eb) in the large gradient regions.

(3) The mesh generation approach has (e) the abilities to preserve (eb) the representation accuracy of the input geometry model, create (eb) a close relationship between geometry modeling and (e&eb) grid generation process, and be automated to a large degree.

(4) Some examples are considered in order to illustrate the method's ability to generate (eb) a surface mesh for (e) a complicated CAD model.

Physics Letters B Volume 442, Issues 1–4, 3 December 1998, Pages 152–158 A remark on the AdS/CFT correspondence and the renormalization group flow E.T. Akhmedov Editor: P.V. Landshoff doi:10.1016/S0370-2693(98)01270-2

(5) The correspondence between the four-dimensional SU (N), SYM taken at large N and (e&eb) the type II B SUGRA on (eb) the AdS5×S5 background is considered.

(6) Authors argue that the classical equations of motion in the SUGRA picture can be interpreted as (=) that of the renormalization group on the SYM side.

(7) In fact, when the D3-brane is (=) slightly excited higher derivative terms in the field theory on (eb) its world-volume deform it from (e) the conformal SYM limit.

(8) They give arguments in favor of that the deformation goes in the way set by (e) the SUGRA equations of motion. Concrete example of the s-wave dilaton is considered. Copyright © 1998 Published by Elsevier B.V.

NOTATION

Module One

This method first distributes the interacting nodes into (e&eb) optimal locations in (eb) the parametric domain of (e) the NURBS surface patch using (e) molecular dynamics simulation

G_{13} : Category one of method first distributes the interacting nodes; optimal locations in (eb) the parametric domain of (e) the NURBS surface patch using (e) molecular dynamics simulation

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of optimal locations in (eb) the parametric domain of (e) the NURBS surface patch using (e) molecular dynamics simulation; method first distributes the interacting nodes

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

method first distributes the interacting nodes into optimal locations in (eb) the parametric domain of (e) the NURBS surface patch using (e) molecular dynamics simulation

G_{16} : Category one of method first distributes the interacting nodes into optimal locations

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of parametric domain of (e) the NURBS surface patch using (e) molecular dynamics simulation

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

method first distributes the interacting nodes into optimal locations in the parametric domain of (e) the NURBS surface patch using (e) molecular dynamics simulation

G_{20} : Category one of NURBS surface patch using (e) molecular dynamics simulation

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of method first distributes the interacting nodes into optimal locations in the parametric domain

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

method first distributes the interacting nodes into optimal locations in the parametric domain of the NURBS surface patch using (e) molecular dynamics simulation

G_{24} : Category one of molecular dynamics simulation

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of method first distributes the interacting nodes into optimal locations in the parametric domain of the NURBS surface patch

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Then, the well-shaped triangles can be created (eb) after connecting the nodes by Delaunay triangulation

G_{28} : Category one of well-shaped triangles

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of connecting the nodes by Delaunay triangulation

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Since the presented interactive algorithm allows to control (e&eb) the distance between (e&eb) a pair of nodes depending on (=) the curvature of the NURBS surface, the method generates (eb) high quality triangular mesh

G_{32} : Category one of presented interactive algorithm; distance between (e&eb) a pair of nodes depending on (=) the curvature of the NURBS surface, the method generates (eb) high quality triangular mesh

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of distance between (e&eb) a pair of nodes depending on (=) the curvature of the NURBS surface, the method generates (eb) high quality triangular mesh ;presented interactive algorithm

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

presented interactive algorithm allows to control the distance between (e&eb) a pair of nodes depending on (=) the curvature of the NURBS surface, the method generates (eb) high quality triangular mesh

G_{36} : Category one of presented interactive algorithm allows to control the distance; pair of nodes depending on (=) the curvature of the NURBS surface, the method generates (eb) high quality triangular mesh

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of pair of nodes depending on (=) the curvature of the NURBS surface, the method generates (eb) high quality triangular mesh; presented interactive algorithm allows to control the distance

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

presented interactive algorithm allows to control the distance between a pair of nodes depending on (=) the curvature of the NURBS surface, the method generates (eb) high quality triangular mesh

G_{40} : Category one of interactive algorithm allows to control the distance between a pair of nodes

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of curvature of the NURBS surface, the method generates (eb) high quality triangular mesh

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

presented interactive algorithm allows to control the distance between a pair of nodes depending on the curvature of the NURBS surface, the method generates (eb) high quality triangular mesh

G_{44} : Category one of presented interactive algorithm allows to control the distance between a pair of nodes depending on the curvature of the NURBS surface

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of high quality triangular mesh

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}),t) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}),t\right)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}),t\right)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}),t\right)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}),t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}),t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt}=(b_{13})^{(1)}T_{14}-\begin{bmatrix}(b'_{13})^{(1)}&-(b''_{13})^{(1)}(G,t)&-(b''_{16})^{(2,2,)}(G_{19},t)&-(b''_{20})^{(3,3,)}(G_{23},t)\\-(b''_{24})^{(4,4,4,4,)}(G_{27},t)&-(b''_{28})^{(5,5,5,5,)}(G_{31},t)&-(b''_{32})^{(6,6,6,6,)}(G_{35},t)\\-(b''_{36})^{(7,7,)}(G_{39},t)&-(b''_{40})^{(8,8)}(G_{43},t)&-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{13} \quad 58$$

$$\frac{dT_{14}}{dt}=(b_{14})^{(1)}T_{13}-\begin{bmatrix}(b'_{14})^{(1)}&-(b''_{14})^{(1)}(G,t)&-(b''_{17})^{(2,2,)}(G_{19},t)&-(b''_{21})^{(3,3,)}(G_{23},t)\\-(b''_{25})^{(4,4,4,4,)}(G_{27},t)&-(b''_{29})^{(5,5,5,5,)}(G_{31},t)&-(b''_{33})^{(6,6,6,6,)}(G_{35},t)\\-(b''_{37})^{(7,7,)}(G_{39},t)&-(b''_{41})^{(8,8)}(G_{43},t)&-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{14} \quad 59$$

$$\frac{dT_{15}}{dt}=(b_{15})^{(1)}T_{14}-\begin{bmatrix}(b'_{15})^{(1)}&-(b''_{15})^{(1)}(G,t)&-(b''_{18})^{(2,2,)}(G_{19},t)&-(b''_{22})^{(3,3,)}(G_{23},t)\\-(b''_{26})^{(4,4,4,4,)}(G_{27},t)&-(b''_{30})^{(5,5,5,5,)}(G_{31},t)&-(b''_{34})^{(6,6,6,6,)}(G_{35},t)\\-(b''_{38})^{(7,7,)}(G_{39},t)&-(b''_{42})^{(8,8)}(G_{43},t)&-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\end{bmatrix}T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt}=(a_{16})^{(2)}G_{17}-\begin{bmatrix}(a'_{16})^{(2)}&+(a''_{16})^{(2)}(T_{17},t)&+(a''_{13})^{(1,1,)}(T_{14},t)&+(a''_{20})^{(3,3,3)}(T_{21},t)\\+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)&+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)&+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)\\+(a''_{36})^{(7,7,7)}(T_{37},t)&+(a''_{40})^{(8,8,8)}(T_{41},t)&+(a''_{44})^{(9,9)}(T_{45},t)\end{bmatrix}G_{16} \quad 61$$

$$\frac{dG_{17}}{dt}=(a_{17})^{(2)}G_{16}-\begin{bmatrix}(a'_{17})^{(2)}&+(a''_{17})^{(2)}(T_{17},t)&+(a''_{14})^{(1,1,)}(T_{14},t)&+(a''_{21})^{(3,3,3)}(T_{21},t)\\+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)&+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)&+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)\\+(a''_{37})^{(7,7,7)}(T_{37},t)&+(a''_{41})^{(8,8,8)}(T_{41},t)&+(a''_{45})^{(9,9)}(T_{45},t)\end{bmatrix}G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition

coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)}\ \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\ \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\ \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{22} \quad (69)$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for

category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)} & \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad 70$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)} & \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}, \boxed{(a''_{25})^{(4)}(T_{25},t)}, \boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
are sixth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
are seventh augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
are eighth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad 78$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$

are fourth detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b_{16}'')^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b_{17}'')^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b_{18}'')^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b_{20}'')^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b_{21}'')^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b_{22}'')^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b_{36}'')^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b_{37}'')^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b_{38}'')^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b_{40}'')^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b_{41}'')^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b_{42}'')^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b_{46}'')^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b_{45}'')^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b_{44}'')^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{ccc} (a_{28}')^{(5)} & \boxed{+(a_{28}'')^{(5)}(T_{29},t)} & \boxed{+(a_{24}'')^{(4,4,)}(T_{25},t)} \quad \boxed{+(a_{32}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{36}'')^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{ccc} (a_{29}')^{(5)} & \boxed{+(a_{29}'')^{(5)}(T_{29},t)} & \boxed{+(a_{25}'')^{(4,4,)}(T_{25},t)} \quad \boxed{+(a_{33}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{ccc} (a_{30}')^{(5)} & \boxed{+(a_{30}'')^{(5)}(T_{29},t)} & \boxed{+(a_{26}'')^{(4,4,)}(T_{25},t)} \quad \boxed{+(a_{34}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{30} \quad (81)$$

Where $\boxed{+(a_{28}'')^{(5)}(T_{29},t)}, \boxed{+(a_{29}'')^{(5)}(T_{29},t)}, \boxed{+(a_{30}'')^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3
And $\boxed{+(a_{24}'')^{(4,4,)}(T_{25},t)}, \boxed{+(a_{25}'')^{(4,4,)}(T_{25},t)}, \boxed{+(a_{26}'')^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3
$\boxed{+(a_{32}'')^{(6,6,6)}(T_{33},t)}, \boxed{+(a_{33}'')^{(6,6,6)}(T_{33},t)}, \boxed{+(a_{34}'')^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3
$\boxed{+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3
$\boxed{+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3
$\boxed{+(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)}, \boxed{+(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)}, \boxed{+(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3
$\boxed{+(a_{36}'')^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3
$\boxed{+(a_{40}'')^{(8,8\,,8,8,8,8)}(T_{41},t)}, \boxed{+(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3
$\boxed{+(a_{46}'')^{(9,9,9,9,9)}(T_{45},t)}, \boxed{+(a_{45}'')^{(9,9,9,9,9)}(T_{45},t)}, \boxed{+(a_{44}'')^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation

coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[\begin{array}{ccc} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) \\ -(b''_{32})^{(6,6,6)}(G_{35},t) & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) \\ -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) \\ -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) & & \end{array}\right] T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[\begin{array}{ccc} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) \\ -(b''_{33})^{(6,6,6)}(G_{35},t) & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) \\ -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) \\ -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) & & \end{array}\right] T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[\begin{array}{ccc} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) \\ -(b''_{34})^{(6,6,6)}(G_{35},t) & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) \\ -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) \\ -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) & & \end{array}\right] T_{30} \quad 84$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \quad 85$$

$$- \left[\begin{array}{ccc} (a'_{32})^{(6)} & +(a''_{32})^{(6)}(T_{33},t) & +(a''_{28})^{(5,5,5)}(T_{29},t) \\ +(a''_{24})^{(4,4,4,)}(T_{25},t) & +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t) \\ +(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t) & +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t) \\ +(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t) & & \end{array}\right] G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)} & +(a''_{33})^{(6)}(T_{33},t) & +(a''_{29})^{(5,5,5)}(T_{29},t) \\ +(a''_{25})^{(4,4,4,)}(T_{25},t) & +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t) \\ +(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) & +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t) \\ +(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) & & \end{array}\right] G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)}\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)}\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)}\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* $1,2$ *and* 3

$-(b_{13}'')^{(1,1,1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1,1,1)}(G,t)$, $-(b_{15}'')^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b_{16}'')^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b_{20}'')^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3

$-(b_{36}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3

$-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2, and 3

$-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a_{36}')^{(7)} & +(a_{36}'')^{(7)}(T_{37},t) & +(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t) & +(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a_{37}')^{(7)} & +(a_{37}'')^{(7)}(T_{37},t) & +(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a_{38}')^{(7)} & +(a_{38}'')^{(7)}(T_{37},t) & +(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15}$$

Where $(a_{36}'')^{(7)}(T_{37},t)$, $(a_{37}'')^{(7)}(T_{37},t)$, $(a_{38}'')^{(7)}(T_{37},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\,-(b''_{36})^{(7)}(G_{39},t) & -(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1)}(G,t) & -(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\,-(b''_{37})^{(7)}(G_{39},t) & -(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1)}(G,t) & -(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\,-(b''_{38})^{(7)}(G_{39},t) & -(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1)}(G,t) & -(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)} G_{41} - \left[\begin{array}{ccc} (a'_{40})^{(8)} \boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array} \right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)} G_{40} - \left[\begin{array}{ccc} (a'_{41})^{(8)} \boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array} \right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)} G_{41} - \left[\begin{array}{ccc} (a'_{42})^{(8)} \boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array} \right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{40}}{dt} =$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt}$$

$$= (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt}$$
$$= (a_{45})^{(9)} G_{44}$$
$$- \left[\begin{array}{c} (a'_{45})^{(9)} \boxed{+(a''_{45})^{(9)}(T_{45},t)} \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array} \right] G_{14}$$

$$\frac{dG_{46}}{dt}$$
$$= (a_{46})^{(9)} G_{45}$$
$$- \left[\begin{array}{c} (a'_{46})^{(9)} \boxed{+(a''_{46})^{(9)}(T_{37},t)} \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array} \right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$
$$(b_{44})^{(9)} T_{45} -$$
$$\left[\begin{array}{c} (b'_{44})^{(9)} \boxed{-(b''_{44})^{(9)}(G_{47},t)} \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{array} \right] T_{13}$$
$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b''_i)^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}', t) - (a_i'')^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}', t)$ and $(a_i'')^{(1)}(T_{14}, t)$. (T_{14}', t) and (T_{14}, t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(\mathrm{p_i})^{(2)}$, $(\mathrm{r_i})^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq \left(\hat{A}_{16}\right)^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}\left((G_{19}), t\right) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}\big((G_{19}), t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, \ (\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq \ (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$\lim_{G\to\infty}(b_i'')^{(3)}\,(G_{23},t) = \ (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$

$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)},$ are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$

$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$

$(b_i'')^{(4)}\big((G_{27}),t\big) \le \ (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$

$lim_{T_2\to\infty}(a_i'')^{(4)}\,(T_{25},t) = (p_i)^{(4)}$ 118

$\lim_{G\to\infty}(b_i'')^{(4)}\,\big((G_{27}),t\big) = \ (r_i)^{(4)}$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}\,(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}\,(G_{31}, t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\,(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ \ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}\,(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}\,\big((G_{35}), t\big) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\big((G_{35}), t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\,(a_i)^{(6)} + (a_i')^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)}\,(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ \ (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(UUU) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(VVV) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq \ (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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(WWW) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(XXX)

$\lim_{G\to\infty}(b_i'')^{(7)}\left((G_{39}),t\right) = \ (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, \ (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}\left((G_{39}),t\right)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $\left[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\right]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(YYY) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(ZZZ) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, \ (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\,(a_i)^{(7)} + (a_i')^{(7)} + \ (\hat{A}_{36})^{(7)} + \ (\hat{P}_{36})^{(7)}\,(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\ (b_i)^{(7)} + (b_i')^{(7)} + \ (\hat{B}_{36})^{(7)} + \ (\hat{Q}_{36})^{(7)}\ \ (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}, \ (r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41},t) \le (p_i)^{(8)} \le (\hat{A}_{40})^{(8)} \quad 138$$

$$(b_i'')^{(8)}((G_{43}),t) \le (r_i)^{(8)} \le (b_i')^{(8)} \le (\hat{B}_{40})^{(8)} \quad 139$$

$$\lim_{T_2\to\infty}(a_i'')^{(8)}(T_{41},t) = (p_i)^{(8)} \quad 140$$

$$\lim_{G\to\infty}(b_i'')^{(8)}((G_{43}),t) = (r_i)^{(8)} \quad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}',t) - (a_i'')^{(8)}(T_{41},t)| \le (\hat{k}_{40})^{(8)}|T_{41} - T_{41}'|e^{-(\hat{M}_{40})^{(8)}t} \quad 142$$

$$|(b_i'')^{(8)}((G_{43})',t) - (b_i'')^{(8)}((G_{43}),t)| < (\hat{k}_{40})^{(8)}||(G_{43}) - (G_{43})'||e^{-(\hat{M}_{40})^{(8)}t} \quad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}',t)$ and $(a_i'')^{(8)}(T_{41},t)$. (T_{41}',t) and (T_{41},t) are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \quad 144$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad 146\,A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t) \le (p_i)^{(9)} \le (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t) \le (r_i)^{(9)} \le (b_i')^{(9)} \le (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty} (a_i'')^{(9)}(T_{45},t) = (p_i)^{(9)}$$

$$\lim_{G\to\infty} (b_i'')^{(9)}(G_{47},t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t) - (a_i'')^{(9)}(T_{45},t)| \le (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t) - (b_i'')^{(9)}\big((G_{47}),t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$ and$(a_i'')^{(9)}(T_{45},t)$. (T_{45}',t) and (T_{45},t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}$, $(r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\,, T_i(0)$

$$G_i(t) \le \left(\hat{P}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t} \;,\quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le \left(\hat{Q}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t} \;,\quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le \left(\hat{P}_{20}\right)^{(3)} e^{(\hat{M}_{20})^{(3)}t} \;,\quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le \left(\hat{Q}_{20}\right)^{(3)} e^{(\hat{M}_{20})^{(3)}t} \;,\quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}$, $T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}$, 155

$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 156

$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0,t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''^{(2)}_{16}\right)\left(T_{17}(s_{(16)}), s_{(16)}\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}\left(T_{17}(s_{(16)}), s_{(17)}\right)\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}\left(T_{17}(s_{(16)}), s_{(16)}\right)\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''^{(3)}_{20}\right)\left(T_{21}(s_{(20)}), s_{(20)}\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}\left(T_{21}(s_{(20)}), s_{(20)}\right)\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}\left(T_{21}(s_{(20)}), s_{(20)}\right)\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)},\ T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^{0} + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^{0} + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^{0} + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^{0} + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^{0} + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^{0} + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G_{27}(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)},\ T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^{0} + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0 \text{ , } T_i(0) = T_i^0 \text{ , } G_i^0 \le (\hat{P}_{40})^{(8)} \text{ , } T_i^0 \le (\hat{Q}_{40})^{(8)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad (169)$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that (170)

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that (171)

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that (172)

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that (173)

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that (174)

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \leq G_{32}^{0}+\int_0^t\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)} =$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(m) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \leq G_{36}^{0}+\int_0^t\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)} =$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \leq G_{40}^{0}+\int_0^t\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)} = \quad 180$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t} \leq \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for G_{41} , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \leq G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \leq \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for G_{45} , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \left\{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\right\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T} : \left(\tilde{G}, \tilde{T}\right) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+\ (a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\ (t)=0\ and\ T_i\ (t)=0$

From 19 to 24 it results

$$G_i\ (t)\geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\geq 0$$

$$T_i\ (t)\geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13}<(\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt}\leq\left((\widehat{M}_{13})^{(1)}\right)_1-(a'_{14})^{(1)}G_{14}$ and by integrating

$$G_{14}\leq\left((\widehat{M}_{13})^{(1)}\right)_2=G_{14}^0+2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15}\leq\left((\widehat{M}_{13})^{(1)}\right)_3=G_{15}^0+2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a'_{15})^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}} \leq$
$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{\mathrm{A}}_{16})^{(2)} + (\widehat{\mathrm{P}}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)\mathrm{d}\left(((G_{19})^{(1)}, (T_{19})^{(1)};\ (G_{19})^{(2)}, (T_{19})^{(2)})\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16{,}17{,}18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $\mathrm{G}_i\,(\mathrm{t}) = 0$ and $\mathrm{T}_i\,(\mathrm{t}) = 0$ 200

it results

$\mathrm{G}_i\,(\mathrm{t}) \geq \mathrm{G}_i^0\mathrm{e}^{\left[-\int_0^{\mathrm{t}}\{(a_i')^{(2)} - (a_i'')^{(2)}(\mathrm{T}_{17}(s_{(16)}), s_{(16)})\}\mathrm{d}s_{(16)}\right]} \geq 0$

$\mathrm{T}_i\,(\mathrm{t}) \geq \mathrm{T}_i^0\mathrm{e}^{(-(b_i')^{(2)}\mathrm{t})} > 0 \quad \text{for t} > 0$

Definition of $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1$, $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2$ and $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$\mathrm{G}_{16} < (\widehat{\mathrm{M}}_{16})^{(2)}$ it follows $\frac{\mathrm{dG}_{17}}{\mathrm{dt}} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)}\mathrm{G}_{17}$ and by integrating

$\mathrm{G}_{17} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 = \mathrm{G}_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$

In the same way , one can obtain

$\mathrm{G}_{18} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3 = \mathrm{G}_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{\mathrm{t}\to\infty}((b_i'')^{(2)}\,((G_{19})(\mathrm{t}), \mathrm{t})) = (b_{17}')^{(2)}$ then $\mathrm{T}_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$: $\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ and\ (\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$G_{24}^{(2)} \left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)}\right) d\left(((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on T_{25} and respectively on (G_{27})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (228)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_2 / (a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\widehat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{32})^{(6)}\right] \leq (\widehat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)}, (T_{35})^{(1)}), ((G_{35})^{(2)}, (T_{35})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})}, \widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})}, \widetilde{(T_{35})}\right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right)\left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)};\ (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ *and* $(\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ (250)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)} \left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)} (T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\,((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1,\left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right),\quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$

From 99 to 44 it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(9)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)}, , (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)} t} - e^{-(a'_{26})^{(4)} t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)} t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28}e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G^0_{28}e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}}G^0_{28}e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}}G^0_{28}e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)}G^0_{28}}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G^0_{30}e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

$$\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_2)^{(5)}((S_1)^{(5)}-(a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t}-e^{-(a'_{30})^{(5)}t}\right]+\ G_{30}^{0}e^{-(a'_{30})^{(5)}t}\Big)$$

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\le T_{30}(t)\le \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$:

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)}\le-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\le-(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)}\le-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\le-(\tau_1)^{(6)}$$

Definition of $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$: 350

By $(v_1)^{(6)}>0$, $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$, $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

Definition of $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)}>0$, $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$, $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

Definition of $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$:-

If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)}=(v_0)^{(6)},(m_1)^{(6)}=(v_1)^{(6)},$ ***if*** $(v_0)^{(6)}<(v_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}$ $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}$ $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}$ $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}$ $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}$ $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})} \left[e^{(S_1)^{(6)} t} - e^{-(a'_{34})^{(6)} t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)} t} \right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^{0}e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}}G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}}G_{36}^{0}e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t}-e^{-(S_2)^{(7)}t}\right]+G_{38}^{0}e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left.\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_2)^{(7)}((S_1)^{(7)}-(a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t}-e^{-(a'_{38})^{(7)}t}\right]+\ G_{38}^{0}e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}}T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}}T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right) - (b''_{41})^{(8)}\left((G_{43}), t\right) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(v_1)^{(8)} > 0$, $(v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$, $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{v}_1)^{(8)}$,, $(\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{v}_1)^{(8)} > 0$, $(\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$, $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$:-

If we define $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$, $\boldsymbol{if}$ $(v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$, $\boldsymbol{if}$ $(v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$,
and $\boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}$, $\boldsymbol{if}$ $(\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, $\boldsymbol{if}$ $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, $\boldsymbol{if}$ $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}$ $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t})$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t}\leq T_{42}(t)\leq \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)},(S_2)^{(8)},(R_1)^{(8)},(R_2)^{(8)}$:- 381

Where $(S_1)^{(8)}=(a_{40})^{(8)}(m_2)^{(8)}-(a'_{40})^{(8)}$

$(S_2)^{(8)}=(a_{42})^{(8)}-(p_{42})^{(8)}$

$(R_1)^{(8)}=(b_{40})^{(8)}(\mu_2)^{(8)}-(b'_{40})^{(8)}$

$(R_2)^{(8)}=(b'_{42})^{(8)}-(r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$:

$(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)}\leq -(a'_{44})^{(9)}+(a'_{45})^{(9)}-(a''_{44})^{(9)}(T_{45},t)+(a''_{45})^{(9)}(T_{45},t)\leq -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)}\leq -(b'_{44})^{(9)}+(b'_{45})^{(9)}-(b''_{44})^{(9)}\big((G_{47}),t\big)-(b''_{45})^{(9)}\big((G_{47}),t\big)\leq -(\tau_1)^{(9)}$

Definition of $(v_1)^{(9)},(v_2)^{(9)},(u_1)^{(9)},(u_2)^{(9)},v^{(9)},u^{(9)}$:

By $(v_1)^{(9)}>0$, $(v_2)^{(9)}<0$ and respectively $(u_1)^{(9)}>0$, $(u_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_1)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)}>0$, $(\bar{v}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$, $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

Definition of $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$:-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)},\ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$
and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \le G_{46}(t) \le \right.$$
$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (v_1)^{(1)} \le (\bar{v}_1)^{(1)} \le \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$, we obtain 386

$$(v_1)^{(1)} \le v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $v^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\right) - (a''_{17})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{\mathrm{G}_{16}}{\mathrm{G}_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (\mathrm{C})^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (\mathrm{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)}\right) \le \frac{dv^{(3)}}{dt} \le -\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \ge \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}}$$

it follows $(v_0)^{(3)} \le v^{(3)}(t) \le (v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \le v^{(3)}(t) \le (\bar{v}_1)^{(3)}$

If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)} \le \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}} \le v^{(3)}(t) \le$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (\bar{v}_1)^{(3)}$$

If $0 < (v_1)^{(3)} \le (\bar{v}_1)^{(3)} \le (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$, we obtain 403

$$(v_1)^{(3)} \le v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

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$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{36}'')^{(7)} = (a_{37}'')^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b_{36}'')^{(7)} = (b_{37}'')^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a_{40}')^{(8)} - (a_{41}')^{(8)} + (a_{40}'')^{(8)}(T_{41}, t)\right) - (a_{41}'')^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(8)} and\ (b''_i)^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\ ]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\ ]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\ ]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0$ 479

$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0$ 480

$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0$ 481

$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0$ 482

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(m) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first

equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T^*_{33} for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$$

Definition and uniqueness of T^*_{37} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T^*_{37} for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$$

Definition and uniqueness of T^*_{41} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{25} such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507
$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{29} such that $\varphi((G_{31})^*) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{33} such that $\varphi({G_{35}}^*) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{37} such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{41} such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right]+(b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{45} such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)} G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)} G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)} T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)} T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)} G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)} G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)} T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)} T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)} G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)} G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)} T_{21}^*}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T_{22}^* = \frac{(b_{22})^{(3)} T_{21}^*}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]} \quad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \qquad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \qquad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \qquad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \qquad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \qquad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \qquad 530$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i \qquad 532$$

$$\frac{\partial (a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial (b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \qquad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \qquad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \qquad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \qquad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \qquad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}\mathrm{T}^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}\mathrm{T}^*_{18}\mathbb{G}_j\right) \quad 539$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}\left((G_{27})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^{*}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^{*}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^{*}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^{*}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^{*}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^{*}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^{*}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^{*}\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\ B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\Big)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\Big]$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\Big)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\Big)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\Big]$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\Big)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\Big)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\Big]$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\Big)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\Big)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\Big)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\Big)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION ELEVEN A

Relationship Between Geometry Modeling And Grid Generation Process

INTRODUCTION—VARIABLES USED

Computer Physics Communications Volume 190, May 2015, Pages 1–14 Molecular dynamics-based triangulation algorithm of free-form parametric surfaces for computer-aided engineering A.L. Zheleznyakova doi:10.1016/j.cpc.2014.12.018

(1) The algorithm enables to produce (eb) uniform mesh, as well as anisotropic adaptive mesh with refinement (e&eb) in the large gradient regions.
(2) The mesh generation approach has (e) the abilities to preserve (eb) the representation accuracy of

the input geometry model, create (eb) a close relationship between geometry modeling and (e&eb) grid generation process, and be automated to a large degree.

(3) Some examples are considered in order to illustrate the method's ability to generate (eb) a surface mesh for (e) a complicated CAD model.

Physics Letters B Volume 442, Issues 1–4, 3 December 1998, Pages 152–158 A remark on the AdS/CFT correspondence and the renormalization group flow E.T. Akhmedov Editor: P.V. Landshoff doi:10.1016/S0370-2693(98)01270-2

(4) The correspondence between the four-dimensional SU (N), SYM taken at large N and (e&eb) the type II B SUGRA on (eb) the AdS5×S5 background is considered.

(5) Authors argue that the classical equations of motion in the SUGRA picture can be interpreted as (=) that of the renormalization group on the SYM side.

(6) In fact, when the D3-brane is (=) slightly excited higher derivative terms in the field theory on (eb) its world-volume deform it from (e) the conformal SYM limit.

(7) They give arguments in favor of that the deformation goes in the way set by (e) the SUGRA equations of motion. Concrete example of the s-wave dilaton is considered. Copyright © 1998 Published by Elsevier B.V.

NOTATION

Module One

Source: Wikipedia

$$\langle A \rangle_{ensemble} = \iint dp^N dr^N A\left(p^N, r^N\right)\rho\left(p^N, r^N\right)$$

The probability density of the ensemble is given by:

$$\rho\left(p^N, r^N\right) = \frac{1}{Q}\exp\left[-H\left(p^N, r^N\right) / k_B T\right]$$

$$Q = \iint dp^N dr^N \exp\left[-H\left(p^N, r^N\right) / k_B T\right]$$

Another way, as done in an MD simulation, is to determine a time average of A, which is expressed as:

$$\langle A \rangle_{time} = \lim_{\tau \to \infty} \frac{1}{\tau} \int_{t=0}^{\tau} A\left(p^N(t), r^N(t)\right) dt \approx \frac{1}{M} \sum_{t=1}^{M} A\left(p^N, r^N\right)$$

$$V = \langle V \rangle = \frac{1}{M} \sum_{i=1}^{M} V_i$$

Average Kinetic energy:

$$K = \langle K \rangle = \frac{1}{M} \sum_{j=1}^{M} \left\{ \sum_{i=1}^{N} \frac{m_i}{2} v_i \cdot v_i \right\}_j$$

The velocities, vi, are often chosen randomly from a Maxwell-Boltzmann or Gaussian distribution at a given temperature, which gives the probability that an atom i has a velocity vx in the x direction at a temperature T.:

$$p(v_{ix}) = \left(\frac{m_i}{2\pi k_B T} \right)^{1/2} \exp\left[-\frac{1}{2} \frac{m_i v_{ix}^2}{k_B T} \right]$$

$$T = \frac{1}{(3N)} \sum_{i=1}^{N} \frac{|p_i|}{2m_i}$$

Bee man's algorithm:

$$r(t+\delta t) = r(t) + v(t)\delta t + \frac{2}{3}a(t)\delta t^2 - \frac{1}{6}a(t-\delta t)\delta t^2$$

$$v(t+\delta t) = v(t) + v(t)\delta t + \frac{1}{3}a(t)\delta t + \frac{5}{6}a(t)\delta t - \frac{1}{6}a(t-\delta t)\delta t$$

The Velocity Verlet algorithm:

$$r(t+\delta t) = r(t) + v(t)\delta t + \frac{1}{2}a(t)\delta t^2$$

$$v(t+\delta t) = v(t) + \frac{1}{2}\left[a(t) + a(t+\delta t)\right]\delta t$$

Discrete Applied Mathematics Volume 127, Issue 1, 1 April 2003, Pages 23–51 Computational Molecular Biology Series - Issue IV Dynamic maintenance and visualization of molecular surfaces Chandrajit L. Bajaja, Valerio Pascuccia, Ariel Shamira, Robert J. Holtb, , , Arun N. Netravalib Under an Elsevier user license

$$B_0^4: \xi_1^2 + \xi_2^2 + \xi_3^2 + \xi_4^2 - 1 \leqslant 0.$$

$$\xi_i = \frac{2x_i}{x_1^2 + x_2^2 + x_3^2 + 1}, \quad i = 1,2,3, \quad \xi_4 = \frac{x_1^2 + x_2^2 + x_3^2 - 1}{x_1^2 + x_2^2 + x_3^2 + 1}.$$

$$x_i = \frac{\xi_i}{1 - \xi_4}, \quad i = 1,2,3$$

$$h: \ a_0 + a_1\xi_1 + a_2\xi_2 + a_3\xi_3 + a_4\xi_4 \leqslant 0,$$

$$b: \ a_0(x_1^2 + x_2^2 + x_3^2 + 1) + a_1 2x_1 + a_2 2x_2 + a_3 2x_3 + a_4(x_1^2 + x_2^2 + x_3^2 - 1) \leqslant 0.$$

$$\dim(\partial b' \cap \partial b'' \cap \partial b''') = 0. \ \Leftrightarrow \ p \in \partial B_0^3.$$

$$x = \frac{2u}{u^2 + v^2 + 1}, \quad y = \frac{2v}{u^2 + v^2 + 1}, \quad z = \frac{u^2 + v^2 - 1}{u^2 + v^2 + 1}.$$

$$\partial B_s \cap h(u_1, u_2, \bar{t};\ u_3) \cap h(u_3, u_1, \bar{t};\ u_2) \cap h(u_2, u_3, \bar{t};\ u_1).$$

$$\pi: \quad x_1 = \frac{l^2 + r_1^2 - r_2^2}{2l} + r\frac{2(r_1 - r_2)}{2l},$$

G_{13} : Category one of **LHS of the above equations**

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of **RHS of the above equations**

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

The algorithm enables to produce (eb) uniform mesh, as well as anisotropic adaptive mesh with refinement (e&eb) in the large gradient regions

G_{16} : Category one of algorithm

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of uniform mesh, as well as anisotropic adaptive mesh with refinement in the large gradient regions

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

The mesh generation approach has (e) the abilities to preserve (eb) the representation accuracy of the input geometry model, create (eb) a close relationship between geometry modeling and (e&eb) grid generation

process, and be automated to a large degree

G_{20} : Category one of abilities to preserve (eb) the representation accuracy of the input geometry model, create (eb) a close relationship between geometry modeling and (e&eb) grid generation process, and be automated to a large degree

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of mesh generation approach

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

mesh generation approach has the abilities to preserve (eb) the representation accuracy of the input geometry model, create (eb) a close relationship between geometry modeling and (e&eb) grid generation process, and be automated to a large degree

G_{24} : Category one of mesh generation approach

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of representation accuracy of the input geometry model, create (eb) a close relationship between geometry modeling and (e&eb) grid generation process, and be automated to a large degree

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

mesh generation approach has the abilities to preserve the representation accuracy of the input geometry model, create (eb) a close relationship between geometry modeling and (e&eb) grid generation process, and be automated to a large degree

G_{28} : Category one of mesh generation approach has the abilities to preserve the representation accuracy of the input geometry model

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of close relationship between geometry modeling and (e&eb) grid generation process, and be automated to a large degree

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

mesh generation approach has the abilities to preserve the representation accuracy of the input geometry model, create a close relationship between geometry modeling and (e&eb) grid generation process, and be automated to a large degree

G_{32} : Category one of mesh generation approach has the abilities to preserve the representation accuracy of the input geometry model, create a close relationship between geometry modeling; grid generation process, and be automated to a large degree

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of grid generation process, and be automated to a large degree ;mesh generation approach has the abilities to preserve the representation accuracy of the input geometry model, create a close relationship between geometry modeling

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Some examples are considered in order to illustrate the method's ability to generate (eb) a surface mesh for (e) a complicated CAD model.

G_{36} : Category one of complicated CAD model

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of surface mesh

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

The correspondence between the four-dimensional SU (N), SYM taken at large N and (e&eb) the type II B SUGRA on (eb) the AdS5×S5 background is considered

G_{40} : Category one of four-dimensional SU (N), SYM taken at large N; type II B SUGRA on (eb) the AdS5×S5 background

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of type II B SUGRA on (eb) the AdS5×S5 background; four-dimensional SU (N), SYM taken at large N

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Authors argue that the classical equations of motion in the SUGRA picture can be interpreted as (=) that of the renormalization group on the SYM side

G_{44} : Category one of classical equations of motion in the SUGRA

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of renormalization group on the SYM side

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION TWELVE

Symmetry Breaking

INTRODUCTION—VARIABLES USED

Nuclear Physics B Volume 556, Issues 1–2, 6 September 1999, Pages 89–114 Ads/CFT correspondence and symmetry breaking Igor R. Klebanov, Edward Witten doi:10.1016/S0550-3213(99)00387-9

(1) **Igor R. Klebanov, Edward Witten** study, using the dual AdS description, (eb) the vacua of field theories where some of the gauge symmetry is broken by (e) expectation values of scalar fields

(2) In such vacua, operators built out of the scalar fields acquire (e) expectation values, and they show how to calculate them from (e) the behavior of perturbations to (e) the AdS background near the boundary.

(3) Specific examples include the N = 4 SYM theory, and (e&eb) theories on D3-branes placed on orbifolds and conifolds.

(4) **Igor R. Klebanov, Edward Witten** also clarify some subtleties of the AdS/CFT correspondence that arise in this analysis. In particular, they explain how scalar fields in (eb) AdS space of sufficiently negative mass-squared can be associated with (e&eb) CFT operators of two possible dimensions.

(5) All dimensions are bounded from (e) below by (d−2)/2

(6) this is the unitarity bound for (e) scalar operators in d-dimensional field theory

(7) They further argue that the generating functional for (e) correlators in the theory with one choice of operator dimension is (=) a Legendre transform of the generating functional in the theory with the other choice.

NOTATION

Module One

Physics Letters B Volume 442, Issues 1–4, 3 December 1998, Pages 152–158 A remark on the AdS/CFT correspondence and the renormalization group flow E.T. Akhmedov Editor: P.V. Landshoff doi:10.1016/S0370-2693(98)01270-2

(1) In fact, when the D3-brane is slightly excited higher derivative terms in the field theory on its world-volume deform (E&EB) it from the conformal SYM limit.

(2) They give arguments in favor of that the deformation goes in the way set by (e) the SUGRA equations of motion. Concrete example of the s-wave dilaton is considered. Copyright © 1998 Published by Elsevier B.V.

G_{13} : Category one of D3-brane is slightly excited higher derivative terms in the field theory on its world-volume; conformal SYM limit

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of conformal SYM limit ;D3-brane is slightly excited higher derivative terms in the field theory on its world-volume

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Deformation goes in the way set by (e) the SUGRA equations of motion.

G_{16} : Category one of SUGRA equations of motion

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of Deformation goes in the way set

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Igor R. Klebanov, Edward Witten study, using the

dual AdS description, (eb) the vacua of field theories where some of the gauge symmetry is broken by (e) expectation values of scalar fields

G_{20} : Category one of dual AdS description

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of vacua of field theories where some of the gauge symmetry is broken by (e) expectation values of scalar fields

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

dual AdS description is used to investigate the vacua of field theories where some of the gauge symmetry is broken by (e) expectation values of scalar fields

G_{24} : Category one of dual AdS description is used to investigate the vacua of field theories where some of the gauge symmetry

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of expectation values of scalar fields

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

In such vacua, operators built out of the scalar fields acquire (e) expectation values, and they show how to calculate them from (e) the behavior of perturbations to (e) the AdS background near the boundary

G_{28} : Category one of expectation values, and they show how to calculate them from (e) the behavior of perturbations to (e) the AdS background near the boundary

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of operators built out of the scalar fields In such vacua

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

expectation values are calculated from (e) the behavior of perturbations to (e) the AdS background near the boundary

G_{32} : Category one of behavior of perturbations to (e) the AdS background near the boundary

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of expectation values

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

expectation values are calculated from the behavior of perturbations to (e) the AdS background near the boundary

G_{36} : Category one of AdS background near the boundary

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of expectation values are calculated from the behavior of perturbations

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

Specific examples include the N = 4 SYM theory, and (e&eb) theories on D3-branes placed on orbifolds and conifolds

G_{40} : Category one of N = 4 SYM theory; theories on D3-branes placed on orbifolds and conifolds

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of theories on D3-branes placed on orbifolds and conifolds; N = 4 SYM theory

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Igor R. Klebanov, Edward Witten also clarify some subtleties of the AdS/CFT correspondence that arise in this analysis. In particular, they explain how
scalar fields in AdS space of sufficiently negative mass-squared can be associated with (e&eb) CFT operators of two possible dimensions

G_{44} : Category one of scalar fields in AdS space of sufficiently negative mass-squared; CFT operators of two possible dimensions

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of CFT operators of two possible dimensions ;scalar fields in AdS space of sufficiently negative mass-squared

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION TWELVE A

Symmetry Breaking And Information Loss

INTRODUCTION—VARIABLES USED

Nuclear Physics B Volume 556, Issues 1–2, 6 September 1999, Pages 89–114 Ads/CFT correspondence and symmetry breaking Igor R. Klebanov, Edward Witten doi:10.1016/S0550-3213(99)00387-9

(1) All dimensions are bounded from (e) below by (d−2)/2
(2) this is the unitarity bound for (e) scalar operators in d-dimensional field theory
(3) They further argue that the generating functional for (e) correlators in the theory with one choice of operator dimension is (=) a Legendre transform of the generating functional in the theory with the other choice.

Information loss in black holes S. W. Hawking Phys. Rev. D 72, 084013 – Published 18 October 2005

(4) The question of whether information is lost in black holes is investigated using (e) Euclidean path integrals.
(5) The formation and evaporation of black holes is regarded as (=) a scattering problem with all measurements being made at infinity.
(6) This seems to be well formulated only in (eb) asymptotically AdS spacetimes.
(7) The path integral over metrics with trivial topology is (=) unitary and information preserving.
(8) On the other hand, the path integral over metrics with nontrivial topologies leads to (eb) correlation functions that decay to zero.
(9) Thus at late times only the unitary information preserving path integrals over trivial topologies will contribute. Elementary quantum gravity interactions do not (e) lose information or quantum coherence.

NOTATION

Module One

All dimensions are bounded from (e) below by (d−2)/2

G_{13} : Category one of(d−2)/2 (take all the three categories as equivalent, in the eventuality of constancy of the factor summarised in the category)

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of All dimensions are bounded

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

this is the unitarity bound for (e) scalar operators in d-dimensional field theory

G_{16} : Category one of scalar operators in d-dimensional field theory

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of unitarity bound

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

They further argue that the generating functional for (e) correlators in the theory with one choice of operator dimension is (=) a Legendre transform of the generating functional in the theory with the other choice

G_{20} : Category one of correlators in the theory with one choice of operator dimension is (=) a Legendre transform of the generating functional in the theory with the other choice

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of generating functional

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

generating functional for correlators in the theory with one choice of operator dimension is (=) a Legendre transform of the generating functional in the theory with the other choice

G_{24} : Category one of generating functional for correlators in the theory with one choice of operator dimension

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of Legendre transform of the generating functional in the theory with the other choice

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

The question of whether information is lost in black holes is investigated using (e) Euclidean path integrals

We assume the proposition and give a model

G_{28} : Category one of information

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of lost in black holes

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

The path integral over metrics with trivial topology is (=) unitary and information preserving

G_{32} : Category one of path integral over metrics with trivial topology

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of unitary and information preserving

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

On the other hand, the path integral over metrics with nontrivial topologies leads to (eb) correlation functions that decay to zero

G_{36} : Category one of path integral over metrics with nontrivial topologies

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of correlation functions that decay to zero

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

Thus at late times only the unitary information preserving path integrals over trivial topologies will contribute.

Elementary quantum gravity interactions do not (e) lose information or quantum coherence.

G_{40} : Category one of unitary information preserving path integrals over trivial topologies

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of contribution

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Elementary quantum gravity interactions do not (e) lose information or quantum coherence

G_{44} : Category one of lose information or quantum coherence; quantum gravity interactions

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of Elementary quantum gravity interactions; lose information or quantum coherence

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION THIRTEEN

Renormalization Group Flows From Holography

INTRODUCTION—VARIABLES USED

Renormalization Group Flows from Holography--Supersymmetry and a c-Theorem D.Z. Freedman, S.S. Gubser, K. Pilch, N.P. Warner

(1) **D.Z. Freedman, S.S. Gubser, K. Pilch, N.P. Warner** obtain first order equations that determine (eb) a supersymmetric kink solution in (eb) five-dimensional N=8 gauged supergravity
(2) The kink interpolates between an exterior anti-de Sitter region with maximal supersymmetry and (e&eb) an interior anti-de Sitter region with one quarter of the maximal supersymmetry.
(3) One eighth of supersymmetry is preserved by (e) the kink as a whole.
(4) They interpret it as describing the renormalization group flow in (eb) N=4 super-Yang-Mills theory broken to an N=1 theory by (e) the addition of a mass term for (e) one of the three adjoint chiral superfields.
(5) A detailed correspondence is obtained between fields of bulk supergravity in the interior anti-de Sitter region and (e&eb) composite operators of the infrared field theory.
(6) **D.Z. Freedman, S.S. Gubser, K. Pilch, N.P. Warner** also point out that the truncation used to find the reduced symmetry critical point can be extended to obtain (eb) a new N=4 gauged supergravity theory holographically dual to a sector of N=2 gauge theories based on (e) quiver diagrams.
(7) They consider more general kink geometries and construct (eb) a c-function that is positive and monotonic if (e) a weak energy condition holds in the bulk gravity theory.
(8) For even-dimensional boundaries, the c-function coincides with (=) the trace anomaly coefficients of the holographically related field theory in (eb) limits where conformal invariance is recovered. Journal reference: Adv.Theor.Math.Phys.3:363-417, 1999 Report number: CERN-TH/99-86,

HUTP-99/A015, USC-99/1, MIT-CTP-2846 Cite as: arXiv: hep-th/9904017

NOTATION

Module One

D.Z. Freedman, S.S. Gubser, K. Pilch, N.P. Warner obtain

first order equations that determine (eb) a supersymmetric kink solution in (eb) five-dimensional N=8 gauged supergravity

G_{13} : Category one of first order equations

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of supersymmetric kink solution in (eb) five-dimensional N=8 gauged supergravity

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

first order equations that determine a supersymmetric kink solution in (eb) five-dimensional N=8 gauged supergravity

G_{16} : Category one of first order equations that determine a supersymmetric kink solution

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of five-dimensional N=8 gauged supergravity

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

The kink interpolates between an exterior anti-de Sitter region with maximal supersymmetry and (e&eb) an interior anti-de Sitter region with one quarter of the maximal supersymmetry

G_{20} : Category one of kink interpolates between an exterior anti-de Sitter region with maximal supersymmetry; interior anti-de Sitter region with one quarter of the maximal supersymmetry

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of interior anti-de Sitter region with one quarter of the maximal supersymmetry; kink interpolates between an exterior anti-de Sitter region with maximal supersymmetry

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

One eighth of supersymmetry is preserved by (e) the kink as a whole

G_{24} : Category one of kink as a whole

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of One eighth of supersymmetry is preserved

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

They interpret it as describing the renormalization group flow in (eb) N=4 super-Yang-Mills theory broken to an N=1 theory by (e) the addition of a mass term for (e) one of the three adjoint chiral superfields

G_{28} : Category one of One eighth of supersymmetry is preserved by (e) the kink as a whole

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of renormalization group flow in (eb) N=4 super-Yang-Mills theory broken to an N=1 theory by (e) the addition of a mass term for (e) one of the three adjoint chiral superfields

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

They interpret it as describing the

renormalization group flow in (eb) N=4 super-Yang-Mills theory broken to an N=1 theory by (e) the addition of a mass term for (e) one of the three adjoint chiral superfields

G_{32} : Category one of renormalization group flow

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of N=4 super-Yang-Mills theory broken to an N=1 theory by (e) the addition of a mass term for (e) one of the three adjoint chiral superfields

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

renormalization group flow in N=4 super-Yang-Mills theory broken to an N=1 theory by (e) the addition of

a mass term for (e) one of the three adjoint chiral superfields

G_{36} : Category one of addition of a mass term for (e) one of the three adjoint chiral superfields

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of renormalization group flow in N=4 super-Yang-Mills theory broken to an N=1 theory

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

renormalization group flow in N=4 super-Yang-Mills theory broken to an N=1 theory by the addition of a mass term for (e) one of the three adjoint chiral superfields

G_{40} : Category one of one of the three adjoint chiral superfields

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of renormalization group flow in N=4 super-Yang-Mills theory broken to an N=1 theory by the addition of a mass term

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

A detailed correspondence is obtained between

fields of bulk supergravity in the interior anti-de Sitter region and (e&eb) composite operators of the infrared field theory.

G_{44} : Category one of fields of bulk supergravity in the interior anti-de Sitter region; composite operators of the infrared field theory.

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of composite operators of the infrared field theory.; fields of bulk supergravity in the interior anti-de Sitter region

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION THIRTEEN A

Fields Of Bulk Supergravity In The Interior Anti-De Sitter Region And Composite Operators Of The Infrared Field Theory

INTRODUCTION—VARIABLES USED

Renormalization Group Flows from Holography--Supersymmetry and a c-Theorem D.Z. Freedman, S.S. Gubser, K. Pilch, N.P. Warner

(1) A detailed correspondence is obtained between fields of bulk supergravity in the interior anti-de Sitter region and (e&eb) composite operators of the infrared field theory.

(2) **D.Z. Freedman, S.S. Gubser, K. Pilch, N.P. Warner** also point out that the truncation used to find the reduced symmetry critical point can be extended to obtain (eb) a new N=4 gauged supergravity theory holographically dual to a sector of N=2 gauge theories based on (e) quiver diagrams.

(3) They consider more general kink geometries and construct (eb) a c-function that is positive and monotonic if (e) a weak energy condition holds in the bulk gravity theory.

(4) For even-dimensional boundaries, the c-function coincides with (=) the trace anomaly coefficients of the holographically related field theory in (eb) limits where conformal invariance is recovered. Journal reference: Adv.Theor.Math.Phys.3:363-417, 1999 Report number: CERN-TH/99-86, HUTP-99/A015, USC-99/1, MIT-CTP-2846 Cite as: arXiv: hep-th/9904017

NOTATION

Module One

D.Z. Freedman, S.S. Gubser, K. Pilch, N.P. Warner also point out that the

Truncation used to find the reduced symmetry critical point can be extended to obtain (eb) a new N=4 gauged supergravity theory holographically dual to a sector of N=2 gauge theories based on (e) quiver diagrams.

G_{13} : Category one of Truncations

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of reduced symmetry critical point can be extended to obtain (eb) a new N=4 gauged supergravity theory holographically dual to a sector of N=2 gauge theories based on (e) quiver diagrams

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Truncation used to find the reduced symmetry critical point can be extended to obtain (eb) a new N=4 gauged supergravity theory holographically dual to a sector of N=2 gauge theories based on (e) quiver diagrams

G_{16} : Category one of Truncation used to find the reduced symmetry critical point

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of a new N=4 gauged supergravity theory holographically dual to a sector of N=2 gauge theories based on (e) quiver diagrams

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Truncation used to find the reduced symmetry critical point can be extended to obtain a new N=4 gauged supergravity theory holographically dual to a sector of N=2 gauge theories based on (e) quiver diagrams

G_{20} : Category one of Truncation used to find the reduced symmetry critical point can be extended to obtain a new N=4 gauged supergravity theory; holographically dual to a sector of N=2 gauge theories based on (e) quiver diagrams

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of holographically dual to a sector of N=2 gauge theories based on (e) quiver diagrams; Truncation used to find the reduced symmetry critical point can be extended to obtain a new N=4 gauged supergravity theory

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Maxwell's equations and gauge invariance Gauge theories Gerard ′t Hooft (2008), Scholarpedia, 3(12):7443. doi:10.4249/scholarpedia.7443 revision #141891 [link to/cite this article] Curator: Gerard ′t Hooft Prof. Gerard ′t Hooft, Institute for Theoretical Physics Universiteit Utrecht: for references please see the Scholarpedia article

The simplest example of a gauge theory is electrodynamics, as described by the Maxwell equations. The electric field strength E→(x→,t) and the magnetic field strength B→(x→,t) obey the homogeneous Maxwell equations (in SI units):

$$\nabla\vec{\to}\times E\vec{\to}+\partial B\vec{\to}\partial t=0 \quad (1)$$

$$\nabla\vec{\to}\cdot B\vec{\to}=0 \quad (2)$$

According to Poincaré's Lemma, Eq. (2) implies that there exists another vector field A→(x→,t) such that

$$B\vec{\to}=\nabla\vec{\to}\times A\vec{\to} \quad (3)$$

Since Eq. (1) now reads

$$\nabla\rightarrow\times(E\rightarrow+\partial A\rightarrow\partial t)=0 \ ,(4)$$

we can also conclude that there is a potential field $\Phi(x\rightarrow,t)$ such that

$$E\rightarrow=-\nabla\rightarrow\Phi-\partial A\rightarrow\partial t \ .(5)$$

The field Φ is the electric potential field; the vector field $A\rightarrow$ is called the vector potential field. The strengths of these potential fields are determined by the inhomogeneous Maxwell equations, which are the equations that relate the strengths of the electromagnetic fields to the electric charges and currents that generate these fields. The use of potential fields often simplifies the problem of solving Maxwell's equations. What turns this theory into a gauge theory is the fact that the values of these potential fields are not completely determined by Maxwell's equations. Consider an electromagnetic field configuration $(E\rightarrow(x\rightarrow,t),B\rightarrow(x\rightarrow,t))$, and suppose that it is described by the potential fields $(\Phi(x\rightarrow,t),A\rightarrow(x\rightarrow,t))$. Then, using any arbitrary scalar function $\Lambda(x\rightarrow,t)$, one can find a different set of potential fields describing the same electric and magnetic fields, by writing

$$\Phi'=\Phi+\partial\Lambda\partial t \ ,A\rightarrow'=A\rightarrow-\nabla\rightarrow\Lambda \ .(6)$$

Inspecting Equations (3) and (5), one easily observes that $E\rightarrow=E\rightarrow'$ and $B\rightarrow=B\rightarrow'$. Thus, the set $(\Phi',A\rightarrow')$ and $(\Phi,A\rightarrow)$ describe the same physical situation. Because of this, we call the transformation (6) a gauge transformation. Since Λ may be chosen to be an arbitrary function of the points $(x\rightarrow,t)$ in space-time, we speak of a local gauge transformation. The fact that the electromagnetic fields are invariant under these local gauge transformations turns Maxwell's theory into a gauge theory.

In relativistic quantum field theory, the field $\psi(x\rightarrow,t)$ of a non-interacting spinless particle would typically obey the equation

$$(\nabla\rightarrow2-\partial2\partial t2)\psi=m2\psi \ ,(7)$$

where units where used such that the velocity of light $c=1$, and Planck's constant $\hbar=1$. This gives the dispersion relation between energy and momentum as dictated by Special Relativity:

$$E=p\rightarrow2+m2-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\sqrt{} \ ,(8)$$

Suppose now that the particle in question carries an electric charge q . How is its equation then affected by the presence of electro-magnetic fields? It turns out that one cannot write the correct equations using the fields $E\rightarrow$ and $B\rightarrow$ directly. Here, one can only choose to add terms depending on the (vector) potential fields instead:

$$(\nabla\rightarrow-iqA\rightarrow)2\psi-(\partial\partial t+iq\Phi)2\psi=m2\psi \ .(9)$$

It can be verified that this equation correctly produces waves that are deflected by the electro-magnetic forces in the way one expects. For instance, the energy $E$is easily seen to be enhanced by an amount $q\Phi(x\rightarrow,t)$, which is the potential energy of a charged particle in an electric potential field.

However, what happens to this equation when performing a gauge transformation? It appears as if the

equation changes, so that the solution for the field ψ should change as well. Indeed, ψ changes in the following way:

$$\psi' = e^{-iq\Lambda}\psi \ , \partial\psi'\partial t = e^{-iq\Lambda}(\partial\psi\partial t - iq\psi\partial\Lambda\partial t) \ . (10)$$

Thus, the field ψ makes a rotation in the complex plane. This is closely related to a 'scale transformation', which would result if one were to remove the 'i' from Eq. (10). It was Hermann Weyl who noted that this symmetry transformation simply redefines the scale of the field ψ , and introduced the word 'gauge' to describe this feature.

The combinations

$$D\vec{\rightarrow}\psi = (\nabla\vec{\rightarrow} - iqA\vec{\rightarrow})\psi, D_t\psi = (\partial\partial t + iq\Phi)\psi \ , (11)$$

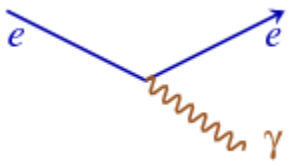


Figure 1: Feynman diagram for electron emitting a photon.

are called covariant derivatives, because they are chosen in such a way that the derivatives of the function $\Lambda(x\vec{\rightarrow},t)$ cancel out in a gauge transformation:

$$(D\vec{\rightarrow}\psi)' = e^{-iq\Lambda}(\nabla\vec{\rightarrow} - iq(\nabla\vec{\rightarrow}\Lambda) - iq(A\vec{\rightarrow} - \nabla\vec{\rightarrow}\Lambda))\psi = e^{-iq\Lambda}(D\vec{\rightarrow}\psi) \ , (12)$$

$$(D_t\psi)' = e^{-iq\Lambda}(\partial\partial t - iq\partial\Lambda\partial t + iq(\Phi + \partial\Lambda\partial t))\psi = e^{-iq\Lambda}(D_t\psi) \ , (13)$$

and this makes it easy to see that Equation (10) correctly describes the way ψ transforms under a local gauge transformation, obeying the same field equation (9) both before and after the transformation (all terms in the equation are multiplied by the same exponential $e^{-iq\Lambda}$, so that that factor is immaterial).

The absolute value, $|\psi(x\vec{\rightarrow},t)|^2$ does not change at all under a gauge transformation, and indeed this is the quantity that corresponds to something that is physically observable: it is the probability that a particle can be found at $(x\vec{\rightarrow},t)$. A rule of thumb is that local gauge invariance requires all derivatives in our equations to be replaced by covariant derivatives.

2. Yang-Mills theory

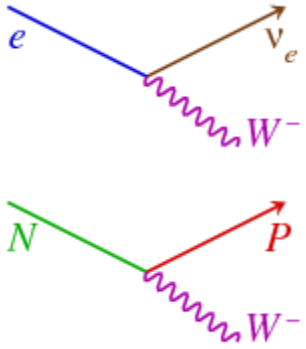


Figure 2: Feynman diagrams for emission of Yang-Mills photons. Above: electron turning into en electron-neutrino; below: neutron turning into proton.

In the 1950s, it was known that the field equations for the field of a proton, $P(x\rightarrow,t)$, and the field of a neutron, $N(x\rightarrow,t)$, are such that one can rotate these fields in a complex two-dimensional space:

$$\begin{pmatrix} P'(x\rightarrow,t) \\ N'(x\rightarrow,t) \end{pmatrix} = (abcd) \begin{pmatrix} P(x\rightarrow,t) \\ N(x\rightarrow,t) \end{pmatrix} , (14)$$

where the matrix $U=(abcd)$ may contain four arbitrary complex numbers, as long as it is unitary ($UU\dagger=I$), and usually, the determinant of U is restricted to be 1. Since these equations resemble the rotations one can perform in ordinary space, to describe spin of a particle, the symmetry in question here was called isospin.

In 1954, C.N. Yang and R.L. Mills published a very important idea. Could one modify the equations in such a way that these isospin rotations could be regarded as local gauge rotations? This would mean that, unlike the case that was known, the matrices U should be allowed to depend on space and time, just like the gauge generator $\Lambda(x\rightarrow,t)$ in electromagnetism. Yang and Mills were also inspired by the observation that Einstein's theory of gravity, General Relativity, also allows for transformations very similar to local gauge transformations: the replacement of the coordinate frame by other coordinates in an arbitrary, space-time dependent way. To write down field equations for protons and neutrons, one needs the derivatives of these fields. The way these derivatives transform under a local gauge transformation implies that there will be terms containing the gradients $\nabla\rightarrow U$ of the matrices U . To make the theory gauge-invariant, these gradients would have to be cancelled out, and in order to do that, Yang and Mills replaced the derivatives $\nabla\rightarrow$ by covariant derivatives $D\rightarrow=\nabla\rightarrow-igA\rightarrow(x\rightarrow,t)$, as was done in electromagnetism, see Equation (11). Here, however, the fields $A\rightarrow$ had to be matrix-valued, just as the isospin U matrices:

$$A\rightarrow = \begin{pmatrix} a\rightarrow 11 & a\rightarrow 12 \\ a\rightarrow 21 & a\rightarrow 22 \end{pmatrix} , (15)$$

$$TrA\rightarrow=a\rightarrow 11+a\rightarrow 22=0 \ , a\rightarrow 11=a\rightarrow *11, a\rightarrow 21=a\rightarrow *12 \ .$$

Since the U matrices contain four coefficients with one constraint (the determinant has to be 1), one ends up with a set of three new vector fields (there are 3 independent real vectors in the matrix (15)). At first sight, they appear to be the fields of a vector particle with isospin one. In practice, this should correspond to particles with one unit of spin (i.e., the particle rotates about its axis), and its electric charge could be neutral or one or minus one unit. Yang-Mills theory therefore predicts and describes a new type of particles with spin one that transmit a force not unlike the electro-magnetic force.

The fields that are equivalent to Maxwell's electric and magnetic fields are obtained by considering

the commutator of two covariant derivatives:

$$[D\mu,Dv]=D\mu Dv-DvD\mu=-ig(\partial\mu Av-\partial vA\mu-ig[A\mu,Av])=-igF\mu v \ ,(16)$$

where the indices take the values $\mu, \nu=0,1,2,3$, with 0 referring to the time-component.

Since $F\mu\nu=-F\nu\mu$, this tensor has 6 independent components, three forming an electric vector field, and three a magnetic field. Each of these components is also a matrix. The commutator, $[A\mu,Av]$ is a new, non-linear term, which makes the Yang-Mills equations a lot more complicated than the Maxwell system. In other respects, the Yang-Mills particles, being the energy quanta of the Yang-Mills fields, are similar to photons, the quanta of light. Yang-Mills particles also carry no intrinsic mass, and travel with the speed of light. Indeed, these features were at first reasons to dismiss this theory, because massless particles of this sort should have been detected long ago, whereas they were conspicuously absent.

3. **The Brout-Englert-Higgs mechanism**

The theory was revived when it was combined with spontaneous breakdown of local gauge symmetry, also known as the Brout-Englert-Higgs mechanism. Consider a scalar (spinless) particle described by a field $\phi(x\rightarrow,t)$. This field is assumed to be a vector field, in the sense that it undergoes some rotation when a gauge transformation is performed. In practice this means that the particle carries one or several kinds of charges that make it sensitive to the Yang-Mills force, and often it has several components, which means there are various species of this particle. Such particles must obey Bose-Einstein statistics, which implies that it can undergo Bose-Einstein condensation. In terms of its field ϕ this means the following:

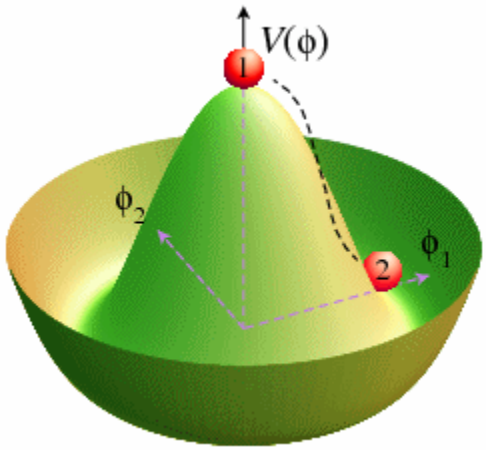


Figure 3: Spontaneous symmetry breaking. An object residing in a rotationally symmetric potential finds a stable, asymmetric position. In the BEH case, it is the Higgs field, $(\phi 1,\phi 2)$ that finds an asymmetric value $(F,0)$.

In the vacuum the field ϕ takes a non-vanishing value F .

This is usually written as

$$\langle\phi(x\rightarrow,t)\rangle=F \ .(17)$$

After a local gauge transformation, this would look like

$\langle\phi'(x\rightarrow,t)\rangle=U(x\rightarrow,t)F$,(18)

G_{24} : Category one of LHS of the above equations (1 to 18)

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of RHS of the above equations (1 to 18)

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

They consider more

general kink geometries and construct (eb) a c-function that is positive and monotonic if (e) a weak energy condition holds in the bulk gravity theory

G_{28} : Category one of general kink geometries

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of c-function that is positive and monotonic if (e) a weak energy condition holds in the bulk gravity theory

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

general kink geometries and construct a c-function that is positive and monotonic if (e) a weak energy condition holds in the bulk gravity theory

G_{32} : Category one of weak energy condition holds in the bulk gravity theory

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of general kink geometries and construct a c-function that is positive and monotonic

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

For even-dimensional boundaries, the c-function coincides with (=) the trace anomaly coefficients of the

holographically related field theory in (eb) limits where conformal invariance is recovered

. Journal reference: Adv.Theor.Math.Phys.3:363-417, 1999 Report number: CERN-TH/99-86, HUTP-99/A015, USC-99/1, MIT-CTP-2846 Cite as: arXiv: hep-th/9904017

G_{36} : Category one of c-function

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of trace anomaly coefficients of the holographically related field theory in (eb) limits where conformal invariance is recovered

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

c-function For even-dimensional boundaries coincides with the trace anomaly coefficients of the holographically related field theory in (eb) limits where conformal invariance is recovered

G_{40} : Category one of c-function For even-dimensional boundaries coincides with the trace anomaly coefficients of the holographically related field theory

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of limits where conformal invariance is recovered

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Maxwell's equations and gauge invariance Gauge theories Gerard ′t Hooft (2008), Scholarpedia, 3(12):7443. doi:10.4249/scholarpedia.7443 revision #141891 [link to/cite this article] Curator: Gerard ′t Hooft Prof. Gerard ′t Hooft, Institute for Theoretical Physics Universiteit Utrecht: for references please see the Scholarpedia article

The weak interactions could now be successfully described by a Yang-Mills theory. The set of local gauge transformations forms the mathematical group SU(2)×U(1) .

This group generates 4 species of photons (3 for SU(2) and 1 for U(1)).

The Brout-Englert-Higgs mechanism breaks this group down in such a way that a subgroup of the form U(1) remains.

This is the electromagnetic theory, with just one photon.

The other three photons become massive; they are responsible for the weak interactions, which in practice

appear to be weak just because these forces have a very short range.

With respect to electromagnetism, two of these intermediate vector bosons, W± , are electrically charged, and a third, Z0 , is electrically neutral.

When the latter's existence was derived from group theoretical arguments, this gave rise to the prediction of a hitherto unnoticed form of the weak interaction: the neutral current interaction.

This theory, that combines electromagnetism and the weak force into one, is called the electro-weak theory, and it was the first fully renormalizable theory for the weak force (see Chapter 5)

G_{44} : Category one of4 species of photons (3 for SU(2) and 1 for U(1)). Subgroup of the form U(1) remains.

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of Brout-Englert-Higgs mechanism

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION FOURTEEN

Renormalization Group Evolution Of Neutrino Parameters And Majorana Phases

INTRODUCTION—VARIABLES USED

Renormalization group evolution of neutrino parameters in presence of seesaw threshold effects and Majorana phases Shivani Gupta, , Sin Kyu Kangb, , C.S. Kima doi:10.1016/j.nuclphysb.2015.01.026 Nuclear Physics B

(1) **Shivani Gupta, , Sin Kyu Kangb, , C.S. Kima** examine the renormalization group evolution (RGE) for (e) different mixing scenarios in (eb) the presence of seesaw threshold effects from high energy scale (GUT) to the low electroweak (EW) scale in the Standard Model (SM) and Minimal Supersymmetric Standard Model (MSSM).

(2) Authors consider four mixing scenarios namely Tri–Bimaximal Mixing (e&eb) Bimaximal Mixing, Hexagonal Mixing and (e&eb) Golden Ratio Mixing which come from (e) different flavor symmetries at (eb) the GUT scale

(3) **Shivani Gupta, , Sin Kyu Kangb, , C.S. Kima** find that the Majorana phases play (e&eb) an important role in the RGE running of these mixing patterns along with (e&eb) the seesaw threshold corrections.

(4) They present a comparative study of the RGE of all these mixing scenarios both with and without (e&eb) Majorana CP phases when (e) seesaw threshold corrections are taken into consideration.

(5) They find that both the RGE running and seesaw effects in the absence of these Majorana phases may lead *to (eb)* $\theta 13<5°$ at low energies both in the SM and MSSM.

(6) However, if the Majorana phases are incorporated into (e&eb) the mixing matrix the running can be enhanced (eb) both in the SM and MSSM.

(7) Even by incorporating non-zero Majorana CP phases in the SM, (eb) we do not get $\theta 13$ in its present 3σ range.

(8) The current values of the two mass squared differences and mixing angles including (e) $\theta 13$ can be

produced in (eb) the MSSM case with tanβ=10 and non-zero Majorana CP phases at low energy.

(9) They also calculate the order of (e) effective Majorana mass and Jarlskog Invariant for (e) each scenario under consideration.

NOTATION

Module One

Shivani Gupta, , Sin Kyu Kangb, , C.S. Kima examine the

renormalization group evolution (RGE) for (e) different mixing scenarios in (eb) the presence of seesaw threshold effects from high energy scale (GUT) to the low electroweak (EW) scale in the Standard Model (SM) and Minimal Supersymmetric Standard Model (MSSM)

G_{13} : Category one of different mixing scenarios in (eb) the presence of seesaw threshold effects from high energy scale (GUT) to the low electroweak (EW) scale in the Standard Model (SM) and Minimal Supersymmetric Standard Model (MSSM

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of renormalization group evolution (RGE)

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Authors consider four mixing scenarios namely
Tri–Bimaximal Mixing (e&eb) Bimaximal Mixing, Hexagonal Mixing and (e&eb) Golden Ratio Mixing which come from (e) different flavor symmetries at (eb) the GUT scale

G_{16} : Category one of Tri–Bimaximal Mixing; Bimaximal Mixing

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of Bimaximal Mixing; Tri–Bimaximal Mixing

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Golden Ratio Mixing which come from (e) different flavor symmetries at the GUT scale

G_{20} : Category one of different flavor symmetries at the GUT scale

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of Golden Ratio Mixing

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Shivani Gupta, Sin Kyu Kangb, , C.S. Kima find that the

Majorana phases play (e&eb) an important role in the RGE running of these mixing patterns along with (e&eb) the seesaw threshold corrections

G_{24} : Category one of Majorana phases ; RGE running of these mixing patterns along with (e&eb) the seesaw threshold corrections

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of RGE running of these mixing patterns along with (e&eb) the seesaw threshold corrections ;Majorana phases

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Majorana phases play an important role in the RGE running of these mixing patterns along with (e&eb) the seesaw threshold corrections

G_{28} : Category one of Majorana phases play an important role in the RGE running of these mixing patterns; seesaw threshold corrections

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of seesaw threshold corrections; Majorana phases play an important role in the RGE running of these mixing patterns

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

They present a comparative study of the

RGE of all these mixing scenarios both with and without (e&eb) Majorana CP phases when (e) seesaw threshold corrections are taken into consideration

G_{32} : Category one of RGE of all these mixing scenarios; Majorana CP phases when (e) seesaw threshold corrections are taken into consideration

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of Majorana CP phases when (e) seesaw threshold corrections are taken into consideration ;RGE of all these mixing scenarios

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

RGE of all these mixing scenarios both with and without Majorana CP phases when (e) seesaw threshold corrections are taken into consideration

G_{36} : Category one of seesaw threshold corrections are taken into consideration

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of RGE of all these mixing scenarios both with and without Majorana CP phases

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

They find that both the

RGE running and seesaw effects in the absence of these Majorana phases may lead *to (eb)* $\theta 13<5°$ at low energies both in the SM and MSSM

G_{40} : Category one of RGE running and seesaw effects in the absence of these Majorana phases

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of $\theta 13<5°$ at low energies both in the SM and MSSM

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

However, if the

Majorana phases are incorporated into (e&eb) the mixing matrix the running can be enhanced (eb) both in the SM and MSSM

G_{44} : Category one of Majorana phases; mixing matrix the running can be enhanced (eb) both in the SM and MSSM

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of mixing matrix the running can be enhanced (eb) both in the SM and MSSM ;Majorana phases

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},\ \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}\ ,$
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}),t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}),t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}),t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}),t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}),t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}),t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}),t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

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$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14}$$

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$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15}$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

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$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t) \end{bmatrix} G_{16}$$

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$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & +(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) & +(a''_{21})^{(3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t) \end{bmatrix} G_{17}$$

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$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & +(a''_{18})^{(2)}(T_{17},t) & +(a''_{15})^{(1,1,)}(T_{14},t) & +(a''_{22})^{(3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{38})^{(7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9)}(T_{45},t) \end{bmatrix} G_{18}$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{13}'')^{(1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a_{40}'')^{(8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a_{44}'')^{(9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b_{16}')^{(2)} & -(b_{16}'')^{(2)}(G_{19},t) & -(b_{13}'')^{(1,1,)}(G,t) & -(b_{20}'')^{(3,3,3,)}(G_{23},t) \\ -(b_{24}'')^{(4,4,4,4,4)}(G_{27},t) & -(b_{28}'')^{(5,5,5,5,5)}(G_{31},t) & -(b_{32}'')^{(6,6,6,6,6)}(G_{35},t) \\ -(b_{36}'')^{(7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8)}(G_{43},t) & -(b_{44}'')^{(9,9)}(G_{47},t) \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b_{17}')^{(2)} & -(b_{17}'')^{(2)}(G_{19},t) & -(b_{14}'')^{(1,1,)}(G,t) & -(b_{21}'')^{(3,3,3,)}(G_{23},t) \\ -(b_{25}'')^{(4,4,4,4,4)}(G_{27},t) & -(b_{29}'')^{(5,5,5,5,5)}(G_{31},t) & -(b_{33}'')^{(6,6,6,6,6)}(G_{35},t) \\ -(b_{37}'')^{(7,7,7)}(G_{39},t) & -(b_{41}'')^{(8,8,8)}(G_{43},t) & -(b_{45}'')^{(9,9)}(G_{47},t) \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b_{18}')^{(2)} & -(b_{18}'')^{(2)}(G_{19},t) & -(b_{15}'')^{(1,1,)}(G,t) & -(b_{22}'')^{(3,3,3,)}(G_{23},t) \\ -(b_{26}'')^{(4,4,4,4,4)}(G_{27},t) & -(b_{30}'')^{(5,5,5,5,5)}(G_{31},t) & -(b_{34}'')^{(6,6,6,6,6)}(G_{35},t) \\ -(b_{38}'')^{(7,7,7)}(G_{39},t) & -(b_{42}'')^{(8,8,8)}(G_{43},t) & -(b_{46}'')^{(9,9)}(G_{47},t) \end{bmatrix} T_{18} \quad 66$$

where $-(b_{16}'')^{(2)}(G_{19},t)$, $-(b_{17}'')^{(2)}(G_{19},t)$, $-(b_{18}'')^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b_{13}'')^{(1,1,)}(G,t)$, $-(b_{14}'')^{(1,1,)}(G,t)$, $-(b_{15}'')^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b_{20}'')^{(3,3,3,)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b_{24}'')^{(4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b_{28}'')^{(5,5,5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b_{32}'')^{(6,6,6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b_{36}'')^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b_{40}'')^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b_{44}'')^{(9,9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{ccc} (a_{20}')^{(3)}\boxed{+(a_{20}'')^{(3)}(T_{21},t)} & \boxed{+(a_{16}'')^{(2,2,2)}(T_{17},t)} & \boxed{+(a_{13}'')^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{36}'')^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a_{44}'')^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{ccc} (a_{21}')^{(3)}\boxed{+(a_{21}'')^{(3)}(T_{21},t)} & \boxed{+(a_{17}'')^{(2,2,2)}(T_{17},t)} & \boxed{+(a_{14}'')^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{37}'')^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a_{45}'')^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{ccc} (a_{22}')^{(3)}\boxed{+(a_{22}'')^{(3)}(T_{21},t)} & \boxed{+(a_{18}'')^{(2,2,2)}(T_{17},t)} & \boxed{+(a_{15}'')^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{38}'')^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a_{46}'')^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{22} \quad (69)$$

$\boxed{+(a_{20}'')^{(3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{13}'')^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a_{14}'')^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{36}'')^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a_{37}'')^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a_{38}'')^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{40}'')^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{44}'')^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a_{46}'')^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[\begin{array}{ccc} (b_{20}')^{(3)}\boxed{-(b_{20}'')^{(3)}(G_{23},t)} & \boxed{-(b_{16}'')^{(2,2,2)}(G_{19},t)} & \boxed{-(b_{13}'')^{(1,1,1,)}(G,t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{36}'')^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b_{40}'')^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b_{44}'')^{(9,9,9)}(G_{47},t)} \end{array}\right] T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad (72)$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad (75)$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}, \boxed{(a''_{25})^{(4)}(T_{25},t)}, \boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* $1,2\ 3$

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* $1,2$ *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* $1,2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category $1,2$ *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category $1,2$ *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
are sixth augmentation coefficients for category $1,2$ *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
are seventh augmentation coefficients for category $1,2$ *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
are eighth augmentation coefficients for category $1,2$ *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad (78)$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* $1,2$ *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* $1,2$ *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* $1,2$ *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
are fourth detrition coefficients for category $1,2$ *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category $1,2$ *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$

are sixth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$

are seventh detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$

are eighth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt}=(a_{28})^{(5)}G_{29}-\left[\begin{array}{ccc}(a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}\end{array}\right]G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt}=(a_{29})^{(5)}G_{28}-\left[\begin{array}{ccc}(a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}\end{array}\right]G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt}=(a_{30})^{(5)}G_{29}-\left[\begin{array}{ccc}(a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}\end{array}\right]G_{30} \quad (81)$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt}=(b_{28})^{(5)}T_{29}-\left[\begin{array}{ccc}(b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}\end{array}\right]T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)} & +(a''_{32})^{(6)}(T_{33},t) & +(a''_{28})^{(5,5,5)}(T_{29},t) & +(a''_{24})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)} & +(a''_{33})^{(6)}(T_{33},t) & +(a''_{29})^{(5,5,5)}(T_{29},t) & +(a''_{25})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)} & +(a''_{34})^{(6)}(T_{33},t) & +(a''_{30})^{(5,5,5)}(T_{29},t) & +(a''_{26})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{34} \quad (87)$$

$+(a''_{32})^{(6)}(T_{33},t)$, $+(a''_{33})^{(6)}(T_{33},t)$, $+(a''_{34})^{(6)}(T_{33},t)$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1,2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1,2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)} & \boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ & \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)} & \boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ & \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & \boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ & \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* $1,2$ *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* $1,2$ *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* $1,2$ *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition

coefficients for category 1, 2, and 3

$-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3

$-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2, and 3

$-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)} & +(a''_{36})^{(7)}(T_{37},t) & +(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)} & +(a''_{37})^{(7)}(T_{37},t) & +(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)} & +(a''_{38})^{(7)}(T_{37},t) & +(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15}$$

Where $(a''_{36})^{(7)}(T_{37},t)$, $(a''_{37})^{(7)}(T_{37},t)$, $(a''_{38})^{(7)}(T_{37},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation

coefficient for 1,2,3

$\frac{dT_{36}}{dt} =$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$\frac{dT_{37}}{dt} =$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$\frac{dT_{38}}{dt} =$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{40}}{dt} =$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt}$$

$$= (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt}$$
$$= (a_{45})^{(9)} G_{44}$$
$$- \left[\begin{array}{ccc} (a'_{45})^{(9)} \boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array} \right] G_{14}$$

$$\frac{dG_{46}}{dt}$$
$$= (a_{46})^{(9)} G_{45}$$
$$- \left[\begin{array}{ccc} (a'_{46})^{(9)} \boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array} \right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$
$$(b_{44})^{(9)} T_{45} -$$
$$\left[\begin{array}{ccc} (b'_{44})^{(9)} \boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{array} \right] T_{13}$$
$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b''_i)^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}', t) - (a_i'')^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}', t)$ and $(a_i'')^{(1)}(T_{14}, t)$. (T_{14}', t) and (T_{14}, t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}, \ (r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = \ (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}\big((G_{19}), t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, \ (\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq \ (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$\lim_{G\to\infty}(b_i'')^{(3)}\,(G_{23},t) = \ (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$

$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)},$ are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$

$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$

$(b_i'')^{(4)}\big((G_{27}),t\big) \le \ (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$

$lim_{T_2\to\infty}(a_i'')^{(4)}\,(T_{25},t) = (p_i)^{(4)}$ 118

$\lim_{G\to\infty}(b_i'')^{(4)}\,\big((G_{27}),t\big) = \ (r_i)^{(4)}$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}\,(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}\,(G_{31}, t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\,(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ \ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}\,(T_{33}, t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G \to \infty}(b_i'')^{(6)}\,\big((G_{35}), t\big) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\big((G_{35}), t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\,(a_i)^{(6)} + (a_i')^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)}\,(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(AAAA) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(BBBB) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq \ (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

132

(CCCC) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$
(DDDD)

$\lim_{G\to\infty}(b_i'')^{(7)}\left((G_{39}),t\right) = \ (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, \ (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}\left((G_{39}),t\right)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $\left[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\right]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(EEEE) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(FFFF) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and \ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, \ (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\,(a_i)^{(7)} + (a_i')^{(7)} + \ (\hat{A}_{36})^{(7)} + \ (\hat{P}_{36})^{(7)}\,(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\ (b_i)^{(7)} + (b_i')^{(7)} + \ (\hat{B}_{36})^{(7)} + \ (\hat{Q}_{36})^{(7)}\ \ (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}, \ (r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41},t) \le (p_i)^{(8)} \le (\hat{A}_{40})^{(8)} \quad 138$$

$$(b_i'')^{(8)}((G_{43}),t) \le (r_i)^{(8)} \le (b_i')^{(8)} \le (\hat{B}_{40})^{(8)} \quad 139$$

$$\lim_{T_2\to\infty}(a_i'')^{(8)}(T_{41},t) = (p_i)^{(8)} \quad 140$$

$$\lim_{G\to\infty}(b_i'')^{(8)}((G_{43}),t) = (r_i)^{(8)} \quad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}',t) - (a_i'')^{(8)}(T_{41},t)| \le (\hat{k}_{40})^{(8)}|T_{41} - T_{41}'|e^{-(\hat{M}_{40})^{(8)}t} \quad 142$$

$$|(b_i'')^{(8)}((G_{43})',t) - (b_i'')^{(8)}((G_{43}),t)| < (\hat{k}_{40})^{(8)}||(G_{43}) - (G_{43})'||e^{-(\hat{M}_{40})^{(8)}t} \quad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}',t)$ and $(a_i'')^{(8)}(T_{41},t)$. (T_{41}',t) and (T_{41},t) are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \quad 144$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad 146\,A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t) \le (p_i)^{(9)} \le (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t) \le (r_i)^{(9)} \le (b_i')^{(9)} \le (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty} (a_i'')^{(9)}(T_{45},t) = (p_i)^{(9)}$$

$$\lim_{G\to\infty} (b_i'')^{(9)}(G_{47},t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t) - (a_i'')^{(9)}(T_{45},t)| \le (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t) - (b_i'')^{(9)}\big((G_{47}),t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$ and$(a_i'')^{(9)}(T_{45},t)$. (T_{45}',t) and (T_{45},t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}} \ , \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}$, $(r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\,, T_i(0)$

$$G_i(t) \le \left(\hat{P}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le \left(\hat{Q}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le \left(\hat{P}_{20}\right)^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le \left(\hat{Q}_{20}\right)^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad , \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}$, $T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}$, 155

$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 156

$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{16})^{(2)}$, $T_i^0 \leq (\hat{Q}_{16})^{(2)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''^{(2)}_{16}\right)\left(T_{17}(s_{(16)}), s_{(16)}\right)\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}\left(T_{17}(s_{(16)}), s_{(17)}\right)\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}\left(T_{17}(s_{(16)}), s_{(16)}\right)\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{20})^{(3)}$, $T_i^0 \leq (\hat{Q}_{20})^{(3)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''^{(3)}_{20}\right)\left(T_{21}(s_{(20)}), s_{(20)}\right)\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}\left(T_{21}(s_{(20)}), s_{(20)}\right)\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}\left(T_{21}(s_{(20)}), s_{(20)}\right)\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)},\ T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)},\ T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{32})^{(6)},\ T_i^0 \le (\hat{Q}_{32})^{(6)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{36})^{(7)},\ T_i^0 \le (\hat{Q}_{36})^{(7)},$

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that 174

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \leq G_{32}^{0}+\int_0^t\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)} =$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(n) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \leq G_{36}^{0}+\int_0^t\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)} =$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \leq G_{40}^{0}+\int_0^t\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)} = \quad 180$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t} \leq \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for G_{41} , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \leq G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \leq \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for G_{45} , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \left\{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\right\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)}\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) - (a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right| \; e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} \} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \qquad (186)$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13{,}14{,}15$ depend only on T_{14} and respectively on G(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: (187)

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. (188)

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}\left|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t} \leq$
$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)d\left(\left((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)}\right)\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\}ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0$ for $t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)}G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$: $\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$G_{24}^{(2)} \left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)}\right) d\left(((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on T_{25} and respectively on (G_{27})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (228)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_2 / (a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)}, (T_{35})^{(1)}), ((G_{35})^{(2)}, (T_{35})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}), (\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}), (\widetilde{T_{35}}) \right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right| \; e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} \} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)} (\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)}; \; (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t} \; and \; (\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$ (250)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2 \; and \; \left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33} \; and \; G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$$
$$sup_i\{max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}, max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)} \left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)} (T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b'_i)^{(7)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1,\left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right),\quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$

From 99 to 44 it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(9)}t)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$
Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)},, (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)}, , (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)} t} - e^{-(a'_{26})^{(4)} t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)} t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G^0_{28} e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G^0_{28}}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G^0_{30} e^{-(S_2)^{(5)} t} \le G_{30}(t) \le \right. \quad 344$$

$$\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_2)^{(5)}((S_1)^{(5)}-(a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t}-e^{-(a'_{30})^{(5)}t}\right]+\ G_{30}^{0}e^{-(a'_{30})^{(5)}t}\Big)$$

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\le T_{30}(t)\le \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$:

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$-(\sigma_2)^{(6)}\le-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\le-(\sigma_1)^{(6)}$

$-(\tau_2)^{(6)}\le-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\le-(\tau_1)^{(6)}$

Definition of $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$: 350

By $(v_1)^{(6)}>0$, $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$, $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

Definition of $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)}>0$, $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$, $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

Definition of $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$:-

If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)}=(v_0)^{(6)},(m_1)^{(6)}=(v_1)^{(6)},$ ***if*** $(v_0)^{(6)}<(v_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}$ $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}$ $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}$ $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}$ $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}$ $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})} \left[e^{(S_1)^{(6)} t} - e^{-(a'_{34})^{(6)} t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)} t} \right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^{0}e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}}G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}}G_{36}^{0}e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t}-e^{-(S_2)^{(7)}t}\right]+G_{38}^{0}e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left.\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_2)^{(7)}((S_1)^{(7)}-(a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t}-e^{-(a'_{38})^{(7)}t}\right]+\ G_{38}^{0}e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}}T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}}T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right) - (b''_{41})^{(8)}\left((G_{43}), t\right) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(v_1)^{(8)} > 0$, $(v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$, $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{v}_1)^{(8)}$,, $(\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{v}_1)^{(8)} > 0$, $(\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$, $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$:-

If we define $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$, $\boldsymbol{if}$ $(v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$, $\boldsymbol{if}$ $(v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$,
and $\boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}$, $\boldsymbol{if}$ $(\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, $\boldsymbol{if}$ $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, $\boldsymbol{if}$ $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}$ $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t})$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t}\leq T_{42}(t)\leq \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)},(S_2)^{(8)},(R_1)^{(8)},(R_2)^{(8)}$:- 381

Where $(S_1)^{(8)}=(a_{40})^{(8)}(m_2)^{(8)}-(a'_{40})^{(8)}$

$(S_2)^{(8)}=(a_{42})^{(8)}-(p_{42})^{(8)}$

$(R_1)^{(8)}=(b_{40})^{(8)}(\mu_2)^{(8)}-(b'_{40})^{(8)}$

$(R_2)^{(8)}=(b'_{42})^{(8)}-(r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$:

$(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)}\leq -(a'_{44})^{(9)}+(a'_{45})^{(9)}-(a''_{44})^{(9)}(T_{45},t)+(a''_{45})^{(9)}(T_{45},t)\leq -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)}\leq -(b'_{44})^{(9)}+(b'_{45})^{(9)}-(b''_{44})^{(9)}\big((G_{47}),t\big)-(b''_{45})^{(9)}\big((G_{47}),t\big)\leq -(\tau_1)^{(9)}$

Definition of $(v_1)^{(9)},(v_2)^{(9)},(u_1)^{(9)},(u_2)^{(9)},v^{(9)},u^{(9)}$:

By $(v_1)^{(9)}>0$, $(v_2)^{(9)}<0$ and respectively $(u_1)^{(9)}>0$, $(u_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_1)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)}>0$, $(\bar{v}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$, $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

Definition of $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$:-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)},\ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$
and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \le G_{46}(t) \le \right.$$
$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (v_1)^{(1)} \le (\bar{v}_1)^{(1)} \le \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$, we obtain 386

$$(v_1)^{(1)} \le v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $v^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\right) - (a''_{17})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{\mathrm{G}_{16}}{\mathrm{G}_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (\mathrm{C})^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (\mathrm{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)}\right) \le \frac{dv^{(3)}}{dt} \le -\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \ge \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}}$$

it follows $(v_0)^{(3)} \le v^{(3)}(t) \le (v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \le v^{(3)}(t) \le (\bar{v}_1)^{(3)}$

If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)} \le \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}} \le v^{(3)}(t) \le$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (\bar{v}_1)^{(3)}$$

If $0 < (v_1)^{(3)} \le (\bar{v}_1)^{(3)} \le (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$, we obtain 403

$$(v_1)^{(3)} \le v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

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$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(8)} and\ (b''_i)^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\,]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\,]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\,]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0$ 479

$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0$ 480

$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0$ 481

$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0$ 482

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(n) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first

equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T^*_{33} for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$$

Definition and uniqueness of T^*_{37} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T^*_{37} for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$$

Definition and uniqueness of T^*_{41} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{25} such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507
$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{29} such that $\varphi((G_{31})^*) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{33} such that $\varphi({G_{35}}^*) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{37} such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{41} such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right]+(b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{45} such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)} G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)} G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)} T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)} T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)} G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)} G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)} T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)} T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)} G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)} G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)} T_{21}^*}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T_{22}^* = \frac{(b_{22})^{(3)} T_{21}^*}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]} \quad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad (525)$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad (526)$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad (527)$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \quad (528)$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \quad (529)$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \quad (530)$$

ASYMPTOTIC STABILITY ANALYSIS (531)

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i \quad (532)$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad (533)$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad (534)$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad (535)$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \quad (536)$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \quad (537)$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}\mathrm{T}^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}\mathrm{T}^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}\left((G_{27})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^{*}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^{*}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^{*}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^{*}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^{*}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^{*}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^{*}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^{*}\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^{*} + \mathbb{G}_i \quad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

ASYMPTOTIC STABILITY ANALYSIS (586 A)

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\Big)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\Big]$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\Big)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\Big)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\Big]$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\Big)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\Big)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\Big]$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\Big)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\Big)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\Big)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\Big)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION FIFTEEN

Large-N Collective Fields And Holography

INTRODUCTION—VARIABLES USED

Large-N collective fields and holography Phys. Rev. D 68, 044011 – Published 12 August 2003 Sumit R. Das and Antal Jevicki

(1) Sumit R. Das and Antal Jevicki propose that the Euclidean bilocal collective field theory of (e) critical large-N vector models provides (eb) a complete definition of the proposed dual theory of higher spin fields in anti–de Sitter spaces.
(2) They show how this bilocal field can be decomposed (e&eb) into an infinite number of even spin

fields in one more dimension.

(3) The collective field has (e) a nontrivial classical solution which leads to (eb) an O (N) thermodynamic entropy characteristic of (e) the lower dimensional theory, as required by (e) general considerations of holography.

(4) A subtle cancellation of the entropy coming from (e) the bulk fields in one higher dimension with O (1) contributions from (e) the classical solution ensures that the subleading terms in thermodynamic quantities are (=) of the expected form.

(5) While the spin components of the collective field transform (e&eb) properly under dilatational, translational, and rotational isometries of AdS, special conformal transformations mix (e&eb) fields of different spins indicating (eb) a need for a nonlocal map between the two sets of fields.

(6) They discuss the nature of the propagating degrees of freedom through (e&eb) a Hamiltonian form of collective field theory and argue that nonsinglet states which are present in a Euclidean version are related to (e&eb) nontrivial backgrounds. DOI: http://dx.doi.org/10.1103/PhysRevD.68.044011

NOTATION

Module One

Renormalization group evolution of neutrino parameters in presence of seesaw threshold effects and Majorana phases Shivani Gupta, , Sin Kyu Kangb, , C.S. Kima doi:10.1016/j.nuclphysb.2015.01.026 Nuclear Physics B

The current values of the two mass squared differences and mixing angles including (e) θ13 can be produced in (eb) the MSSM case with tanβ=10 and non-zero Majorana CP phases at low energy.

They also calculate the order of (e) effective Majorana mass and Jarlskog Invariant for (e) each scenario under consideration

G_{13} : Category one of current values of the two mass squared differences and mixing angles including (e) θ13 can be produced

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of MSSM case with tanβ=10 and non-zero Majorana CP phases at low energy

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

They also calculate the order of

effective Majorana mass and Jarlskog Invariant for (e) each scenario under consideration

G_{16} : Category one of each scenario under consideration

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of effective Majorana mass and Jarlskog Invariant

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Sumit R. Das and Antal Jevicki propose that the

Euclidean bilocal collective field theory of (e) critical large-N vector models provides (eb) a complete definition of the proposed dual theory of higher spin fields in anti–de Sitter spaces

G_{20} : Category one of critical large-N vector models provides (eb) a complete definition of the proposed dual theory of higher spin fields in anti–de Sitter spaces

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of Euclidean bilocal collective field theory

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Euclidean bilocal collective field theory of critical large-N vector models provides (eb) a complete definition of the proposed dual theory of higher spin fields in anti–de Sitter spaces

G_{24} : Category one of Euclidean bilocal collective field theory of critical large-N vector models

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of complete definition of the proposed dual theory of higher spin fields in anti–de Sitter spaces

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

They show how this

bilocal field can be decomposed (e&eb) into an infinite number of even spin fields in one more dimension

G_{28} : Category one of bilocal field; infinite number of even spin fields in one more dimension

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of infinite number of even spin fields in one more dimension; bilocal field

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

collective field has (e) a nontrivial classical solution which leads to (eb) an O (N) thermodynamic entropy characteristic of (e) the lower dimensional theory, as required by (e) general considerations of holography

G_{32} : Category one of nontrivial classical solution which leads to (eb) an O (N) thermodynamic entropy characteristic of (e) the lower dimensional theory, as required by (e) general considerations of holography

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of collective field

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

collective field has a nontrivial classical solution which leads to (eb) an O (N) thermodynamic entropy characteristic of (e) the lower dimensional theory, as required by (e) general considerations of holography

G_{36} : Category one of collective field has a nontrivial classical solution

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of O (N) thermodynamic entropy characteristic of (e) the lower dimensional theory, as required by (e) general considerations of holography

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

collective field has a nontrivial classical solution which leads to an O (N) thermodynamic entropy characteristic of (e) the lower dimensional theory, as required by (e) general considerations of holography

G_{40} : Category one of lower dimensional theory, as required by general considerations of holography

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of collective field has a nontrivial classical solution which leads to an O (N) thermodynamic entropy characteristic

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

While the spin components of the collective field transform (e&eb) properly under dilatational, translational, and rotational isometries of AdS, special conformal transformations mix (e&eb) fields of different spins indicating (eb) a need for a nonlocal map between the two sets of fields

G_{44} : Category one of spin components of the collective field; dilatational, translational, and rotational isometries of AdS, special conformal transformations mix (e&eb) fields of different spins indicating (eb) a need for a nonlocal map between the two sets of fields

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of dilatational, translational, and rotational isometries of AdS, special conformal transformations mix (e&eb) fields of different spins indicating (eb) a need for a nonlocal map between the two sets of fields; spin components of the collective field

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION SIXTEEN

Perturbative Effects In Ads5 × S5 String Theory

INTRODUCTION—VARIABLES USED

Tom Banks and Michael B. Green JHEP05 (1998)002 doi:10.1088/1126-6708/1998/05/002 Non-perturbative effects in AdS5 × S5 string theory and d = 4 SUSY Yang-Mills

(1) Tom Banks1 and Michael B. Green1 show that five-dimensional anti de-Sitter space remains (eb) a solution to low-energy type IIB supergravity when (e) the leading higher-derivative corrections to the classical supergravity (which are non-perturbative in the string coupling) are included.
(2) Furthermore, at this order in the low energy expansion of the IIB theory the graviton two-point and (e&eb) three-point functions in AdS5 × S5 are shown (=) not to be renormalized and a precise expression is obtained for (e) the four-graviton and related S-matrix elements.
(3) By invoking Maldacena's conjectured connection between IIB superstring theory and (e&eb) supersymmetric Yang-Mills theory corresponding statements are obtained concerning correlation functions of (e) the energy-momentum tensor and related operators in (eb) the large-N Yang-Mills theory.
(4) This leads to (eb) interesting non-perturbative statements and insights into the r?le of instantons in the gauge theory.

NOTATION

Module One

spin components of the collective field transform properly under dilatational, translational, and rotational isometries of AdS, special conformal transformations mix (e&eb) fields of different spins indicating (eb) a

need for a nonlocal map between the two sets of fields

G_{13} : Category one of spin components of the collective field transform properly under dilatational, translational, and rotational isometries of AdS, special conformal transformations ; fields of different spins indicating (eb) a need for a nonlocal map between the two sets of fields

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of fields of different spins indicating (eb) a need for a nonlocal map between the two sets of fields ;spin components of the collective field transform properly under dilatational, translational, and rotational isometries of AdS, special conformal transformations

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

spin components of the collective field transform properly under dilatational, translational, and rotational isometries of AdS, special conformal transformations mix fields of different spins indicating (eb) a need for a nonlocal map between the two sets of fields

G_{16} : Category one of spin components of the collective field transform properly under dilatational, translational, and rotational isometries of AdS, special conformal transformations mix fields of different spins

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of need for a nonlocal map between the two sets of fields

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

They discuss the nature of the propagating degrees of freedom through (e&eb) a Hamiltonian form of collective field theory and argue that nonsinglet states which are present in a Euclidean version are related to (e&eb) nontrivial backgrounds

DOI: http://dx.doi.org/10.1103/PhysRevD.68.044011

G_{20} : Category one of nature of the propagating degrees of freedom; Hamiltonian form of collective field theory and argue that nonsinglet states which are present in a Euclidean version are related to (e&eb) nontrivial backgrounds

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of Hamiltonian form of collective field theory and argue that nonsinglet states which are present in a Euclidean version are related to (e&eb) nontrivial backgrounds; nature of the propagating degrees of freedom

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

nature of the propagating degrees of freedom through a Hamiltonian form of collective field theory and argue that nonsinglet states which are present in a Euclidean version are related to (e&eb) nontrivial backgrounds

DOI: http://dx.doi.org/10.1103/PhysRevD.68.044011

G_{24} : Category one of nature of the propagating degrees of freedom through a Hamiltonian form of collective field theory and argue that nonsinglet states which are present in a Euclidean version; nontrivial backgrounds

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of nontrivial backgrounds; nature of the propagating degrees of freedom through a Hamiltonian form of collective field theory and argue that nonsinglet states which are present in a Euclidean version

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Tom Banks1 and Michael B. Green1 show that

five-dimensional anti de-Sitter space remains (eb) a solution to low-energy type IIB supergravity when (e) the leading higher-derivative corrections to the classical supergravity (which are non-perturbative in the string coupling) are included

G_{28} : Category one of five-dimensional anti de-Sitter space

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of solution to low-energy type IIB supergravity when (e) the leading higher-derivative corrections to the classical supergravity (which are non-perturbative in the string coupling) are included

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

five-dimensional anti de-Sitter space remains a solution to low-energy type IIB supergravity when (e) the leading higher-derivative corrections to the classical supergravity (which are non-perturbative in the string coupling) are included

G_{32} : Category one of leading higher-derivative corrections to the classical supergravity (which are non-

perturbative in the string coupling) are included

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of five-dimensional anti de-Sitter space remains a solution to low-energy type IIB supergravity

T_{33} : Category two of SAS

T_{34} : Category three of SAS

;Module seven

Furthermore, the

graviton at this order in the low energy expansion of the IIB theory two-point and (e&eb) three-point functions in AdS5 × S5 are shown (=) not to be renormalized and a precise expression is obtained for (e) the four-graviton and related S-matrix elements

G_{36} : Category one of graviton at this order in the low energy expansion of the IIB theory two-point; three-point functions in AdS5 × S5 are shown (=) not to be renormalized and a precise expression is obtained for (e) the four-graviton and related S-matrix elements

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of three-point functions in AdS5 × S5 are shown (=) not to be renormalized and a precise expression is obtained for (e) the four-graviton and related S-matrix elements graviton at this order in the low energy expansion of the IIB theory two-point

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

graviton at this order in the low energy expansion of the IIB theory two-point and three-point functions in AdS5 × S5 are shown not to be renormalized and a precise expression is obtained for (e) the four-graviton and related S-matrix elements

G_{40} : Category one of graviton at this order in the low energy expansion of the IIB theory two-point and three-point functions in AdS5 × S5

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of unrenormalized and a precise expression is obtained for the four-graviton and related S-matrix elements

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

By invoking Maldacena's conjectured connection between IIB superstring theory and (e&eb) supersymmetric Yang-Mills theory corresponding statements are obtained concerning correlation functions of (e) the energy-momentum tensor and related operators in (eb) the large-N Yang-Mills theory

G_{44} : Category one of Maldacena's conjectured connection between IIB superstring theory and supersymmetric Yang-Mills theory

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of correlation functions of (e) the energy-momentum tensor and related operators in (eb) the large-N Yang-Mills theory

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION SEVENTEEN

Matrix Theory And Heterotic Strings

INTRODUCTION—VARIABLES USED

Nuclear Physics B Volume 505, Issues 1–2, 10 November 1997, Pages 84–108 Matrix theory and heterotic strings on tori Petr Hořava doi:10.1016/S0550-3213(97)00509-9

(1) Petr Hořava considers compactifications of the matrix model of M-theory on, and interprets them as (=) orbifolds of the supersymmetric U (N) Yang-Mills theory on (eb) R × Td+1.
(2) The orbifold group acts both on (e&eb) the gauge group and on the Td+1, reduces (e) the gauge group to (eb) O (N) over 1 + 1-dimensional fixed-point submanifolds, and breaks (e) half of the supersymmetry.
(3) He clarifies some puzzling aspects of the gauge anomaly cancellation in (eb) the presence of spacetime Wilson lines; in general, the Yang-Mills theory requires (e) certain Chern-Simons couplings to (e&eb) supergravity background fields.
(4) They discuss the possibility that D8-branes are present as (=) certain matrix configurations in (eb) the yang-Mills theory, and the fundamental fermions emerge as (eb) zero-modes.
(5) Finally, they point out that the correspondence between matrix theory and (e&eb) string theory suggests (eb) the existence of a multitude of (e) non-trivial RG fixed points and dualities in (eb) orbifold Yang-Mills theories with (e&eb) eight supercharges in various dimensions. Copyright © 1997 Published by Elsevier B.V.

NOTATION

Module One

By invoking Maldacena's conjectured connection between IIB superstring theory and (e&eb) supersymmetric Yang-Mills theory corresponding statements are obtained concerning correlation functions of (e) the energy-momentum tensor and related operators in (eb) the large-N Yang-Mills theory

G_{13} : Category one of IIB superstring theory; supersymmetric Yang-Mills theory

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of supersymmetric Yang-Mills theory; IIB superstring theory

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

By invoking Maldacena's conjectured connection between IIB superstring theory and supersymmetric Yang-Mills theory corresponding statements are obtained concerning correlation functions of (e) the energy-momentum tensor and related operators in (eb) the large-N Yang-Mills theory

G_{16} : Category one of energy-momentum tensor and related operators in (eb) the large-N Yang-Mills theory

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of connection between IIB superstring theory and supersymmetric Yang-Mills theory corresponding statements are obtained concerning correlation functions

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

energy-momentum tensor and related operators in (eb) the large-N Yang-Mills theory

G_{20} : Category one of energy-momentum tensor and related operators

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of large-N Yang-Mills theory

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Rotations of quantum states

Quantum rotation operator

The rotation operator about the unit vector n (defining the axis of rotation) through angle θ is

$$U[R(\theta,\hat{\mathbf{n}})] = \exp\left(-\frac{i\theta}{\hbar}\hat{\mathbf{n}}\cdot\hat{\mathbf{J}}\right)$$

where J = (Jx, Jy, Jz) are the rotation generators (also the angular momentum matrices):

$$J_x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad J_y = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix} \quad J_z = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

and let $\widehat{R} = \widehat{R}(\theta,\hat{\mathbf{n}})$ be a rotation matrix. Taylor expanding to first order in θ gives the infinitesimal rotation operator:

$$U[R(\theta,\hat{\mathbf{n}})] = 1 - \frac{i\theta}{\hbar}\hat{\mathbf{n}}\cdot\hat{\mathbf{J}}$$

An operator $\widehat{\Omega}$ is invariant under a unitary transformation U if:

$$\widehat{\Omega} = U^{\dagger}\widehat{\Omega}U$$

in this case for the rotation $\widehat{U}(R)$:

$$\widehat{\Omega} = U(R)^{\dagger}\widehat{\Omega}U(R) = \exp\left(\frac{i\theta}{\hbar}\hat{\mathbf{n}}\cdot\hat{\mathbf{J}}\right)\widehat{\Omega}\exp\left(-\frac{i\theta}{\hbar}\hat{\mathbf{n}}\cdot\hat{\mathbf{J}}\right)$$

Angular momentum eigenkets

The orthonormal basis set for total angular momentum is $|j,m\rangle$, where j is the total angular momentum quantum number and m is the magnetic angular momentum quantum number, which takes values −j, −j + 1, ..., j − 1, j. A general state

$$|\psi\rangle = \sum_m |j,m\rangle$$

in the space rotates to a new state $|j,m\rangle$ by:

$$|\bar{\psi}\rangle = U(R)|\psi\rangle$$

Using the completeness condition:

$$I = \sum_{m'} |j,m'\rangle\langle j,m'|$$

we have

$$|\bar{\psi}\rangle = IU(R)|\psi\rangle = \sum_{mm'} |j,m'\rangle\langle j,m'|U(R)|j,m\rangle$$

Introducing the Wigner D matrix elements:

$$D(R)^{(j)}_{m'm} = \langle j,m'|U(R)|j,m\rangle$$

gives the matrix multiplication:

$$|\bar{\psi}\rangle = \sum_{mm'} D^{(j)}_{m'm}|j,m\rangle \quad \Rightarrow \quad |\bar{\psi}\rangle = D^{(j)}|\psi\rangle$$

For one basis ket:

$$|\overline{j,m}\rangle = \sum_{m'} D(R)^{(j)}_{m'm}|j,m'\rangle$$

For the case of orbital angular momentum, the eigenstates $|l,m\rangle$ of the orbital angular momentum operator L and solutions of Laplace's equation on a 3d sphere are spherical harmonics:

$$Y^m_\ell(\theta,\varphi) = \langle\theta,\phi|\ell,m\rangle = \sqrt{\frac{(2\ell+1)}{4\pi}\frac{(\ell-m)!}{(\ell+m)!}}\,P^m_\ell(\cos\theta)\,e^{im\phi}$$

where Pℓm is an associated Legendre polynomial, ℓ is the orbital angular momentum quantum number, and m is the orbital magnetic quantum number which takes the values −ℓ, −ℓ + 1, ... ℓ − 1, ℓ The formalism of spherical harmonics have wide applications in applied mathematics, and are closely related to the formalism of spherical tensors, as shown below.

Spherical harmonics are functions of the polar and azimuthal angles, ϕ and θ respectively, which can be conveniently collected into a unit vector n(θ, ϕ) pointing in the direction of those angles, in the Cartesian basis it is:

$$\hat{\mathbf{n}}(\theta,\phi) = \cos\phi\sin\theta\mathbf{e}_x + \sin\phi\sin\theta\mathbf{e}_y + \cos\theta\mathbf{e}_z$$

So a spherical harmonic can also be written $Y^m_l = \langle\mathbf{n}|lm\rangle$. Spherical harmonic states $|m,l\rangle$ rotate according to the inverse rotation matrix U(R−1), while rotates by the initial rotation matrix $\hat{U}(R)$.

$$|\overline{\ell,m}\rangle = \sum_{m'} D^{(\ell)}_{m'm}U(R^{-1})|\ell,m'\rangle\,, \quad |\bar{\hat{\mathbf{n}}}\rangle = U(R)|\hat{\mathbf{n}}\rangle$$

Rotation of tensor operators

We define the Rotation of an operator by requiring that the expectation value of the original operator $\hat{\mathbf{A}}$ with respect to the initial state be equal to the expectation value of the rotated operator with respect to the rotated state,

$$\langle\psi'|\widehat{A'}|\psi'\rangle = \langle\psi|\widehat{A}|\psi\rangle$$

Now as,

$$|\psi\rangle \to |\psi'\rangle = U(R)|\psi\rangle\,, \quad \langle\psi| \to \langle\psi'| = \langle\psi|\widehat{U}^{\dagger}(R)$$

we have,

$$\langle\psi|U^{\dagger}(R)\widehat{A}'U(R)|\psi\rangle = \langle\psi|\widehat{A}|\psi\rangle$$

since, $|\psi\rangle$ is arbitrary,

$$U^{\dagger}(R)\widehat{A}'U(R) = \widehat{A}$$

Scalar operators

A scalar operator is invariant under rotations

$$U(R)^{\dagger}\widehat{S}U(R) = \widehat{S}$$

and we have a simple result, that the scalar operator commutes with the rotation generators:

$$\left[\widehat{S}, \widehat{\mathbf{J}}\right] = 0$$

Examples of scalar operators include

the energy operator:

$$\widehat{E}\psi = i\hbar\frac{\partial}{\partial t}\psi$$

potential energy V

$$\widehat{V}(\mathbf{r}, t)\psi = V(\mathbf{r}, t)\psi$$

kinetic energy T:

$$\widehat{T}(\mathbf{r}, t)\psi = -\frac{\hbar^2}{2m}\nabla^2(\mathbf{r}, t)$$

as well as the operator coupling term in spin-orbit coupling:

$$\widehat{\mathbf{L}}\cdot\widehat{\mathbf{S}} = \widehat{L}_x\widehat{S}_x + \widehat{L}_y\widehat{S}_y + \widehat{L}_z\widehat{S}_z\,.$$

G_{24} : Category one of LHS of the above equations

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of RHS of the above equations

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Petr Hořava considers

compactifications of the matrix model of M-theory on, and interprets them as (=) orbifolds of the supersymmetric U (N) Yang-Mills theory on (eb) R × Td+1

Calabi-Yau manifold Shing-Tung Yau (2009), Scholarpedia, 4(8):6524.doi:10.4249/scholarpedia.6524 revision #91093 [link to/cite this article] Curator: Shing-Tung Yau Prof. Shing-Tung Yau, Mathematics, Harvard University, and Cambridge, MA

(1) Calabi-Yau manifolds are compact, complex Kähler manifolds that have trivial first Chern classes (over R).
(2) In most cases, we assume that they have finite fundamental groups.
(3) By the conjecture of Calabi (1957) proved by Yau (1977; 1979), there exists on every Calabi-Yau manifold a Kähler metric with vanishing Ricci curvature.

The Ricci tensor of Calabi-Yau manifolds

(1) A complex manifold is a topological space covered by complex coordinate charts such that the transition between overlapping charts are holomorphic; Hermitian metric on a complex manifold is a smooth assignment of Hermitian inner product structures on the holomorphic tangent spaces of the manifold; a Hermitian metric is called a Kähler metric if near every point the Hermitian metric is approximated by a flat metric up to second order. In a holomorphic coordinate chart with coordinate variables (z1,⋯,zn) , a Hermitian metric has its associated Hermitian form

$\omega=-1---\sqrt{2}\sum g_{ij}^{-}dz_i\wedge dz_j^{-}$.

(2) The second order approximation property is that for each holomorphic co-ordinates so that the derivatives of the functions g_{ij}^{-} vanish at that point. It can be shown that this is equivalent to saying that the Hermitian form is closed. In such a case, we call the Hermitian form the Kähler form of the Kähler metric.
(3) Given any Kähler metric, one defines its full curvature tensor by certain expressions of covariant derivatives of the metric; the Ricci curvature is a partial contraction of the full curvature tensor. This gives a tensor which is of the same type as the Kähler form. In local coordinates,

$Ric=-1---\sqrt{}\sum R_{ij}^{-}dz_i\wedge dz_j^{-}$. We call a Kähler metric Ricci-flat if its Ricci tensor vanishes identically.

The Calabi conjecture

(4) According to a well-known theorem of Chern, the Ricci form divided by 2π is a (1,1)-form that represents the first Chern class of a compact complex manifold. Rooted in his attempt to find canonical Kähler metrics for a Kähler manifold, in 1954, E. Calabi (Calabi, 1957) proposed his

celebrated conjecture.

(5) Conjecture. To every closed (1,1)-form 12πC1(X) representing the first Chern class c1(X) of a Kähler manifold X , there is a unique Kähler metric in the same Kähler class whose Ricci tensor (form) is the closed (1,1)-form C1(X) .

(6) In case the complex manifold has vanishing first Chern class, the zero form represents the first Chern class of the manifold. The Calabi conjecture implies the existence of a unique Ricci-flat Kähler metric in every Kähler class.

(7) Early on, Calabi realized that his conjecture can be reduced to a complex Monge-Ampère equation.

Yau's theorem

By the late 1960s, many were doubtful of the Calabi conjecture. Some attempted to use a reduction theorem of Cheeger and Gromoll (1971) to construct counterexamples to the Conjecture. Using the reduction theorem and assuming the conjecture, Yau announced the following splitting theorem in his 1973 lecture at the Stanford geometry conference: Every compact Kähler manifold with vanishing Ricci curvature can be covered by a metric product of a torus and a simply connected manifold with a Ricci-flat Kähler metric. He then used this theorem to produce a "counterexample" to the conjecture. The "counterexample" was soon discovered to be flawed; Yau withdrew his Stanford lecture. (The flaw was due to the mistaken assumption that manifolds with numerically non-negative anti-canonical divisor admits a first Chern form which is pointwise non-negative.)

In 1976, Yau (Yau, 1977; Yau, 1979) proved the Calabi conjecture by solving the complex Monge-Ampère equation for a real valued function ϕ

det(gij¯+∂2ϕ∂zi∂z¯j)=efdet(gij¯),

where ef is any smooth function of average 1 and gij¯+∂i∂j¯ϕ is required to be positive definite. The solution ϕ of the above equation ensures that the new Kähler metric

ω+−1−−−√∂∂¯ϕ

can attain any Ricci (curvature) form in the class referred to in the Calabi conjecture.

G_{28} : Category one of compactifications of the matrix model of M-theory: LHS of above equations

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of orbifolds of the supersymmetric U (N) Yang-Mills theory on (eb) R × Td+1; RHS of above equations

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

orbifold group acts both on (e&eb) the gauge group and on the Td+1, reduces (e) the gauge group to (eb) O (N) over 1 + 1-dimensional fixed-point submanifolds, and breaks (e) half of the supersymmetry

G_{32} : Category one of orbifold group; gauge group and on the Td+1, reduces (e) the gauge group to (eb) O

(N) over 1 + 1-dimensional fixed-point submanifolds, and breaks (e) half of the supersymmetry

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of gauge group and on the Td+1, reduces (e) the gauge group to (eb) O (N) over 1 + 1-dimensional fixed-point submanifolds, and breaks (e) half of the supersymmetry ;orbifold group

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

orbifold group acts both on the gauge group and on the Td+1, reduces (e) the gauge group to (eb) O (N) over 1 + 1-dimensional fixed-point submanifolds, and breaks (e) half of the supersymmetry

G_{36} : Category one of gauge group to (eb) O (N) over 1 + 1-dimensional fixed-point submanifolds, and breaks (e) half of the supersymmetry

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of orbifold group acts both on the gauge group and on the Td+1

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

orbifold group acts both on the gauge group and on the Td+1, reduces the gauge group to (eb) O (N) over 1 + 1-dimensional fixed-point submanifolds, and breaks (e) half of the supersymmetry

G_{40} : Category one of orbifold group acts both on the gauge group and on the Td+1, reduces the gauge group

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of O (N) over 1 + 1-dimensional fixed-point submanifolds, and breaks (e) half of the supersymmetry

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

orbifold group acts both on the gauge group and on the Td+1, reduces the gauge group to O (N) over 1 + 1-

dimensional fixed-point submanifolds, and breaks (e) half of the supersymmetry

G_{44} : Category one of half of the supersymmetry

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of orbifold group acts both on the gauge group and on the Td+1, reduces the gauge group to O (N) over 1 + 1-dimensional fixed-point submanifolds

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}),t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}),t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}),t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}),t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}), t\right)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}), t\right)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\left((G_{39}), t\right)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}), t\right)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}), t\right)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}),t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}),t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

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$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14}$$

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$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15}$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

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$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t) \end{bmatrix} G_{16}$$

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$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & +(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) & +(a''_{21})^{(3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t) \end{bmatrix} G_{17}$$

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$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & +(a''_{18})^{(2)}(T_{17},t) & +(a''_{15})^{(1,1,)}(T_{14},t) & +(a''_{22})^{(3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{38})^{(7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9)}(T_{45},t) \end{bmatrix} G_{18}$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{13}'')^{(1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a_{40}'')^{(8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a_{44}'')^{(9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b_{16}')^{(2)} & -(b_{16}'')^{(2)}(G_{19},t) & -(b_{13}'')^{(1,1,)}(G,t) & -(b_{20}'')^{(3,3,3,)}(G_{23},t) \\ -(b_{24}'')^{(4,4,4,4,4)}(G_{27},t) & -(b_{28}'')^{(5,5,5,5,5)}(G_{31},t) & -(b_{32}'')^{(6,6,6,6,6)}(G_{35},t) \\ -(b_{36}'')^{(7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8)}(G_{43},t) & -(b_{44}'')^{(9,9)}(G_{47},t) \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b_{17}')^{(2)} & -(b_{17}'')^{(2)}(G_{19},t) & -(b_{14}'')^{(1,1,)}(G,t) & -(b_{21}'')^{(3,3,3,)}(G_{23},t) \\ -(b_{25}'')^{(4,4,4,4,4)}(G_{27},t) & -(b_{29}'')^{(5,5,5,5,5)}(G_{31},t) & -(b_{33}'')^{(6,6,6,6,6)}(G_{35},t) \\ -(b_{37}'')^{(7,7,7)}(G_{39},t) & -(b_{41}'')^{(8,8,8)}(G_{43},t) & -(b_{45}'')^{(9,9)}(G_{47},t) \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b_{18}')^{(2)} & -(b_{18}'')^{(2)}(G_{19},t) & -(b_{15}'')^{(1,1,)}(G,t) & -(b_{22}'')^{(3,3,3,)}(G_{23},t) \\ -(b_{26}'')^{(4,4,4,4,4)}(G_{27},t) & -(b_{30}'')^{(5,5,5,5,5)}(G_{31},t) & -(b_{34}'')^{(6,6,6,6,6)}(G_{35},t) \\ -(b_{38}'')^{(7,7,7)}(G_{39},t) & -(b_{42}'')^{(8,8,8)}(G_{43},t) & -(b_{46}'')^{(9,9)}(G_{47},t) \end{bmatrix} T_{18} \quad 66$$

where $-(b_{16}'')^{(2)}(G_{19},t)$, $-(b_{17}'')^{(2)}(G_{19},t)$, $-(b_{18}'')^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b_{13}'')^{(1,1,)}(G,t)$, $-(b_{14}'')^{(1,1,)}(G,t)$, $-(b_{15}'')^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b_{20}'')^{(3,3,3,)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b_{24}'')^{(4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b_{28}'')^{(5,5,5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b_{32}'')^{(6,6,6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b_{36}'')^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b_{40}'')^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b_{44}'')^{(9,9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{ccc} (a_{20}')^{(3)}\boxed{+(a_{20}'')^{(3)}(T_{21},t)} & \boxed{+(a_{16}'')^{(2,2,2)}(T_{17},t)} & \boxed{+(a_{13}'')^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{36}'')^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a_{44}'')^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{ccc} (a_{21}')^{(3)}\boxed{+(a_{21}'')^{(3)}(T_{21},t)} & \boxed{+(a_{17}'')^{(2,2,2)}(T_{17},t)} & \boxed{+(a_{14}'')^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{37}'')^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a_{45}'')^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{ccc} (a_{22}')^{(3)}\boxed{+(a_{22}'')^{(3)}(T_{21},t)} & \boxed{+(a_{18}'')^{(2,2,2)}(T_{17},t)} & \boxed{+(a_{15}'')^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{38}'')^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a_{46}'')^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{22} \quad (69)$$

$\boxed{+(a_{20}'')^{(3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{13}'')^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a_{14}'')^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{36}'')^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a_{37}'')^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a_{38}'')^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{40}'')^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{44}'')^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a_{46}'')^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[\begin{array}{ccc} (b_{20}')^{(3)}\boxed{-(b_{20}'')^{(3)}(G_{23},t)} & \boxed{-(b_{16}'')^{(2,2,2)}(G_{19},t)} & \boxed{-(b_{13}'')^{(1,1,1,)}(G,t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{36}'')^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b_{40}'')^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b_{44}'')^{(9,9,9)}(G_{47},t)} \end{array}\right] T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad (72)$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad (75)$$

$(a''_{24})^{(4)}(T_{25},t)$, $(a''_{25})^{(4)}(T_{25},t)$, $(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$ *are fourth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$ *are fifth detrition coefficients for category* 1, 2 *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$

are sixth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$

are seventh detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$

are eighth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt}=(a_{28})^{(5)}G_{29}-\left[\begin{array}{ccc}(a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}\end{array}\right]G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt}=(a_{29})^{(5)}G_{28}-\left[\begin{array}{ccc}(a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}\end{array}\right]G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt}=(a_{30})^{(5)}G_{29}-\left[\begin{array}{ccc}(a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}\end{array}\right]G_{30} \quad (81)$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt}=(b_{28})^{(5)}T_{29}-\left[\begin{array}{ccc}(b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}\end{array}\right]T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)} & +(a''_{32})^{(6)}(T_{33},t) & +(a''_{28})^{(5,5,5)}(T_{29},t) & +(a''_{24})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)} & +(a''_{33})^{(6)}(T_{33},t) & +(a''_{29})^{(5,5,5)}(T_{29},t) & +(a''_{25})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)} & +(a''_{34})^{(6)}(T_{33},t) & +(a''_{30})^{(5,5,5)}(T_{29},t) & +(a''_{26})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{34} \quad (87)$$

$+(a''_{32})^{(6)}(T_{33},t)$, $+(a''_{33})^{(6)}(T_{33},t)$, $+(a''_{34})^{(6)}(T_{33},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5)}(T_{29},t)$ *are second augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{24})^{(4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,)}(T_{25},t)$ *are third augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)$ - are fourth augmentation coefficients

$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)$ - fifth augmentation coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)$ Eighth augmentation coefficients

$+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)} & -(b''_{32})^{(6)}(G_{35},t) & -(b''_{28})^{(5,5,5)}(G_{31},t) & -(b''_{24})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{13})^{(1,1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)} & -(b''_{33})^{(6)}(G_{35},t) & -(b''_{29})^{(5,5,5)}(G_{31},t) & -(b''_{25})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{14})^{(1,1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & -(b''_{34})^{(6)}(G_{35},t) & -(b''_{30})^{(5,5,5)}(G_{31},t) & -(b''_{26})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{15})^{(1,1,1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{34} \quad (90)$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,)}(G_{27},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition

coefficients for category 1, 2, and 3

$-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3

$-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2, and 3

$-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)} & +(a''_{36})^{(7)}(T_{37},t) & +(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)} & +(a''_{37})^{(7)}(T_{37},t) & +(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)} & +(a''_{38})^{(7)}(T_{37},t) & +(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15}$$

Where $(a''_{36})^{(7)}(T_{37},t)$, $(a''_{37})^{(7)}(T_{37},t)$, $(a''_{38})^{(7)}(T_{37},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation

coefficient for 1,2,3

$\frac{dT_{36}}{dt} =$ 94

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$\frac{dT_{37}}{dt} =$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$\frac{dT_{38}}{dt} =$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{40}}{dt} =$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt}$$

$$= (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt}$$
$$= (a_{45})^{(9)} G_{44}$$
$$- \left[\begin{array}{ccc} (a'_{45})^{(9)} & \boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} \quad \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array} \right] G_{14}$$

$$\frac{dG_{46}}{dt}$$
$$= (a_{46})^{(9)} G_{45}$$
$$- \left[\begin{array}{ccc} (a'_{46})^{(9)} & \boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} \quad \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array} \right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$
$$(b_{44})^{(9)} T_{45} -$$
$$\left[\begin{array}{ccc} (b'_{44})^{(9)} & \boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} \quad \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{array} \right] T_{13}$$
$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b''_i)^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}', t) - (a_i'')^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}', t)$ and $(a_i'')^{(1)}(T_{14}, t)$. (T_{14}', t) and (T_{14}, t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}\big((G_{19}), t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, \ (\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq \ (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$\lim_{G\to\infty}(b_i'')^{(3)}\,(G_{23},t) = \ (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$

$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)},$ are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$

$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$

$(b_i'')^{(4)}\big((G_{27}),t\big) \le \ (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$

$lim_{T_2\to\infty}(a_i'')^{(4)}\,(T_{25},t) = (p_i)^{(4)}$ 118

$\lim_{G\to\infty}(b_i'')^{(4)}\,\big((G_{27}),t\big) = \ (r_i)^{(4)}$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}\,(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}\,(G_{31}, t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\,(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ \ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}\,(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}\,\big((G_{35}), t\big) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\big((G_{35}), t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\,(a_i)^{(6)} + (a_i')^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)}\,(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(GGGG) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(HHHH)The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq \ (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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(IIII) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(JJJJ)

$\lim_{G\to\infty}(b_i'')^{(7)}\left((G_{39}),t\right) = \ (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, \ (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \leq (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}\left((G_{39}),t\right)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$. (T_{37}',t) and (T_{37},t) are points belonging to the interval $\left[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\right]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(KKKK) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(LLLL) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, \ (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\,(a_i)^{(7)} + (a_i')^{(7)} + \ (\hat{A}_{36})^{(7)} + \ (\hat{P}_{36})^{(7)}\,(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\ (b_i)^{(7)} + (b_i')^{(7)} + \ (\hat{B}_{36})^{(7)} + \ (\hat{Q}_{36})^{(7)}\ \ (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}, \ (r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41},t) \le (p_i)^{(8)} \le (\hat{A}_{40})^{(8)} \quad 138$$

$$(b_i'')^{(8)}((G_{43}),t) \le (r_i)^{(8)} \le (b_i')^{(8)} \le (\hat{B}_{40})^{(8)} \quad 139$$

$$\lim_{T_2\to\infty}(a_i'')^{(8)}(T_{41},t) = (p_i)^{(8)} \quad 140$$

$$\lim_{G\to\infty}(b_i'')^{(8)}((G_{43}),t) = (r_i)^{(8)} \quad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}',t) - (a_i'')^{(8)}(T_{41},t)| \le (\hat{k}_{40})^{(8)}|T_{41} - T_{41}'|e^{-(\hat{M}_{40})^{(8)}t} \quad 142$$

$$|(b_i'')^{(8)}((G_{43})',t) - (b_i'')^{(8)}((G_{43}),t)| < (\hat{k}_{40})^{(8)}||(G_{43}) - (G_{43})'||e^{-(\hat{M}_{40})^{(8)}t} \quad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}',t)$ and $(a_i'')^{(8)}(T_{41},t)$. (T_{41}',t) and (T_{41},t) are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \quad 144$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad 146\,A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t) \le (p_i)^{(9)} \le (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t) \le (r_i)^{(9)} \le (b_i')^{(9)} \le (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty} (a_i'')^{(9)}(T_{45},t) = (p_i)^{(9)}$$

$$\lim_{G\to\infty} (b_i'')^{(9)}(G_{47},t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t) - (a_i'')^{(9)}(T_{45},t)| \le (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t) - (b_i'')^{(9)}\big((G_{47}),t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$ and$(a_i'')^{(9)}(T_{45},t)$. (T_{45}',t) and (T_{45},t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}$, $(r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\,, T_i(0)$

$$G_i(t) \le \left(\hat{P}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t} \;,\quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le \left(\hat{Q}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t} \;,\quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le \left(\hat{P}_{20}\right)^{(3)} e^{(\hat{M}_{20})^{(3)}t} \;,\quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le \left(\hat{Q}_{20}\right)^{(3)} e^{(\hat{M}_{20})^{(3)}t} \;,\quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq \left(\hat{P}_{13}\right)^{(1)}$, $T_i^0 \leq \left(\hat{Q}_{13}\right)^{(1)}$, 155

$0 \leq G_i(t) - G_i^0 \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 156

$0 \leq T_i(t) - T_i^0 \leq \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0,t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''^{(2)}_{16}\right)\left(T_{17}(s_{(16)}), s_{(16)}\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}\left(T_{17}(s_{(16)}), s_{(17)}\right)\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}\left(T_{17}(s_{(16)}), s_{(16)}\right)\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''^{(3)}_{20}\right)\left(T_{21}(s_{(20)}), s_{(20)}\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}\left(T_{21}(s_{(20)}), s_{(20)}\right)\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}\left(T_{21}(s_{(20)}), s_{(20)}\right)\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)},\ T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^{0} + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^{0} + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^{0} + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^{0} + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^{0} + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^{0} + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)},\ T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^{0} + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that 174

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)} =$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(o) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \leq G_{36}^{0}+\int_{0}^{t}\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)} =$$
$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \leq G_{40}^{0}+\int_{0}^{t}\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)} = \quad 180$$
$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t} \leq \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for G_{41} , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \leq G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \leq \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for G_{45} , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \left\{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\right\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+\ (a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\ (t)=0\ and\ T_i\ (t)=0$

From 19 to 24 it results

$$G_i\ (t)\geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\geq 0$$

$$T_i\ (t)\geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13}<(\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt}\leq\left((\widehat{M}_{13})^{(1)}\right)_1-(a'_{14})^{(1)}G_{14}$ and by integrating

$$G_{14}\leq\left((\widehat{M}_{13})^{(1)}\right)_2=G_{14}^0+2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15}\leq\left((\widehat{M}_{13})^{(1)}\right)_3=G_{15}^0+2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a'_{15})^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}\left|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t} \leq$
$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)d\left(\left((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)}\right)\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\}ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0$ for $t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)}G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$: $\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$G_{20}^{(2)} \left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ *and* $(\widehat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a'_{22})^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$G_{24}^{(2)} \left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)}\right) d\left(((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on T_{25} and respectively on (G_{27})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (228)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_2 / (a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)}, (T_{35})^{(1)}), ((G_{35})^{(2)}, (T_{35})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})}, \widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})}, \widetilde{(T_{35})}\right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right)\left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)};\ (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ *and* $(\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ (250)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$$
$$sup_i\{max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}, max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)} \left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)} (T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}),(\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}),(\widetilde{T_{43}})\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1,\left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right),\quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$

From 99 to 44 it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(9)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)},\ if\ (v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}, if\ (v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)},$

and $\boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)},\ if\ (\bar{v}_1)^{(1)} < (v_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)},, (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)}, , (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)} t} - e^{-(a'_{26})^{(4)} t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)} t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G^0_{28} e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G^0_{28}}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G^0_{30} e^{-(S_2)^{(5)} t} \le G_{30}(t) \le \right. \quad 344$$

$$\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_2)^{(5)}((S_1)^{(5)}-(a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t}-e^{-(a'_{30})^{(5)}t}\right]+\ G_{30}^{0}e^{-(a'_{30})^{(5)}t}\Big)$$

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\le T_{30}(t)\le \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$:

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$-(\sigma_2)^{(6)}\le-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\le-(\sigma_1)^{(6)}$

$-(\tau_2)^{(6)}\le-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\le-(\tau_1)^{(6)}$

Definition of $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$: 350

By $(v_1)^{(6)}>0$, $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$, $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

Definition of $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)}>0$, $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$, $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

Definition of $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$:-

If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)}=(v_0)^{(6)},(m_1)^{(6)}=(v_1)^{(6)},$ ***if*** $(v_0)^{(6)}<(v_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}$ $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}$ $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}$ $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}$ $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}$ $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})} \left[e^{(S_1)^{(6)} t} - e^{-(a'_{34})^{(6)} t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)} t} \right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)},, (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^{0}e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}}G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}}G_{36}^{0}e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t}-e^{-(S_2)^{(7)}t}\right]+G_{38}^{0}e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left.\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_2)^{(7)}((S_1)^{(7)}-(a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t}-e^{-(a'_{38})^{(7)}t}\right]+\ G_{38}^{0}e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}}T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}}T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right) - (b''_{41})^{(8)}\left((G_{43}), t\right) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(v_1)^{(8)} > 0$, $(v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$, $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{v}_1)^{(8)}$,, $(\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{v}_1)^{(8)} > 0$, $(\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$, $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$:-

If we define $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$, $\boldsymbol{if}$ $(v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$, $\boldsymbol{if}$ $(v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$,
and $\boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}$, $\boldsymbol{if}$ $(\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, $\boldsymbol{if}$ $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, $\boldsymbol{if}$ $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}$ $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t})$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \qquad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \qquad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t}\leq T_{42}(t)\leq \qquad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)},(S_2)^{(8)},(R_1)^{(8)},(R_2)^{(8)}$:- 381

Where $(S_1)^{(8)}=(a_{40})^{(8)}(m_2)^{(8)}-(a'_{40})^{(8)}$

$(S_2)^{(8)}=(a_{42})^{(8)}-(p_{42})^{(8)}$

$(R_1)^{(8)}=(b_{40})^{(8)}(\mu_2)^{(8)}-(b'_{40})^{(8)}$

$(R_2)^{(8)}=(b'_{42})^{(8)}-(r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$:

$(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)}\leq -(a'_{44})^{(9)}+(a'_{45})^{(9)}-(a''_{44})^{(9)}(T_{45},t)+(a''_{45})^{(9)}(T_{45},t)\leq -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)}\leq -(b'_{44})^{(9)}+(b'_{45})^{(9)}-(b''_{44})^{(9)}\big((G_{47}),t\big)-(b''_{45})^{(9)}\big((G_{47}),t\big)\leq -(\tau_1)^{(9)}$

Definition of $(v_1)^{(9)},(v_2)^{(9)},(u_1)^{(9)},(u_2)^{(9)},v^{(9)},u^{(9)}$:

By $(v_1)^{(9)}>0$, $(v_2)^{(9)}<0$ and respectively $(u_1)^{(9)}>0$, $(u_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_1)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)}>0$, $(\bar{v}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$, $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

Definition of $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$:-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)},\ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$
and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \le G_{46}(t) \le \right.$$
$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (v_1)^{(1)} \le (\bar{v}_1)^{(1)} \le \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$, we obtain 386

$$(v_1)^{(1)} \le v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $v^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\right) - (a''_{17})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{\mathrm{G}_{16}}{\mathrm{G}_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (\mathrm{C})^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (\mathrm{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)}\right) \le \frac{dv^{(3)}}{dt} \le -\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \ge \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}}$$

it follows $(v_0)^{(3)} \le v^{(3)}(t) \le (v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \le v^{(3)}(t) \le (\bar{v}_1)^{(3)}$

If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)} \le \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}} \le v^{(3)}(t) \le$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (\bar{v}_1)^{(3)}$$

If $0 < (v_1)^{(3)} \le (\bar{v}_1)^{(3)} \le (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$, we obtain 403

$$(v_1)^{(3)} \le v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

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$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{36}'')^{(7)} = (a_{37}'')^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b_{36}'')^{(7)} = (b_{37}'')^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a_{40}')^{(8)} - (a_{41}')^{(8)} + (a_{40}'')^{(8)}(T_{41}, t)\right) - (a_{41}'')^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(8)} and\ (b''_i)^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\,]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\,]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\,]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0$ 479

$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0$ 480

$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0$ 481

$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0$ 482

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(o) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first

equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T^*_{33} for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$$

Definition and uniqueness of T^*_{37} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T^*_{37} for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$$

Definition and uniqueness of T^*_{41} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{25} such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507
$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{29} such that $\varphi((G_{31})^*) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{33} such that $\varphi({G_{35}}^*) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{37} such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{41} such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right]+(b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{45} such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)} G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)} G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \qquad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \qquad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \qquad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \qquad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \qquad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \qquad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \qquad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i \qquad 532$$

$$\frac{\partial (a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial (b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \qquad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \qquad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \qquad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \qquad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \qquad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}\mathrm{T}^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}\mathrm{T}^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}\left((G_{27})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)}$, $\frac{\partial(b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\ B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\Big)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\Big]$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\Big)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\Big)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\Big]$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\Big)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\Big)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\Big]$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\Big)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\Big)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\Big)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\Big)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION EIGHTEEN

Holographic Matrix Models And Quantum Space And Time

INTRODUCTION—VARIABLES USED

Source: Wikipedia (for references please see Wikipedia)

Being

On the first page of Being and Time, Heidegger describes the project in the following way: "our aim in the following treatise is to work out the question of the sense of being and to do so concretely."[2] Heidegger claims that traditional ontology has prejudicially overlooked this question, dismissing it as overly general,

undefinable, or obvious.[3]

Instead Heidegger proposes to understand being itself, as distinguished from any specific entities (beings).[4] "'Being' is not something like a being."[5] Being, Heidegger claims, is "what determines beings as beings, that in terms of which beings are already understood."[6] Heidegger is seeking to identify the criteria or conditions by which any specific entity can show up at all (see world disclosure).[7]

If we grasp Being, we will clarify the meaning of being, or "sense" of being ("Sinn des Seins"), where by "sense" Heidegger means that "in terms of which something becomes intelligible as something."[8] According to Heidegger, as this sense of being precedes any notions of how or in what manner any particular being or beings exist, it is pre-conceptual, non-propositional, and hence pre-scientific.[9] Thus, in Heidegger's view, fundamental ontology would be an explanation of the understanding preceding any other way of knowing, such as the use of logic, theory, specific ontology[10] or act of reflective thought. At the same time, there is no access to being other than via beings themselves—hence pursuing the question of being inevitably means asking about a being with regard to its being.[11] Heidegger argues that a true understanding of being (Seinsverständnis) can only proceed by referring to particular beings, and that the best method of pursuing being must inevitably, he says, involve a kind of hermeneutic circle, that is (as he explains in his critique of prior work in the field of hermeneutics), it must rely upon repetitive yet progressive acts of interpretation. "The methodological sense of phenomenological description is interpretation."[12]

Dasein

Thus the question Heidegger asks in the introduction to Being and Time is: what is the being that will give access to the question of the meaning of Being? Heidegger's answer is that it can only be that being for whom the question of Being is important, the being for whom Being matters.[10] As this answer already indicates, the being for whom Being is a question is not a what, but a who. Heidegger calls this being Dasein (an ordinary German word literally meaning "being-there" i.e. existence), and the method pursued in Being and Time consists in the attempt to delimit the characteristics of Dasein, in order thereby to approach the meaning of Being itself through an interpretation of the temporality of Dasein. Dasein is not "man," but is nothing other than "man"—it is this distinction that enables Heidegger to claim that Being and Time is something other than philosophical anthropology.

Heidegger's account of Dasein passes through a dissection of the experiences of Angst and mortality, and then through an analysis of the structure of "care" as such. From there he raises the problem of "authenticity," that is, the potentiality or otherwise for mortal Dasein to exist fully enough that it might actually understand being. Heidegger is clear throughout the book that nothing makes certain that Dasein is capable of this understanding.

Time

Finally, this question of the authenticity of individual Dasein cannot be separated from the "historicality" of Dasein. On the one hand, Dasein, as mortal, is "stretched along" between birth and death, and thrown into its world, that is, thrown into its possibilities, possibilities which Dasein is charged with the task of assuming. On the other hand, Dasein's access to this world and these possibilities is always via a history and a tradition—this is the question of "world historicality," and among its consequences is Heidegger's argument that Dasein's potential for authenticity lies in the possibility of choosing a "hero."

Thus, more generally, the outcome of the progression of Heidegger's argument is the thought that the being of Dasein is time. Nevertheless, Heidegger concludes his work with a set of enigmatic questions

foreshadowing the necessity of a destruction (that is, a transformation) of the history of philosophy in relation to temporality—these were the questions to be taken up in the never completed continuation of his project:

The existential and ontological constitution of the totality of Dasein is grounded in temporality. Accordingly, a primordial mode of temporalizing of ecstatic temporality itself must make the ecstatic project of being in general possible. How is this mode of temporalizing of temporality to be interpreted? Is there a way leading from primordial time to the meaning of being? Does time itself reveal itself as the horizon of being?[13]

Phenomenology in Heidegger and Husserl

Although Heidegger describes his method in Being and Time as phenomenological, the question of its relation to the phenomenology of Husserl is complex. The fact that Heidegger believes that ontology includes an irreducible hermeneutic (interpretative) aspect, for example, might be thought to run counter to Husserl's claim that phenomenological description is capable of a form of scientific positivity. On the other hand, however, several aspects of the approach and method of Being and Time seem to relate more directly to Husserl's work.

The central Husserlian concept of the directedness of all thought—intentionality—for example, while scarcely mentioned in Being and Time, has been identified by some with Heidegger's central notion of "Sorge" (Cura, care or concern). However, for Heidegger, theoretical knowledge represents only one kind of intentional behaviour, and he asserts that it is grounded in more fundamental modes of behaviour and forms of practical engagement with the surrounding world. Whereas a theoretical understanding of things grasps them according to "presence," for example, this may conceal that our first experience of a being may be in terms of its being "ready-to-hand." Thus, for instance, when someone reaches for a tool such as a hammer, their understanding of what a hammer is is not determined by a theoretical understanding of its presence, but by the fact that it is something we need at the moment we wish to do hammering. Only a later understanding might come to contemplate a hammer as an object.

Hermeneutics

The total understanding of being results from an explication of the implicit knowledge of being that inheres in Dasein. Philosophy thus becomes a form of interpretation, but since there is no external reference point outside being from which to begin this interpretation, the question becomes to know in which way to proceed with this interpretation. This is the problem of the "hermeneutic circle," and the necessity for the interpretation of the meaning of being to proceed in stages: this is why Heidegger's technique in Being and Time is sometimes referred to as hermeneutical phenomenology.This interpretative aspect of Heidegger's project had a profound influence on the hermeneutic approach of his student Hans-Georg Gadamer.

Destruction of metaphysics

As part of his ontological project, Heidegger undertakes a reinterpretation of previous Western philosophy. He wants to explain why and how theoretical knowledge came to seem like the most fundamental relation to being. This explanation takes the form of a destructuring (Destruktion) of the philosophical tradition, an interpretative strategy that reveals the fundamental experience of being at the base of previous philosophies that had become entrenched and hidden within the theoretical attitude of the metaphysics of presence. This Destruktion is not simply a negative operation but rather a positive transformation, or recovery.

In Being and Time Heidegger briefly undertakes a destructuring of the philosophy of René Descartes, but the second volume, which was intended to be a Destruktion of Western philosophy in all its stages, was never written. In later works Heidegger uses this approach to interpret the philosophies of Aristotle, Kant,

Georg Wilhelm Friedrich Hegel, and Plato, among others. This aspect of Heidegger's work exerted a profound influence on Jacques Derrida, although there are also important differences between Heidegger's Destruktion and Derrida's deconstruction.

Holographic Matrix Models and Quantum Space and Time The world as a hologram Leonard Susskind1 J. Math Phys. 36, 6377 (1995); http://dx.doi.org/10.1063/1.531249

(1) According to't Hooft the combination of quantum mechanics and (e&eb) gravity requires the three-dimensional world to be (=) an image of data that can be stored on (eb) a two-dimensional projection much like a holographic image.
(2) The two-dimensional description only requires (e) one discrete degree of freedom per Planck area and yet it is rich enough to describe all three-dimensional phenomena.
(3) After outlining't Hooft's proposal authors give a preliminary informal description of how it may be implemented. One finds a basic requirement that particles must grow in size as (e&eb) their momenta are increased far above the Planck scale.
(4) The consequences for high-energy particle collisions are described. The phenomenon of particle growth with momentum was previously discussed in the context of string theory and was related to (e&eb) information spreading near black hole horizons.
(5) The considerations of this paper indicate that the effect is much more rapid at all but the earliest times. In fact the rate of spreading is found to saturate (e&eb) the bound from causality.
(6) Finally authors consider string theory as (=) a possible realization of't Hooft's idea.
(7) The light front lattice string model of Klebanov and Susskind is reviewed and its similarities with (e&eb) the holographic theory are demonstrated. The agreement between the two requires unproven but plausible assumptions about the nonperturbative behavior of (e) string theory. Very similar ideas to those in this paper have long been held by Charles Thorn. © 1995 American Institute of Physics

NOTATION

Module One

The Kaluza Hypothesis

In his 1921 paper,[2] Kaluza established all the elements of the classical 5-dimensional theory: the metric, the field equations, the equations of motion, the stress-energy tensor, and the cylinder condition. The theory has no free parameters; it merely extends general relativity to five dimensions. One starts by hypothesizing a form of the 5-dimensional metric $\widetilde{g}_{ab}$, where roman indices span 5 dimensions. Let us also introduce the 4-dimensional spacetime metric $g_{\mu\nu}$, where Greek indices span the usual 4 dimensions of space and time; a 4-vector A^{μ} which will be identified with the electromagnetic vector potential; and a scalar field ϕ. Then decompose the 5D metric so that the 4D metric is framed by the electromagnetic vector potential, with the scalar field at the fifth diagonal. This can be visualized as:

$$\widetilde{g}_{ab} \equiv \begin{bmatrix} g_{\mu\nu} + \phi^2 A_\mu A_\nu & \phi^2 A_\mu \\ \phi^2 A_\nu & \phi^2 \end{bmatrix}.$$

More precisely, we can write

$$\widetilde{g}_{\mu\nu} \equiv g_{\mu\nu} + \phi^2 A_\mu A_\nu, \qquad \widetilde{g}_{5\nu} \equiv \widetilde{g}_{\nu 5} \equiv \phi^2 A_\nu, \qquad \widetilde{g}_{55} \equiv \phi^2$$

where the index 5 indicates the fifth coordinate by convention even though the first four coordinates are

indexed with 0, 1, 2, and 3. The associated inverse metric is

$$\widetilde{g}^{ab} \equiv \begin{bmatrix} g^{\mu\nu} & -A^{\mu} \\ -A^{\nu} & g_{\alpha\beta}A^{\alpha}A^{\beta} + \frac{1}{\phi^2} \end{bmatrix}.$$

So far, this decomposition is quite general and all terms are dimensionless. Kaluza then applies the machinery of standard general relativity to this metric. The field equations are obtained from 5-dimensional Einstein equations, and the equations of motion are obtained from the 5-dimensional geodesic hypothesis. The resulting field equations provide both the equations of general relativity and of electrodynamics; the equations of motion provide the 4-dimensional geodesic equation and theLorentz force law. And one finds that electric charge is identified with motion in the fifth dimension.

The hypothesis for the metric implies an invariant 5-dimensional length element ds:

$$ds^2 \equiv \widetilde{g}_{ab}dx^a dx^b = g_{\mu\nu}dx^{\mu}dx^{\nu} + \phi^2(A_{\nu}dx^{\nu} + dx^5)^2$$

Field Equations from the Kaluza Hypothesis

The field equations of the 5-dimensional theory were never adequately provided by Kaluza or Klein, mainly regarding the scalar field. The full Kaluza field equations are generally attributed to Thiry,[7] who most famously obtained vacuum field equations, although Kaluza [2] originally provided a stress-energy tensor for his theory and Thiry included a stress-energy tensor in his thesis. But as described by Gonner,[5] several independent groups worked on the field equations in the 1940s and earlier. Thiry is perhaps best known only because an English translation was provided by Applequist, Chodos, & Freund in their review book.[19] Applequist et al. also provided an English translation of Kaluza's paper. There are no English translations of the Jordan papers.[9][10][12]

To obtain the 5D field equations, the 5D connections $\widetilde{\Gamma}^{a}_{bc}$ are calculated from the 5D metric $\widetilde{g}_{ab}$, and the 5D Ricci tensor $\widetilde{R}_{ab}$ is calculated from the 5D connections.

The classic results of Thiry and other authors presume the cylinder condition:

$$\frac{\partial \widetilde{g}_{ab}}{\partial x^5} = 0.$$

Without this assumption, the field equations become much more complex, providing many more degrees of freedom that can be identified with various new fields. Paul Wesson and colleagues have pursued relaxation of the cylinder condition to gain extra terms that can be identified with the matter fields,[20] for which Kaluza [2]otherwise inserted a stress-energy tensor by hand.

It has been an objection to the original Kaluza hypothesis to invoke the fifth dimension only to negate its dynamics. But Thiry argued [5] that the interpretation of the Lorentz force law in terms of a 5-dimensional geodesic mitigates strongly for a fifth dimension irrespective of the cylinder condition. Most authors have therefore employed the cylinder condition in deriving the field equations. Furthermore, vacuum equations are typically assumed for which

$$\widetilde{R}_{ab} = 0$$

where

$$\tilde{R}_{ab} \equiv \partial_c \tilde{\Gamma}^c_{ab} - \partial_b \tilde{\Gamma}^c_{ca} + \tilde{\Gamma}^c_{cd}\tilde{\Gamma}^d_{ab} - \tilde{\Gamma}^c_{bd}\tilde{\Gamma}^d_{ac}$$

and

$$\tilde{\Gamma}^a_{bc} \equiv \frac{1}{2}\tilde{g}^{ad}(\partial_b \tilde{g}_{dc} + \partial_c \tilde{g}_{db} - \partial_d \tilde{g}_{bc})$$

The vacuum field equations obtained in this way by Thiry [7] and Jordan's group [9][10][12] are as follows.

The field equation for ϕ is obtained from

$$\tilde{R}_{55} = 0 \Rightarrow \Box\phi = \frac{1}{4}\phi^3 F^{\alpha\beta}F_{\alpha\beta}$$

where $F_{\alpha\beta} \equiv \partial_\alpha A_\beta - \partial_\beta A_\alpha$, where $\Box \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu$, and where ∇_μ is a standard, 4D covariant derivative. It shows that the electromagnetic field is a source for the scalar field. Note that the scalar field cannot be set to a constant without constraining the electromagnetic field. The earlier treatments by Kaluza and Klein did not have an adequate description of the scalar field, and did not realize the implied constraint on the electromagnetic field by assuming the scalar field to be constant.

The field equation for A^ν is obtained from

$$\tilde{R}_{5\alpha} = 0 = \frac{1}{2}g^{\beta\mu}\nabla_\mu(\phi^3 F_{\alpha\beta})$$

It has the form of the vacuum Maxwell equations if the scalar field is constant.

The field equation for the 4D Ricci tensor $R_{\mu\nu}$ is obtained from

$$\tilde{R}_{\mu\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{R} = 0 \Rightarrow R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{2}\phi^2\left(g^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta} - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right) + \frac{1}{\phi}\left(\nabla_\mu\nabla_\nu\phi - g_{\mu\nu}\Box\phi\right)$$

where R is the standard 4D Ricci scalar.

This equation shows the remarkable result, called the "Kaluza miracle", that the precise form for the electromagnetic stress-energy tensor emerges from the 5D vacuum equations as a source in the 4D equations: field from the vacuum. This relation allows the definitive identification of A^μ with the electromagnetic vector potential. Therefore the field needs to be rescaled with a conversion constant k such that $A^\mu \rightarrow kA^\mu$.

The relation above shows that we must have

$$\frac{k^2}{2} = \frac{8\pi G}{c^4}\frac{1}{\mu_0}$$

where G is the gravitational constant and μ_0 is the permeability of free space. In the Kaluza theory, the gravitational constant can be understood as an electromagnetic coupling constant in the metric. There is also a stress-energy tensor for the scalar field. The scalar field behaves like a variable gravitational constant, in terms of modulating the coupling of electromagnetic stress energy to spacetime curvature. The sign of ϕ^2 in the metric is fixed by correspondence with 4D theory so that electromagnetic energy densities are positive. This turns out to imply that the 5th coordinate is spacelike in its signature in the metric.

In the presence of matter, the 5D vacuum condition can not be assumed. Indeed, Kaluza did not assume it. The full field equations require evaluation of the 5D Einstein tensor

$$\tilde{G}_{ab} \equiv \tilde{R}_{ab} - \frac{1}{2}\tilde{g}_{ab}\tilde{R}$$

as seen in the recovery of the electromagnetic stress-energy tensor above. The 5D curvature tensors are complex, and most English-language reviews contain errors in either $\tilde{G}_{ab}$ or $\tilde{R}_{ab}$, as does the English translation of.[7] See [18] for a complete set of 5D curvature tensors under the cylinder condition, evaluated using tensor algebra software.

Equations of Motion from the Kaluza Hypothesis

The equations of motion are obtained from the 5-dimensional geodesic hypothesis [2] in terms of a 5-velocity $\tilde{U}^a \equiv dx^a/ds$:

$$\tilde{U}^b\tilde{\nabla}_b\tilde{U}^a = \frac{d\tilde{U}^a}{ds} + \tilde{\Gamma}^a_{bc}\tilde{U}^b\tilde{U}^c = 0$$

This equation can be recast in several ways, and it has been studied in various forms by authors including Kaluza,[2] Pauli,[21] Gross & Perry,[22] Gegenberg & Kunstatter,[23] and Wesson & Ponce de Leon,[24] but it is instructive to convert it back to the usual 4-dimensional length element $c^2d\tau^2 \equiv g_{\mu\nu}dx^\mu dx^\nu$, which is related to the 5-dimensional length element ds as given above:

$$ds^2 = c^2d\tau^2 + \phi^2(kA_\nu dx^\nu + dx^5)^2$$

Then the 5D geodesic equation can be written [25] for the spacetime components of the 4velocity, $U^\nu \equiv dx^\nu/d\tau$

$$\frac{dU^\nu}{d\tau} + \tilde{\Gamma}^\mu_{\alpha\beta}U^\alpha U^\beta + 2\tilde{\Gamma}^\mu_{5\alpha}U^\alpha U^5 + \tilde{\Gamma}^\mu_{55}(U^5)^2 + U^\mu\frac{d}{d\tau}\ln\left(\frac{cd\tau}{ds}\right) = 0$$

:

The term quadratic in U^ν provides the 4D geodesic equation plus some electromagnetic terms:

$$\tilde{\Gamma}^\mu_{\alpha\beta} = \Gamma^\mu_{\alpha\beta} + \frac{1}{2}g^{\mu\nu}k^2\phi^2(A_\alpha F_{\beta\nu} + A_\beta F_{\alpha\nu} + A_\alpha A_\beta\partial_\nu \ln\phi^2)$$

The term linear in U^ν provides the Lorentz force law:

$$\tilde{\Gamma}^{\mu}_{5\alpha} = \frac{1}{2} g^{\mu\nu} k\phi^2 (F_{\alpha\nu} - A_\alpha \partial_\nu \ln \phi^2)$$

This is another expression of the "Kaluza miracle". The same hypothesis for the 5D metric that provides electromagnetic stress-energy in the Einstein equations, also provides the Lorentz force law in the equation of motions along with the 4D geodesic equation. Yet correspondence with the Lorentz force law requires that we identify the component of 5-velocity along the 5th dimension with electric charge:

$$kU^5 = k\frac{dx^5}{d\tau} \rightarrow \frac{q}{mc}$$

where m is particle mass and q is particle electric charge. Thus, electric charge is understood as motion along the 5th dimension. The fact that the Lorentz force law could be understood as a geodesic in 5 dimensions was to Kaluza a primary motivation for considering the 5-dimensional hypothesis, even in the presence of the aesthetically-unpleasing cylinder condition.

Yet there is a problem: the term quadratic in U^5.

$$\tilde{\Gamma}^{\mu}_{55} = -\frac{1}{2} g^{\mu\alpha} \partial_\alpha \phi^2$$

If there is no gradient in the scalar field, the term quadratic in U^5 vanishes. But otherwise the expression above implies

$$U^5 \sim c\frac{q/m}{G^{1/2}}$$

For elementary particles, $U^5 > 10^{20} c$. The term quadratic in U^5 should dominate the equation, perhaps in contradiction to experience. This was the main shortfall of the 5-dimensional theory as Kaluza saw it,[2] and he gives it some discussion in his original article.

The equation of motion for U^5 is particularly simple under the cylinder condition. Start with the alternate form of the geodesic equation, written for the covariant 5-velocity:

$$\frac{d\tilde{U}_a}{ds} = \frac{1}{2} \tilde{U}^b \tilde{U}^c \frac{\partial \tilde{g}_{bc}}{\partial x^a}$$

This means that under the cylinder condition, $\tilde{U}_5$ is a constant of the 5-dimensional motion:

$$\tilde{U}_5 = \tilde{g}_{5a} \tilde{U}^a = \phi^2 \frac{c d\tau}{ds} (kA_\nu U^\nu + U^5) = \text{constant}$$

Kaluza's Hypothesis for the Matter Stress-Energy Tensor

Kaluza [2] proposed a 5D matter stress tensor $\tilde{T}^{ab}_M$ of the form

$$\tilde{T}_M^{ab} = \rho \frac{dx^a}{ds} \frac{dx^b}{ds}$$

where ρ is a density and the length element ds is as defined above.

Then, the spacetime component gives a typical "dust" stress energy tensor:

$$\tilde{T}_M^{\mu\nu} = \rho \frac{dx^\mu}{ds} \frac{dx^\nu}{ds}$$

The mixed component provides a 4-current source for the Maxwell equations:

$$\tilde{T}_M^{5\mu} = \rho \frac{dx^\mu}{ds} \frac{dx^5}{ds} = \rho U^\mu \frac{q}{kmc}$$

Just as the 5-dimensional metric comprises the 4-D metric framed by the electromagnetic vector potential, the 5-dimensional stress-energy tensor comprises the 4-D stress-energy tensor framed by the vector 4-current.

Quantum Interpretation of Klein

Kaluza's original hypothesis was purely classical and extended discoveries of general relativity. By the time of Klein's contribution, the discoveries of Heisenberg, Schroedinger, and de Broglie were receiving a lot of attention. Klein's Nature paper [4] suggested that the fifth dimension is closed and periodic, and that the identification of electric charge with motion in the fifth dimension be interpreted as standing waves of wavelength λ^5, much like the electrons around a nucleus in the Bohr model of the atom. The quantization of electric charge could then be nicely understood in terms of integer multiples of fifth-dimensional momentum. Combining the previous Kaluza result for U^5 in terms of electric charge, and a de Broglie relation for momentum $p^5 = h/\lambda^5$, Klein [4] obtained an expression for the 0th mode of such waves:

$$mU^5 = \frac{cq}{G^{1/2}} = \frac{h}{\lambda^5} \rightarrow \lambda^5 \sim \frac{hG^{1/2}}{cq}$$

where h is the Planck constant. Klein found $\lambda^5 \sim 10^{-30}$ cm, and thereby an explanation for the cylinder condition in this small value.

Klein's Zeitschrift für Physik paper of the same year,[3] gave a more-detailed treatment that explicitly invoked the techniques of Schroedinger and de Broglie. It recapitulated much of the classical theory of Kaluza described above, and then departed into Klein's quantum interpretation. Klein solved a Schroedinger-like wave equation using an expansion in terms of fifth-dimensional waves resonating in the closed, compact fifth dimension.

The Kaluza–Klein geometry

To build the Kaluza–Klein theory, one picks an invariant metric on the circle S1 that is the fiber of the U(1)-bundle of electromagnetism. In this discussion, an invariant metric is simply one that is invariant under rotations of the circle. Suppose this metric gives the circle a total length of Λ. One then considers metrics $\hat{g}$

on the bundleP that are consistent with both the fiber metric, and the metric on the underlying manifold M. The consistency conditions are:

The projection of $\hat{g}$ to the vertical subspace $\mathrm{Vert}_p P \subset T_p P$ needs to agree with metric on the fiber over a point in the manifold M.

The projection of $\hat{g}$ to the horizontal subspace $\mathrm{Hor}_p P \subset T_p P$ of the tangent space at point p ∈ P must be isomorphic to the metric g on M at π(p).

The Kaluza–Klein action for such a metric is given by

$$S(\hat{g}) = \int_P R(\hat{g})\ \mathrm{vol}(\hat{g})$$

The scalar curvature, written in components, then expands to

$$R(\hat{g}) = \pi^* \left(R(g) - \frac{\Lambda^2}{2}|F|^2 \right)$$

where π* is the pullback of the fiber bundle projection π: P → M. The connection A on the fiber bundle is related to the electromagnetic field strength as

$$\pi^* F = \mathrm{d}A$$

That there always exists such a connection, even for fiber bundles of arbitrarily complex topology, is a result from homology and specifically, K-theory. ApplyingFubini's theorem and integrating on the fiber, one gets

$$S(\hat{g}) = \Lambda \int_M \left(R(g) - \frac{1}{\Lambda^2}|F|^2 \right)\ \mathrm{vol}(g)$$

Varying the action with respect to the component A, one regains the Maxwell equations. Applying the variational principle to the base metric g, one gets the Einstein equations

$$R_{ij} - \frac{1}{2} g_{ij} R = \frac{1}{\Lambda^2} T_{ij}$$

with the stress–energy tensor being given by

$$T^{ij} = F^{ik} F^{jl} g_{kl} - \frac{1}{4} g^{ij} |F|^2,$$

Sometimes called the Maxwell stress tensor

The original theory identifies Λ with the fiber metric g55, and allows Λ to vary from fiber to fiber. In this case, the coupling between gravity and the electromagnetic field is not constant, but has its own dynamical field, the radion.

Motivation and background

In canonical gravity, spacetime is foliated into spacelike submanifolds. The three-metric (i.e., metric on the hypersurface) is γ_{ij} and given by

$$g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = (-N^2 + \beta_k\beta^k)\,\mathrm{d}t^2 + 2\beta_k\,\mathrm{d}x^k\,\mathrm{d}t + \gamma_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j.$$

In that equation the Roman indices run over the values 1, 2, 3 and the Greek indices run over the values 1, 2, 3, 4. The three-metric γ_{ij} is the field, and we denote its conjugate momenta as π^{kl}. The Hamiltonian is a constraint (characteristic of most relativistic systems)

$$\mathcal{H} = \frac{1}{2\sqrt{\gamma}} G_{ijkl}\pi^{ij}\pi^{kl} - \sqrt{\gamma}\,{}^{(3)}R = 0$$

where $\gamma = \det(\gamma_{ij})$ and $G_{ijkl} = (\gamma_{ik}\gamma_{jl} + \gamma_{il}\gamma_{jk} - \gamma_{ij}\gamma_{kl})$ is the Wheeler-DeWitt metric.

Quantization "puts hats" on the momenta and field variables; that is, the functions of numbers in the classical case become operators that modify the state function in the quantum case. Thus we obtain the operator

$$\widehat{\mathcal{H}} = \frac{1}{2\sqrt{\gamma}} \widehat{G}_{ijkl}\widehat{\pi}^{ij}\widehat{\pi}^{kl} - \sqrt{\gamma}\,{}^{(3)}\widehat{R}.$$

Working in "position space", these operators are

$$\hat{\gamma}_{ij}(t, x^k) \to \gamma_{ij}(t, x^k)$$

$$\hat{\pi}^{ij}(t, x^k) \to -i\frac{\delta}{\delta\gamma_{ij}(t, x^k)}.$$

Derivation from path integral:

The Wheeler–DeWitt equation can be derived from a path integral using the gravitational action in the Euclidean quantum gravity paradigm:[3]

$$Z = \int_C \mathrm{e}^{-I[g_{\mu\nu},\phi]}\mathcal{D}\mathbf{g}\,\mathcal{D}\phi$$

where one integrates over a class of Riemannian four-metrics and matter fields matching certain boundary conditions. Because the concept of a universal time coordinate seems unphysical, and at odds with the principles of general relativity, the action is evaluated around a 3-metric which we take as the boundary of the classes of four-metrics and on which a certain configuration of matter fields exists. This latter might for example be the current configuration of matter in our universe as we observe it today. Evaluating the action so that it only depends on the 3-metric and the matter fields is sufficient to remove the need for a time coordinate as it effectively fixes a point in the evolution of the universe.

We obtain the Hamiltonian constraint from

$$\frac{\delta I_{EH}}{\delta N} = 0$$

where I_{EH} is the Einstein-Hilbert action, and N is the lapse function (i.e., the Lagrange multiplier for the Hamiltonian constraint). This is purely classical so far. We can recover the Wheeler–DeWitt equation from

$$\frac{\delta Z}{\delta N} = 0 = \int \left.\frac{\delta I[g_{\mu\nu}, \phi]}{\delta N}\right|_{\Sigma} \exp\left(-I[g_{\mu\nu}, \phi]\right) \mathcal{D}\mathbf{g}\, \mathcal{D}\phi$$

where Σ is the three-dimensional boundary. Observe that this expression vanishes, implying that the functional derivative also vanishes, giving us the Wheeler–DeWitt equation. A similar statement may be made for the diffeomorphism constraint (take functional derivative with respect to the shift functions instead).

Mathematical formalism

The Wheeler–DeWitt equation [1] is a functional differential equation. It is ill defined in the general case, but very important in theoretical physics, especially inquantum gravity. It is a functional differential equation on the space of three dimensional spatial metrics. The Wheeler–DeWitt equation has the form of an operator acting on a wave functional; the functional reduces to a function in cosmology. Contrary to the general case, the Wheeler–DeWitt equation is well defined in mini-superspaces like the configuration space of cosmological theories. An example of such a wave function is the Hartle–Hawking state. Bryce DeWitt first published this equation in 1967 under the name "Einstein–Schrödinger equation"; it was later renamed the "Wheeler–DeWitt equation".[4]

Simply speaking, the Wheeler–DeWitt equation says

$$\hat{H}(x)|\psi\rangle = 0$$

where $\hat{H}(x)$ is the Hamiltonian constraint in quantized general relativity and $|\psi\rangle$ stands for the wave function of the universe. Unlike ordinary quantum field theory or quantum mechanics, the Hamiltonian is a first class constraint on physical states. We also have an independent constraint for each point in space.

Although the symbols $\hat{H}$ and $|\psi\rangle$ may appear familiar, their interpretation in the Wheeler–DeWitt equation is substantially different from non-relativistic quantum mechanics. $|\psi\rangle$ is no longer a spatial wave function in the traditional sense of a complex-valued function that is defined on a 3-dimensional space-like surface and normalized to unity. Instead it is a functional of field configurations on all of spacetime. This wave function contains all of the information about the geometry and matter content of the universe. $\hat{H}$ is still an operator that acts on the Hilbert space of wave functions, but it is not the same Hilbert space as in the nonrelativistic case, and the Hamiltonian no longer determines evolution of the system, so the Schrödinger equation $\hat{H}|\psi\rangle = i\hbar\partial/\partial t|\psi\rangle$ no longer applies. This property is known as timelessness. The reemergence of time requires the tools of decoherence and clock operators (or the use of a scalar field).

We also need to augment the Hamiltonian constraint with momentum constraints

$$\vec{\mathcal{P}}(x)\,|\psi\rangle = 0$$

associated with spatial diffeomorphism invariance.

In minisuperspace approximations, we only have one Hamiltonian constraint (instead of infinitely many of them).

In fact, the principle of general covariance in general relativity implies that global evolution per se does not exist; the time t is just a label we assign to one of the coordinate axes. Thus, what we think about as time evolution of any physical system is just a gauge transformation, similar to that of QED induced by U(1) local gauge transformation $\psi \to e^{i\theta(\vec{r})}\psi$ where $\theta(\vec{r})$ plays the role of local time. The role of a Hamiltonian is simply to restrict the space of the "kinematic" states of the Universe to that of "physical" states - the ones that follow gauge orbits. For this reason we call it a "Hamiltonian constraint." Upon quantization, physical states become wave functions that lie in the kernel of the Hamiltonian operator.

In general, the Hamiltonian vanishes for a theory with general covariance or time-scaling invariance.

$$\left(\beta mc^2 + c(\alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3)\right)\psi(x,t) = i\hbar\frac{\partial\psi(x,t)}{\partial t}$$

$$-\frac{\hbar^2}{2m}\nabla^2\phi = i\hbar\frac{\partial}{\partial t}\phi.$$

$$\nabla\cdot J + \frac{\partial\rho}{\partial t} = 0.$$

$$\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} = \left(A\partial_x + B\partial_y + C\partial_z + \frac{i}{c}D\partial_t\right)\left(A\partial_x + B\partial_y + C\partial_z + \frac{i}{c}D\partial_t\right).$$

$$i\hbar\gamma^\mu\partial_\mu\psi - mc\psi = 0$$

$$\left[\nabla^2 - \frac{m^2c^2}{\hbar^2}\right]\psi(\mathbf{r}) = 0$$

$$\Box\psi + \frac{\partial V}{\partial\psi} = 0$$

$$\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\psi = \frac{m^2c^2}{\hbar^2}\psi$$ (Klein Gordon equation)

$$T^{\mu\nu} = \frac{\hbar^2}{m}\left(\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha} - \eta^{\mu\nu}\eta^{\alpha\beta}\right)\partial_\alpha\bar{\psi}\partial_\beta\psi - \eta^{\mu\nu}mc^2\bar{\psi}\psi.$$

Poison equation:

$$\left[\nabla^2 - \frac{m^2c^2}{\hbar^2}\right]\psi(\mathbf{r}) = 0$$

Proca action:

$$\mathcal{L} = -\frac{1}{16\pi}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) + \frac{m^2c^2}{8\pi\hbar^2}A^\nu A_\nu.$$

$$\partial_\mu(\partial^\mu A^\nu - \partial^\nu A^\mu) + \left(\frac{mc}{\hbar}\right)^2 A^\nu = 0$$

Equations in vector calculus notation:

$$\Box\phi - \frac{\partial}{\partial t}\left(\frac{1}{c^2}\frac{\partial\phi}{\partial t} + \nabla\cdot\mathbf{A}\right) = -\left(\frac{mc}{\hbar}\right)^2\phi$$

$$\Box\mathbf{A} + \nabla\left(\frac{1}{c^2}\frac{\partial\phi}{\partial t} + \nabla\cdot\mathbf{A}\right) = -\left(\frac{mc}{\hbar}\right)^2\mathbf{A}$$

Bargmann–Wigner equations From Wikipedia, the free encyclopedia

$$(-\gamma^\mu\hat{P}_\mu + mc)_{\alpha_r\alpha'_r}\psi_{\alpha_1\cdots\alpha'_r\cdots\alpha_{2j}} = 0$$

N = 1 SUSY in 4D (with 4 real generators); Supersymmetric Gauge Theory (Wikipedia)

In four dimensions, the minimal N = 1 supersymmetry may be written using a superspace. This superspace involves four extra fermionic coordinates $\theta^1, \theta^2, \bar{\theta}^1, \bar{\theta}^2$, transforming as a two-component spinor and its conjugate.

Every superfield, i.e. a field that depends on all coordinates of the superspace, may be expanded with respect to the new fermionic coordinates. There exists a special kind of superfields, the so-called chiral superfields, that only depend on the variables θ but not their conjugates (more precisely, $\overline{D}f = 0$). However, avector superfield depends on all coordinates. It describes a gauge field and its superpartner, namely a Weyl fermion that obeys a Dirac equation.

$$V = C + i\theta\chi - i\bar{\theta}\bar{\chi} + \tfrac{i}{2}\theta^2(M+iN) - \tfrac{i}{2}\bar{\theta}^2(M-iN) - \theta\sigma^\mu\bar{\theta}v_\mu + i\theta^2\bar{\theta}\left(\bar{\lambda} + \tfrac{1}{2}\bar{\sigma}^\mu\partial_\mu\chi\right) - i\bar{\theta}^2\theta\left(\lambda + \tfrac{i}{2}\sigma^\mu\partial_\mu\bar{\chi}\right) + \tfrac{1}{2}\theta^2\bar{\theta}^2\left(D + \tfrac{1}{2}\Box C\right)$$

V is the vector superfield (prepotential) and is real (V = V). The fields on the right hand side are component fields.

The gauge transformations act as

$$V \to V + \Lambda + \overline{\Lambda}$$

where Λ is any chiral superfield.

It's easy to check that the chiral superfield

$$W_\alpha \equiv -\tfrac{1}{4}\overline{D}^2 D_\alpha V$$

is gauge invariant. So is its complex conjugate $\overline{W}_{\dot{\alpha}}$.

A nonSUSY covariant gauge which is often used is the Wess–Zumino gauge. Here, C, χ, M and N are all set to zero. The residual gauge symmetries are gauge transformations of the traditional bosonic type.

A chiral superfield X with a charge of q transforms as

$$X \to e^{q\Lambda} X, \qquad \overline{X} \to e^{q\overline{\Lambda}} X$$

Therefore Xe−qVX is gauge invariant. Here e−qV is called a bridge since it "bridges" a field which transforms under Λ only with a field which transforms under Λonly.

More generally, if we have a real gauge group G that we wish to supersymmetrized, we first have to complexify it to Gc · e−qV then acts a compensator for the complex gauge transformations in effect absorbing them leaving only the real parts. This is what's being done in the Wess–Zumino gauge.

Eucledian Quantum Gravity:

$$\int \mathcal{D}\mathbf{g}\,\mathcal{D}\phi\,\exp\left(\int d^4x\sqrt{|\mathbf{g}|}(R+\mathcal{L}_{\text{matter}})\right)$$

Relation to ADM Formalism

Euclidean Quantum Gravity does relate back to ADM formalism used in canonical quantum gravity and recovers the Wheeler–DeWitt equation under various circumstances. If we have some matter field \phi, then the path integral reads

$$Z = \int \mathcal{D}\mathbf{g}\,\mathcal{D}\phi\,\exp\left(\int d^4x\sqrt{|\mathbf{g}|}(R+\mathcal{L}_{\text{matter}})\right)$$

$$\frac{\delta Z}{\delta N} = 0 = \int \mathcal{D}\mathbf{g}\,\mathcal{D}\phi\;\left.\frac{\delta S}{\delta N}\right|_\Sigma \exp\left(\int d^4x\sqrt{|\mathbf{g}|}(R+\mathcal{L}_{\text{matter}})\right)$$

In physics, spinfoam or spin foam is a topological structure made out of two-dimensional faces that represents one of the configurations that must be summed to obtain a Feynman's path integral (functional integration) description of quantum gravity. It is closely related to loop quantum gravity.

The partition function for a spin foam model is, in general

$$Z := \sum_\Gamma w(\Gamma)\left[\sum_{j_f,i_e}\prod_f A_f(j_f)\prod_e A_e(j_f,i_e)\prod_v A_v(j_f,i_e)\right]$$

G_{13} : Category one of LHS of above equations

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of RHS of above equations

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Nuclear Physics B Volume 505, Issues 1–2, 10 November 1997, Pages 84–108 Matrix theory and heterotic strings on tori Petr Hořava doi:10.1016/S0550-3213(97)00509-9

He clarifies some puzzling aspects of the

gauge anomaly cancellation in (eb) the presence of spacetime Wilson lines; in general, the Yang-Mills theory requires (e) certain Chern-Simons couplings to (e&eb) supergravity background fields.

(1) They discuss the possibility that D8-branes are present as (=) certain matrix configurations in (eb) the yang-Mills theory and the fundamental fermions emerge as (eb) zero-modes.
(2) Finally, they point out that the correspondence between matrix theory and (e&eb) string theory suggests (eb) the existence of a multitude of (e) non-trivial RG fixed points and dualities in (eb) orbifold Yang-Mills theories with (e&eb) eight supercharges in various dimensions. Copyright © 1997 Published by Elsevier B.V.

G_{16} : Category one of gauge anomaly cancellation

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of presence of spacetime Wilson lines; in general, the Yang-Mills theory requires (e) certain Chern-Simons couplings to (e&eb) supergravity background fields

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

gauge anomaly cancellation in the presence of spacetime Wilson lines; in general, the Yang-Mills theory requires (e) certain Chern-Simons couplings to (e&eb) supergravity background fields

G_{20} : Category one of Chern-Simons couplings to (e&eb) supergravity background fields

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of gauge anomaly cancellation in the presence of spacetime Wilson lines; in general, the Yang-Mills theory

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Chern-Simons couplings to (e&eb) supergravity background fields

G_{24} : Category one of Chern-Simons couplings; supergravity background fields

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of supergravity background fields ;Chern-Simons couplings

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

They discuss the possibility that D8-branes are present as (=) certain matrix configurations in (eb) the yang-Mills theory and the fundamental fermions emerge as (eb) zero-modes

G_{28} : Category one of D8-branes

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of certain matrix configurations in (eb) the yang-Mills theory and the fundamental fermions emerge as (eb) zero-modes

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

D8-branes are present as certain matrix configurations in (eb) the yang-Mills theory and the fundamental fermions emerge as (eb) zero-modes

G_{32} : Category one of D8-branes are present as certain matrix configurations

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of yang-Mills theory and the fundamental fermions emerge as (eb) zero-modes

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

D8-branes are present as certain matrix configurations in the yang-Mills theory and the fundamental fermions emerge as (eb) zero-modes

G_{36} : Category one of D8-branes are present as certain matrix configurations in the yang-Mills theory and the fundamental fermions

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of zero-modes

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

Finally, they point out that the correspondence between

matrix theory and (e&eb) string theory suggests (eb) the existence of a multitude of (e) non-trivial RG fixed points and dualities in (eb) orbifold Yang-Mills theories with (e&eb) eight supercharges in various dimensions. Copyright © 1997 Published by Elsevier B.V.

G_{40} : Category one of matrix theory; string theory suggests (eb) the existence of a multitude of (e) non-trivial RG fixed points and dualities in (eb) orbifold Yang-Mills theories with (e&eb) eight supercharges in various dimensions. Copyright © 1997 Published by Elsevier B.V.

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of string theory suggests (eb) the existence of a multitude of (e) non-trivial RG fixed points and dualities in (eb) orbifold Yang-Mills theories with (e&eb) eight supercharges in various dimensions. Copyright © 1997 Published by Elsevier B.V.; matrix theory

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

matrix theory and string theory suggests (eb) the existence of a multitude of (e) non-trivial RG fixed points and dualities in (eb) orbifold Yang-Mills theories with (e&eb) eight supercharges in various dimensions. Copyright © 1997 Published by Elsevier B.V.

G_{44} : Category one of matrix theory and string theory suggests

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of existence of a multitude of (e) non-trivial RG fixed points and dualities in (eb) orbifold Yang-Mills theories with (e&eb) eight supercharges in various dimensions

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},\ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**
$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)}\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad (60)$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t) \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & +(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) & +(a''_{21})^{(3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t) \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & +(a''_{18})^{(2)}(T_{17},t) & +(a''_{15})^{(1,1,)}(T_{14},t) & +(a''_{22})^{(3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{38})^{(7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9)}(T_{45},t) \end{bmatrix} G_{18} \quad (63)$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)}\ \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\ \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\ \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[\begin{array}{c} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\ \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{array}\right] T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[\begin{array}{c} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\ \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\ \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\ \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\ \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{array}\right] T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* $1, 2\ 3$

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$

$are\ fourth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$
$are\ fifth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$
$are\ sixth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$
$are\ seventh\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$
$are\ eighth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

$Where\ -(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ $are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ $are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ $are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
$are\ eighth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad (81)$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \quad (84)$$

$where\ \boxed{-(b''_{28})^{(5)}(G_{31},t)}\ ,\boxed{-(b''_{29})^{(5)}(G_{31},t)}\ ,\boxed{-(b''_{30})^{(5)}(G_{31},t)}\ \ are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}\ ,\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}\ ,\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}\ are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}\ ,\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}\ ,\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}\ are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}\ ,\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}\ ,\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}\ ,\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}\ ,\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}\ ,\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}\ ,\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}\ ,\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}\ ,\boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}\ ,\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}\ ,\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}\ ,\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}\ ,\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32} \quad 85$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}\ ,\boxed{+(a''_{33})^{(6)}(T_{33},t)}\ ,\boxed{+(a''_{34})^{(6)}(T_{33},t)}\ are\ first\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\ ,\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\ ,\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\ are\ second\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}\ ,\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}\ ,\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}\ \ are\ third\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\ ,\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\ ,\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\ ,\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\ ,\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation

coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$ Eighth augmentation coefficients

$+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)} & -(b''_{32})^{(6)}(G_{35},t) & -(b''_{28})^{(5,5,5)}(G_{31},t) & -(b''_{24})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{13})^{(1,1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)} & -(b''_{33})^{(6)}(G_{35},t) & -(b''_{29})^{(5,5,5)}(G_{31},t) & -(b''_{25})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{14})^{(1,1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & -(b''_{34})^{(6)}(G_{35},t) & -(b''_{30})^{(5,5,5)}(G_{31},t) & -(b''_{26})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{15})^{(1,1,1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{34} \quad (90)$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,)}(G_{27},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3

$-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3

$-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2, and 3

$-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \qquad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt}$$ *95*

$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt}$$

$$= (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt}$$
$$= (a_{42})^{(8)} G_{41}$$
$$- \left[\begin{array}{ccc} (a'_{42})^{(8)} \boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array} \right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$
$$(b_{40})^{(8)} T_{41} - \left[\begin{array}{ccc} (b'_{40})^{(8)} \boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{13}$$

$$\frac{dT_{41}}{dt} =$$
$$(b_{41})^{(8)} T_{40} - \left[\begin{array}{ccc} (b'_{41})^{(8)} \boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{14}$$

$$\frac{dT_{42}}{dt} =$$
$$(b_{42})^{(8)} T_{41} - \left[\begin{array}{ccc} (b'_{42})^{(8)} \boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{38}'')^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

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$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a_{44}')^{(9)}\boxed{+(a_{44}'')^{(9)}(T_{45},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a_{45}')^{(9)}\boxed{+(a_{45}'')^{(9)}(T_{45},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a_{46}')^{(9)}\boxed{+(a_{46}'')^{(9)}(T_{37},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{44}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{46}'')^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second

augmentation coefficient for category 1, 2 and 3
$+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3
$+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3
$+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3
$+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$
$$(b_{44})^{(9)}T_{45} -$$
$$\begin{bmatrix} (b_{44}')^{(9)}\left[-(b_{44}'')^{(9)}(G_{47},t)\right] & \left[-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{13}$$
$$\frac{dT_{45}}{dt} =$$
$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}\left[-(b_{45}'')^{(9)}(G_{47},t)\right] & \left[-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{14}$$
$$\frac{dT_{46}}{dt} =$$
$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}\left[-(b_{46}'')^{(9)}(G_{47},t)\right] & \left[-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{15}$$

Where $-(b_{44}'')^{(9)}(G_{47},t)$, $-(b_{45}'')^{(9)}(G_{47},t)$, $-(b_{46}'')^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3
$-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\ ,\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}), t\big) \le \ (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}\,(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\,\big((G_{27}), t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - \ T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30$ 122

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \leq (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)}$$ 123

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}\left((G_{35}), t\right) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(MMMM) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(NNNN) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

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(OOOO) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(PPPP)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and $(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(QQQQ) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(RRRR) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)} \qquad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be

noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\ ,\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}<1 \qquad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+\ (\hat{A}_{40})^{(8)}+(\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \qquad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[\ (b_i)^{(8)}+(b_i')^{(8)}+\ (\hat{B}_{40})^{(8)}+(\hat{Q}_{40})^{(8)}\ (\hat{k}_{40})^{(8)}]<1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \qquad 146A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t)-(a_i'')^{(9)}(T_{45},t)|\le(\hat{k}_{44})^{(9)}|T_{45}-\ T_{45}'|e^{-(\widehat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t)-(b_i'')^{(9)}\big((G_{47}),t\big)|<(\hat{k}_{44})^{(9)}||(G_{47})-(G_{47})'||e^{-(\widehat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$

and $(a_i'')^{(9)}(T_{45},t)$. (T_{45}',t) and (T_{45},t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)},(\widehat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{44})^{(9)}=1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$:

$(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},$ are positive constants

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\ ,\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}<1$$

Definition of $(\hat{P}_{44})^{(9)},(\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},(\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)},(a_i')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(p_i)^{(9)},\ (r_i)^{(9)},i=44,45,46,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)}+(a_i')^{(9)}+\ (\hat{A}_{44})^{(9)}+\ (\hat{P}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)}+(b_i')^{(9)}+\ (\hat{B}_{44})^{(9)}+\ (\hat{Q}_{44})^{(9)}\ \ (\hat{k}_{44})^{(9)}]<1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\ ,T_i(0)$:

$$G_i(t)\leq\ \left(\hat{P}_{13}\right)^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{G_i(0)=G_i^0>0}$$

$$T_i(t)\leq\ (\hat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{T_i(0)=T_i^0>0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\ ,T_i(0)$

$$G_i(t)\leq\ (\hat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ ,\ \ G_i(0)=G_i^0>0$$

$$T_i(t)\leq\ (\hat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\ ,\ \ T_i(0)=T_i^0>0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t)\leq\ (\hat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ ,\ \ G_i(0)=G_i^0>0$$

$$T_i(t)\leq\ (\hat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ ,\ \ T_i(0)=T_i^0>0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0)\ ,T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{24}\right)^{(4)} e^{\left(\hat{M}_{24}\right)^{(4)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{24}\right)^{(4)} e^{\left(\hat{M}_{24}\right)^{(4)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{28}\right)^{(5)} e^{\left(\hat{M}_{28}\right)^{(5)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{28}\right)^{(5)} e^{\left(\hat{M}_{28}\right)^{(5)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{32}\right)^{(6)} e^{\left(\hat{M}_{32}\right)^{(6)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{32}\right)^{(6)} e^{\left(\hat{M}_{32}\right)^{(6)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{36}\right)^{(7)} e^{\left(\hat{M}_{36}\right)^{(7)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{36}\right)^{(7)} e^{\left(\hat{M}_{36}\right)^{(7)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{40}\right)^{(8)} e^{\left(\hat{M}_{40}\right)^{(8)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{40}\right)^{(8)} e^{\left(\hat{M}_{40}\right)^{(8)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{44}\right)^{(9)} e^{\left(\hat{M}_{44}\right)^{(9)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{44}\right)^{(9)} e^{\left(\hat{M}_{44}\right)^{(9)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{13})^{(1)}, T_i^0 \le (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{16})^{(2)}, T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\tilde{M}_{16})^{(2)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\tilde{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{32})^{(6)}$, $T_i^0 \leq (\hat{Q}_{32})^{(6)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0,t)$

Proof: 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(p) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left(e^{(\tilde{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\tilde{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left(e^{(\tilde{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\tilde{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0\right) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\tilde{M}_{44})^{(9)}} \left(e^{(\tilde{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\tilde{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\tilde{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\tilde{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\tilde{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[(\widehat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on

$G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 19 to 24 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(1)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If $G_{13}\ \ is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}\,(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on

(G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)} t)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

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$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)}$$

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$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)}$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),\left((G_{23})^{(2)},(T_{23})^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

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$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\,(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on T_{21} and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \quad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 227

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24{,}25{,}26$ depend only on T_{25} and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(4)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\,((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\widehat{P}_{28})^{(5)}$ and $(\widehat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\widehat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \tag{234}$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \tag{235}$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric (236)

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})},\widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})},\widetilde{(T_{31})}\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\leq \tag{237}$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\ (G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(5)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)}+\left((\widehat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\widehat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\widehat{Q}_{32})^{(6)}\right]\leq(\widehat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(6)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose

$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)}+\left((\widehat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\widehat{P}_{36})^{(7)} \tag{256}$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\widehat{Q}_{36})^{(7)}\right]\leq(\widehat{Q}_{36})^{(7)} \tag{257}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric (258)

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{39}}),(\widetilde{T_{39}}):\left((\widetilde{G_{39}}),(\widetilde{T_{39}})\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$

It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\leq \tag{259}$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. (260)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})}, \widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by 274

$(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$. The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right)) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)}, , (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}$, $if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}$, $if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}$, $if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13} e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13} e^{(S_1)^{(1)} t}$

where $(p_i)^{(1)}$ is defined by equation

$\frac{1}{(m_1)^{(1)}} G^0_{13} e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G^0_{13} e^{(S_1)^{(1)} t}$

$$\left(\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t}-e^{-(S_2)^{(1)}t}\right]+G_{15}^{0}e^{-(S_2)^{(1)}t}\le G_{15}(t)\le\right. \quad (286)$$

$$\left.\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}\right]+G_{15}^{0}e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation (291)

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: (292)

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(T_{17},t)+(a''_{17})^{(2)}(T_{17},t)\le-(\sigma_1)^{(2)} \quad (293)$$

$$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)} \quad (294)$$

Definition of $(\nu_1)^{(2)},(\nu_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: (295)

By $(\nu_1)^{(2)}>0$, $(\nu_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0$, $(u_2)^{(2)}<0$ the roots (296)

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_1)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ (297)

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and (298)

Definition of $(\bar{\nu}_1)^{(2)},,(\bar{\nu}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: (299)

By $(\bar{\nu}_1)^{(2)}>0$, $(\bar{\nu}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0$, $(\bar{u}_2)^{(2)}<0$ the (300)

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_2)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ (301)

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t}\right] + T_{18}^0 e^{-(R_2)^{(2)}t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, ***if*** $(\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, ***if*** $(\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, ***if*** $(\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}\left((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)}\right)}\left[e^{\left((S_1)^{(3)} - (p_{20})^{(3)}\right)t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}\left((S_1)^{(3)} - (a'_{22})^{(3)}\right)}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{\left((R_1)^{(3)} + (r_{20})^{(3)}\right)t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{\left((R_1)^{(3)} + (r_{20})^{(3)}\right)t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}\left((R_1)^{(3)} - (b'_{22})^{(3)}\right)}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}\left((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)}\right)}\left[e^{\left((R_1)^{(3)} + (r_{20})^{(3)}\right)t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$$

$$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right) - (b''_{25})^{(4)}\left((G_{27}), t\right) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$

and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)},,(\bar{v}_2)^{(4)},(\bar{u}_1)^{(4)},(\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$ and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}$, $(\mu_2)^{(4)}$, $(v_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}$, $(\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, ***if*** $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)}t} - e^{-(a'_{26})^{(4)}t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)}t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^{0}}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t}-e^{-(R_2)^{(4)}t}\right]+T_{26}^{0}e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)},(S_2)^{(4)},(R_1)^{(4)},(R_2)^{(4)}$:- 337

Where $(S_1)^{(4)}=(a_{24})^{(4)}(m_2)^{(4)}-(a'_{24})^{(4)}$

$(S_2)^{(4)}=(a_{26})^{(4)}-(p_{26})^{(4)}$

$(R_1)^{(4)}=(b_{24})^{(4)}(\mu_2)^{(4)}-(b'_{24})^{(4)}$

$(R_2)^{(4)}=(b'_{26})^{(4)}-(r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)},(\sigma_2)^{(5)},(\tau_1)^{(5)},(\tau_2)^{(5)}$:

$(\sigma_1)^{(5)},(\sigma_2)^{(5)},(\tau_1)^{(5)},(\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)}\le-(a'_{28})^{(5)}+(a'_{29})^{(5)}-(a''_{28})^{(5)}(T_{29},t)+(a''_{29})^{(5)}(T_{29},t)\le-(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)}\le-(b'_{28})^{(5)}+(b'_{29})^{(5)}-(b''_{28})^{(5)}\big((G_{31}),t\big)-(b''_{29})^{(5)}\big((G_{31}),t\big)\le-(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)},(\nu_2)^{(5)},(u_1)^{(5)},(u_2)^{(5)},\nu^{(5)},u^{(5)}$: 339

By $(\nu_1)^{(5)}>0$, $(\nu_2)^{(5)}<0$ and respectively $(u_1)^{(5)}>0$, $(u_2)^{(5)}<0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2+(\sigma_1)^{(5)}\nu^{(5)}-(a_{28})^{(5)}=0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2+(\tau_1)^{(5)}u^{(5)}-(b_{28})^{(5)}=0$ and

Definition of $(\bar{\nu}_1)^{(5)},,(\bar{\nu}_2)^{(5)},(\bar{u}_1)^{(5)},(\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)}>0$, $(\bar{\nu}_2)^{(5)}<0$ and respectively $(\bar{u}_1)^{(5)}>0$, $(\bar{u}_2)^{(5)}<0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2+(\sigma_2)^{(5)}\nu^{(5)}-(a_{28})^{(5)}=0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2+(\tau_2)^{(5)}u^{(5)}-(b_{28})^{(5)}=0$

Definition of $(m_1)^{(5)},(m_2)^{(5)},(\mu_1)^{(5)},(\mu_2)^{(5)},(\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)},(m_2)^{(5)},(\mu_1)^{(5)},(\mu_2)^{(5)}$ by

$(m_2)^{(5)}=(\nu_0)^{(5)},(m_1)^{(5)}=(\nu_1)^{(5)}$, $\boldsymbol{if}\ (\nu_0)^{(5)}<(\nu_1)^{(5)}$

$(m_2)^{(5)}=(\nu_1)^{(5)},(m_1)^{(5)}=(\bar{\nu}_1)^{(5)}$, $\boldsymbol{if}\ (\nu_1)^{(5)}<(\nu_0)^{(5)}<(\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)}=\frac{G_{28}^{0}}{G_{29}^{0}}}$

$(m_2)^{(5)}=(\nu_1)^{(5)},(m_1)^{(5)}=(\nu_0)^{(5)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(5)}<(\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)}=(u_0)^{(5)},(\mu_1)^{(5)}=(u_1)^{(5)}$, $\boldsymbol{if}\ (u_0)^{(5)}<(u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, $\boldsymbol{if}$ $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}$ $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{28}(t) \leq G_{28}^0 e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{29}(t) \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \leq G_{30}(t) \leq \right. \quad 344$$

$$\left. \frac{(a_{30})^{(5)} G_{28}^0}{(m_2)^{(5)}((S_1)^{(5)} - (a'_{30})^{(5)})}\left[e^{(S_1)^{(5)} t} - e^{-(a'_{30})^{(5)} t}\right] + G_{30}^0 e^{-(a'_{30})^{(5)} t}\right)$$

$$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \leq T_{28}(t) \leq T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \leq T_{28}(t) \leq \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \leq T_{30}(t) \leq \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \leq -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \leq -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \leq -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \leq -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}$, $(m_2)^{(6)}$, $(\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}$, $(m_2)^{(6)}$, $(\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad (354)$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad (355)$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t} - e^{-(a'_{34})^{(6)}t}\right] + G_{34}^0 e^{-(a'_{34})^{(6)}t}\right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad (356)$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}}T_{32}^0 e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(v_1)^{(7)}, (v_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, v^{(7)}, u^{(7)}$: 361

By $(v_1)^{(7)} > 0, (v_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{v}_1)^{(7)}, , (\bar{v}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)} > 0, (\bar{v}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)},\ \boldsymbol{if}\ (\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)},\ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}),t\big) - (b''_{45})^{(9)}\big((G_{47}),t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)},, (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, ***if*** $(\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, ***if*** $(\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, ***if*** $(\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,
and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44}e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44}e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G^0_{44}e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G^0_{44}e^{(S_1)^{(9)}t}$$

(

$$\frac{(a_{46})^{(9)}G^0_{44}}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})}\Big[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t}\Big] + G^0_{46}e^{-(S_2)^{(9)}t} \le G_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}G^0_{44}}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})}[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G^0_{46}e^{-(a'_{46})^{(9)}t})$$

$$\boxed{T_{44}^{0}e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^{0}e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^{0}e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^{0}e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^{0}}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t}-e^{-(b'_{46})^{(9)}t}\right]+T_{46}^{0}e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^{0}}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t}-e^{-(R_2)^{(9)}t}\right]+T_{46}^{0}e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^{0}}{G_{14}^{0}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \le \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \le \nu^{(1)}(t) \le (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \le \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \le \nu^{(1)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \le (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \le (\bar{\nu}_1)^{(1)} \le \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \le \nu^{(1)}(t) \le \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \le (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \le \nu^{(1)}(t) \le (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \le u^{(1)}(t) \le (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{dv^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)v^{(2)} - (a_{17})^{(2)}v^{(2)}$$

Definition of $v^{(2)}$:- $\boxed{v^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(v^{(2)}\right)^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)}\right) \leq \frac{dv^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(v^{(2)}\right)^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{v}_1)^{(2)}, (v_0)^{(2)}$:-

For $0 < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (v_1)^{(2)} < (\bar{v}_1)^{(2)}$

$$v^{(2)}(t) \geq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(v_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (v_2)^{(2)}}}$$

it follows $(v_0)^{(2)} \leq v^{(2)}(t) \leq (v_1)^{(2)}$

In the same manner , we get 391

$$v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{v}_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (\bar{v}_2)^{(2)}}}$$

From which we deduce $(v_0)^{(2)} \leq v^{(2)}(t) \leq (\bar{v}_1)^{(2)}$ 392

If $0 < (v_1)^{(2)} < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{v}_1)^{(2)}$ we find like in the previous case, 393

$$(v_1)^{(2)} \leq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_2)^{(2)}\right)t\right]}} \leq v^{(2)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \leq (\bar{v}_1)^{(2)}$$

If $0 < (v_1)^{(2)} \leq (\bar{v}_1)^{(2)} \leq (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(v_1)^{(2)} \leq v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \leq (v_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(2)}(t)$:- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then
$(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

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If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\quad \boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$v^{(6)}(t) \le \frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)}=\frac{(\bar{v}_1)^{(6)}-(v_0)^{(6)}}{(v_0)^{(6)}-(\bar{v}_2)^{(6)}}}$$

From which we deduce $(v_0)^{(6)} \le v^{(6)}(t) \le (\bar{v}_1)^{(6)}$

If $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{v}_1)^{(6)}$ we find like in the previous case, 414

$$(v_1)^{(6)} \le \frac{(v_1)^{(6)}+(C)^{(6)}(v_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)}-(v_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)}-(v_2)^{(6)}\right)t\right]}} \le v^{(6)}(t) \le$$

$$\frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \le (\bar{v}_1)^{(6)}$$

If $0 < (v_1)^{(6)} \le (\bar{v}_1)^{(6)} \le \boxed{(v_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(v_1)^{(6)} \le v^{(6)}(t) \le \frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \le (v_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(6)}(t)$:-

$$(m_2)^{(6)} \le v^{(6)}(t) \le (m_1)^{(6)}, \quad \boxed{v^{(6)}(t)=\frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \le u^{(6)}(t) \le (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t)=\frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(v_1)^{(6)} = (\bar{v}_1)^{(6)}$ if in addition $(v_0)^{(6)} = (v_1)^{(6)}$ then $v^{(6)}(t) = (v_0)^{(6)}$ and as a consequence $G_{32}(t) = (v_0)^{(6)}G_{33}(t)$ **this also defines** $(v_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(v_1)^{(6)}$ and $(\bar{v}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)}=\frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}\right)\leq\frac{dv^{(7)}}{dt}\leq-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0<\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}<(\nu_1)^{(7)}<(\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t)\geq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}\quad,\quad\boxed{(C)^{(7)}=\frac{(\nu_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)}\leq\nu^{(7)}(t)\leq(\nu_1)^{(7)}$

In the same manner , we get 417

$$\nu^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(7)}=\frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)}\leq\nu^{(7)}(t)\leq(\bar{\nu}_1)^{(7)}$

If $0<(\nu_1)^{(7)}<(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}<(\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)}\leq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}\leq\nu^{(7)}(t)\leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\bar{\nu}_1)^{(7)}$$

If $0<(\nu_1)^{(7)}\leq(\bar{\nu}_1)^{(7)}\leq\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)}\leq\nu^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)}\leq\nu^{(7)}(t)\leq(m_1)^{(7)},\quad\boxed{\nu^{(7)}(t)=\frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

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If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case,

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

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If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$ if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\quad \boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \leq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \leq \nu^{(9)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \leq (\bar{\nu}_1)^{(9)} \leq \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \leq \nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \leq \nu^{(9)}(t) \leq (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for**

the special case .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a_i'')^{(1)} and\ (b_i'')^{(1)}$ are independent on t , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0$ 447

$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0$ 448

has a unique positive solution , which is an equilibrium solution

$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0$ 449

$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0$ 450

$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0$ 451

$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0$ 452

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(p) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T_{33}^{*} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^{*} for which $f(T_{33}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

Definition and uniqueness of T_{37}^{*} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^{*} for which $f(T_{37}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^{*})]}$$

Definition and uniqueness of T_{41}^{*} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^{*} for which $f(T_{41}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^{*})]}$$

Definition and uniqueness of T_{45}^{*} :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^{*} for which $f(T_{45}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^{*})]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi(G^{*}) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that

there exists a unique G_{37}^{*} such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)}G_{17}^{*}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)}G_{17}^{*}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T_{17}^{*})]} \quad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)}T_{17}^{*}}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)}T_{17}^{*}}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^{*})]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)}G_{21}^{*}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)}G_{21}^{*}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$$T_{28}^* = \frac{(b_{28})^{(5)} T_{29}^*}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^*)]} \quad , \quad T_{30}^* = \frac{(b_{30})^{(5)} T_{29}^*}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^*)]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^* given by $\varphi((G_{35})^*) = 0$, T_{33}^* given by $f(T_{33}^*) = 0$ and

$$G_{32}^* = \frac{(a_{32})^{(6)} G_{33}^*}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34}^* = \frac{(a_{34})^{(6)} G_{33}^*}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

$$T_{32}^* = \frac{(b_{32})^{(6)} T_{33}^*}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^*)]} \quad , \quad T_{34}^* = \frac{(b_{34})^{(6)} T_{33}^*}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^*)]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^* given by $\varphi((G_{39})^*) = 0$, T_{37}^* given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

G_{41}^* given by $\varphi((G_{43})^*) = 0$, T_{41}^* given by $f(T_{41}^*) = 0$ and

$$G_{40}^{*}=\frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}\quad,\quad G_{42}^{*}=\frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

$$T_{40}^{*}=\frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}\quad,\quad T_{42}^{*}=\frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*})=0$, T_{45}^{*} given by $f(T_{45}^{*})=0$ and

$$G_{44}^{*}=\frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}\quad,\quad G_{46}^{*}=\frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$$

$$T_{44}^{*}=\frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}\quad,\quad T_{46}^{*}=\frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}\ and\ (b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i=G_i^{*}+\mathbb{G}_i\qquad,T_i=T_i^{*}+\mathbb{T}_i$$

$$\frac{\partial(a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^{*})=(q_{14})^{(1)}\quad,\ \frac{\partial(b_i'')^{(1)}}{\partial G_j}(\,G^{*}\,)=s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt}=-\big((a'_{13})^{(1)}+(p_{13})^{(1)}\big)\mathbb{G}_{13}+(a_{13})^{(1)}\mathbb{G}_{14}-(q_{13})^{(1)}G_{13}^{*}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt}=-\big((a'_{14})^{(1)}+(p_{14})^{(1)}\big)\mathbb{G}_{14}+(a_{14})^{(1)}\mathbb{G}_{13}-(q_{14})^{(1)}G_{14}^{*}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt}=-\big((a'_{15})^{(1)}+(p_{15})^{(1)}\big)\mathbb{G}_{15}+(a_{15})^{(1)}\mathbb{G}_{14}-(q_{15})^{(1)}G_{15}^{*}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt}=-\big((b'_{13})^{(1)}-(r_{13})^{(1)}\big)\mathbb{T}_{13}+(b_{13})^{(1)}\mathbb{T}_{14}+\textstyle\sum_{j=13}^{15}\big(s_{(13)(j)}T_{13}^{*}\mathbb{G}_j\big) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt}=-\big((b'_{14})^{(1)}-(r_{14})^{(1)}\big)\mathbb{T}_{14}+(b_{14})^{(1)}\mathbb{T}_{13}+\textstyle\sum_{j=13}^{15}\big(s_{(14)(j)}T_{14}^{*}\mathbb{G}_j\big) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt}=-\big((b'_{15})^{(1)}-(r_{15})^{(1)}\big)\mathbb{T}_{15}+(b_{15})^{(1)}\mathbb{T}_{14}+\textstyle\sum_{j=13}^{15}\big(s_{(15)(j)}T_{15}^{*}\mathbb{G}_j\big) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions

$(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \tag{532}$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \ , \ \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \tag{533}$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \tag{534}$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \tag{535}$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \tag{536}$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \tag{537}$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}')^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T_{17}^*\mathbb{G}_j\right) \tag{538}$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}')^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T_{18}^*\mathbb{G}_j\right) \tag{539}$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \ , \ \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}')^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \tag{541}$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}')^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \tag{542}$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}')^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \tag{543}$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}')^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T_{20}^*\mathbb{G}_j\right) \tag{544}$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^{*}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T_{22}^{*}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^{*}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^{*}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^{*}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^{*}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^{*}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^{*}\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}(\,(G_{31})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^{*}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^{*}) = (q_{41})^{(8)} \quad , \frac{\partial(b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^{*}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^{*}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^{*}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^{*}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \ , \ \frac{\partial(b''_i)^{(9)}}{\partial G_j}(\ (G_{47})^*\ ) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\ B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\ C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\ D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\ E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\ F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\ G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14}+(b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\Big]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION EIGHTEEN A

World As A Hologram

INTRODUCTION—VARIABLES USED

Holographic Matrix Models and Quantum Space and Time The world as a hologram Leonard Susskind1 J. Math Phys. 36, 6377 (1995); http://dx.doi.org/10.1063/1.531249

(1) According to't Hooft the combination of quantum mechanics and (e&eb) gravity requires the three-dimensional world to be (=) an image of data that can be stored on (eb) a two-dimensional projection much like a holographic image.
(2) The two-dimensional description only requires (e) one discrete degree of freedom per Planck area and yet it is rich enough to describe all three-dimensional phenomena.
(3) After outlining't Hooft's proposal authors give a preliminary informal description of how it may be implemented. One finds a basic requirement that particles must grow in size as (e&eb) their momenta are increased far above the Planck scale.
(4) The consequences for high-energy particle collisions are described. The phenomenon of particle growth with momentum was previously discussed in the context of string theory and was related to (e&eb) information spreading near black hole horizons.
(5) The considerations of this paper indicate that the effect is much more rapid at all but the earliest times. In fact the rate of spreading is found to saturate (e&eb) the bound from causality.
(6) Finally authors consider string theory as (=) a possible realization of't Hooft's idea.
(7) The light front lattice string model of Klebanov and Susskind is reviewed and its similarities with (e&eb) the holographic theory are demonstrated. The agreement between the two requires unproven but plausible assumptions about the nonperturbative behavior of (e) string theory. Very similar ideas to those in this paper have long been held by Charles Thorn. © 1995 American Institute of Physics

NOTATION

Module One

According to't Hooft the combination of quantum mechanics and gravity requires the three-dimensional world to be (=) an image of data that can be stored on (eb) a two-dimensional projection much like a holographic image

G_{13} : Category one of combination of quantum mechanics and gravity requires the three-dimensional world

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of image of data that can be stored on (eb) a two-dimensional projection much like a holographic image

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

combination of quantum mechanics and gravity requires the three-dimensional world to be an image of data that can be stored on (eb) a two-dimensional projection much like a holographic image

G_{16} : Category one of combination of quantum mechanics and gravity requires the three-dimensional world to be an image of data that can be stored

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of two-dimensional projection much like a holographic image

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

The two-dimensional description only requires (e) one discrete degree of freedom per Planck area and yet it is rich enough to describe all three-dimensional phenomena

G_{20} : Category one of one discrete degree of freedom per Planck area and yet it is rich enough to describe all three-dimensional phenomena

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of two-dimensional description

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

After outlining't Hooft's proposal authors give a preliminary informal description of how it may be implemented. One finds a basic requirement that

particles must grow in size as (e&eb) their momenta are increased far above the Planck scale

G_{24} : Category one of particles must grow in size; momenta are increased far above the Planck scale

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of momenta are increased far above the Planck scale; particles must grow in size

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

The consequences for high-energy particle collisions are described.

The phenomenon of particle growth with momentum was previously discussed in the context of string theory and was related to (e&eb) information spreading near black hole horizons

G_{28} : Category one of particle growth with momentum in the context of string theory; information spreading near black hole horizons

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of information spreading near black hole horizons; particle growth with momentum in the context of string theory

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

The considerations of this paper indicate that the effect is much more rapid at all but the earliest times. In fact the
rate of spreading is found to saturate (e&eb) the bound from causality.

G_{32} : Category one of rate of spreading; bound from causality

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of bound from causality; rate of spreading

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Finally authors consider string theory as (=) a possible realization of't Hooft's idea

G_{36} : Category one of string theory

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of realization of't Hooft's idea

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

The light front lattice string model of Klebanov and Susskind is reviewed and its similarities with (e&eb) the holographic theory are demonstrated. The agreement between the two requires unproven but plausible assumptions about the nonperturbative behavior of (e) string theory. Very similar ideas to those in this paper have long been held by Charles Thorn. © 1995 American Institute of Physics

G_{40} : Category one of light front lattice string model of Klebanov and Susskind; holographic theory

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of holographic theory ;light front lattice string model of Klebanov and Susskind

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Yang–Mills action (Wikipedia):

The Yang–Mills action is written on the lattice using Wilson loops (named after Kenneth G. Wilson), so that the limit $a \to 0$ formally reproduces the original continuum action. Given a faithful irreducible representation ρ of G, the lattice Yang-Mills action is the sum over all lattice sites of the (real component of the) trace over the n links e1, ..., en in the Wilson loop,

$$S = \sum_F -\Re\{\chi^{(\rho)}(U(e_1)\cdots U(e_n))\}.$$

Here, χ is the character. If ρ is a real (or pseudoreal) representation, taking the real component is redundant, because even if the orientation of a Wilson loop is flipped, its contribution to the action remains unchanged.

There are many possible lattice Yang-Mills actions, depending on which Wilson loops are used in the action. The simplest "Wilson action" uses only the 1×1 Wilson loop, and differs from the continuum action by "lattice artifacts" proportional to the small lattice spacing a. By using more complicated Wilson loops to construct "improved actions", lattice artifacts can be reduced to be proportional to a^2, making computations more accurate.

In physics, lattice field theory is the study of lattice models of quantum field theory, that is, of field theory on a spacetime that has been discretized onto a lattice. Although most lattice field theories are not exactly solvable, they are of tremendous appeal because they can be studied by simulation on a computer. One hopes that, by performing simulations on larger and larger lattices, while making the lattice spacing smaller and smaller, one will be able to recover the behaviour of the continuum theory. Just as in all lattice models, numerical simulation gives access to field configurations that are not accessible to perturbation theory, such

as solitons. Likewise, non-trivial vacuum states can be discovered and probed. The method is particularly appealing for the quantization of a gauge theory. Most quantization methods keep Poincaré invariance manifest but sacrifice manifest gauge symmetry by requiring gauge fixing. Only after renormalization can gauge invariance is recovered. Lattice field theory differs from these in that it keeps manifest gauge invariance, but sacrifices manifest Poincaré invariance— recovering it only after renormalization. The articles on lattice gauge theory and lattice QCD explore these issues in greater detail.

G_{44} : Category one of first term on the RHS

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of second term on RHS and so on……..

T_{45} : Category two of SAS

T_{46} : Category three of SAS

ECTION NINETEEN

Quantum Gravity On Negatively Curved Space-Times To Conformal Quantum Field Theories

INTRODUCTION—VARIABLES USED

Holographic Matrix Models and Quantum Space and Time The world as a hologram Leonard Susskind1 J. Math Phys. 36, 6377 (1995); http://dx.doi.org/10.1063/1.531249

(1) From the present standpoint, the collection of ideas relating theories of **quantum gravity on (eb) negatively curved space-times to conformal quantum field theories** on (eb) their boundaries is a coherent and firmly established theoretical framework for fundamental physics.
(2) The AdS/CFT conjecture postulates (eb) an exact equivalence between, specifically, type IIB string theory on ${\it AdS} _5\times {S}^5$ and maximally supersymmetric Yang-Mills theory on (eb)the boundary, and gives(eb) a concrete realization of the holographic principle
(3) It is supported by a large number of explicit calculations of spectra and other physical observables, and has led to (eb) a number of new insights into both quantum gravity and field theory.
(4) At the same time, there are also so many open questions (apart from the mathematical proof of the conjecture itself, which likely would also lead to profound new mathematical insights) that physicists and mathematicians will be occupied for a long time to come. Integrability has emerged in the last decade as a most powerful tool for making progress into (e&eb) the genuinely strongly coupled regime of the duality. This has not always been so.
(5) The idea that the mathematical problems of Quantum Gravity might ultimately be intertwined with (e&eb) the difficulties of Quantum Field Theory hatched about forty years ago when the early dual resonance models for (e) strong interactions developed into string theory as (=) a candidate for a theory of all of fundamental physics.
(6) An early hint that the experimentally successful theory of the strong force, QCD, remained related to (e&eb) strings was the observation of 't Hooft that perturbative Feynman diagrams re-organize (e&eb) into a planar diagram expansion in (eb) the large-N limit.
(7) Concurrently, what might now be interpreted as the first hints that quantum gravity should be holographic began to slowly emerge in the 1970's, starting with the discovery of the area law for (e)

the entropy of black holes by Bekenstein and Hawking, and developing in work of Polyakov, 't Hooft, Susskind and others.

(8) For a long time, general acceptance was hampered by the lack of an explicitly computable model in which the reduction of the number of degrees of freedom could be checked quantitatively. This situation changed (e&eb) in a dramatic way with Maldacena's epoch making work \cite {lambda}.

(9) Inspired in part by the discovery of D-branes and by the microscopic explanation of black hole entropy in string theory, Maldacena proposed **an {\it exact quantum equivalence} between type IIB string theory in anti-de Sitter space** and (e&eb)maximally supersymmetric Yang-Mills theory on its boundary.

NOTATION

Module One

From the present standpoint, the collection of ideas relating theories of

quantum gravity on (eb) negatively curved space-times to conformal quantum field theories on (eb) their boundaries is a coherent and firmly established theoretical framework for fundamental physics

G_{13} : Category one of **quantum gravity**

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of **negatively curved space-times to conformal quantum field theories** on (eb) their boundaries

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

quantum gravity on negatively curved space-times to conformal quantum field theories on (eb) their boundaries is a coherent and firmly established theoretical framework for fundamental physics

G_{16} : Category one of **quantum gravity on negatively curved space-times to conformal quantum field theories**

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of their boundaries

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

$$\begin{aligned}\dot{\omega} = \omega^2 \frac{96}{5}\eta (M\omega)^{5/3}\Big\{1 &- \frac{743+924\eta}{336}(M\omega)^{2/3} \\ &- \left(\frac{1}{12}\sum_{i=1,2}\left(\chi_i \hat{\mathbf{L}}_n \cdot \hat{\mathbf{S}}_i \left(\frac{113 m_i^2}{M^2}+75\eta\right)\right) - 4\pi\right) M\omega \\ &+ \left(\frac{34\,103}{18\,144}+\frac{13\,661}{2016}\eta+\frac{59}{18}\eta^2\right)(M\omega)^{4/3} \\ &- \frac{1}{48}\eta\chi_1\chi_2(247(\hat{\mathbf{S}}_1\cdot\hat{\mathbf{S}}_2) - 721(\hat{\mathbf{L}}_n\cdot\hat{\mathbf{S}}_1)(\hat{\mathbf{L}}_n\cdot\hat{\mathbf{S}}_2))(M\omega)^{4/3} \\ &- \frac{1}{672}(4159+15\,876\eta)\pi(M\omega)^{5/3} + \left(\left(\frac{16\,447\,322\,263}{139\,708\,800} - \frac{1712}{105}\gamma_E + \frac{16}{3}\pi^2\right)\right. \\ &+ \left(-\frac{273\,811\,877}{1088\,640}+\frac{451}{48}\pi^2 - \frac{88}{3}\hat{\theta}\eta\right)\eta \\ &\left. + \frac{541}{896}\eta^2 - \frac{5605}{2592}\eta^3 - \frac{856}{105}\log(16(M\omega)^{2/3})\right)(M\omega)^2 \\ &+ \left(-\frac{4415}{4032}+\frac{358\,675}{6048}\eta+\frac{91\,495}{1512}\eta^2\right)\pi(M\omega)^{7/3}\Big\} \end{aligned} \tag{1}$$

$$\begin{aligned}\dot{K}_x = (N^x K_x)' &+ \frac{N\alpha K_\varphi^2 E^\varphi}{2(E^x)^{3/2}} - \frac{NK_\varphi^2 E^\varphi}{\sqrt{E^x}}\frac{\delta\alpha}{\delta E^x} - \frac{N\bar{\alpha}K_\varphi K_x}{\sqrt{E^x}} - 2NK_\varphi K_x\sqrt{E^x}\frac{\delta\bar{\alpha}}{\delta E^x} \\ &+ \frac{N\alpha E^\varphi}{2(E^x)^{3/2}} - \frac{NE^\varphi}{\sqrt{E^x}}\frac{\delta\alpha}{\delta E^x} + \frac{N\alpha(E^{x'})^2}{8E^\varphi(E^x)^{3/2}} - \frac{N(E^{x'})^2}{4E^\varphi\sqrt{E^x}}\frac{\delta\alpha}{\delta E^x} - \frac{N'\alpha E^{x'}}{2E^\varphi\sqrt{E^x}} - \frac{N\alpha E^{x''}}{2E^\varphi\sqrt{E^x}} \\ &+ \frac{N\alpha E^{\varphi'}E^{x'}}{2(E^\varphi)^2\sqrt{E^x}} + \frac{N''\bar{\alpha}\sqrt{E^x}}{E^\varphi} - \frac{N'\bar{\alpha}E^{\varphi'}\sqrt{E^x}}{(E^\varphi)^2} + \frac{2N'E^{x'}\sqrt{E^x}}{E^\varphi}\frac{\delta\bar{\alpha}}{\delta E^x} - \frac{2NE^{\varphi'}E^{x'}\sqrt{E^x}}{(E^\varphi)^2}\frac{\delta\bar{\alpha}}{\delta E^x} \\ &- \frac{N\bar{\alpha}(E^{x'})^2}{4E^\varphi(E^x)^{3/2}} + \frac{N(E^{x'})^2}{E^\varphi\sqrt{E^x}}\frac{\delta\bar{\alpha}}{\delta E^x} + \frac{N'\bar{\alpha}E^{x'}}{E^\varphi\sqrt{E^x}} + \frac{N\bar{\alpha}E^{x''}}{E^\varphi\sqrt{E^x}} - \frac{N\bar{\alpha}E^{\varphi'}E^{x'}}{(E^\varphi)^2\sqrt{E^x}} + \frac{NE^{x''}\sqrt{E^x}}{E^\varphi}\frac{\delta\bar{\alpha}}{\delta E^x} \\ &+ \frac{N\bar{\alpha}''\sqrt{E^x}}{E^\varphi} - 6\pi G\frac{N\bar{\alpha}E^\varphi(p^x)^2}{(E^x)^{5/2}} + 4\pi G\frac{NE^\varphi(p^x)^2}{(E^x)^{3/2}}\frac{\delta\bar{\bar{\alpha}}}{\delta E^x}, \end{aligned} \tag{43}$$

$$\begin{aligned}\mathcal{F}(\hat{E}^i \triangleright f)(x) = \mathcal{F}\left(\sum_\epsilon (\hat{E}^i \triangleright f)_\epsilon\right)(x)\,\xi_\epsilon &= \mathrm{i}t\sum_\epsilon\int \mathrm{d}g\,\frac{\mathrm{d}}{\mathrm{d}s} f(\mathrm{e}^{\mathrm{i}s\sigma^i}g)\,I_\epsilon(g)\,e_g(x)\,\xi_{\epsilon(g)}\Big|_{s=0} \\ &= \mathrm{i}t\sum_\epsilon\frac{\mathrm{d}}{\mathrm{d}s}\int \mathrm{d}g I_\epsilon(\mathrm{e}^{-\mathrm{i}s\sigma^i}g)\,e_{\mathrm{e}^{-\mathrm{i}s\sigma^i}g}(x)\xi_{\epsilon(\mathrm{e}^{-\mathrm{i}s\sigma^i}g)}\,f(g)\Big|_{s=0} \\ &= \mathrm{i}t\sum_\epsilon\int \mathrm{d}g\; I_\epsilon(\mathrm{e}^{-\mathrm{i}s\sigma^i}g)\frac{\mathrm{d}}{\mathrm{d}s}e_{\mathrm{e}^{-\mathrm{i}s\sigma^i}g}(x)\xi_{\epsilon(\mathrm{e}^{-\mathrm{i}s\sigma^i}g)}\Big|_{s=0} f(g) \\ &= \mathrm{i}t\sum_\epsilon\int \mathrm{d}g\; I_\epsilon(\mathrm{e}^{-\mathrm{i}s\sigma^i}g)\frac{\mathrm{d}}{\mathrm{d}s}e_{\mathrm{e}^{-\mathrm{i}s\sigma^i}}(x)\,\xi_{\epsilon(\mathrm{e}^{-\mathrm{i}s\sigma^i})}\Big|_{s=0} \star e_g(x)\,\xi_{\epsilon(g)}\,f(g) \\ &= \mathrm{i}\frac{t}{K}\sum_\epsilon\int \mathrm{d}g\,I_\epsilon(g)\,(-\mathrm{i}x^j)\star e_g(x)f(g)\xi_{\epsilon(g)} \\ &= \frac{t}{K}\sum_\epsilon x^j\star\mathcal{F}(f_\epsilon)(x)\,\xi_\epsilon = \frac{t}{K}x^j\star\mathcal{F}(f)(x), \end{aligned} \tag{35}$$

$$\left[A_a^i(x), A_b^j(y)\right]_{\mathrm{DB}} = \left(K\left(A_b^i(x)\,{}^0E_a^j(y)\right.\right.$$
$$-\left.{}^0E_b^i(x)A_a^j(y) + 2\delta^{ij}\delta_{kl}\,{}^0E^k_{[b}(y)A^l_{a]}(x)\right) + (K_i$$
$$+K_j)\left({}^0E_b^i(x)A_a^j(y) - A_b^i(x)\,{}^0E_a^j(y)\right) + 2\,(K_i$$
$$+K_k)\,\delta^{ij}\delta_{kl}\,{}^0E^k_{[a}(y)A^l_{b]}(x)\Big)\,\delta^{(3)}(x-y) - \frac{\epsilon^{ij}{}_k}{\gamma}\,(K_i$$
$$+K_j)\left({}^0E_b^k(x)\frac{\partial\delta^{(3)}(x-y)}{\partial y^a} + {}^0E_a^k(y)\frac{\partial\delta^{(3)}(x-y)}{\partial x^b}\right), \tag{14}$$

$$\delta S[g] = \int \frac{1}{2\kappa}\left(\delta f(R)\sqrt{-g} + f(R)\delta\sqrt{-g}\right)\mathrm{d}^4x$$
$$= \int \frac{1}{2\kappa}\left(F(R)\delta R\sqrt{-g} - \frac{1}{2}\sqrt{-g}g_{\mu\nu}\delta g^{\mu\nu}f(R)\right)\mathrm{d}^4x$$
$$= \int \frac{1}{2\kappa}\sqrt{-g}\left(F(R)(R_{\mu\nu}\delta g^{\mu\nu} + g_{\mu\nu}\Box\delta g^{\mu\nu} - \nabla_\mu\nabla_\nu\delta g^{\mu\nu}) - \frac{1}{2}g_{\mu\nu}\delta g^{\mu\nu}f(R)\right)\mathrm{d}^4x$$

Quantum gravity Hamilton-Jacobi equations

$$H + \frac{\partial S}{\partial t} = 0$$

$$S_H = \frac{1}{2\kappa}\int \sqrt{-g}R d^4x$$

$$\hat{g}_{ij}(t, x^k) \to g_{ij}(t, x^k)$$

$$\hat{\pi}_{ij}(t, x^k) \to -i\frac{\delta}{\delta g_{ij}(t, x^k)}$$

Schrödinger's Equation

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r},t) + V(\mathbf{r},t)\psi(\mathbf{r},t)$$

i is the imaginary number, $\sqrt{-1}$.
$\hbar$ is Planck's constant divided by 2π: 1.05459×10^{-34} joule-second.
$\psi(\mathbf{r},t)$ is the wave function, defined over space and time.
m is the mass of the particle.
∇^2 is the Laplacian operator, $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.
$V(\mathbf{r},t)$ is the potential energy influencing the particle.

$$\begin{aligned}
0 &= \frac{-F'(R_0) + 2\Lambda^2 F''(R_0)}{2\Lambda^2\cosh^2 x}\delta R - \frac{F(R_0)}{\Lambda^2\cosh^2 x}\delta\rho - F'(R_0)(-\delta\ddot{\rho} + 2\delta\ddot{\varphi} + \delta\rho'' + 2\tanh x\delta\varphi') \\
&\quad + \tanh x F''(R_0)\delta R' + F''(R_0)\delta R'', \\
0 &= -\frac{-F'(R_0) + 2\Lambda^2 F''(R_0)}{2\Lambda^2\cosh^2 x}\delta R + \frac{F(R_0)}{\Lambda^2\cosh^2 x}\delta\rho - F'(R_0)(\delta\ddot{\rho} + 2\delta\varphi'' - \delta\rho'' + 2\tanh x\delta\varphi') \\
&\quad + F''(R_0)\delta\ddot{R} + \tanh x F''(R_0)\delta R', \\
0 &= -2(\delta\dot{\varphi}' + \tanh x\delta\dot{\varphi}) + \frac{F''(R_0)}{F'(R_0)}(\delta\dot{R}' + \tanh x\delta\dot{R}), \\
0 &= -\frac{-F'(R_0) + 2\Lambda^2 F''(R_0)}{2\Lambda^2}\delta R - \frac{F(R_0)}{\Lambda^2}\delta\varphi - \cosh^2 x F'(R_0)(-\delta\ddot{\varphi} + \delta\varphi'') \\
&\quad - \cosh^2 x F''(R_0)(-\delta\ddot{R} + \delta R''). \qquad (9)
\end{aligned}$$

$$\begin{aligned}
P_{\mathrm{I}}: &\quad y'' = 6y^2 + t \\
P_{\mathrm{II}}: &\quad y'' = 2y^3 + ty + \alpha \\
P_{\mathrm{III}}: &\quad y'' = \frac{1}{y}(y')^2 - \frac{1}{t}y' + \frac{1}{t}(\alpha y^2 + \beta) + \gamma y^3 + \frac{\delta}{y} \\
P_{\mathrm{IV}}: &\quad y'' = \frac{1}{2y}(y')^2 + \frac{3}{2}y^3 + 4ty^2 + 2(t^2 - \alpha)y + \frac{\beta}{y} \\
P_{\mathrm{V}}: &\quad y'' = \left(\frac{1}{2y} + \frac{1}{y-1}\right)(y')^2 - \frac{1}{t}y' + \frac{(y-1)^2}{t^2}\left(\alpha y + \frac{\beta}{y}\right) + \frac{\gamma}{t}y + \delta\frac{y(y+1)}{y-1} \\
P_{\mathrm{VI}}: &\quad y'' = \frac{1}{2}\left(\frac{1}{y} + \frac{1}{y-1} + \frac{1}{y-t}\right)(y')^2 - \left(\frac{1}{t} + \frac{1}{t-1} + \frac{1}{y-t}\right)y' \\
&\qquad + \frac{y(y-1)(y-t)}{t^2(t-1)^2}\left(\alpha + \beta\frac{t}{y^2} + \gamma\frac{t-1}{(y-1)^2} + \delta\frac{t(t-1)}{(y-t)^2}\right).
\end{aligned}$$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$i\hbar\frac{\partial\Psi(t,\mathbf{x}_1,\ldots,\mathbf{x}_N)}{\partial t}=\left(-\sum_{i=1}^{N}\frac{\hbar^2}{2m_i}\nabla_i^2+\sum_{i\neq j}V_{ij}(|\mathbf{x}_i-\mathbf{x}_j|)\right.$$
$$\left.-G\sum_{i,j=1}^{N}m_im_j\int \mathrm{d}^3\mathbf{y}_1\cdots\mathrm{d}^3\mathbf{y}_N\frac{|\Psi(t,\mathbf{y}_1,\ldots,\mathbf{y}_N)|^2}{|\mathbf{x}_i-\mathbf{y}_j|}\right)\Psi(t,\mathbf{x}_1,\ldots,\mathbf{x}_N).$$

$$\begin{aligned}
\frac{\delta S}{\delta\sigma}&=-\left(\Box+m_3^2+\frac{\lambda_2}{2}v^2+\frac{\lambda_3}{6}u^2\right)\sigma\\
&\quad-\sqrt{N}v\left(m_3^2+\frac{\lambda_2}{6}v^2+\frac{\lambda_3}{6}u^2\right)\\
&\quad-\frac{\lambda_2}{6N}(\sigma^3+\phi_j^2\sigma)-\frac{\lambda_3}{6N}\psi_i^2\sigma-\frac{\lambda_2}{2\sqrt{N}}v\sigma^2\\
&\quad-\frac{\lambda_3}{6\sqrt{N}}v\psi_i^2-\frac{\lambda_3}{3\sqrt{N}}u\psi_1\sigma-\frac{\lambda_3}{3}uv\psi_1,\\
\frac{\delta S}{\delta\psi_1}&=-\left(\Box+m_2^2+\frac{\lambda_1}{2}u^2+\frac{\lambda_3}{6}v^2\right)\psi_1\\
&\quad-\sqrt{N}u\left(m_2^2+\frac{\lambda_1}{6}u^2+\frac{\lambda_3}{6}v^2\right)-\frac{\lambda_3}{3}uv\sigma\\
&\quad-\frac{\lambda_1}{6N}\psi_i^2\psi_1-\frac{\lambda_3}{6N}\psi_1(\sigma^2+\phi_j^2)-\frac{\lambda_3}{3\sqrt{N}}v\psi_1\sigma\\
&\quad-\frac{\lambda_1}{6\sqrt{N}}u(3\psi_1^2+\psi_\alpha^2)-\frac{\lambda_3}{6\sqrt{N}}u(\sigma^2+\phi_j^2),
\end{aligned}\tag{4}$$

$$S[g]=\int_M d^4x\sqrt{-g}\left\{-\frac{1}{6}{}^{(4)}R+\frac{\Lambda}{3}\right\}$$
$$+S_\phi-\frac{1}{3}\int_{\partial M}d^3x\sqrt{h}K,$$

$$\begin{aligned}
|\Psi\rangle&=|O_0\rangle\left(\tfrac{1}{\sqrt{2}}|\uparrow\rangle+\tfrac{1}{\sqrt{2}}|\downarrow\rangle\right)|A_0\rangle|e_0\rangle\\
&\to|O_0\rangle\left(\tfrac{1}{\sqrt{2}}|\uparrow\rangle|A_\uparrow\rangle+\tfrac{1}{\sqrt{2}}|\downarrow\rangle|A_\downarrow\rangle\right)|e_0\rangle\quad\text{(apparatus measures)}\\
&\to|O_0\rangle\left(\tfrac{1}{\sqrt{2}}|\uparrow\rangle|A_\uparrow\rangle|e_\uparrow\rangle+\tfrac{1}{\sqrt{2}}|\downarrow\rangle|A_\downarrow\rangle|e_\downarrow\rangle\right)\quad\text{(decoherence)}\\
&=\tfrac{1}{\sqrt{2}}|O_0\rangle|\uparrow\rangle|A_\uparrow\rangle|e_\uparrow\rangle+\tfrac{1}{\sqrt{2}}|O_0\rangle|\downarrow\rangle|A_\downarrow\rangle|e_\downarrow\rangle\quad\text{(self-locating uncertainty)}\\
&\to\tfrac{1}{\sqrt{2}}|O_\uparrow\rangle|\uparrow\rangle|A_\uparrow\rangle|e_\uparrow\rangle+\tfrac{1}{\sqrt{2}}|O_\downarrow\rangle|\downarrow\rangle|A_\downarrow\rangle|e_\downarrow\rangle\quad\text{(measurement complete)}
\end{aligned}$$

$$\bar{\alpha}_u=\bar{\alpha}_3+\bar{\alpha}_4+\frac{1}{\delta}\ln\left(\frac{M_u}{M_x}\right)$$

$$E = \left(\phi^5/2\right)mc^2 \simeq mc^2/22$$

$$\nabla^2\Phi = 4\pi G\rho + \frac{\kappa_g}{4}\nabla^2\rho + \alpha_g\epsilon^{ij}\nabla_i\Phi\nabla_j\rho + \eta\rho^2 + \gamma\nabla\rho\cdot\nabla\rho + \epsilon_1\nabla\Phi\cdot\nabla\rho + \epsilon_2\Phi\nabla^2\rho + \epsilon_3\rho\nabla^2\Phi + \cdots$$

$$B_{n6k} = k\int_{h_1}^{h_2}\left[D_1^2 + \frac{(h-H_1)(D_2^2-D_1^2)}{H_2-H_1}\right]\left[\frac{SG_0}{e^{-\phi_0(1.57)}}e^{-\phi_0(h)}\right]dh$$

$$= \frac{kSG_0He^{(-\phi_0(-1.57+H_2+H_1)/H)}}{\Phi_2^2(-H_2+H_1)}$$

$$\times\left[\begin{array}{l} e^{(\phi_0H_2/H)}D_2^2H_2\phi_2 + e^{(\phi_0H_1/H)}D_2^2H - e^{(\phi_0H_1/H)}D_2^2H \\ -e^{(\phi_0H_1/H)}D_2^2H_1\phi_2 - e^{(\phi_0H_2/H)}D_1^2H_2\phi_2 + e^{(\phi_0H_2/H)}D_1^2H_1\phi_2 \\ -e^{(\phi_0H_1/H)}D_2^2H + e^{(\phi_0H_2/H)}D_1^2H \end{array}\right] + e \quad (22)$$

$$c^2 \to c^2\left(\frac{1-\beta}{1+\beta}\right)^2 = v^2$$

$$S = \frac{\pi\, Akc^3}{2hG}$$

SOURCE: WIKIPEDIA

G_{20} : Category one of LHS of the above equations

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of RHS of the above equations

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

The AdS/CFT conjecture postulates (eb) an exact equivalence between, specifically, type IIB string theory on ${\it AdS} _5\times {S}^5$ and maximally supersymmetric Yang-Mills theory on (eb)the boundary, and gives(eb) a concrete realization of the holographic principle

G_{24} : Category one of AdS/CFT conjecture

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of exact equivalence between, specifically, type IIB string theory on ${\it AdS} _5\times {S}^5$ and maximally supersymmetric Yang-Mills theory on (eb)the boundary, and gives(eb) a concrete realization of the holographic principle

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

exact equivalence between, specifically,

type IIB string theory on ${\it AdS} _5\times {S}^5$ and maximally supersymmetric Yang-Mills theory on (eb)the boundary, and gives(eb) a concrete realization of the holographic principle

G_{28} : Category one of type IIB string theory on ${\it AdS} _5\times {S}^5$

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of maximally supersymmetric Yang-Mills theory

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

G_{32} : Category one of AdS/CFT conjecture

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of realization of the holographic principle

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

The idea that the mathematical problems of Quantum Gravity might ultimately be intertwined with (e&eb) the difficulties of Quantum Field Theory hatched about forty years ago when the early dual resonance models for (e) strong interactions developed into string theory as (=) a candidate for a theory of all of fundamental physics

G_{36} : Category one of mathematical problems of Quantum Gravity; Quantum Field Theory

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of Quantum Field Theory; mathematical problems of Quantum Gravity

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

dual resonance models for (e) strong interactions developed into string theory as (=) a candidate for a theory of all of fundamental physics

G_{40} : Category one of strong interactions developed into string theory as (=) a candidate for a theory of all of fundamental physics

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of dual resonance models

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

dual resonance models for strong interactions developed into string theory as (=) a candidate for a theory of all of fundamental physics

G_{44} : Category one of dual resonance models for strong interactions developed into string theory

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of candidate for a theory of all of fundamental physics

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)}\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad (60)$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t) \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & +(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) & +(a''_{21})^{(3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t) \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & +(a''_{18})^{(2)}(T_{17},t) & +(a''_{15})^{(1,1,)}(T_{14},t) & +(a''_{22})^{(3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{38})^{(7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9)}(T_{45},t) \end{bmatrix} G_{18} \quad (63)$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{20})^{(3)}\,+(a''_{20})^{(3)}(T_{21},t)\,+(a''_{16})^{(2,2,2)}(T_{17},t)\,+(a''_{13})^{(1,1,1,)}(T_{14},t) \\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)\,+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)\,+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7,7)}(T_{37},t)\,+(a''_{40})^{(8,8,8,8)}(T_{41},t)\,+(a''_{44})^{(9,9,9)}(T_{45},t) \end{array}\right] G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c} (a'_{21})^{(3)}\,+(a''_{21})^{(3)}(T_{21},t)\,+(a''_{17})^{(2,2,2)}(T_{17},t)\,+(a''_{14})^{(1,1,1,)}(T_{14},t) \\ +(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)\,+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)\,+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{37})^{(7,7,7,7)}(T_{37},t)\,+(a''_{41})^{(8,8,8,8)}(T_{41},t)\,+(a''_{45})^{(9,9,9)}(T_{45},t) \end{array}\right] G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{22})^{(3)}\,+(a''_{22})^{(3)}(T_{21},t)\,+(a''_{18})^{(2,2,2)}(T_{17},t)\,+(a''_{15})^{(1,1,1,)}(T_{14},t) \\ +(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)\,+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)\,+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{38})^{(7,7,7,7)}(T_{37},t)\,+(a''_{42})^{(8,8,8,8)}(T_{41},t)\,+(a''_{46})^{(9,9,9)}(T_{45},t) \end{array}\right] G_{22} \quad (69)$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[\begin{array}{c} (b'_{20})^{(3)}\,-(b''_{20})^{(3)}(G_{23},t)\,-(b''_{16})^{(2,2,2)}(G_{19},t)\,-(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)\,-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)\,-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t)\,-(b''_{40})^{(8,8,8,8)}(G_{43},t)\,-(b''_{44})^{(9,9,9)}(G_{47},t) \end{array}\right] T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[\begin{array}{c} (b'_{21})^{(3)}\,-(b''_{21})^{(3)}(G_{23},t)\,-(b''_{17})^{(2,2,2)}(G_{19},t)\,-(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)\,-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)\,-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t)\,-(b''_{41})^{(8,8,8,8)}(G_{43},t)\,-(b''_{45})^{(9,9,9)}(G_{47},t) \end{array}\right] T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)} \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}, \boxed{(a''_{25})^{(4)}(T_{25},t)}, \boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* $1, 2\ 3$

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$

$are\ fourth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$
$are\ fifth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$
$are\ sixth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$
$are\ seventh\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$
$are\ eighth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

$Where\ -(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ $are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ $are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ $are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
$are\ eighth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad (81)$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \quad (84)$$

$where\ \boxed{-(b''_{28})^{(5)}(G_{31},t)}\ ,\boxed{-(b''_{29})^{(5)}(G_{31},t)}\ ,\boxed{-(b''_{30})^{(5)}(G_{31},t)}\ \ are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}\ ,\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}\ ,\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}\ are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}\ ,\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}\ ,\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}\ are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}\ ,\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}\ ,\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}\ ,\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}\ ,\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}\ ,\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}\ ,\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}\ ,\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}\ ,\boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}\ ,\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}\ ,\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}\ ,\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}\ ,\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad (87)$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}\ ,\boxed{+(a''_{33})^{(6)}(T_{33},t)}\ ,\boxed{+(a''_{34})^{(6)}(T_{33},t)}\ are\ first\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\ ,\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\ ,\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\ are\ second\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}\ ,\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}\ ,\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}\ \ are\ third\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\ ,\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\ ,\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\ ,\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\ ,\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation

coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$ Eighth augmentation coefficients

$+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)} & -(b''_{32})^{(6)}(G_{35},t) & -(b''_{28})^{(5,5,5)}(G_{31},t) & -(b''_{24})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{13})^{(1,1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)} & -(b''_{33})^{(6)}(G_{35},t) & -(b''_{29})^{(5,5,5)}(G_{31},t) & -(b''_{25})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{14})^{(1,1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & -(b''_{34})^{(6)}(G_{35},t) & -(b''_{30})^{(5,5,5)}(G_{31},t) & -(b''_{26})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{15})^{(1,1,1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{34} \quad 90$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,)}(G_{27},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3

$-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3

$-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2, and 3

$-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \quad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt}$$

$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt}$$

$$= (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt}$$
$$= (a_{42})^{(8)} G_{41}$$
$$- \left[\begin{array}{ccc} (a'_{42})^{(8)} \boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array} \right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$
$$(b_{40})^{(8)} T_{41} - \left[\begin{array}{ccc} (b'_{40})^{(8)} \boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{13}$$

$$\frac{dT_{41}}{dt} =$$
$$(b_{41})^{(8)} T_{40} - \left[\begin{array}{ccc} (b'_{41})^{(8)} \boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{14}$$

$$\frac{dT_{42}}{dt} =$$
$$(b_{42})^{(8)} T_{41} - \left[\begin{array}{ccc} (b'_{42})^{(8)} \boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{38}'')^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

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$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a_{44}')^{(9)}\boxed{+(a_{44}'')^{(9)}(T_{45},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a_{45}')^{(9)}\boxed{+(a_{45}'')^{(9)}(T_{45},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a_{46}')^{(9)}\boxed{+(a_{46}'')^{(9)}(T_{37},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{44}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{46}'')^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second

augmentation coefficient for category 1, 2 and 3
$+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3
$+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3
$+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3
$+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$
$$(b_{44})^{(9)}T_{45} -$$
$$\begin{bmatrix} (b_{44}')^{(9)}\left[-(b_{44}'')^{(9)}(G_{47},t)\right] & \left[-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{13}$$
$$\frac{dT_{45}}{dt} =$$
$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}\left[-(b_{45}'')^{(9)}(G_{47},t)\right] & \left[-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{14}$$
$$\frac{dT_{46}}{dt} =$$
$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}\left[-(b_{46}'')^{(9)}(G_{47},t)\right] & \left[-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{15}$$

Where $-(b_{44}'')^{(9)}(G_{47},t)$, $-(b_{45}'')^{(9)}(G_{47},t)$, $-(b_{46}'')^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3
$-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\ ,\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)},\ (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)},\ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}), t\big) \le \ (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}\,(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\,\big((G_{27}), t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)},\ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - \ T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30$ 122

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)}$$ 123

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}\left((G_{35}), t\right) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(SSSS) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(TTTT) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

132

(UUUU) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(VVVV)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(WWWW) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(XXXX) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)} \qquad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be

noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\ ,\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}<1 \qquad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+\ (\hat{A}_{40})^{(8)}+(\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \qquad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[\ (b_i)^{(8)}+(b_i')^{(8)}+\ (\hat{B}_{40})^{(8)}+(\hat{Q}_{40})^{(8)}\ (\hat{k}_{40})^{(8)}]<1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \qquad 146\text{A}$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t)-(a_i'')^{(9)}(T_{45},t)|\le(\hat{k}_{44})^{(9)}|T_{45}-\ T_{45}'|e^{-(\widehat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t)-(b_i'')^{(9)}\big((G_{47}),t\big)|<(\hat{k}_{44})^{(9)}||(G_{47})-(G_{47})'||e^{-(\widehat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$

and $(a_i'')^{(9)}(T_{45},t)$. (T_{45}',t) and (T_{45},t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)},(\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)}=1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},$ are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\ ,\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}<1$$

Definition of $(\hat{P}_{44})^{(9)},(\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},(\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)},(a_i')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(p_i)^{(9)},\ (r_i)^{(9)},i=44,45,46,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)}+(a_i')^{(9)}+\ (\hat{A}_{44})^{(9)}+\ (\hat{P}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)}+(b_i')^{(9)}+\ (\hat{B}_{44})^{(9)}+\ (\hat{Q}_{44})^{(9)}\ \ (\hat{k}_{44})^{(9)}]<1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\ ,T_i(0)$:

$$G_i(t)\leq\ \left(\hat{P}_{13}\right)^{(1)}e^{(\hat{M}_{13})^{(1)}t}\ ,\ \boxed{G_i(0)=G_i^0>0}$$

$$T_i(t)\leq\ (\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}\quad ,\quad \boxed{T_i(0)=T_i^0>0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\ ,T_i(0)$

$$G_i(t)\leq\ (\hat{P}_{16})^{(2)}e^{(\hat{M}_{16})^{(2)}t}\quad ,\quad G_i(0)=G_i^0>0$$

$$T_i(t)\leq\ (\hat{Q}_{16})^{(2)}e^{(\hat{M}_{16})^{(2)}t}\quad ,\quad T_i(0)=T_i^0>0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t)\leq\ (\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}t}\quad ,\quad G_i(0)=G_i^0>0$$

$$T_i(t)\leq\ (\hat{Q}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}t}\quad ,\quad T_i(0)=T_i^0>0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0)\ ,T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{\left(\hat{M}_{24}\right)^{(4)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{\left(\hat{M}_{24}\right)^{(4)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{\left(\hat{M}_{28}\right)^{(5)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{\left(\hat{M}_{28}\right)^{(5)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{\left(\hat{M}_{32}\right)^{(6)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{\left(\hat{M}_{32}\right)^{(6)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{\left(\hat{M}_{36}\right)^{(7)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{\left(\hat{M}_{36}\right)^{(7)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{\left(\hat{M}_{40}\right)^{(8)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{\left(\hat{M}_{40}\right)^{(8)} t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{\left(\hat{M}_{44}\right)^{(9)} t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{\left(\hat{M}_{44}\right)^{(9)} t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{13})^{(1)}, T_i^0 \le (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{16})^{(2)}, T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\tilde{M}_{16})^{(2)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\tilde{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^{0} + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{32})^{(6)}$, $T_i^0 \leq (\hat{Q}_{32})^{(6)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0,t)$

Proof: 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(q) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left(e^{(\tilde{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\tilde{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left(e^{(\tilde{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\tilde{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0\right) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\tilde{M}_{44})^{(9)}} \left(e^{(\tilde{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\tilde{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\tilde{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\tilde{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\tilde{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[(\widehat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on

$G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 19 to 24 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(1)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If $G_{13}\ \ is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}\,(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on

(G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)} t)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\,(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on T_{21} and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \quad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 227

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on T_{25} and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(4)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25}\ \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t\ > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25}\ \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25}\ \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for $G_{29}\ , G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\ , \frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\widehat{P}_{28})^{(5)}$ and $(\widehat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\widehat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \tag{234}$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \tag{235}$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric (236)

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{31}}),(\widetilde{T_{31}})$: $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\leq \tag{237}$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\ (G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(5)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} is bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)}+\left((\widehat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\widehat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\widehat{Q}_{32})^{(6)}\right]\leq(\widehat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(6)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose

$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})},\widetilde{(T_{39})}:\left(\widetilde{(G_{39})},\widetilde{(T_{39})}\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$

It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\leq \quad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\widehat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{40})^{(8)}\right] \leq (\widehat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})}, \widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by 274

$(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$. The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right)) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)}, , (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}, \ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}, \ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13} e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13} e^{(S_1)^{(1)} t}$

where $(p_i)^{(1)}$ is defined by equation

$\frac{1}{(m_1)^{(1)}} G^0_{13} e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G^0_{13} e^{(S_1)^{(1)} t}$

$$\left(\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t}-e^{-(S_2)^{(1)}t}\right]+G_{15}^{0}e^{-(S_2)^{(1)}t}\le G_{15}(t)\le\right. \quad (286)$$

$$\left.\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}\right]+G_{15}^{0}e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation (291)

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: (292)

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(T_{17},t)+(a''_{17})^{(2)}(T_{17},t)\le-(\sigma_1)^{(2)} \quad (293)$$

$$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)} \quad (294)$$

Definition of $(\nu_1)^{(2)},(\nu_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: (295)

By $(\nu_1)^{(2)}>0$, $(\nu_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0$, $(u_2)^{(2)}<0$ the roots (296)

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_1)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ (297)

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and (298)

Definition of $(\bar{\nu}_1)^{(2)},,(\bar{\nu}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: (299)

By $(\bar{\nu}_1)^{(2)}>0$, $(\bar{\nu}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0$, $(\bar{u}_2)^{(2)}<0$ the (300)

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_2)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ (301)

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t}\right] + T_{18}^0 e^{-(R_2)^{(2)}t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations
$(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$
and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by
$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, ***if*** $(\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, ***if*** $(\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, ***if*** $(\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$$

$$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(v^{(4)}\big)^2 + (\sigma_1)^{(4)} v^{(4)} - (a_{24})^{(4)} = 0$

and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)},,(\bar{v}_2)^{(4)},(\bar{u}_1)^{(4)},(\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$ and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)},(\mu_2)^{(4)},(v_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)},(\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, ***if*** $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)}G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)}G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)}t} - e^{-(a'_{26})^{(4)}t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)}t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^{0}}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t}-e^{-(R_2)^{(4)}t}\right]+T_{26}^{0}e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)},(S_2)^{(4)},(R_1)^{(4)},(R_2)^{(4)}$:- 337

Where $(S_1)^{(4)}=(a_{24})^{(4)}(m_2)^{(4)}-(a'_{24})^{(4)}$

$(S_2)^{(4)}=(a_{26})^{(4)}-(p_{26})^{(4)}$

$(R_1)^{(4)}=(b_{24})^{(4)}(\mu_2)^{(4)}-(b'_{24})^{(4)}$

$(R_2)^{(4)}=(b'_{26})^{(4)}-(r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)},(\sigma_2)^{(5)},(\tau_1)^{(5)},(\tau_2)^{(5)}$:

$(\sigma_1)^{(5)},(\sigma_2)^{(5)},(\tau_1)^{(5)},(\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)}\le-(a'_{28})^{(5)}+(a'_{29})^{(5)}-(a''_{28})^{(5)}(T_{29},t)+(a''_{29})^{(5)}(T_{29},t)\le-(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)}\le-(b'_{28})^{(5)}+(b'_{29})^{(5)}-(b''_{28})^{(5)}\big((G_{31}),t\big)-(b''_{29})^{(5)}\big((G_{31}),t\big)\le-(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)},(\nu_2)^{(5)},(u_1)^{(5)},(u_2)^{(5)},\nu^{(5)},u^{(5)}$: 339

By $(\nu_1)^{(5)}>0$, $(\nu_2)^{(5)}<0$ and respectively $(u_1)^{(5)}>0$, $(u_2)^{(5)}<0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2+(\sigma_1)^{(5)}\nu^{(5)}-(a_{28})^{(5)}=0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2+(\tau_1)^{(5)}u^{(5)}-(b_{28})^{(5)}=0$ and

Definition of $(\bar{\nu}_1)^{(5)},,(\bar{\nu}_2)^{(5)},(\bar{u}_1)^{(5)},(\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)}>0$, $(\bar{\nu}_2)^{(5)}<0$ and respectively $(\bar{u}_1)^{(5)}>0$, $(\bar{u}_2)^{(5)}<0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2+(\sigma_2)^{(5)}\nu^{(5)}-(a_{28})^{(5)}=0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2+(\tau_2)^{(5)}u^{(5)}-(b_{28})^{(5)}=0$

Definition of $(m_1)^{(5)},(m_2)^{(5)},(\mu_1)^{(5)},(\mu_2)^{(5)},(\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)},(m_2)^{(5)},(\mu_1)^{(5)},(\mu_2)^{(5)}$ by

$(m_2)^{(5)}=(\nu_0)^{(5)},(m_1)^{(5)}=(\nu_1)^{(5)}$, $\boldsymbol{if}\ (\nu_0)^{(5)}<(\nu_1)^{(5)}$

$(m_2)^{(5)}=(\nu_1)^{(5)},(m_1)^{(5)}=(\bar{\nu}_1)^{(5)}$, $\boldsymbol{if}\ (\nu_1)^{(5)}<(\nu_0)^{(5)}<(\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)}=\frac{G_{28}^{0}}{G_{29}^{0}}}$

$(m_2)^{(5)}=(\nu_1)^{(5)},(m_1)^{(5)}=(\nu_0)^{(5)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(5)}<(\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)}=(u_0)^{(5)},(\mu_1)^{(5)}=(u_1)^{(5)}$, $\boldsymbol{if}\ (u_0)^{(5)}<(u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, $\boldsymbol{if}$ $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}$ $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{28}(t) \leq G_{28}^0 e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{29}(t) \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \leq G_{30}(t) \leq \right. \quad 344$$

$$\left. \frac{(a_{30})^{(5)} G_{28}^0}{(m_2)^{(5)}((S_1)^{(5)} - (a'_{30})^{(5)})}\left[e^{(S_1)^{(5)} t} - e^{-(a'_{30})^{(5)} t}\right] + G_{30}^0 e^{-(a'_{30})^{(5)} t}\right)$$

$$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \leq T_{28}(t) \leq T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \leq T_{28}(t) \leq \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \leq T_{30}(t) \leq \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \leq -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \leq -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \leq -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}((G_{35}), t) - (b''_{33})^{(6)}((G_{35}), t) \leq -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}$, $(m_2)^{(6)}$, $(\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}$, $(m_2)^{(6)}$, $(\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t} - e^{-(a'_{34})^{(6)}t}\right] + G_{34}^0 e^{-(a'_{34})^{(6)}t}\right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}}T_{32}^0 e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(v_1)^{(7)}, (v_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, v^{(7)}, u^{(7)}$: 361

By $(v_1)^{(7)} > 0, (v_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{v}_1)^{(7)}, , (\bar{v}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)} > 0, (\bar{v}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)},\ \boldsymbol{if}\ (\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)},\ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}),t\big) - (b''_{45})^{(9)}\big((G_{47}),t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)},, (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, $\boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, $\boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, $\boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, $\boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,
and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44}e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44}e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G^0_{44}e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G^0_{44}e^{(S_1)^{(9)}t}$$

(

$$\frac{(a_{46})^{(9)}G^0_{44}}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})}\Big[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t}\Big] + G^0_{46}e^{-(S_2)^{(9)}t} \le G_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}G^0_{44}}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})}[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G^0_{46}e^{-(a'_{46})^{(9)}t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)} e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{dv^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)v^{(2)} - (a_{17})^{(2)}v^{(2)}$$

Definition of $v^{(2)}$:- $\boxed{v^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(v^{(2)}\right)^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)}\right) \leq \frac{dv^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(v^{(2)}\right)^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{v}_1)^{(2)}, (v_0)^{(2)}$:-

For $0 < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (v_1)^{(2)} < (\bar{v}_1)^{(2)}$

$$v^{(2)}(t) \geq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(v_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (v_2)^{(2)}}}$$

it follows $(v_0)^{(2)} \leq v^{(2)}(t) \leq (v_1)^{(2)}$

In the same manner , we get 391

$$v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{v}_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (\bar{v}_2)^{(2)}}}$$

From which we deduce $(v_0)^{(2)} \leq v^{(2)}(t) \leq (\bar{v}_1)^{(2)}$ 392

If $0 < (v_1)^{(2)} < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{v}_1)^{(2)}$ we find like in the previous case, 393

$$(v_1)^{(2)} \leq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_2)^{(2)}\right)t\right]}} \leq v^{(2)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \leq (\bar{v}_1)^{(2)}$$

If $0 < (v_1)^{(2)} \leq (\bar{v}_1)^{(2)} \leq (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(v_1)^{(2)} \leq v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \leq (v_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(2)}(t)$:- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then
$(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

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If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\quad \boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$v^{(6)}(t) \le \frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)}=\frac{(\bar{v}_1)^{(6)}-(v_0)^{(6)}}{(v_0)^{(6)}-(\bar{v}_2)^{(6)}}}$$

From which we deduce $(v_0)^{(6)} \le v^{(6)}(t) \le (\bar{v}_1)^{(6)}$

If $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{v}_1)^{(6)}$ we find like in the previous case, 414

$$(v_1)^{(6)} \le \frac{(v_1)^{(6)}+(C)^{(6)}(v_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)}-(v_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)}-(v_2)^{(6)}\right)t\right]}} \le v^{(6)}(t) \le$$

$$\frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \le (\bar{v}_1)^{(6)}$$

If $0 < (v_1)^{(6)} \le (\bar{v}_1)^{(6)} \le \boxed{(v_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(v_1)^{(6)} \le v^{(6)}(t) \le \frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \le (v_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(6)}(t)$:-

$$(m_2)^{(6)} \le v^{(6)}(t) \le (m_1)^{(6)}, \quad \boxed{v^{(6)}(t)=\frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \le u^{(6)}(t) \le (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t)=\frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(v_1)^{(6)} = (\bar{v}_1)^{(6)}$ if in addition $(v_0)^{(6)} = (v_1)^{(6)}$ then $v^{(6)}(t) = (v_0)^{(6)}$ and as a consequence $G_{32}(t) = (v_0)^{(6)}G_{33}(t)$ **this also defines** $(v_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(v_1)^{(6)}$ and $(\bar{v}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)}=\frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}\right)\leq\frac{dv^{(7)}}{dt}\leq-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0<\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}<(\nu_1)^{(7)}<(\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t)\geq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}\quad,\quad\boxed{(C)^{(7)}=\frac{(\nu_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)}\leq\nu^{(7)}(t)\leq(\nu_1)^{(7)}$

In the same manner , we get 417

$$\nu^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(7)}=\frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)}\leq\nu^{(7)}(t)\leq(\bar{\nu}_1)^{(7)}$

If $0<(\nu_1)^{(7)}<(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}<(\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)}\leq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}\leq\nu^{(7)}(t)\leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\bar{\nu}_1)^{(7)}$$

If $0<(\nu_1)^{(7)}\leq(\bar{\nu}_1)^{(7)}\leq\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)}\leq\nu^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)}\leq\nu^{(7)}(t)\leq(m_1)^{(7)},\quad\boxed{\nu^{(7)}(t)=\frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

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If $0 < (v_1)^{(8)} < (v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{v}_1)^{(8)}$ we find like in the previous case,

$$(v_1)^{(8)} \leq \frac{(v_1)^{(8)}+(C)^{(8)}(v_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((v_1)^{(8)}-(v_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((v_1)^{(8)}-(v_2)^{(8)}\right)t\right]}} \leq v^{(8)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(8)}+(\bar{C})^{(8)}(\bar{v}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}} \leq (\bar{v}_1)^{(8)}$$

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If $0 < (v_1)^{(8)} \leq (\bar{v}_1)^{(8)} \leq \boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain

$$(v_1)^{(8)} \leq v^{(8)}(t) \leq \frac{(\bar{v}_1)^{(8)}+(\bar{C})^{(8)}(\bar{v}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}} \leq (v_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(8)}(t)$:-

$$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(v_1)^{(8)} = (\bar{v}_1)^{(8)}$ if in addition $(v_0)^{(8)} = (v_1)^{(8)}$ then $v^{(8)}(t) = (v_0)^{(8)}$ and as a consequence $G_{40}(t) = (v_0)^{(8)}G_{41}(t)$ **this also defines** $(v_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(v_1)^{(8)}$ and $(\bar{v}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\quad \boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \leq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \leq \nu^{(9)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \leq (\bar{\nu}_1)^{(9)} \leq \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \leq \nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \leq \nu^{(9)}(t) \leq (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for**

the special case .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(v_1)^{(9)}$ and $(\bar{v}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a_i'')^{(1)} and\ (b_i'')^{(1)}$ are independent on t , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0$ 447

$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0$ 448

has a unique positive solution , which is an equilibrium solution

$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0$ 449

$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0$ 450

$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0$ 451

$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0$ 452

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(q) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T_{33}^{*} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^{*} for which $f(T_{33}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

Definition and uniqueness of T_{37}^{*} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^{*} for which $f(T_{37}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^{*})]}$$

Definition and uniqueness of T_{41}^{*} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^{*} for which $f(T_{41}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^{*})]}$$

Definition and uniqueness of T_{45}^{*} :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^{*} for which $f(T_{45}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^{*})]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi(G^{*}) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that

there exists a unique G_{37}^{*} such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)}G_{17}^{*}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)}G_{17}^{*}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T_{17}^{*})]} \quad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)}T_{17}^{*}}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)}T_{17}^{*}}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^{*})]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)}G_{21}^{*}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)}G_{21}^{*}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$$T_{28}^* = \frac{(b_{28})^{(5)} T_{29}^*}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^*)]} \quad , \quad T_{30}^* = \frac{(b_{30})^{(5)} T_{29}^*}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^*)]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^* given by $\varphi((G_{35})^*) = 0$, T_{33}^* given by $f(T_{33}^*) = 0$ and

$$G_{32}^* = \frac{(a_{32})^{(6)} G_{33}^*}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34}^* = \frac{(a_{34})^{(6)} G_{33}^*}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

$$T_{32}^* = \frac{(b_{32})^{(6)} T_{33}^*}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^*)]} \quad , \quad T_{34}^* = \frac{(b_{34})^{(6)} T_{33}^*}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^*)]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^* given by $\varphi((G_{39})^*) = 0$, T_{37}^* given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

G_{41}^* given by $\varphi((G_{43})^*) = 0$, T_{41}^* given by $f(T_{41}^*) = 0$ and

$$G_{40}^{*}=\frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}\quad,\quad G_{42}^{*}=\frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

$$T_{40}^{*}=\frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}\quad,\quad T_{42}^{*}=\frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$$

Finally we obtain the unique solution of 89 to 99 523
A

G_{45}^{*} given by $\varphi((G_{47})^{*})=0$, T_{45}^{*} given by $f(T_{45}^{*})=0$ and

$$G_{44}^{*}=\frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}\quad,\quad G_{46}^{*}=\frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$$

$$T_{44}^{*}=\frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}\quad,\quad T_{46}^{*}=\frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}\ and\ (b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i=G_i^{*}+\mathbb{G}_i\qquad,T_i=T_i^{*}+\mathbb{T}_i$$

$$\frac{\partial(a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^{*})=(q_{14})^{(1)}\quad,\ \frac{\partial(b_i'')^{(1)}}{\partial G_j}(\,G^{*}\,)=s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt}=-\big((a'_{13})^{(1)}+(p_{13})^{(1)}\big)\mathbb{G}_{13}+(a_{13})^{(1)}\mathbb{G}_{14}-(q_{13})^{(1)}G_{13}^{*}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt}=-\big((a'_{14})^{(1)}+(p_{14})^{(1)}\big)\mathbb{G}_{14}+(a_{14})^{(1)}\mathbb{G}_{13}-(q_{14})^{(1)}G_{14}^{*}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt}=-\big((a'_{15})^{(1)}+(p_{15})^{(1)}\big)\mathbb{G}_{15}+(a_{15})^{(1)}\mathbb{G}_{14}-(q_{15})^{(1)}G_{15}^{*}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt}=-\big((b'_{13})^{(1)}-(r_{13})^{(1)}\big)\mathbb{T}_{13}+(b_{13})^{(1)}\mathbb{T}_{14}+\textstyle\sum_{j=13}^{15}\big(s_{(13)(j)}T_{13}^{*}\mathbb{G}_j\big) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt}=-\big((b'_{14})^{(1)}-(r_{14})^{(1)}\big)\mathbb{T}_{14}+(b_{14})^{(1)}\mathbb{T}_{13}+\textstyle\sum_{j=13}^{15}\big(s_{(14)(j)}T_{14}^{*}\mathbb{G}_j\big) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt}=-\big((b'_{15})^{(1)}-(r_{15})^{(1)}\big)\mathbb{T}_{15}+(b_{15})^{(1)}\mathbb{T}_{14}+\textstyle\sum_{j=13}^{15}\big(s_{(15)(j)}T_{15}^{*}\mathbb{G}_j\big) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions

$(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \tag{532}$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \ , \ \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \tag{533}$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \tag{534}$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \tag{535}$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \tag{536}$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \tag{537}$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}')^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T_{17}^*\mathbb{G}_j\right) \tag{538}$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}')^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T_{18}^*\mathbb{G}_j\right) \tag{539}$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \ , \ \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}')^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \tag{541}$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}')^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \tag{542}$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}')^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \tag{543}$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}')^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T_{20}^*\mathbb{G}_j\right) \tag{544}$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^{*}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T_{22}^{*}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^{*}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^{*}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^{*}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^{*}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^{*}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^{*}\mathbb{G}_j\right) \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}(\,(G_{31})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^{*}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}(\,(G_{35})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^{*}) = (q_{41})^{(8)} \quad , \frac{\partial(b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^{*}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^{*}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^{*}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^{*}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \ , \ \frac{\partial(b''_i)^{(9)}}{\partial G_j}(\ (G_{47})^*\ ) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\ B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\ C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\ D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\ E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\ F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\ G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14}+(b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\Big]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION TWENTY

Virasoro And Super-Virasoro Algebra As The Gauge Algebra

INTRODUCTION—VARIABLES USED

Holographic Matrix Models and Quantum Space and Time The world as a hologram Leonard Susskind1 J. Math Phys. 36, 6377 (1995); http://dx.doi.org/10.1063/1.531249

(1) This duality has passed a large number of quantitative tests (including spectra of chiral and (e&eb) non-BPS operators,(e&eb) correlation functions, (e&eb) expectation values of Wilson loops, thermodynamic properties, and many more).

(2) The discovery that integrable structures are present in (eb) certain physical systems that occur in nature has led, since the late 1960's, to (eb) a tremendous amount of theoretical activity on (eb) these types of systems that are (=) both exactly solvable and display (e&eb) a remarkable stability, yet are genuinely non-linear interacting systems often with (e&eb) an infinite number of degrees of freedom.

(3) The main technique applicable to classical systems is (=) the inverse scattering-Riemann-Hilbert method, while quantum mechanical systems are typically solved by (e) factorization techniques (Bethe ansatz, etc.). Both of those groups of techniques have entered in an important way in recent developments in AdS/CFT.

(4) In fact, infinite symmetry algebras have always played (e&eb) an important role in string theory, and have shaped important parallel developments in field theory along the way.

(5) For example, the study of the **Virasoro and super-Virasoro algebra as (=) the gauge algebra** of perturbative string theory led to (eb) the development of vertex operator algebras, and 2-dimensional rational conformal field theories remain (eb) classic examples of exactly solvable models in quantum field theory.

(6) In a different way, integrable systems started playing a fundamental role (e&eb) in topological field and string theory after Witten and Kontsevich's work on (eb) intersection theory on the moduli space of algebraic curves.

NOTATION

Module One

Inspired in part by the discovery of D-branes and by the microscopic explanation of black hole entropy in string theory, Maldacena proposed **an**

{\it exact quantum equivalence} between type IIB string theory in anti-de Sitter space and (e&eb)maximally supersymmetric Yang-Mills theory on its boundary

G_{13} : Category one of**{\it exact quantum equivalence} between type IIB string theory in anti-de Sitter**

space

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of maximally supersymmetric Yang-Mills theory on its boundary

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

This duality has passed a large number of quantitative tests (including spectra of chiral and (e&eb) non-BPS operators,(e&eb) correlation functions, (e&eb) expectation values of Wilson loops, thermodynamic properties, and many more)

G_{16} : Category one of chiral; non-BPS operators

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of non-BPS operators ;chiral

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

The discovery that integrable structures are present in (eb) certain physical systems that occur in nature has led, since the late 1960's, to (eb) a tremendous amount of theoretical activity on (eb) these types of systems that are (=) both exactly solvable and display (e&eb) a remarkable stability, yet are genuinely non-linear interacting systems often with (e&eb) an infinite number of degrees of freedom

G_{20} : Category one of integrable structures

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of certain physical systems that occur in nature

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

non-linear interacting systems often with (e&eb) an infinite number of degrees of freedom

G_{24} : Category one of non-linear interacting systems; infinite number of degrees of freedom

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of infinite number of degrees of freedom; non-linear interacting systems

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

For example, the study of the
Virasoro and super-Virasoro algebra as (=) the gauge algebra of perturbative string theory led to (eb) the development of vertex operator algebras, and 2-dimensional rational conformal field theories remain (eb) classic examples of exactly solvable models in quantum field theory.

G_{28} : Category one of **Virasoro and super-Virasoro algebra**

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of **gauge algebra** of perturbative string theory

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Virasoro and super-Virasoro algebra as the gauge algebra of perturbative string theory led to (eb) the development of vertex operator algebras, and 2-dimensional rational conformal field theories remain (eb) classic examples of exactly solvable models in quantum field theory

G_{32} : Category one of **Virasoro and super-Virasoro algebra as the gauge algebra** of perturbative string theory

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of vertex operator algebras, and 2-dimensional rational conformal field theories remain (eb) classic examples of exactly solvable models in quantum field theory

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

de Sitter Space in Non-Critical String Theory Alexander Maloney, Eva Silverstein, Andrew Strominger

(1) Supercritical string theories in D>10 dimensions with no moduli are described, generalizing (e&eb) the asymmetric orientifold construction of one of the authors.
(2) By taking the number of dimensions to be large and turning on fluxes, dilaton potentials are (=) generated with nontrivial minima at (eb) arbitrarily small cosmological constant and D-dimensional

string coupling, separated by (e&eb) a barrier from a flat-space linear dilaton region, but possibly suffering from strong coupling problems.

(3) The general issue of the decay of a de Sitter vacuum to flat space is discussed. For relatively small barriers, such decays are described by (e) gravitational instantons.

(4) It is shown that for a sufficiently large potential barrier, the bubble wall crosses (e&eb) the horizon.

(5) At the same time the instanton decay time exceeds (e) the Poincare recurrence time.

(6) It is argued that the inclusion of such instantons is (=) neither physically meaningful nor consistent with basic principles such as causality.

(7) This raises the possibility that such de Sitter vacua are (=) effectively stable.

(8) In the case of the supercritical flux models, decays to (e&eb) the linear dilaton region can be forbidden by (e) such large barriers, but decays to lower flux vacua including AdS minima nevertheless proceed (eb) consistently with this criterion.

(9) These models provide concrete examples in which cosmological constant reduction by (e) flux relaxation can be explored Subjects: High Energy Physics - Theory (hep-th) Report number: SLAC-PUB-9228, HUTP-02/A019 Cite as: arXiv: hep-th/0205316

Supercritical string theories in D>10 dimensions with no moduli are described, generalizing (e&eb) the asymmetric orientifold construction of one of the authors

G_{36} : Category one of Supercritical string theories in D>10 dimensions with no moduli; asymmetric orientifold construction

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of asymmetric orientifold construction ;Supercritical string theories in D>10 dimensions with no moduli

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

number of dimensions to be large and turning on fluxes dilaton potentials are (=) generated with nontrivial minima at (eb) arbitrarily small cosmological constant and D-dimensional string coupling, separated by (e&eb) a barrier from a flat-space linear dilaton region, but possibly suffering from strong coupling problems

G_{40} : Category one of number of dimensions to be large and turning on fluxes

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of dilaton potentials

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

number of dimensions to be large and turning on fluxes dilaton potentials are generated with nontrivial minima at (eb) arbitrarily small cosmological constant and D-dimensional string coupling, separated by (e&eb) a barrier from a flat-space linear dilaton region, but possibly suffering from strong coupling problems

G_{44} : Category one of number of dimensions to be large and turning on fluxes dilaton potentials are generated; nontrivial minima at (eb) arbitrarily small cosmological constant and D-dimensional string coupling, separated by (e&eb) a barrier from a flat-space linear dilaton region, but possibly suffering from strong coupling problems

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of nontrivial minima at (eb) arbitrarily small cosmological constant and D-dimensional string coupling, separated by (e&eb) a barrier from a flat-space linear dilaton region, but possibly suffering from strong coupling problems; number of dimensions to be large and turning on fluxes dilaton potentials are generated

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION TWENTY A

de Sitter Space And Non-Critical String Theory

INTRODUCTION—VARIABLES USED

de Sitter Space in Non-Critical String Theory Alexander Maloney, Eva Silverstein, Andrew Strominger

(1) The general issue of the decay of a de Sitter vacuum to flat space is discussed. For relatively small barriers, such decays are described by (e) gravitational instantons.
(2) It is shown that for a sufficiently large potential barrier, the bubble wall crosses (e&eb) the horizon.
(3) At the same time the instanton decay time exceeds (e) the Poincare recurrence time.
(4) It is argued that the inclusion of such instantons is (=) neither physically meaningful nor consistent with basic principles such as causality.
(5) This raises the possibility that such de Sitter vacua are (=) effectively stable.
(6) In the case of the supercritical flux models, decays to (e&eb) the linear dilaton region can be forbidden by (e) such large barriers, but decays to lower flux vacua including AdS minima nevertheless proceed (eb) consistently with this criterion.
(7) These models provide concrete examples in which cosmological constant reduction by (e) flux relaxation can be explored Subjects: High Energy Physics - Theory (hep-th) Report number: SLAC-

PUB-9228, HUTP-02/A019 Cite as: arXiv: hep-th/0205316

NOTATION

Module One

number of dimensions to be large and turning on fluxes dilaton potentials are generated with nontrivial minima at arbitrarily small cosmological constant and D-dimensional string coupling, separated by (e&eb) a barrier from a flat-space linear dilaton region, but possibly suffering from strong coupling problems

G_{13} : Category one of number of dimensions to be large and turning on fluxes dilaton potentials are generated with nontrivial minima at arbitrarily small cosmological constant and D-dimensional string coupling ; barrier from a flat-space linear dilaton region, but possibly suffering from strong coupling problems

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of barrier from a flat-space linear dilaton region, but possibly suffering from strong coupling problems ;number of dimensions to be large and turning on fluxes dilaton potentials are generated with nontrivial minima at arbitrarily small cosmological constant and D-dimensional string coupling

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

The general issue of the decay of a de Sitter vacuum to flat space is discussed. For relatively

such decays for small barriers are described by (e) gravitational instantons

G_{16} : Category one of gravitational instantons

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of such decays for small barriers

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

It is shown that for a sufficiently large potential barrier, the bubble wall crosses (e&eb) the horizon

G_{20} : Category one of bubble wall for a sufficiently large potential barrier; horizon

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of horizon; bubble wall for a sufficiently large potential barrier

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

. At the same time the

instanton decay time exceeds (e) the Poincare recurrence time

G_{24} : Category one of Poincare recurrence time

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of instanton decay time

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

It is argued that the

inclusion of such instantons is (=) neither physically meaningful nor consistent with basic principles such as causality

G_{28} : Category one of inclusion of such instantons

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of neither physically meaningful nor consistent with basic principles such as causality

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

This raises the possibility that

such de Sitter vacua are (=) effectively stable

G_{32} : Category one of such de Sitter vacua

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of effectively stable

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

decays In the case of the supercritical flux models to (e&eb) the linear dilaton region can be forbidden by (e) such large barriers, but decays to lower flux vacua including AdS minima nevertheless proceed (eb) consistently with this criterion

G_{36} : Category one of decays In the case of the supercritical flux models; linear dilaton region can be forbidden by (e) such large barriers, but decays to lower flux vacua including AdS minima nevertheless proceed (eb) consistently with this criterion

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of linear dilaton region can be forbidden by (e) such large barriers, but decays to lower flux vacua including AdS minima nevertheless proceed (eb) consistently with this criterion; decays In the case of the supercritical flux models

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

decays In the case of the supercritical flux models to the linear dilaton region can be forbidden by (e) such large barriers, but decays to lower flux vacua including AdS minima nevertheless proceed (eb) consistently with this criterion

G_{40} : Category one of such large barriers, but decays to lower flux vacua including AdS minima nevertheless proceed (eb) consistently with this criterion

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of decays In the case of the supercritical flux models to the linear dilaton region

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

decays In the case of the supercritical flux models to the linear dilaton region can be forbidden by such large barriers, but decays to lower flux vacua including AdS minima nevertheless proceed (eb) consistently with this criterion

G_{44} : Category one of decays In the case of the supercritical flux models to the linear dilaton region can be forbidden by such large barriers, but decays to lower flux vacua including AdS minima

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of consistently with this criterion

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION TWENTY ONE

Four-Dimensional Supersymmetric And Two-Dimensional Integrable Quantum Field Theories

INTRODUCTION—VARIABLES USED

Holographic Matrix Models and Quantum Space and Time The world as a hologram Leonard Susskind1 J. Math Phys. 36, 6377 (1995); http://dx.doi.org/10.1063/1.531249

(1) The appearance of integrable systems on the two sides of (e) the AdS/CFT correspondence, as well as the remarkable relation between **four-dimensional supersymmetric and (e&eb)two-dimensional integrable quantum field theories i**nitiated by Alday-Gaiotto-Tachikawa and Nekrasov-Shatashvili are but the most recent in a long chain of connections.

(2) For a comprehensive review of integrable structures in AdS/CFT, see \cite{beisertetal}. In field theory, the calculation of the spectrum of anomalous dimensions in (eb) the planar limit of ${\mathcal N} =4$ super-Yang-Mills is identical to the (=) determination of the spectrum of the Hamiltonian of an integrable spin chain, generalizing (eb) the Heisenberg spin chain (Minahan-Zarembo, Beisert-Staudacher).

(3) On the gravity side, the string worldsheet theory inherits (e) infinite-dimensional symmetry algebras from (e) the highly symmetric target space of (e) the sigma-model (Metsaev-Tseytlin, Bena-Polchinski-Roiban).

(4) Exploitation of integrability allows (eb) testing the correspondence in (eb) regimes of parameter space that extend beyond (eb) standard approximation schemes of either theory.

(5) Exploitation of integrability allows (eb) testing the correspondence in (eb) regimes of parameter space that extend beyond (eb) standard approximation schemes of either theory

(6) Later advances in this field have led to the formulation of exact equations for (e) the full spectrum of anomalous dimensions of (e) ${\mathcal N} =4$ SYM and ABJM models (AdS/CFT Y-system, thermodynamic Bethe Ansatz, and computation of the spectral curve) as well as to considerable advances in the computation of various physical quantities: correlation functions, Wilson loops, and scattering amplitudes. For the first time, we are close to a rather complete solution of (e) a gauge theory in a physical dimension.

(7) Integrable structures have also surfaced during the calculation of scattering amplitudes in (eb) maximally supersymmetric Yang-Mills theory, following Witten's reformulation of (e) perturbative Yang-Mills theory as (=) a string theory in twistor space. **Holographic Matrix Models and Quantum Space and Time The world as a hologram Leonard Susskind1 J. Math Phys. 36, 6377 (1995); http://dx.doi.org/10.1063/1.531249**

NOTATION

Module One

number of dimensions to be large and turning on fluxes dilaton potentials are generated with nontrivial minima at arbitrarily small cosmological constant and D-dimensional string coupling, separated by (e&eb) a barrier from a flat-space linear dilaton region, but possibly suffering from strong coupling problems

G_{13} : Category one of number of dimensions to be large and turning on fluxes dilaton potentials are generated with nontrivial minima at arbitrarily small cosmological constant and D-dimensional string coupling; barrier from a flat-space linear dilaton region, but possibly suffering from strong coupling problems

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of barrier from a flat-space linear dilaton region, but possibly suffering from strong coupling problems; number of dimensions to be large and turning on fluxes dilaton potentials are generated with nontrivial minima at arbitrarily small cosmological constant and D-dimensional string coupling

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

The appearance of integrable systems on the two sides of (e) the AdS/CFT correspondence, as well as the remarkable relation between **four-dimensional supersymmetric and (e&eb)two-dimensional integrable quantum field theories i**nitiated by Alday-Gaiotto-Tachikawa and Nekrasov-Shatashvili are but the most recent in a long chain of connections

G_{16} : Category one of Ads/CFT correspondence, as well as the remarkable relation between **four-dimensional supersymmetric and (e&eb)two-dimensional integrable quantum field theories i**nitiated by Alday-Gaiotto-Tachikawa and Nekrasov-Shatashvili are but the most recent in a long chain of connections

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of appearance of integrable systems on the two sides

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

appearance of integrable systems on the two sides of the AdS/CFT correspondence, as well as the remarkable relation between **four-dimensional supersymmetric and (e&eb) two-dimensional integrable quantum field theories i**nitiated by Alday-Gaiotto-Tachikawa and Nekrasov-Shatashvili are but the most recent in a long chain of connections

G_{20} : Category one of appearance of integrable systems on the two sides of the AdS/CFT correspondence, as well as the relation between **four-dimensional supersymmetric; two-dimensional integrable quantum field theories i**nitiated by Alday-Gaiotto-Tachikawa and Nekrasov-Shatashvili are but the most recent in a long chain of connections

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of **two-dimensional integrable quantum field theories i**nitiated by Alday-Gaiotto-Tachikawa and Nekrasov-Shatashvili are but the most recent in a long chain of connections ;appearance of integrable systems on the two sides of the AdS/CFT correspondence, as well as the remarkable relation between **four-dimensional supersymmetric**

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

For a comprehensive review of integrable structures in AdS/CFT, see \cite{beisertetal}. In field theory, the

calculation of the spectrum of anomalous dimensions in (eb) the planar limit of ${\mathcal N} =4$ super-Yang-Mills is identical to the (=) determination of the spectrum of the Hamiltonian of an integrable spin chain, generalizing (eb) the Heisenberg spin chain (Minahan-Zarembo, Beisert-Staudacher)

G_{24} : Category one of calculation of the spectrum of anomalous dimensions

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of planar limit of ${\mathcal N} =4$ super-Yang-Mills is identical to the (=) determination of the spectrum of the Hamiltonian of an integrable spin chain, generalizing (eb) the Heisenberg spin chain (Minahan-Zarembo, Beisert-Staudacher

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

calculation of the spectrum of anomalous dimensions in the planar limit of ${\mathcal N} =4$ super-Yang-Mills is identical to the (=) determination of the spectrum of the Hamiltonian of an integrable spin chain, generalizing (eb) the Heisenberg spin chain (Minahan-Zarembo, Beisert-Staudacher

G_{28} : Category one of calculation of the spectrum of anomalous dimensions in the planar limit of ${\mathcal N} =4$ super-Yang-Mills

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of determination of the spectrum of the Hamiltonian of an integrable spin chain, generalizing (eb) the Heisenberg spin chain (Minahan-Zarembo, Beisert-Staudacher

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

calculation of the spectrum of anomalous dimensions in the planar limit of ${\mathcal N} =4$ super-Yang-Mills is identical to the determination of the spectrum of the Hamiltonian of an integrable spin chain, generalizing (eb) the Heisenberg spin chain (Minahan-Zarembo, Beisert-Staudacher

G_{32} : Category one of calculation of the spectrum of anomalous dimensions in the planar limit of ${\mathcal N} =4$ super-Yang-Mills is identical to the determination of the spectrum of the Hamiltonian of an integrable spin chain; Heisenberg spin chain (Minahan-Zarembo, Beisert-Staudacher

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of Heisenberg spin chain (Minahan-Zarembo, Beisert-Staudacher ;calculation of the spectrum of anomalous dimensions in the planar limit of ${\mathcal N} =4$ super-Yang-Mills is identical to the determination of the spectrum of the Hamiltonian of an integrable spin chain

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Exploitation of integrability allows (eb) testing the correspondence in (eb) regimes of parameter space that extend beyond (eb) standard approximation schemes of either theory

G_{36} : Category one of Exploitation of integrability

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of testing the correspondence in (eb) regimes of parameter space that extend beyond (eb) standard approximation schemes of either theory

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

Exploitation of integrability allows testing the correspondence in (eb) regimes of parameter space that extend beyond (eb) standard approximation schemes of either theory

G_{40} : Category one of Exploitation of integrability allows testing the correspondence

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of regimes of parameter space that extend beyond (eb) standard approximation schemes of either theory

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Exploitation of integrability allows testing the correspondence in regimes of parameter space that extend beyond (eb) standard approximation schemes of either theory

G_{44} : Category one of standard approximation schemes of either theory

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of Exploitation of integrability allows testing the correspondence in regimes of parameter space

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION TWENTY TWO

Scattering Amplitudes In Maximally Supersymmetric Yang-Mills Theory

INTRODUCTION—VARIABLES USED

Holographic Matrix Models and Quantum Space and Time The world as a hologram Leonard Susskind1 J. Math Phys. 36, 6377 (1995); http://dx.doi.org/10.1063/1.531249

(1) Integrable structures have also surfaced during the calculation of scattering amplitudes in (eb) maximally supersymmetric Yang-Mills theory, following Witten's reformulation of (e) perturbative Yang-Mills theory as (=) a string theory in twistor space

(2) They are also instrumental in the exact calculation of expectation values of (e) Wilson loops and the corresponding dual minimal world-sheets.

(3) Work on AdS/CFT is motivated by (e) some of the most fundamental questions in (eb) contemporary theoretical physics: the solution of the conundrums associated with (e&eb) black holes and early universe cosmology on the quantum gravity side, and (e&eb) the elevation of quantum Yang-Mills theory to a mathematically rigorous framework on the field theory side

(4) The problem is phrased as follows: Yang–Mills Existence and Mass Gap. Prove that a non-trivial quantum Yang–Mills theory exists on (eb) \mathbb{R}^4 for (e) any compact simple gauge group G, and has (e) a mass gap $\Delta > 0$. Existence includes establishing axiomatic properties at least as strong as those cited in Streater & Wightman (1964), Osterwalder & Schrader (1973) and Osterwalder & Schrader (1975). ***We assume the correctness of the proposition and give a model. Note that there will be contradiction in the dovetailing mathematical exposition, in the eventuality of nonconformal ity to the assumption.***

(5) The importance of these problems can be succinctly summarized in the attention they have caught as targets of some prestigious modern Science Prizes: ``Yang-Mills existence and mass gap'' is one

of the 6 unsolved Clay Millenium Prize problems, and the list of Laureates of the new ``Fundamental Physics Prize" includes many of the pioneers and original contributors in the field of quantum gravity, AdS/CFT, black holes, and quantum cosmology. Since the initial proposal by Maldacena, AdS/CFT has had a number of stunning partial successes in addressing these problems (apart from the development and the tests of the conjecture itself), and has fundamentally changed the way in which they are approached. Among other things, AdS/CFT has \\ $\bullet$ shown (eb) new ways to calculate microscopically the entropy of black holes \\ $\bullet$ provided (e) a new perspective on (eb) confinement and chiral symmetry breaking \\ $\bullet$ opened (eb) calculational windows into properties of strongly coupled gauge theory plasmas \\ $\bullet$ given a non-perturbative definition of string theory on (eb) Anti-de-Sitter space \\ $\bullet$ essentially settled the debate about (e) unitarity of black hole evaporation \\ $\bullet$ indicated (eb) new ``holographic" approaches to (e) early universe cosmology \\ $\bullet$ sharpened definitions of (e) the notion of the ``wave function of the universe" \\ $\bullet$ triggered new ideas for beyond the standard model particle physics (extra dimensions, etc.)

(6) Physicists' interest in AdS/CFT has also triggered a number of exciting developments in pure mathematics, for instance in conformal geometry and the spectral geometry of certain Einstein manifolds, in both Minkowski and Euclidean signature. Moreover, thanks to the familiar fact that supersymmetric solutions of supergravity can be characterized (eb) algebraically (Calabi-Yau manifolds as cones over Sasaki-Einstein manifolds being the prototypical example), the subject has also connected up with (e&eb) a number of developments in algebraic geometry, the representation theory of certain quiver algebras, and many more. Finally, versions of the correspondence that operate purely within the realm of topological quantum theory have been put forward, such as the relation between topological strings and Chern-Simons gauge theory or matrix models.

(7) These correspondences in fact connect up with (e&eb) the circle of ideas relating moduli problems, intersection theory on (eb) moduli spaces, (less than maximal) supersymmetric gauge theory with (e&eb) algebraic integrable systems, classical and quantum.

NOTATION

Module One

Integrable structures have also surfaced during the calculation of scattering amplitudes in (eb) maximally supersymmetric Yang-Mills theory, following Witten's reformulation of (e) perturbative Yang-Mills theory as (=) a string theory in twistor space.

Holographic Matrix Models and Quantum Space and Time The world as a hologram Leonard Susskind1 J. Math Phys. 36, 6377 (1995); http://dx.doi.org/10.1063/1.531249

G_{13} : Category one of Integrable structures have also surfaced during the calculation of scattering amplitudes

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of maximally supersymmetric Yang-Mills theory, following Witten's reformulation of (e) perturbative Yang-Mills theory as (=) a string theory in twistor space.

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Integrable structures have also surfaced during the calculation of scattering amplitudes in (eb) maximally supersymmetric Yang-Mills theory, following Witten's reformulation of (e)

perturbative Yang-Mills theory as (=) a string theory in twistor space

G_{16} : Category one of perturbative Yang-Mills theory

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of string theory in twistor space

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

They are also instrumental in the exact calculation of expectation values of (e) Wilson loops and the corresponding dual minimal world-sheets.

G_{20} : Category one of Wilson loops and the corresponding dual minimal world-sheets.

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of exact calculation of expectation values

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Work on AdS/CFT is motivated by (e) some of the most fundamental questions in (eb) contemporary theoretical physics: the

solution of the conundrums associated with (e&eb) black holes and early universe cosmology on the quantum gravity side, and (e&eb) the elevation of quantum Yang-Mills theory to a mathematically rigorous framework on the field theory side

G_{24} : Category one of solution of the conundrums; black holes and early universe cosmology on the quantum gravity side, and (e&eb) the elevation of quantum Yang-Mills theory to a mathematically rigorous framework on the field theory side

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of black holes and early universe cosmology on the quantum gravity side, and (e&eb) the elevation of quantum Yang-Mills theory to a mathematically rigorous framework on the field theory side ;solution of the conundrums

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

solution of the conundrums associated with black holes and early universe cosmology on the quantum gravity side, and (e&eb) the elevation of quantum Yang-Mills theory to a mathematically rigorous framework on the field theory side

G_{28} : Category one of solution of the conundrums associated with black holes and early universe cosmology on the quantum gravity side; elevation of quantum Yang-Mills theory to a mathematically rigorous framework on the field theory side

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of elevation of quantum Yang-Mills theory to a mathematically rigorous framework on the field theory side; solution of the conundrums associated with black holes and early universe cosmology on the quantum gravity side

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

The problem is phrased as follows: Yang–Mills Existence and Mass Gap. Prove that a

non-trivial quantum Yang–Mills theory exists on (eb) \mathbb{R}^4 for (e) any compact simple gauge group G, and has (e) a mass gap $\Delta > 0$. Existence includes establishing axiomatic properties at least as strong as those cited in Streater & Wightman (1964), Osterwalder & Schrader (1973) and Osterwalder & Schrader (1975).

We assume the correctness of the proposition and give a model. Note that there will be contradiction in the dovetailing mathematical exposition, in the eventuality of nonconformal ity to the assumption

G_{32} : Category one of non-trivial quantum Yang–Mills theory exists

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of\mathbb{R}^4 for (e) any compact simple gauge group G, and has (e) a mass gap $\Delta > 0$.

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Non-trivial quantum Yang–Mills theory exists on \mathbb{R}^4 for (e) any compact simple gauge group G, and has (e) a mass gap $\Delta > 0$.

Existence includes establishing axiomatic properties at least as strong as those cited in Streater & Wightman (1964), Osterwalder & Schrader (1973) and Osterwalder & Schrader (1975).

G_{36} : Category one of mass gap $\Delta > 0$.

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of non-trivial quantum Yang–Mills theory exists on \mathbb{R}^4 for (e) any compact simple gauge group G,

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

The importance of these problems can be succinctly summarized in the attention they have caught as targets of some prestigious modern Science Prizes: `
`Yang-Mills existence and mass gap" is one of the 6 unsolved Clay Millenium Prize problems, and the list of Laureates of the new ``Fundamental Physics Prize" includes many of the pioneers and original contributors in the field of quantum gravity, AdS/CFT, black holes, and quantum cosmology.

Since the initial proposal by Maldacena, AdS/CFT has had a number of stunning partial successes in addressing these problems (apart from the development and the tests of the conjecture itself), and has fundamentally changed the way in which they are approached.

Among other things, AdS/CFT has \\ $\bullet$ shown (eb) new ways to calculate microscopically the entropy of black holes \\ $\bullet$ provided (e) a new perspective on (eb) confinement and chiral symmetry breaking \\ $\bullet$ opened (eb) calculational windows into properties of strongly coupled gauge theory plasmas \\ $\bullet$ given a non-perturbative definition of string theory on (eb) Anti-de-Sitter space \\ $\bullet$ essentially settled the debate about (e) unitarity of black hole evaporation

\\ $\bullet$ indicated (eb) new ``holographic" approaches to (e) early universe cosmology \\ $\bullet$ sharpened definitions of (e) the notion of the ``wave function of the universe" \\ $\bullet$ triggered new ideas for beyond the standard model particle physics (extra dimensions, etc.)

G_{40} : Category one of AdS/CFT has \\ $\bullet$

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of new ways to calculate microscopically the entropy of black holes \\ $\bullet$ provided (e) a new perspective on (eb) confinement and chiral symmetry breaking \\ $\bullet$ opened (eb) calculational windows into properties of strongly coupled gauge theory plasmas \\ $\bullet$ given a non-perturbative definition of string theory on (eb) Anti-de-Sitter space \\ $\bullet$ essentially settled the debate about (e) unitarity of black hole evaporation

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Among other things,

AdS/CFT has \\ $\bullet$ shown new ways to calculate microscopically the entropy of black holes \\ $\bullet$ provided (e) a new perspective on (eb) confinement and chiral symmetry breaking \\ $\bullet$ opened (eb) calculational windows into properties of strongly coupled gauge theory plasmas \\ $\bullet$ given a non-perturbative definition of string theory on (eb) Anti-de-Sitter space \\ $\bullet$ essentially settled the debate about (e) unitarity of black hole evaporation

G_{44} : Category one of new perspective on (eb) confinement and chiral symmetry breaking \\ $\bullet$ opened (eb) calculational windows into properties of strongly coupled gauge theory plasmas \\ $\bullet$ given a non-perturbative definition of string theory on (eb) Anti-de-Sitter space \\ $\bullet$ essentially settled the debate about (e) unitarity of black hole evaporation

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of AdS/CFT has \\ $\bullet$ shown new ways to calculate microscopically the entropy of black holes \\ $\bullet$

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,

are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right]G_{13} \quad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14},t)\right]G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14},t)\right]G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G,t)\right]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t)\right]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}),t\big)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}),t\big)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}),t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\big((G_{19}),t\big) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}), t)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}), t)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}), t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}), t) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}), t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & \boxed{+(a''_{13})^{(1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & \boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & \boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation

coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\left[-(b''_{13})^{(1)}(G,t)\right] & \left[-(b''_{16})^{(2,2,)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,)}(G_{35},t)\right] \\ \left[-(b''_{36})^{(7,7,)}(G_{39},t)\right] & \left[-(b''_{40})^{(8,8)}(G_{43},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\left[-(b''_{14})^{(1)}(G,t)\right] & \left[-(b''_{17})^{(2,2,)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,)}(G_{35},t)\right] \\ \left[-(b''_{37})^{(7,7,)}(G_{39},t)\right] & \left[-(b''_{41})^{(8,8)}(G_{43},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\left[-(b''_{15})^{(1)}(G,t)\right] & \left[-(b''_{18})^{(2,2,)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,)}(G_{35},t)\right] \\ \left[-(b''_{38})^{(7,7,)}(G_{39},t)\right] & \left[-(b''_{42})^{(8,8)}(G_{43},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & \boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)} & \boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} & \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)} & \boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} & \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)} & \boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} & \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)} & \boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{24} \quad 73$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)} & \boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{25} \quad 74$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)} & \boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
are sixth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
are seventh augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
are eighth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)} & \boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad 76$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)} & \boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad (78)$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
are fourth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad (81)$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation

coefficients for category 1,2, and 3
$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3
$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3
$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3
$+(a''_{40})^{(8,8\ ,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3
$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3
$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3
$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3
$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3
$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3
$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3
$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3
$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3
$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \quad 85$$

$$- \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}, \boxed{(a''_{37})^{(7)}(T_{37},t)}, \boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b_{36}')^{(7)}\boxed{-(b_{36}'')^{(7)}(G_{39},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b_{37}')^{(7)}\boxed{-(b_{37}'')^{(7)}(G_{39},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b_{38}')^{(7)}\boxed{-(b_{38}'')^{(7)}(G_{39},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{matrix} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{matrix}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{matrix} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{matrix}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{matrix} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{matrix}\right] G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\,-(b''_{40})^{(8)}(G_{43},t) & -(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\,-(b''_{41})^{(8)}(G_{43},t) & -(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\,-(b''_{42})^{(8)}(G_{43},t) & -(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)}\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)}\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)}\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq \ (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = \ (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \ , \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (a_i)^{(1)} + (a_i')^{(1)} + \ (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i')^{(1)} + \ (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \leq (p_i)^{(2)} \leq \left(\hat{A}_{16}\right)^{(2)} \qquad 102$$

$$(b_i'')^{(2)}(G_{19},t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\big((G_{19}),t\big) = \ (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\big((G_{19}),t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\big[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\big]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}} \ , \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\,(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + \ (\hat{P}_{16})^{(2)}\,(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + \ (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \le (p_i)^{(3)} \le (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \le (r_i)^{(3)} \le (b_i')^{(3)} \le (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\widehat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\widehat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\widehat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[\ (a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\,(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and \ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\,(a_i)^{(6)} + (a_i')^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)}\,(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(YYYY) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(ZZZZ) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$(a_i'')^{(7)}(T_{37},t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$

$(b_i'')^{(7)}(G_{39},t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$

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(AAAAA) $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

(BBBBB)

$\lim_{G\to\infty}(b_i'')^{(7)}((G_{39}),t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$

$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}((G_{39}),t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$. (T_{37}',t) and (T_{37},t) are points belonging to the interval $\left[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\right]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(CCCCC) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(DDDDD) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$

$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$

Where we suppose

$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$ 138

$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$ 139

$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$ 140

$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t}$ 142

$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t}$ 143

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ 144

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}$, $(r_i)^{(8)}, i = 40,41,42$,
Satisfy the inequalities

$\frac{1}{(\hat{M}_{40})^{(8)}} [(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 145

$\frac{1}{(\hat{M}_{40})^{(8)}} [(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 146

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\hat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)} ||(G_{47}) - (G_{47})'|| e^{-(\hat{M}_{44})^{(9)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}\left[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)} (\hat{k}_{44})^{(9)}\right] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t} \ , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t} \ , \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \ , \ T_i(0) = T_i^0 \ , \ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} , T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\overline{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''^{(9)}_{44} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad (169)$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that (170)

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that (171)

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that (172)

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that (173)

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that (174)

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \leq G_{32}^{0}+\int_0^t\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)} =$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(r) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \leq G_{36}^{0}+\int_0^t\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)} =$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \leq G_{40}^{0}+\int_0^t\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)} = \quad 180$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t} \leq \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for G_{41} , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \leq G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \leq \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for G_{45} , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \left\{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\right\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)} \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} +$$

$$(a''_{13})^{(1)} \left(T_{14}^{(1)}, s_{(13)}\right) \left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}} +$$

$$G_{13}^{(2)} \left|(a''_{13})^{(1)} \left(T_{14}^{(1)}, s_{(13)}\right) - (a''_{13})^{(1)} \left(T_{14}^{(2)}, s_{(13)}\right)\right| \; e^{-(\widehat{M}_{13})^{(1)} s_{(13)}} e^{(\widehat{M}_{13})^{(1)} s_{(13)}} \} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)} t} \leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}} \left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)} (\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)} (T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(1)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)} \left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}\left|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right| \; e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\} ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}t} \leq$
$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right) d\left(\left((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)}\right)\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$: $\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$G_{24}^{(2)} \left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)}\right) d\left(((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on T_{25} and respectively on (G_{27})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (228)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_2 / (a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\widehat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{32})^{(6)}\right] \leq (\widehat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}), (\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}), (\widetilde{T_{35}}) \right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right| \; e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} \} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)} (\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)}; \; (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t} \; and \; (\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$ (250)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2 \; and \; \left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33} \; and \; G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)} \left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)}; (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t} \; and \; (\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)} (T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2 \; and \; \left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37} \; and \; G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \le \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \le \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\,((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \ge (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$$T_{37} \ge \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$$T_{37} \ge \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_7}$$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \le (\hat{P}_{40})^{(8)}$$ 267

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \le (\hat{Q}_{40})^{(8)}$$ 268

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1,\left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right),\quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$

From 99 to 44 it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(9)}t)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$: 281

By $(v_1)^{(1)} > 0$, $(v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{v}_1)^{(1)},, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{v}_1)^{(1)} > 0$, $(\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)},, (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)}, , (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)} t} - e^{-(a'_{26})^{(4)} t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)} t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G^0_{28} e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G^0_{28}}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G^0_{30} e^{-(S_2)^{(5)} t} \le G_{30}(t) \le \right. \quad 344$$

$$\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_2)^{(5)}((S_1)^{(5)}-(a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t}-e^{-(a'_{30})^{(5)}t}\right]+\ G_{30}^{0}e^{-(a'_{30})^{(5)}t}\Big)$$

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\le T_{30}(t)\le \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$:

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)}\le-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\le-(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)}\le-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\le-(\tau_1)^{(6)}$$

Definition of $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$: 350

By $(v_1)^{(6)}>0$, $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$, $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

Definition of $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)}>0$, $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$, $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

Definition of $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$:-

If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)}=(v_0)^{(6)},(m_1)^{(6)}=(v_1)^{(6)}$, $\boldsymbol{if}\ (v_0)^{(6)}<(v_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}$ $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}$ $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}$ $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}$ $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}$ $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})} \left[e^{(S_1)^{(6)} t} - e^{-(a'_{34})^{(6)} t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)} t} \right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^{0}e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}}G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}}G_{36}^{0}e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t}-e^{-(S_2)^{(7)}t}\right]+G_{38}^{0}e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left.\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_2)^{(7)}((S_1)^{(7)}-(a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t}-e^{-(a'_{38})^{(7)}t}\right]+\ G_{38}^{0}e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}}T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}}T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right) - (b''_{41})^{(8)}\left((G_{43}), t\right) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(v_1)^{(8)} > 0$, $(v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$, $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{v}_1)^{(8)}$,, $(\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{v}_1)^{(8)} > 0$, $(\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$, $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$:-

If we define $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$, $\boldsymbol{if}\ (v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$, $\boldsymbol{if}\ (v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$,
and $\boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}$, $\boldsymbol{if}\ (\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, $\boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, $\boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t})$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \qquad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \qquad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t}\leq T_{42}(t)\leq \qquad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)},(S_2)^{(8)},(R_1)^{(8)},(R_2)^{(8)}$:- 381

Where $(S_1)^{(8)}=(a_{40})^{(8)}(m_2)^{(8)}-(a'_{40})^{(8)}$

$(S_2)^{(8)}=(a_{42})^{(8)}-(p_{42})^{(8)}$

$(R_1)^{(8)}=(b_{40})^{(8)}(\mu_2)^{(8)}-(b'_{40})^{(8)}$

$(R_2)^{(8)}=(b'_{42})^{(8)}-(r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$:

$(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)}\leq -(a'_{44})^{(9)}+(a'_{45})^{(9)}-(a''_{44})^{(9)}(T_{45},t)+(a''_{45})^{(9)}(T_{45},t)\leq -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)}\leq -(b'_{44})^{(9)}+(b'_{45})^{(9)}-(b''_{44})^{(9)}\big((G_{47}),t\big)-(b''_{45})^{(9)}\big((G_{47}),t\big)\leq -(\tau_1)^{(9)}$

Definition of $(v_1)^{(9)},(v_2)^{(9)},(u_1)^{(9)},(u_2)^{(9)},v^{(9)},u^{(9)}$:

By $(v_1)^{(9)}>0$, $(v_2)^{(9)}<0$ and respectively $(u_1)^{(9)}>0$, $(u_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_1)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)}>0$, $(\bar{v}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$, $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

Definition of $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$:-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)},\ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$
and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \le G_{46}(t) \le \right.$$
$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (v_1)^{(1)} \le (\bar{v}_1)^{(1)} \le \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$, we obtain 386

$$(v_1)^{(1)} \le v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $v^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\right) - (a''_{17})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{\mathrm{G}_{16}}{\mathrm{G}_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (\mathrm{C})^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (\mathrm{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)}\right) \le \frac{dv^{(3)}}{dt} \le -\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \ge \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}}$$

it follows $(v_0)^{(3)} \le v^{(3)}(t) \le (v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \le v^{(3)}(t) \le (\bar{v}_1)^{(3)}$

If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)} \le \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}} \le v^{(3)}(t) \le$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (\bar{v}_1)^{(3)}$$

If $0 < (v_1)^{(3)} \le (\bar{v}_1)^{(3)} \le (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$, we obtain 403

$$(v_1)^{(3)} \le v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

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$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{36}'')^{(7)} = (a_{37}'')^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b_{36}'')^{(7)} = (b_{37}'')^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a_{40}')^{(8)} - (a_{41}')^{(8)} + (a_{40}'')^{(8)}(T_{41}, t)\right) - (a_{41}'')^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(8)} and\ (b''_i)^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\ ]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\ ]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\ ]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0$ 479

$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0$ 480

$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0$ 481

$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0$ 482

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(r) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first

equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T^*_{33} for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$$

Definition and uniqueness of T^*_{37} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T^*_{37} for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$$

Definition and uniqueness of T^*_{41} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{25} such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507
$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{29} such that $\varphi((G_{31})^*) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{33} such that $\varphi({G_{35}}^*) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{37} such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{41} such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right]+(b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{45} such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)} G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)} G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)} T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)} T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)} G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)} G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)} T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)} T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)} G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)} G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)} T_{21}^*}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T_{22}^* = \frac{(b_{22})^{(3)} T_{21}^*}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]} \quad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \qquad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \qquad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \qquad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \qquad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \qquad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \qquad 530$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i \qquad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \qquad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \qquad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \qquad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \qquad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \qquad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}\mathrm{T}^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}\mathrm{T}^*_{18}\mathbb{G}_j\right) \quad 539$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}\left((G_{27})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^{*}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^{*}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^{*}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^{*}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^{*}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^{*}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^{*}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^{*}\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\ B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\Big)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\Big]$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\Big)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\Big)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\Big]$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\Big)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\Big)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\Big]$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\Big)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\Big)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\Big)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\Big)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION TWENTY TWO A

Unitarity Of Black Hole Evaporation

INTRODUCTION—VARIABLES USED

NOTATION

Module One

AdS/CFT has \\ $\bullet$ shown new ways to calculate microscopically the entropy of black holes \\ $\bullet$ provided a new perspective on (eb) confinement and chiral symmetry breaking \\ $\bullet$ opened (eb) calculational windows into properties of strongly coupled gauge theory plasmas \\ $\bullet$ given a non-perturbative definition of string theory on (eb) Anti-de-Sitter space \\ $\bullet$ essentially settled the

debate about (e) unitarity of black hole evaporation

G_{13} : Category one of AdS/CFT has \\ $\bullet$ shown new ways to calculate microscopically the entropy of black holes \\ $\bullet$ provided a new perspective

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of confinement and chiral symmetry breaking \\ $\bullet$ opened (eb) calculational windows into properties of strongly coupled gauge theory plasmas \\ $\bullet$ given a non-perturbative definition of string theory on (eb) Anti-de-Sitter space \\ $\bullet$ essentially settled the debate about (e) unitarity of black hole evaporation

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

AdS/CFT has \\ $\bullet$ shown new ways to calculate microscopically the entropy of black holes \\ $\bullet$ provided a new perspective on confinement and chiral symmetry breaking \\ $\bullet$ opened (eb) calculational windows into properties of strongly coupled gauge theory plasmas \\ $\bullet$ given a non-perturbative definition of string theory on (eb) Anti-de-Sitter space \\ $\bullet$ essentially settled the debate about (e) unitarity of black hole evaporation

G_{16} : Category one of AdS/CFT has \\ $\bullet$ shown new ways to calculate microscopically the entropy of black holes \\ $\bullet$ provided a new perspective on confinement and chiral symmetry breaking \\ $\bullet$

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of calculational windows into properties of strongly coupled gauge theory plasmas \\ $\bullet$ given a non-perturbative definition of string theory on (eb) Anti-de-Sitter space \\ $\bullet$ essentially settled the debate about (e) unitarity of black hole evaporation

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

AdS/CFT has \\ $\bullet$ shown new ways to calculate microscopically the entropy of black holes \\ $\bullet$ provided a new perspective on confinement and chiral symmetry breaking \\ $\bullet$ opened calculational windows into properties of strongly coupled gauge theory plasmas \\ $\bullet$ given a non-perturbative definition of string theory on (eb) Anti-de-Sitter space \\ $\bullet$ essentially settled the debate about (e) unitarity of black hole evaporation

G_{20} : Category one of AdS/CFT has \\ $\bullet$ shown new ways to calculate microscopically the entropy of black holes \\ $\bullet$ provided a new perspective on confinement and chiral symmetry breaking \\ $\bullet$ opened calculational windows into properties of strongly coupled gauge theory plasmas \\ $\bullet$ given a non-perturbative definition of string theory

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of Anti-de-Sitter space \\ $\bullet$ essentially settled the debate about (e) unitarity of black hole evaporation

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

AdS/CFT has \\ $\bullet$ shown new ways to calculate microscopically the entropy of black holes \\ $\bullet$ provided a new perspective on confinement and chiral symmetry breaking \\ $\bullet$ opened calculational windows into properties of strongly coupled gauge theory plasmas \\ $\bullet$ given a non-perturbative definition of string theory on Anti-de-Sitter space \\ $\bullet$ essentially settled the debate about (e) unitarity of black hole evaporation

G_{24} : Category one of AdS/CFT has \\ $\bullet$ shown new ways to calculate microscopically the entropy of black holes \\ $\bullet$ provided a new perspective on confinement and chiral symmetry breaking \\ $\bullet$ opened calculational windows into properties of strongly coupled gauge theory plasmas \\ $\bullet$ given a non-perturbative definition of string theory on Anti-de-Sitter space \\ $\bullet$ essentially; unitarity of black hole evaporation

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of unitarity of black hole evaporation; AdS/CFT has \\ $\bullet$ shown new ways to calculate microscopically the entropy of black holes \\ $\bullet$ provided a new perspective on confinement and chiral symmetry breaking \\ $\bullet$ opened calculational windows into properties of strongly coupled gauge theory plasmas \\ $\bullet$ given a non-perturbative definition of string theory on Anti-de-Sitter space \\ $\bullet$ essentially

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Physicists' interest in AdS/CFT has also triggered a number of exciting developments in pure mathematics, for instance in conformal geometry and the spectral geometry of certain Einstein manifolds, in both Minkowski and Euclidean signature

Moreover, thanks to the familiar fact that

Supersymmetric solutions of supergravity can be characterized (eb) algebraically (Calabi-Yau manifolds as cones over Sasaki-Einstein manifolds being the prototypical example), the subject has also connected up with (e&eb) a number of developments in algebraic geometry, the representation theory of certain quiver algebras, and many more.

Finally, versions of the correspondence that operate purely within the realm of topological quantum theory have been put forward, such as the relation between topological strings and Chern-Simons gauge theory or matrix models

G_{28} : Category one of supersymmetric solutions of supergravity

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of algebraically (Calabi-Yau manifolds as cones over Sasaki-Einstein manifolds being the prototypical example), the subject has also connected up with (e&eb) a number of developments in algebraic geometry, the representation theory of certain quiver algebras, and many more.

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Supersymmetric solutions of supergravity can be characterized algebraically (Calabi-Yau manifolds as cones over Sasaki-Einstein manifolds being the prototypical example), the subject has also connected up with (e&eb) a number of developments in algebraic geometry, the representation theory of certain quiver algebras, and many more

G_{32} : Category one of Supersymmetric solutions of supergravity can be characterized algebraically (Calabi-Yau manifolds as cones over Sasaki-Einstein manifolds being the prototypical example); algebraic geometry, the representation theory of certain quiver algebras,

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of algebraic geometry, the representation theory of certain quiver algebras; Supersymmetric solutions of supergravity can be characterized algebraically (Calabi-Yau manifolds as cones over Sasaki-Einstein manifolds being the prototypical example)

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Calabi-Yau manifolds as cones over Sasaki-Einstein manifolds being the prototypical example

G_{36} : Category one of Calabi-Yau manifolds

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of cones over Sasaki-Einstein manifolds

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

These correspondences in fact connect up with (e&eb) the circle of ideas relating moduli problems, intersection theory on moduli spaces, (less than maximal) supersymmetric gauge theory with (e&eb) algebraic integrable systems, classical and quantum

N = 1 SUSY in 4D (with 4 real generators) (Wikipedia)

In four dimensions, the minimal N = 1 supersymmetry may be written using a superspace. This superspace involves four extra fermionic coordinates $\theta^1, \theta^2, \bar{\theta}^1, \bar{\theta}^2$, transforming as a two-component spinor and its conjugate.

Every superfield, i.e. a field that depends on all coordinates of the superspace, may be expanded with respect to the new fermionic coordinates. There exists a special kind of superfields, the so-called chiral superfields, that only depend on the variables θ but not their conjugates (more precisely, $\overline{D}f = 0$). However, avector superfield depends on all coordinates. It describes a gauge field and its superpartner, namely a Weyl fermion that obeys a Dirac equation.

$$V = C + i\theta\chi - i\bar{\theta}\bar{\chi} + \tfrac{i}{2}\theta^2(M + iN) - \tfrac{i}{2}\bar{\theta}^2(M - iN) - \theta\sigma^\mu\bar{\theta}v_\mu + i\theta^2\bar{\theta}\left(\bar{\lambda} + \tfrac{1}{2}\bar{\sigma}^\mu\partial_\mu\chi\right) - i\bar{\theta}^2\theta\left(\lambda + \tfrac{i}{2}\sigma^\mu\partial_\mu\bar{\chi}\right) + \tfrac{1}{2}\theta^2\bar{\theta}^2\left(D + \tfrac{1}{2}\Box C\right)$$

V is the vector superfield (prepotential) and is real (V = V). The fields on the right hand side are component fields.

The gauge transformations act as

$$V \to V + \Lambda + \overline{\Lambda}$$

where Λ is any chiral superfield.

It's easy to check that the chiral superfield

$$W_\alpha \equiv -\tfrac{1}{4}\overline{D}^2 D_\alpha V$$

is gauge invariant. So is its complex conjugate $\overline{W}_{\dot{\alpha}}$.

A nonSUSY covariant gauge which is often used is the Wess–Zumino gauge. Here, C, χ, M and N are all set to zero. The residual gauge symmetries are gauge transformations of the traditional bosonic type.

A chiral superfield X with a charge of q transforms as

$$X \to e^{q\Lambda}X, \qquad \overline{X} \to e^{q\overline{\Lambda}}X$$

Therefore Xe−qVX is gauge invariant. Here e−qV is called a bridge since it "bridges" a field which transforms under Λ only with a field which transforms under Λonly.

More generally, if we have a real gauge group G that we wish to supersymmetrize, we first have to complexify it to Gc · e−qV then acts a compensator for the complex gauge transformations in effect absorbing them leaving only the real parts. This is what's being done in the Wess–Zumino gauge.

Differential super forms

Let's rephrase everything to look more like a conventional Yang–Mills gauge theory. We have a U(1) gauge symmetry acting upon full superspace with a 1-superform gauge connection A. In the analytic basis for the tangent space, the covariant derivative is given by $D_M = d_M + iqA_M$. Integrability conditions for chiral superfields with the chiral constraint

$$\overline{D}_{\dot{\alpha}} X = 0$$

leave us with

$$\left\{\overline{D}_{\dot{\alpha}}, \overline{D}_{\dot{\beta}}\right\} = F_{\dot{\alpha}\dot{\beta}} = 0.$$

A similar constraint for antichiral superfields leaves us with Fαβ = 0. This means that we can either gauge fix $A_{\dot{\alpha}} = 0$ or Aα = 0 but not both simultaneously. Call the two different gauge fixing schemes I and II respectively. In gauge I, $\overline{d}_{\dot{\alpha}} X = 0$ and in gauge II, dα X = 0. Now, the trick is to use two different gauges simultaneously; gauge I for chiral superfields and gauge II for antichiral superfields. In order to bridge between the two different gauges, we need a gauge transformation. Call it e−V(by convention). If we were using one gauge for all fields, XX would be gauge invariant. However, we need to convert gauge I to gauge II, transforming X to (e−V)qX. So, the gauge invariant quantity is Xe−qVX.

In gauge I, we still have the residual gauge eΛ where $\overline{d}_{\dot{\alpha}} \Lambda = 0$ and in gauge II, we have the residual gauge eΛ satisfying dα Λ = 0. Under the residual gauges, the bridge transforms as

$$e^{-V} \rightarrow e^{-\overline{\Lambda} - V - \Lambda}.$$

Without any additional constraints, the bridge e−V wouldn't give all the information about the gauge field. However, with the additional constraint $F_{\dot{\alpha}\beta}$, there's only one unique gauge field which is compatible with the bridge modulo gauge transformations. Now, the bridge gives exactly the same information content as the gauge field.

Korteweg–de Vries equation From Wikipedia, the free encyclopedia

$$\partial_t \phi + \partial_x^3 \phi + 6\,\phi\,\partial_x \phi = 0$$

$$-c\frac{df}{dX} + \frac{d^3 f}{dX^3} + 6f\frac{df}{dX} = 0,$$

Sine-Gordon equation

$$\mathcal{L}_{\mathrm{SG}}(\varphi) = \frac{1}{2}(\varphi_t^2 - \varphi_x^2) - 1 + \cos\varphi.$$

$$\mathcal{L}_{\mathrm{SG}}(\varphi) = \frac{1}{2}(\varphi_t^2 - \varphi_x^2) - \frac{\varphi^2}{2} + \sum_{n=2}^{\infty} \frac{(-\varphi^2)^n}{(2n)!}$$
$$= \mathcal{L}_{\mathrm{KG}}(\varphi) + \sum_{n=2}^{\infty} \frac{(-\varphi^2)^n}{(2n)!}.$$

G_{40} : Category one of LHS of the above equations

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of RHS of above equations

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Moduli space From Wikipedia, the free encyclopedia

(1) In algebraic geometry, a moduli space is a geometric space (usually a scheme or an algebraic stack) whose points represent (eb) algebro-geometric objects of some fixed kind, or isomorphism classes of such objects.
(2) Such spaces frequently arise as (=) solutions to classification problems: If one can show that a collection of interesting objects (e.g., the smooth algebraic curves of a fixed genus) can be given the structure of a geometric space, then one can parametrize (e&eb) such objects by introducing coordinates on the resulting space.
(3) In this context, the term "modulus" is used synonymously with "parameter"; moduli spaces were (=) first understood as spaces of parameters rather than as spaces of objects.

Non linear Schrodinger equation:

$$i\partial_t\psi = -\frac{1}{2}\partial_x^2\psi + \kappa|\psi|^2\psi$$

$$H = \int \mathrm{d}x \left[\frac{1}{2}|\partial_x\psi|^2 + \frac{\kappa}{2}|\psi|^4\right]$$

Boussinesq approximation:

$$\frac{\partial \eta}{\partial t} + u\,\frac{\partial \eta}{\partial x} - w = 0$$
$$\frac{\partial \varphi}{\partial t} + \frac{1}{2}\left(u^2 + w^2\right) + g\,\eta = 0,$$

Landau–Lifshitz equation:

$$H = \frac{1}{2}\int \left[\sum_i \left(\frac{\partial \mathbf{S}}{\partial x_i}\right)^2 - J(\mathbf{S})\right] dx \qquad (1)$$

$$\frac{\partial \mathbf{S}}{\partial t} = \mathbf{S} \wedge \sum_i \frac{\partial^2 \mathbf{S}}{\partial x_i^2} + \mathbf{S} \wedge J\mathbf{S}. \qquad (2)$$

$$\frac{\partial \mathbf{S}}{\partial t} = \mathbf{S} \wedge \frac{\partial^2 \mathbf{S}}{\partial x^2} + \mathbf{S} \wedge J\mathbf{S}. \qquad (3)$$

$$\frac{\partial \mathbf{S}}{\partial t} = \mathbf{S} \wedge \left(\frac{\partial^2 \mathbf{S}}{\partial x^2} + \frac{\partial^2 \mathbf{S}}{\partial y^2}\right) + \mathbf{S} \wedge J\mathbf{S} \qquad (4)$$

Definition (Representation Theory) : Wikipedia

See also: group representation, algebra representation and Lie algebra representation

There are two ways to say what a representation is The first uses the idea of an action, generalizing the way that matrices act on column vectors by matrix multiplication. A representation of a group G or (associative or Lie) algebra A on a vector space V is a map

$$\Phi : G \times V \to V \quad \text{or} \quad \Phi : A \times V \to V$$

with two properties. First, for any g in G (or a in A), the map

$$\begin{aligned} \varphi(g) : V &\to V \\ v &\mapsto \Phi(g, v) \end{aligned}$$

is linear (over F). Second, if we introduce the notation g · v for Φ (g, v), then for any g1, g2 in G and v in V:

$$(1) \quad e \cdot v = v$$

$$(2) \quad g_1 \cdot (g_2 \cdot v) = (g_1 g_2) \cdot v$$

where e is the identity element of G and g1g2 is product in G. The requirement for associative algebras is analogous, except that associative algebras do not always have an identity element, in which case equation (1) is ignored. Equation (2) is an abstract expression of the associativity of matrix multiplication. This doesn't hold for the matrix commutator and also there is no identity element for the commutator. Hence for Lie algebras, the only requirement is that for any x1, x2 in A and v in V:

$$(2') \quad x_1 \cdot (x_2 \cdot v) - x_2 \cdot (x_1 \cdot v) = [x_1, x_2] \cdot v$$

where [x1, x2] is the Lie bracket, which generalizes the matrix commutator MN − NM.

The second way to define a representation focuses on the map φ sending g in G to a linear map φ(g): V → V, which satisfies

$$\varphi(g_1 g_2) = \varphi(g_1) \circ \varphi(g_2) \quad \text{for all } g_1, g_2 \in G$$

G_{44} : Category one of LHS of above equations

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of RHS of above equations

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},\ \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}\ ,$
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)} G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right] G_{13} \quad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)} G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right] G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)} G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right] G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)} T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right] T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)} T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right] T_{14} \quad 5$$

$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15}$ 6

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16}$ 7

$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17}$ 8

$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18}$ 9

$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}),t\right)\right]T_{16}$ 10

$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}),t\right)\right]T_{17}$ 11

$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}),t\right)\right]T_{18}$ 12

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}),t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20}$ 13

$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21}$ 14

$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22}$ 15

$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20}$ 16

$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21}$ 17

$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22}$ 18

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24}$ 19

$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25}$ 20

$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26}$ 21

$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24}$ 22

$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25}$ 23

$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26}$ 24

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$
$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$
$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$
$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)\right]T_{28} \quad 28$$
$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)\right]T_{29} \quad 29$$
$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}), t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}), t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}), t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}), t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}),t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}),t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad (60)$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition

coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)}\ \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\ \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\ \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{array}\right] G_{22} \quad (69)$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for

category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)} & \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad 70$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)} & \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}, \boxed{(a''_{25})^{(4)}(T_{25},t)}, \boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
are sixth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
are seventh augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
are eighth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad 78$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$

are fourth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$

are fifth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$

are sixth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$

are seventh detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$

are eighth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{ccc} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{ccc} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{ccc} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{30} \quad 81$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation

coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)} & +(a''_{32})^{(6)}(T_{33},t) & +(a''_{28})^{(5,5,5)}(T_{29},t) & +(a''_{24})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)} & +(a''_{33})^{(6)}(T_{33},t) & +(a''_{29})^{(5,5,5)}(T_{29},t) & +(a''_{25})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)}\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)}\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)}\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* $1,2$ *and* 3

$-(b_{13}'')^{(1,1,1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1,1,1)}(G,t)$, $-(b_{15}'')^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b_{16}'')^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b_{20}'')^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3

$-(b_{36}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3

$-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2, and 3

$-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a_{36}')^{(7)} & +(a_{36}'')^{(7)}(T_{37},t) & +(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t) & +(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a_{37}')^{(7)} & +(a_{37}'')^{(7)}(T_{37},t) & +(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a_{38}')^{(7)} & +(a_{38}'')^{(7)}(T_{37},t) & +(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15}$$

Where $(a_{36}'')^{(7)}(T_{37},t)$, $(a_{37}'')^{(7)}(T_{37},t)$, $(a_{38}'')^{(7)}(T_{37},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\,-(b''_{36})^{(7)}(G_{39},t) & -(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1)}(G,t) & -(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\,-(b''_{37})^{(7)}(G_{39},t) & -(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1)}(G,t) & -(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\,-(b''_{38})^{(7)}(G_{39},t) & -(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1)}(G,t) & -(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{40}}{dt} =$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt}$$

$$= (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt}$$
$$= (a_{45})^{(9)} G_{44}$$
$$- \begin{bmatrix} (a'_{45})^{(9)} \boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt}$$
$$= (a_{46})^{(9)} G_{45}$$
$$- \begin{bmatrix} (a'_{46})^{(9)} \boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$
$$(b_{44})^{(9)} T_{45} -$$
$$\begin{bmatrix} (b'_{44})^{(9)} \boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$
$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b''_i)^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}', t) - (a_i'')^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}', t)$ and $(a_i'')^{(1)}(T_{14}, t)$. (T_{14}', t) and (T_{14}, t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(\mathrm{p_i})^{(2)}$, $(\mathrm{r_i})^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq \left(\hat{A}_{16}\right)^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}\left((G_{19}), t\right) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}\big((G_{19}), t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, \ (\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$\lim_{G\to\infty}(b_i'')^{(3)}\,(G_{23},t) = \ (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$

$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)},$ are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$

$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$

$(b_i'')^{(4)}\big((G_{27}),t\big) \le \ (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$

$lim_{T_2\to\infty}(a_i'')^{(4)}\,(T_{25},t) = (p_i)^{(4)}$ 118

$\lim_{G\to\infty}(b_i'')^{(4)}\,\big((G_{27}),t\big) = \ (r_i)^{(4)}$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}\,(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}\,(G_{31}, t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\,(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ \ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}\,(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}\,\big((G_{35}), t\big) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\big((G_{35}), t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\,(a_i)^{(6)} + (a_i')^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)}\,(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ \ (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(EEEEE) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(FFFFF) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq \ (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

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(GGGGG) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(HHHHH)

$\lim_{G\to\infty}(b_i'')^{(7)}\left((G_{39}),t\right) = \ (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, \ (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36{,}37{,}38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}\left((G_{39}),t\right)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $\left[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\right]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(IIIII) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(JJJJJ) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, \ (r_i)^{(7)}, i = 36{,}37{,}38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\,(a_i)^{(7)} + (a_i')^{(7)} + \ (\hat{A}_{36})^{(7)} + \ (\hat{P}_{36})^{(7)}\,(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\ (b_i)^{(7)} + (b_i')^{(7)} + \ (\hat{B}_{36})^{(7)} + \ (\hat{Q}_{36})^{(7)}\ (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40{,}41{,}42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}, \ (r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41},t) \le (p_i)^{(8)} \le (\hat{A}_{40})^{(8)} \quad 138$$

$$(b_i'')^{(8)}((G_{43}),t) \le (r_i)^{(8)} \le (b_i')^{(8)} \le (\hat{B}_{40})^{(8)} \quad 139$$

$$\lim_{T_2\to\infty}(a_i'')^{(8)}(T_{41},t) = (p_i)^{(8)} \quad 140$$

$$\lim_{G\to\infty}(b_i'')^{(8)}((G_{43}),t) = (r_i)^{(8)} \quad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}',t) - (a_i'')^{(8)}(T_{41},t)| \le (\hat{k}_{40})^{(8)}|T_{41} - T_{41}'|e^{-(\hat{M}_{40})^{(8)}t} \quad 142$$

$$|(b_i'')^{(8)}((G_{43})',t) - (b_i'')^{(8)}((G_{43}),t)| < (\hat{k}_{40})^{(8)}||(G_{43}) - (G_{43})'||e^{-(\hat{M}_{40})^{(8)}t} \quad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}',t)$ and $(a_i'')^{(8)}(T_{41},t)$. (T_{41}',t) and (T_{41},t) are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \quad 144$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad 146A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t) \le (p_i)^{(9)} \le (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t) \le (r_i)^{(9)} \le (b_i')^{(9)} \le (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty} (a_i'')^{(9)}(T_{45},t) = (p_i)^{(9)}$$

$$\lim_{G\to\infty} (b_i'')^{(9)}(G_{47},t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t) - (a_i'')^{(9)}(T_{45},t)| \le (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t) - (b_i'')^{(9)}\big((G_{47}),t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$ and$(a_i'')^{(9)}(T_{45},t)$. (T_{45}',t) and (T_{45},t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}$, $(r_i)^{(9)}, i = 44,45,46,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\,, T_i(0)$

$$G_i(t) \le \left(\hat{P}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t} \;,\quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le \left(\hat{Q}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t} \;,\quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le \left(\hat{P}_{20}\right)^{(3)} e^{(\hat{M}_{20})^{(3)}t} \;,\quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le \left(\hat{Q}_{20}\right)^{(3)} e^{(\hat{M}_{20})^{(3)}t} \;,\quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq \left(\hat{P}_{13}\right)^{(1)}$, $T_i^0 \leq \left(\hat{Q}_{13}\right)^{(1)}$, 155

$0 \leq G_i(t) - G_i^0 \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 156

$0 \leq T_i(t) - T_i^0 \leq \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{16})^{(2)}$, $T_i^0 \leq (\hat{Q}_{16})^{(2)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''^{(2)}_{16}\right)\left(T_{17}(s_{(16)}), s_{(16)}\right)\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}\left(T_{17}(s_{(16)}), s_{(17)}\right)\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}\left(T_{17}(s_{(16)}), s_{(16)}\right)\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{20})^{(3)}$, $T_i^0 \leq (\hat{Q}_{20})^{(3)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''^{(3)}_{20}\right)\left(T_{21}(s_{(20)}), s_{(20)}\right)\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}\left(T_{21}(s_{(20)}), s_{(20)}\right)\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}\left(T_{21}(s_{(20)}), s_{(20)}\right)\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)},\ T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^{0} + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^{0} + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^{0} + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^{0} + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^{0} + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^{0} + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)},\ T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^{0} + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that 174

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \leq G_{32}^{0}+\int_0^t\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(s) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \leq G_{36}^{0}+\int_0^t\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$
$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \leq G_{40}^{0}+\int_0^t\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}= \quad 180$$
$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t} \leq \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \leq G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \leq \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+\ (a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\ (t)=0\ and\ T_i\ (t)=0$

From 19 to 24 it results

$$G_i\ (t)\geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\geq 0$$

$$T_i\ (t)\geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13}<(\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt}\leq\left((\widehat{M}_{13})^{(1)}\right)_1-(a'_{14})^{(1)}G_{14}$ and by integrating

$$G_{14}\leq\left((\widehat{M}_{13})^{(1)}\right)_2=G_{14}^0+2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15}\leq\left((\widehat{M}_{13})^{(1)}\right)_3=G_{15}^0+2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a'_{15})^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}} \leq$
$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{\mathrm{A}}_{16})^{(2)} + (\widehat{\mathrm{P}}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)\mathrm{d}\left(\left((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)}\right)\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $\mathrm{G}_i\,(\mathrm{t}) = 0$ and $\mathrm{T}_i\,(\mathrm{t}) = 0$ 200

it results

$\mathrm{G}_i\,(\mathrm{t}) \geq \mathrm{G}_i^0\mathrm{e}^{\left[-\int_0^{\mathrm{t}}\{(a_i')^{(2)} - (a_i'')^{(2)}(\mathrm{T}_{17}(s_{(16)}), s_{(16)})\}\mathrm{d}s_{(16)}\right]} \geq 0$

$\mathrm{T}_i\,(\mathrm{t}) \geq \mathrm{T}_i^0\mathrm{e}^{(-(b_i')^{(2)}\mathrm{t})} > 0 \quad \text{for t} > 0$

Definition of $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1$, $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2$ and $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$\mathrm{G}_{16} < (\widehat{\mathrm{M}}_{16})^{(2)}$ it follows $\frac{\mathrm{dG}_{17}}{\mathrm{dt}} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)}\mathrm{G}_{17}$ and by integrating

$\mathrm{G}_{17} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 = \mathrm{G}_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$

In the same way , one can obtain

$\mathrm{G}_{18} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3 = \mathrm{G}_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{\mathrm{t}\to\infty}((b_i'')^{(2)}\,((G_{19})(\mathrm{t}), \mathrm{t})) = (b_{17}')^{(2)}$ then $\mathrm{T}_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$: $\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0$ *and* $T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$G_{24}^{(2)} \left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)}\right) d\left(((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on T_{25} and respectively on (G_{27})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (228)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_2 / (a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\tilde{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\tilde{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\tilde{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\tilde{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)}, (T_{35})^{(1)}), ((G_{35})^{(2)}, (T_{35})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})}, \widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})}, \widetilde{(T_{35})}\right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right)\left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)};\ (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ *and* $(\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ (250)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)} \left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)} (T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_7}$$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)}$$ 267

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\widehat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{40})^{(8)}\right] \leq (\widehat{Q}_{40})^{(8)}$$ 268

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1,\left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right),\quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{47}}),(\widetilde{T_{47}}):\left((\widetilde{G_{47}}),(\widetilde{T_{47}})\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$

From 99 to 44 it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(9)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$: 281

By $(v_1)^{(1)} > 0$, $(v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{v}_1)^{(1)},, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{v}_1)^{(1)} > 0$, $(\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)},\ if\ (v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}, if\ (v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)},$

and $\boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)},\ if\ (\bar{v}_1)^{(1)} < (v_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)}, , (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)} t} - e^{-(a'_{26})^{(4)} t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)} t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G^0_{28} e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G^0_{28}}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G^0_{30} e^{-(S_2)^{(5)} t} \le G_{30}(t) \le \right. \quad 344$$

$$\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_2)^{(5)}((S_1)^{(5)}-(a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t}-e^{-(a'_{30})^{(5)}t}\right]+\ G_{30}^{0}e^{-(a'_{30})^{(5)}t}\Big)$$

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \qquad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le \frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \qquad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\le T_{30}(t)\le \qquad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$:

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)}\le -(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\le -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)}\le -(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\le -(\tau_1)^{(6)}$$

Definition of $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$: 350

By $(v_1)^{(6)}>0$, $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$, $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

Definition of $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)}>0$, $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$, $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

Definition of $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$:-

If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)}=(v_0)^{(6)},(m_1)^{(6)}=(v_1)^{(6)},$ ***if*** $(v_0)^{(6)}<(v_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}$ $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}$ $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}$ $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}$ $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}$ $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})} \left[e^{(S_1)^{(6)} t} - e^{-(a'_{34})^{(6)} t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)} t} \right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)},, (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^{0}e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}}G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}}G_{36}^{0}e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t}-e^{-(S_2)^{(7)}t}\right]+G_{38}^{0}e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left.\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_2)^{(7)}((S_1)^{(7)}-(a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t}-e^{-(a'_{38})^{(7)}t}\right]+\ G_{38}^{0}e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}}T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}}T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right) - (b''_{41})^{(8)}\left((G_{43}), t\right) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(v_1)^{(8)} > 0$, $(v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$, $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{v}_1)^{(8)}$,, $(\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{v}_1)^{(8)} > 0$, $(\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$, $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$:-

If we define $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$, $\boldsymbol{if}$ $(v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$, $\boldsymbol{if}$ $(v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$,
and $\boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}$, $\boldsymbol{if}$ $(\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, $\boldsymbol{if}$ $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, $\boldsymbol{if}$ $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}$ $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t})$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \qquad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \qquad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t}\leq T_{42}(t)\leq \qquad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)},(S_2)^{(8)},(R_1)^{(8)},(R_2)^{(8)}$:- 381

Where $(S_1)^{(8)}=(a_{40})^{(8)}(m_2)^{(8)}-(a'_{40})^{(8)}$

$(S_2)^{(8)}=(a_{42})^{(8)}-(p_{42})^{(8)}$

$(R_1)^{(8)}=(b_{40})^{(8)}(\mu_2)^{(8)}-(b'_{40})^{(8)}$

$(R_2)^{(8)}=(b'_{42})^{(8)}-(r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$:

$(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)}\leq -(a'_{44})^{(9)}+(a'_{45})^{(9)}-(a''_{44})^{(9)}(T_{45},t)+(a''_{45})^{(9)}(T_{45},t)\leq -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)}\leq -(b'_{44})^{(9)}+(b'_{45})^{(9)}-(b''_{44})^{(9)}\big((G_{47}),t\big)-(b''_{45})^{(9)}\big((G_{47}),t\big)\leq -(\tau_1)^{(9)}$

Definition of $(v_1)^{(9)},(v_2)^{(9)},(u_1)^{(9)},(u_2)^{(9)},v^{(9)},u^{(9)}$:

By $(v_1)^{(9)}>0$, $(v_2)^{(9)}<0$ and respectively $(u_1)^{(9)}>0$, $(u_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_1)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)}>0$, $(\bar{v}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$, $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

Definition of $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$:-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)},\ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$
and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \le G_{46}(t) \le \right.$$
$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (v_1)^{(1)} \le (\bar{v}_1)^{(1)} \le \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$, we obtain 386

$$(v_1)^{(1)} \le v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $v^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\right) - (a''_{17})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{\mathrm{G}_{16}}{\mathrm{G}_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (\mathrm{C})^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (\mathrm{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \le \nu^{(2)}(t) \le (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \le u^{(2)}(t) \le (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)}\right) \le \frac{dv^{(3)}}{dt} \le -\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \ge \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}}$$

it follows $(v_0)^{(3)} \le v^{(3)}(t) \le (v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \le v^{(3)}(t) \le (\bar{v}_1)^{(3)}$

If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)} \le \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}} \le v^{(3)}(t) \le$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (\bar{v}_1)^{(3)}$$

If $0 < (v_1)^{(3)} \le (\bar{v}_1)^{(3)} \le (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$, we obtain 403

$$(v_1)^{(3)} \le v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

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If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case,

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

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If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

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$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{36}'')^{(7)} = (a_{37}'')^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b_{36}'')^{(7)} = (b_{37}'')^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a_{40}')^{(8)} - (a_{41}')^{(8)} + (a_{40}'')^{(8)}(T_{41}, t)\right) - (a_{41}'')^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(8)} and\ (b''_i)^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\,]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\,]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\,]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0$ 479

$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0$ 480

$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0$ 481

$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0$ 482

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(s) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first

equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T^*_{33} for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$$

Definition and uniqueness of T^*_{37} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T^*_{37} for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$$

Definition and uniqueness of T^*_{41} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{25} such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507
$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{29} such that $\varphi((G_{31})^*) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{33} such that $\varphi({G_{35}}^*) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{37} such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{41} such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right]+(b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{45} such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)} G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)} G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)} T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)} T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)} G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)} G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)} T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)} T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)} G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)} G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)} T_{21}^*}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T_{22}^* = \frac{(b_{22})^{(3)} T_{21}^*}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]} \quad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$$T_{28}^{*}=\frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*}=\frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*})=0$, T_{33}^{*} given by $f(T_{33}^{*})=0$ and

$$G_{32}^{*}=\frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*}=\frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*}=\frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*}=\frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*})=0$, T_{37}^{*} given by $f(T_{37}^{*})=0$ and

$$G_{36}^{*}=\frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}^{*}=\frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*}=\frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]} \quad , \quad T_{38}^{*}=\frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*})=0$, T_{41}^{*} given by $f(T_{41}^{*})=0$ and

$$G_{40}^{*}=\frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42}^{*}=\frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

$$T_{40}^{*}=\frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]} \quad , \quad T_{42}^{*}=\frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*})=0$, T_{45}^{*} given by $f(T_{45}^{*})=0$ and

$$G_{44}^{*}=\frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46}^{*}=\frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$$

$$T_{44}^{*}=\frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]} \quad , \quad T_{46}^{*}=\frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \qquad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \qquad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \qquad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \qquad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \qquad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \qquad 530$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i \qquad 532$$

$$\frac{\partial (a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial (b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \qquad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \qquad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \qquad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \qquad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \qquad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}\mathrm{T}^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}\mathrm{T}^*_{18}\mathbb{G}_j\right) \quad 539$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}\left((G_{27})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)}$, $\frac{\partial(b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\Big)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\Big]$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\Big)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\Big)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\Big]$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\Big)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\Big)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\Big]$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\Big)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\Big)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\Big)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\Big)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\Big)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\Big)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION TWENTY THREE

Ads/CFT And Its Extensions Provide Holographic Duals

INTRODUCTION—VARIABLES USED

Quantum spacetime From Wikipedia, the free encyclopedia

(1) In mathematical physics, the concept of quantum spacetime is a generalization of the usual concept of spacetime in which some variables that ordinarily commute are assumed not to commute and form a different Lie algebra. The choice of that algebra still varies from theory to theory. As a result of this change some variables that are usually continuous may become discrete. Often only such discrete variables are called "quantized"; usage varies. The idea of quantum spacetime was

proposed in the early days of quantum theory by Heisenberg and Ivanenko as a way to eliminate infinities from quantum field theory. The germ of the idea passed from Heisenberg to Rudolf Peierls, who noted that electrons in a magnetic field can be regarded as moving in a quantum space-time, and to Robert Oppenheimer, who carried it to Hartland Snyder, who published the first concrete example.[1] Snyder's Lie algebra was made simple by C. N. Yang in the same year.

(2) Physical reasons have been given to believe that physical spacetime is a quantum spacetime. In quantum mechanics position and momentum variables x, p are already noncommutative, obey the Heisenberg uncertainty principle, and are continuous. Because of the Heisenberg uncertainty relations, greater energy is needed to probe smaller distances. Ultimately, according to gravity theory, the probing particles form black holes and destroy what is to be measured. Then the process cannot be repeated, and so does not count as a measurement. This limited measurability led many to expect that our usual picture of continuous commutative spacetime breaks down at Planck scale distances, if not sooner.

(3) Again, physical spacetime is expected to be quantum because physical coordinates are already slightly noncommutative. The astronomical coordinates of a star are modified by gravitational fields between us and the star, as in the deflection of light by the sun, one of the classic tests of general relativity. Therefore the coordinates actually depend on gravitational field variables. According to quantum theories of gravity these field variables do not commute; therefore coordinates that depend on them likely do not commute.Any noncommutative algebra with at least four generators could be interpreted as a quantum spacetime, but the following desiderata have been suggested:

Local Lorentz group and Poincaré group symmetries should be retained, possibly in a generalised form. Their generalisation often takes the form of a quantum group acting on the quantum spacetime algebra.

The algebra might plausibly arise in an effective description of quantum gravity effects in some regime of that theory. For example, a physical parameter \lambda, perhaps the Planck length, might control the deviation from commutative classical spacetime, so that ordinary Lorentzian spacetime arises as \lambda\to 0.

There might be a notion of quantum differential calculus on the quantum spacetime algebra, compatible with the (quantum) symmetry and preferably reducing to the usual differential calculus as \lambda\to 0.

This would permit wave equations for particles and fields and facilitate predictions for experimental deviations from classical spacetime physics that can then be tested experimentally.

The Lie algebra should be semisimple (Yang, I. E. Segal 1947). This makes it easier to formulate a finite theory.

Bi cross product of space time:

$$[x_i, x_j] = 0, \quad [x_i, t] = i\lambda x_i$$

$$\sqrt{p_1^2 + p_2^2 + p_3^2} < \lambda^{-1}$$

q-Deformed spacetime (Wikipedia)

This model was introduced independently by a team working under Julius Wess in 1990 and by Majid and coworkers in a series of papers on braided matrices starting a year later. The point of view in the second approach is that usual Minkowski spacetime has a nice description via Pauli matrices as the space of 2 x 2

hermitian matrices. In quantum group theory and using braided monoidal category methods one has a natural q-version of this defined here for real values of q as a `braided hermitian matrix' of generators and relations

$$\begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix} = \begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix}^{\dagger}, \quad \beta\alpha = q^2\alpha\beta,\ [\alpha,\delta] = 0,\ [\beta,\gamma] = (1-q^{-2})\alpha(\delta-\alpha),\ [\delta,\beta] = (1-q^{-2})\alpha\beta$$

These relations say that the generators commute as $q \to 1$ thereby recovering usual Minkowski space. One can work with more familiar variables x, y, z, t as linear combinations of these. In particular, time

$$t = \mathrm{Trace}_q\begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix} = q\delta + q^{-1}\alpha$$

is given by a natural braided trace of the matrix and commutes with the other generators (so this model has a very different flavour from the bicrossproduct one). The braided-matrix picture also leads naturally to a quantity

$$\det_q\begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix} = \alpha\delta - q^2\gamma\beta$$

which as $q \to 1$ returns us the usual Minkowski distance (this translates to a metric in the quantum differential geometry). The parameter $q = e^{\lambda}$ or $q = e^{i\lambda}$ is dimensionless and λ is thought to be a ratio of the Planck scale and the cosmological length. That is, there are indications that that this model relates to quantum gravity with non-zero cosmological constant, the choice of q depending on whether this is positive or negative. We have described the mathematically better understood but perhaps less physically justified positive case here.

Entropic force:

$$\mathbf{F}(\mathbf{X_0}) = T\nabla_{\mathbf{X}}S(\mathbf{X})|_{\mathbf{X_0}}$$

Vijay Balasubramanian et al 2005 Holographic Matrix Models and Quantum Space and Time The world as a hologram Leonard Susskind1 J. Math Phys. 36, 6377 (1995); http://dx.doi.org/10.1063/1.531249

(1) Perhaps the only object that is commensurate to this impressive amount of progress is the list of open problems that one hopes to address using AdS/CFT correspondence. While in principle providing answers to such fundamental questions as the resolution the singularities of (e) classical General Relativity, unitarity of (e) black hole formation and evaporation, the detailed mechanisms behind (eb) remain poorly understood.

(2) One of the central questions is the emergence of bulk locality from (e) the perspective of the boundary theory, as underlined very recently by the headline generating so-called ``firewall paradox".

(3) A related major challenge is to extend the correspondence such that it applies (e&eb) not only to (asymptotically) AdS backgrounds, which are (=) not realistic from the point of view of (e) cosmological observations.

(4) Conversely, while AdS/CFT and its extensions provide (eb) holographic duals for (e) several

phenomena characteristic of (e) the gauge theories describing (eb) our real world (the QCD and electroweak theory), there is no exact dual capturing (e) all their features in a calculable fashion.

(5) The hope in applying methods of integrable systems to (e&eb) the maximally supersymmetric situation (and its variants, most notably the correspondence relating type IIA string theory on (eb)${\it AdS}_4\times {{\mathbb C}{\mathbb P}}^3$ to Chern-Simons-Matter theories (ABJM) in 3 dimensions) is to eventually obtain (eb) an exact solution of the simplest non-trivial case as a starting point for further

NOTATION

Module One

Why does the predicted mass of the quantum vacuum have little effect on the expansion of the universe?

G_{13} : Category one of predicted mass of quantum vacuum

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of expansion of universe (both accentuation and attrition coefficients are taken as zero)

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

$$\mathrm{d}S = \frac{\mathrm{d}M}{T}.$$

G_{16} : Category one of LHS of the above equation

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of RHS of the above equation

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

(1) Physical reasons have been given to believe that physical spacetime is a quantum spacetime.
(2) In quantum mechanics position and momentum variables x, p are already noncommutative, obey (eb) the Heisenberg uncertainty principle, and are continuous.
(3) Because of the Heisenberg uncertainty relations, greater energy is (=) needed to probe smaller distances.
(4) Ultimately, according to gravity theory, the probing particles form (eb) black holes and destroy (e) what is to be measured.
(5) Then (eb) the process cannot be repeated, and so does not count as a measurement.
(6) This limited measurability led many to expect that our usual picture of continuous commutative spacetime breaks down (e) at Planck scale distances, if not sooner.

G_{20} : Category one of position and momentum variables x, p are already noncommutative

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of Heisenberg uncertainty principle, and are continuous

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Because of the Heisenberg uncertainty relations, greater energy is (=) needed to probe smaller distances

G_{24} : Category one of Heisenberg uncertainty relations imply that greater energy

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of needed to probe smaller distances

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Ultimately, according to gravity theory, the probing particles form (eb) black holes and destroy (e) what is to be measured

G_{28} : Category one of probing particles according to gravity theory

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of black holes and destroy (e) what is to be measured

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

probing particles according to gravity theory form black holes and destroy (e) what is to be measured

G_{32} : Category one of probing particles according to gravity theory

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of black holes and destroy (e) what is to be measured

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

probing particles according to gravity theory form black holes and destroy (e) what is to be measured

G_{36} : Category one of what is to be measured

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of probing particles according to gravity theory form black holes

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

G_{40} : Category one of probing particles according to gravity theory form black holes and destroy what is to be measured

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of process cannot be repeated, and so does not count as a measurement

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

This limited measurability led many to expect that our usual picture of continuous commutative spacetime breaks down (e) at Planck scale distances, if not sooner.

G_{44} : Category one of limited measurability led many to expect that our usual picture of continuous commutative spacetime

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of Planck scale distances

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION TWENTY FOUR

Massless Black Holes And Black Rings

INTRODUCTION—VARIABLES USED

Class. Quantum Grav 22 4803 doi:10.1088/0264-9381/22/22/010 Massless black holes and black rings as effective geometries of the D1–D5 system Vijay Balasubramanian1, Per Kraus2 and Masaki Shigemori3

(1) **Vijay Balasubramanian1, Per Kraus2 and Masaki Shigemori3** compute correlation functions in (eb) the AdS3/CFT2 correspondence to study the emergence of (e) effective spacetime geometries describing (eb) complex underlying microstates.
(2) The basic argument is that almost all microstates of fixed charges lie close to (eb) certain 'typical' configurations.
(3) These give a universal response to (e) generic probes, which is captured by (e) an emergent geometry.

Vijay Balasubramanian et al 2005 Class. Quantum Grav 22 4803 doi:10.1088/0264-9381/22/22/010 Massless black holes and black rings as effective geometries of the D1–D5 system Vijay Balasubramanian1, Per Kraus2 and Masaki Shigemori3

(4) The details of the microstates can only be observed by (e) atypical probes.
(5) Authors compute two-point functions in (eb) typical ground states of the Ramond sector of (e) the D1–D5 CFT, and compare with (e&eb) bulk two-point functions computed in (eb) asymptotically AdS3 geometries.
(6) Authors compute two-point functions in (eb) typical ground states of the Ramond sector of (e) the D1–D5 CFT, and compare with (e&eb) bulk two-point functions computed in (eb) asymptotically AdS3 geometries.
(7) For large central charge (which leads to (eb) a good semiclassical limit), and sufficiently small time separation, a typical Ramond ground state of (e) vanishing R-charge has the M = 0 BTZ black hole as its effective description.
(8) At large time separation this effective description breaks down. The CFT correlators authors compute take over, and give (eb) a response whose details depend on (=) the microstate.
(9) They also discuss typical states with (e&eb) nonzero R-charge, and argue that the effective geometry should be (=) a singular black ring.
(10) Results support the argument that black hole geometry should be understood as (=) an effective coarse-grained description that accurately describes (eb) the results of certain typical measurements, but breaks down (e) in general.

NOTATION

Module One

Vijay Balasubramanian1, Per Kraus2 and Masaki Shigemori3 compute

correlation functions in (eb) the AdS3/CFT2 correspondence to study the emergence of (e) effective spacetime geometries describing (eb) complex underlying microstates

G_{13} : Category one of correlation functions

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of AdS3/CFT2 correspondence to study the emergence of (e) effective spacetime geometries describing (eb) complex underlying microstates

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

correlation functions in the AdS3/CFT2 correspondence to study the emergence of (e) effective spacetime geometries describing (eb) complex underlying microstates

G_{16} : Category one of correlation functions in the AdS3/CFT2 correspondence

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of effective spacetime geometries describing (eb) complex underlying microstates

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

correlation functions in the AdS3/CFT2 correspondence to study the emergence of effective spacetime geometries describing (eb) complex underlying microstates

G_{20} : Category one of correlation functions in the AdS3/CFT2 correspondence to study the emergence of effective spacetime geometries

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of complex underlying microstates

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

The basic argument is that almost

all microstates of fixed charges lie close to (eb) certain 'typical' configurations

G_{24} : Category one of all microstates of fixed charges

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of certain 'typical' configurations

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

The details of the microstates can only be observed by (e) atypical probes

G_{28} : Category one of atypical probes

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of details of the microstates

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Authors compute

two-point functions in (eb) typical ground states of the Ramond sector of (e) the D1–D5 CFT, and compare with (e&eb) bulk two-point functions computed in (eb) asymptotically AdS3 geometries

G_{32} : Category one of two-point functions

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of typical ground states of the Ramond sector of (e) the D1–D5 CFT, and compare with (e&eb) bulk two-point functions computed in (eb) asymptotically AdS3 geometries

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

two-point functions in typical ground states of the Ramond sector of (e) the D1–D5 CFT, and compare with (e&eb) bulk two-point functions computed in (eb) asymptotically AdS3 geometries

G_{36} : Category one of D1–D5 CFT, and compare with (e&eb) bulk two-point functions computed in (eb) asymptotically AdS3 geometries

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of two-point functions in typical ground states of the Ramond sector

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

two-point functions in typical ground states of the Ramond sector of the D1–D5 CFT, and compare with (e&eb) bulk two-point functions computed in (eb) asymptotically AdS3 geometries

G_{40} : Category one of two-point functions in typical ground states of the Ramond sector of the D1–D5 CFT; bulk two-point functions computed in (eb) asymptotically AdS3 geometries

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of bulk two-point functions computed in (eb) asymptotically AdS3 geometries; two-point functions in typical ground states of the Ramond sector of the D1–D5 CFT

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

two-point functions in typical ground states of the Ramond sector of the D1–D5 CFT, and compare with bulk two-point functions computed in (eb) asymptotically AdS3 geometries

G_{44} : Category one of two-point functions in typical ground states of the Ramond sector of the D1–D5 CFT, and compare with bulk two-point functions computed

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of asymptotically AdS3 geometries

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION TWENTY FIVE

Quantum Causal Histories (QCHs)

INTRODUCTION—VARIABLES USED

Eli Hawkins et al 2003 Class. Quantum Grav 20 3839 doi:10.1088/0264-9381/20/16/320 Evolution in quantum causal histories Eli Hawkins1, Fotini Markopoulou2,3 and Hanno Sahlmann4

(1) Eli Hawkins1, Fotini Markopoulou2,3 and Hanno Sahlmann4 provide a precise definition and analysis of quantum causal histories (QCHs).

(2) A QCH consists of (e) a discrete, locally finite, causal pre-spacetime with matrix algebras encoding (e) the quantum structure at each event.

(3) The evolution of quantum states and observables is described by (e) completely positive maps between the algebras at (eb) causally related events.

(4) The evolution of quantum states and observables is described by (e) completely positive maps between the algebras at (eb) causally related events

(5) They show that this local description of evolution is (=) sufficient and that unitary evolution can be recovered (eb) wherever it should actually be expected.

(6) This formalism may describe a quantum cosmology without (e) an assumption of global hyperbolicity; it is thus more general than the Wheeler–De Witt approach.

(7) The structure of a QCH is also closely related to (e&eb) quantum information theory and (e&eb) algebraic quantum field theory on a causal set.

Daniele Oriti 2001 Rep. Prog Phys. 64 1703 doi:10.1088/0034-4885/64/12/203 Spacetime geometry from algebra: spin foam models for non-perturbative quantum gravity

(8) This is an introduction to spin foam models for (e) non-perturbative quantum gravity, an approach that lies at (eb) the point of convergence of (e) many different research areas, including (e) loop quantum gravity, topological quantum field theories, (e&eb) path integral quantum gravity, lattice field theory, matrix models, category theory and statistical mechanics.

(9) This is an introduction to spin foam models for (e) non-perturbative quantum gravity, an approach that lies at (eb) the point of convergence of (e) many different research areas, including (e) loop quantum gravity, topological quantum field theories, (e&eb) path integral quantum gravity, lattice field theory, matrix models, category theory and statistical mechanics

(10) Authors describe the general formalism and ideas of spin foam models, the picture of quantum geometry emerging from (e) them, and give a review of the results obtained so far, in both the Euclidean and Lorentzian cases. They focus in particular on the Barrett-Crane model for four-dimensional quantum gravity.

NOTATION

Module One

A QCH consists of (e) a discrete, locally finite, causal pre-spacetime with matrix algebras encoding (e) the quantum structure at each event

G_{13} : Category one of discrete, locally finite, causal pre-spacetime with matrix algebras encoding (e) the quantum structure at each event

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of quantum causal histories

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

QCH consists of a discrete, locally finite, causal pre-spacetime with matrix algebras encoding (e) the quantum structure at each event

G_{16} : Category one of quantum structure at each event

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of QCH consists of a discrete, locally finite, causal pre-spacetime with matrix algebras

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

evolution of quantum states and observables is described by (e) completely positive maps between the algebras at (eb) causally related events

G_{20} : Category one of completely positive maps between the algebras at (eb) causally related events

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of evolution of quantum states and observables

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

evolution of quantum states and observables is described by completely positive maps between the algebras at (eb) causally related events

G_{24} : Category one of evolution of quantum states and observables is described by completely positive maps between the algebras

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of causally related events

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

This formalism may describe a quantum cosmology without (e) an assumption of global hyperbolicity; it is thus more general than the Wheeler–De Witt approach

G_{28} : Category one of assumption of global hyperbolicity; it is thus more general than the Wheeler–De Witt approach

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of quantum cosmology

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

The structure of a QCH is also closely related to (e&eb) quantum information theory and (e&eb) algebraic quantum field theory on a causal set

G_{32} : Category one of structure of a QCH; quantum information theory and algebraic quantum field theory on a causal set

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of quantum information theory and algebraic quantum field theory on a causal set; structure of a QCH

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

This is an introduction to

spin foam models for (e) non-perturbative quantum gravity, an approach that lies at (eb) the point of convergence of (e) many different research areas, including (e) loop quantum gravity, topological quantum field theories, (e&eb) path integral quantum gravity, lattice field theory, matrix models, category theory and statistical mechanics

G_{36} : Category one ofnon-perturbative quantum gravity, an approach that lies at (eb) the point of convergence of (e) many different research areas, including (e) loop quantum gravity, topological quantum field theories, (e&eb) path integral quantum gravity, lattice field theory, matrix models, category theory and statistical mechanics

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of spin foam models

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

spin foam models for non-perturbative quantum gravity, an approach that lies at (eb) the point of convergence of (e) many different research areas, including (e) loop quantum gravity, topological quantum

field theories, (e&eb) path integral quantum gravity, lattice field theory, matrix models, category theory and statistical mechanics

G_{40} : Category one of spin foam models for non-perturbative quantum gravity, an approach that lies

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of point of convergence of (e) many different research areas, including (e) loop quantum gravity, topological quantum field theories, (e&eb) path integral quantum gravity, lattice field theory, matrix models, category theory and statistical mechanics

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

spin foam models for non-perturbative quantum gravity, an approach that lies at the point of convergence of (e) many different research areas, including (e) loop quantum gravity, topological quantum field theories, (e&eb) path integral quantum gravity, lattice field theory, matrix models, category theory and statistical mechanics

G_{44} : Category one of many different research areas, including (e) loop quantum gravity, topological quantum field theories, (e&eb) path integral quantum gravity, lattice field theory, matrix models, category theory and statistical mechanics

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of spin foam models for non-perturbative quantum gravity, an approach that lies at the point of convergence

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}$ $(a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$ $(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients

$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad (1)$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad (2)$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad (3)$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad (4)$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad (5)$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad (6)$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad (7)$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad (8)$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad (9)$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big)\right]T_{16} \quad (10)$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}), t\big)\right]T_{17} \quad (11)$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}), t\big)\right]T_{18} \quad (12)$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\big((G_{19}), t\big) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad (13)$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad (14)$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad (15)$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad (16)$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad (17)$$
$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad (18)$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor
$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$
$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$
$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$
$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}),t)\right]T_{24} \quad 22$$
$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}),t)\right]T_{25} \quad 23$$
$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}),t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**
$-(b''_{24})^{(4)}((G_{27}),t) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$
$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$
$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$
$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28} \quad 28$$
$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$
$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**
$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}),t\right)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}),t\right)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\left((G_{39}),t\right)\right]T_{38} \quad 42$$
$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}),t\right)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}),t\right)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}),t\right)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}),t\right)\right]T_{46} \quad 54$$
$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}),t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for

category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\left[-(b''_{13})^{(1)}(G,t)\right] & \left[-(b''_{16})^{(2,2,)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,)}(G_{35},t)\right] \\ \left[-(b''_{36})^{(7,7,)}(G_{39},t)\right] & \left[-(b''_{40})^{(8,8)}(G_{43},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\left[-(b''_{14})^{(1)}(G,t)\right] & \left[-(b''_{17})^{(2,2,)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,)}(G_{35},t)\right] \\ \left[-(b''_{37})^{(7,7,)}(G_{39},t)\right] & \left[-(b''_{41})^{(8,8)}(G_{43},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\left[-(b''_{15})^{(1)}(G,t)\right] & \left[-(b''_{18})^{(2,2,)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,)}(G_{35},t)\right] \\ \left[-(b''_{38})^{(7,7,)}(G_{39},t)\right] & \left[-(b''_{42})^{(8,8)}(G_{43},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15} \quad (60)$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b_{40}'')^{(8,8)}(G_{43},t)$ $-(b_{41}'')^{(8,8)}(G_{43},t)$ $-(b_{42}'')^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt}=(a_{16})^{(2)}G_{17}-\begin{bmatrix}(a_{16}')^{(2)} & +(a_{16}'')^{(2)}(T_{17},t) & +(a_{13}'')^{(1,1,)}(T_{14},t) & +(a_{20}'')^{(3,3,3)}(T_{21},t)\\ +(a_{24}'')^{(4,4,4,4,4)}(T_{25},t) & +(a_{28}'')^{(5,5,5,5,5)}(T_{29},t) & +(a_{32}'')^{(6,6,6,6,6)}(T_{33},t)\\ +(a_{36}'')^{(7,7,7)}(T_{37},t) & +(a_{40}'')^{(8,8,8)}(T_{41},t) & +(a_{44}'')^{(9,9)}(T_{45},t)\end{bmatrix}G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt}=(a_{17})^{(2)}G_{16}-\begin{bmatrix}(a_{17}')^{(2)} & +(a_{17}'')^{(2)}(T_{17},t) & +(a_{14}'')^{(1,1,)}(T_{14},t) & +(a_{21}'')^{(3,3,3)}(T_{21},t)\\ +(a_{25}'')^{(4,4,4,4,4)}(T_{25},t) & +(a_{29}'')^{(5,5,5,5,5)}(T_{29},t) & +(a_{33}'')^{(6,6,6,6,6)}(T_{33},t)\\ +(a_{37}'')^{(7,7,7)}(T_{37},t) & +(a_{41}'')^{(8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9)}(T_{45},t)\end{bmatrix}G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt}=(a_{18})^{(2)}G_{17}-\begin{bmatrix}(a_{18}')^{(2)} & +(a_{18}'')^{(2)}(T_{17},t) & +(a_{15}'')^{(1,1,)}(T_{14},t) & +(a_{22}'')^{(3,3,3)}(T_{21},t)\\ +(a_{26}'')^{(4,4,4,4,4)}(T_{25},t) & +(a_{30}'')^{(5,5,5,5,5)}(T_{29},t) & +(a_{34}'')^{(6,6,6,6,6)}(T_{33},t)\\ +(a_{38}'')^{(7,7,7)}(T_{37},t) & +(a_{42}'')^{(8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9)}(T_{45},t)\end{bmatrix}G_{18} \quad (63)$$

Where $+(a_{16}'')^{(2)}(T_{17},t)$, $+(a_{17}'')^{(2)}(T_{17},t)$, $+(a_{18}'')^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{13}'')^{(1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a_{40}'')^{(8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a_{44}'')^{(9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt}=(b_{16})^{(2)}T_{17}-\begin{bmatrix}(b_{16}')^{(2)} & -(b_{16}'')^{(2)}(G_{19},t) & -(b_{13}'')^{(1,1,)}(G,t) & -(b_{20}'')^{(3,3,3,)}(G_{23},t)\\ -(b_{24}'')^{(4,4,4,4,4)}(G_{27},t) & -(b_{28}'')^{(5,5,5,5,5)}(G_{31},t) & -(b_{32}'')^{(6,6,6,6,6)}(G_{35},t)\\ -(b_{36}'')^{(7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8)}(G_{43},t) & -(b_{44}'')^{(9,9)}(G_{47},t)\end{bmatrix}T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients

for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{26} \quad (75)$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
are sixth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
are seventh augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
are eighth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad 78$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
are fourth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad 81$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation

coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8\ ,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \quad 85$$

$$- \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}, \boxed{(a''_{37})^{(7)}(T_{37},t)}, \boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\,-(b''_{40})^{(8)}(G_{43},t) & -(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\,-(b''_{41})^{(8)}(G_{43},t) & -(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\,-(b''_{42})^{(8)}(G_{43},t) & -(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq \ (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = \ (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \ , \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\,(a_i)^{(1)} + (a_i')^{(1)} + \ (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}\,(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i')^{(1)} + \ (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \leq (p_i)^{(2)} \leq \left(\hat{A}_{16}\right)^{(2)} \qquad 102$$

$$(b_i'')^{(2)}(G_{19},t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\big((G_{19}),t\big) = \ (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - \ T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\big((G_{19}),t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\big[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\big]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}} \ , \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\,(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + \ (\hat{P}_{16})^{(2)}\,(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + \ (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \le (p_i)^{(3)} \le (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \le (r_i)^{(3)} \le (b_i')^{(3)} \le (\hat{B}_{20})^{(3)}$$

$$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with (116)
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \quad (117)$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\widehat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\widehat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\widehat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24,25,26,$ satisfy the inequalities

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[\ (a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\,(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32{,}33{,}34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and \ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32{,}33{,}34,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\,(a_i)^{(6)} + (a_i')^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)}\,(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(KKKKK) $\qquad (a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36{,}37{,}38$ 131

(LLLLL) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37},t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39},t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

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(MMMMM) $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

(NNNNN)

$\lim_{G\to\infty}(b_i'')^{(7)}\big((G_{39}),t\big) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}\big((G_{39}),t\big)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$. (T_{37}',t) and (T_{37},t) are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(OOOOO) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(PPPPP) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \tag{136}$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: (137)

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \tag{138}$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \tag{139}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \tag{140}$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\left((G_{43}), t\right) = (r_i)^{(8)} \tag{141}$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \tag{142}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\left((G_{43}), t\right)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \tag{143}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \tag{144}$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}$, $(r_i)^{(8)}, i = 40,41,42$,
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \tag{145}$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \tag{146}$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\hat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)} ||(G_{47}) - (G_{47})'|| e^{-(\hat{M}_{44})^{(9)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}\left[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)} (\hat{k}_{44})^{(9)}\right] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$, $T_i(0) = T_i^0 > 0$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t}$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t}$, $T_i(0) = T_i^0 > 0$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \ , \ T_i(0) = T_i^0 \ , \ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} , T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\overline{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad (169)$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that (170)

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that (171)

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that (172)

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that (173)

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that (174)

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \leq G_{32}^{0}+\int_0^t\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(t) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \leq G_{36}^{0}+\int_0^t\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$
$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \leq G_{40}^{0}+\int_0^t\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}= \quad 180$$
$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t} \leq \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for G_{41} , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \leq G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \leq \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for G_{45} , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \left\{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\right\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+\ (a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\ (t)=0\ and\ T_i\ (t)=0$

From 19 to 24 it results

$$G_i\ (t)\geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\geq 0$$

$$T_i\ (t)\geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1,\left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13}<(\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt}\leq\left((\widehat{M}_{13})^{(1)}\right)_1-(a'_{14})^{(1)}G_{14}$ and by integrating

$$G_{14}\leq\left((\widehat{M}_{13})^{(1)}\right)_2=G_{14}^0+2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15}\leq\left((\widehat{M}_{13})^{(1)}\right)_3=G_{15}^0+2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a'_{15})^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}\left|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t} \leq$
$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)d\left(\left((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)}\right)\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\}ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0$ for $t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)}G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$: $\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0$ *and* $T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+\ (a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right) d\left(\left((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on T_{25} and respectively on (G_{27})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (228)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(4)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)}G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\widehat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{32})^{(6)}\right] \leq (\widehat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})}, \widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})}, \widetilde{(T_{35})}\right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right)\left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)};\ (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ *and* $(\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ (250)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)} \left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)} (T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b'_i)^{(7)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\,((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1,\left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right),\quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$

From 99 to 44 it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(9)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)}, , (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)} t} - e^{-(a'_{26})^{(4)} t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)} t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28}e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G^0_{28}e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}}G^0_{28}e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}}G^0_{28}e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)}G^0_{28}}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G^0_{30}e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

$$\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_2)^{(5)}((S_1)^{(5)}-(a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t}-e^{-(a'_{30})^{(5)}t}\right]+\ G_{30}^{0}e^{-(a'_{30})^{(5)}t}\Big)$$

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\le T_{30}(t)\le \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$:

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$-(\sigma_2)^{(6)}\le-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\le-(\sigma_1)^{(6)}$

$-(\tau_2)^{(6)}\le-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\le-(\tau_1)^{(6)}$

Definition of $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$: 350

By $(v_1)^{(6)}>0$, $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$, $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

Definition of $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)}>0$, $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$, $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

Definition of $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$:-

If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)}=(v_0)^{(6)},(m_1)^{(6)}=(v_1)^{(6)},$ ***if*** $(v_0)^{(6)}<(v_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}$ $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}$ $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}$ $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}$ $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}$ $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})} \left[e^{(S_1)^{(6)} t} - e^{-(a'_{34})^{(6)} t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)} t} \right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)},, (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^{0}e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}}G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}}G_{36}^{0}e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t}-e^{-(S_2)^{(7)}t}\right]+G_{38}^{0}e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left.\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_2)^{(7)}((S_1)^{(7)}-(a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t}-e^{-(a'_{38})^{(7)}t}\right]+\ G_{38}^{0}e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}}T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}}T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right) - (b''_{41})^{(8)}\left((G_{43}), t\right) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(v_1)^{(8)} > 0$, $(v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$, $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{v}_1)^{(8)}$,, $(\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{v}_1)^{(8)} > 0$, $(\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$, $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$:-

If we define $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$, $\boldsymbol{if}$ $(v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$, $\boldsymbol{if}$ $(v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$,
and $\boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}$, $\boldsymbol{if}$ $(\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, $\boldsymbol{if}$ $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, $\boldsymbol{if}$ $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}$ $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t})$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t}\leq T_{42}(t)\leq \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)},(S_2)^{(8)},(R_1)^{(8)},(R_2)^{(8)}$:- 381

Where $(S_1)^{(8)}=(a_{40})^{(8)}(m_2)^{(8)}-(a'_{40})^{(8)}$

$(S_2)^{(8)}=(a_{42})^{(8)}-(p_{42})^{(8)}$

$(R_1)^{(8)}=(b_{40})^{(8)}(\mu_2)^{(8)}-(b'_{40})^{(8)}$

$(R_2)^{(8)}=(b'_{42})^{(8)}-(r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$:

$(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)}\leq -(a'_{44})^{(9)}+(a'_{45})^{(9)}-(a''_{44})^{(9)}(T_{45},t)+(a''_{45})^{(9)}(T_{45},t)\leq -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)}\leq -(b'_{44})^{(9)}+(b'_{45})^{(9)}-(b''_{44})^{(9)}\big((G_{47}),t\big)-(b''_{45})^{(9)}\big((G_{47}),t\big)\leq -(\tau_1)^{(9)}$

Definition of $(v_1)^{(9)},(v_2)^{(9)},(u_1)^{(9)},(u_2)^{(9)},v^{(9)},u^{(9)}$:

By $(v_1)^{(9)}>0$, $(v_2)^{(9)}<0$ and respectively $(u_1)^{(9)}>0$, $(u_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_1)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)}>0$, $(\bar{v}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$, $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

Definition of $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$:-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)},\ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$
and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \le G_{46}(t) \le \right.$$
$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (v_1)^{(1)} \le (\bar{v}_1)^{(1)} \le \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$, we obtain 386

$$(v_1)^{(1)} \le v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $v^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\right) - (a''_{17})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{\mathrm{G}_{16}}{\mathrm{G}_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (\mathrm{C})^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (\mathrm{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \le \nu^{(2)}(t) \le (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \le u^{(2)}(t) \le (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)}\right) \le \frac{dv^{(3)}}{dt} \le -\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \ge \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}}$$

it follows $(v_0)^{(3)} \le v^{(3)}(t) \le (v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \le v^{(3)}(t) \le (\bar{v}_1)^{(3)}$

If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)} \le \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}} \le v^{(3)}(t) \le$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (\bar{v}_1)^{(3)}$$

If $0 < (v_1)^{(3)} \le (\bar{v}_1)^{(3)} \le (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$, we obtain 403

$$(v_1)^{(3)} \le v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

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$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{36}'')^{(7)} = (a_{37}'')^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b_{36}'')^{(7)} = (b_{37}'')^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a_{40}')^{(8)} - (a_{41}')^{(8)} + (a_{40}'')^{(8)}(T_{41}, t)\right) - (a_{41}'')^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(8)} and\ (b''_i)^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\ ]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\ ]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\ ]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0$ 479

$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0$ 480

$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0$ 481

$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0$ 482

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(t) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first

equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T^*_{33} for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$$

Definition and uniqueness of T^*_{37} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T^*_{37} for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$$

Definition and uniqueness of T^*_{41} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{25} such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507
$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{29} such that $\varphi((G_{31})^*) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{33} such that $\varphi({G_{35}}^*) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{37} such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{41} such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right]+(b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{45} such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)} G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)} G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)} T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)} T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)} G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)} G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)} T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)} T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)} G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)} G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)} T_{21}^*}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T_{22}^* = \frac{(b_{22})^{(3)} T_{21}^*}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]} \quad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)} G_{37}^{*}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)} G_{37}^{*}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)} T_{37}^{*}}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)} T_{37}^{*}}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)} G_{41}^{*}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)} G_{41}^{*}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)} T_{41}^{*}}{[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)} T_{41}^{*}}{[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)} G_{45}^{*}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)} G_{45}^{*}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)} T_{45}^{*}}{[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)} T_{45}^{*}}{[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47})^{*})]}$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \qquad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \qquad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \qquad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \qquad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \qquad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \qquad 530$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i \qquad 532$$

$$\frac{\partial (a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial (b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \qquad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \qquad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \qquad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \qquad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \qquad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}\mathrm{T}^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}\mathrm{T}^*_{18}\mathbb{G}_j\right) \quad 539$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}\left((G_{27})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^{*}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^{*}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^{*}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^{*}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^{*}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^{*}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^{*}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^{*}\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\ B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\Big)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\Big]$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\Big)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\Big)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\Big]$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\Big)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\Big)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\right)$$

$$\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\Big)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\right)$$

$$\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\Big)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION TWENTY SIX

de Sitter Holography

INTRODUCTION—VARIABLES USED

Maulik Parikh and Erik Verlinde JHEP01 (2005)054 doi:10.1088/1126-6708/2005/01/054 de Sitter holography with a finite number of states

(1) Maulik Parikh and Erik Verlinde investigate the possibility that, in a combined theory of quantum mechanics and gravity, de Sitter space is described by(e) finitely many states.

(2) The notion of observer complementarity, which states (eb) that each observer has (e) complete but complementary information, implies that (eb) for a single observer; the complete Hilbert space

describes (eb) one side of the horizon.

(3) Observer complementarity is implemented by (e) identifying antipodal states with (=) outgoing states.

(4) The de Sitter group acts on (e&eb) S-matrix elements.

(5) Despite the fact that the de Sitter group has (e) no nontrivial finite-dimensional unitary representations, author show that (eb) it is possible to construct (eb) an S-matrix that is finite-dimensional, unitary, and de Sitter-invariant.

(6) Despite the fact that the de Sitter group has (e) no nontrivial finite-dimensional unitary representations, author show that (eb) it is possible to construct (eb) an S-matrix that is finite-dimensional, unitary, and de Sitter-invariant

(7) **Maulik Parikh and Erik Verlinde** present classes of examples that realize (eb) this idea holographically in terms of (e&eb) spinor fields on the boundary sphere.

(8) The finite dimensionality is due to (e) Fermi statistics and an ``exclusion principle" that truncates (e) the orthonormal basis in (eb) which the spinor fields can be expanded.

(9) The finite dimensionality is due to (e) Fermi statistics and an ``exclusion principle" that truncates (e) the orthonormal basis in (eb) which the spinor fields can be expanded

NOTATION

Module One

Maulik Parikh and Erik Verlinde investigate the possibility that, in a combined theory of quantum mechanics and gravity,
de Sitter space is described by(e) finitely many states.

G_{13} : Category one of finitely many states

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of de Sitter space

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

The notion of observer complementarity, which states (eb) that each observer has (e) complete but complementary information, implies that (eb) for a single observer; the complete Hilbert space describes (eb) one side of the horizon

G_{16} : Category one of notion of observer complementarity

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of each observer has (e) complete but complementary information, implies that (eb) for a single observer; the complete Hilbert space describes (eb) one side of the horizon

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

notion of observer complementarity, which states that each observer has (e) complete but complementary information, implies that (eb) for a single observer; the complete Hilbert space describes (eb) one side of the horizon

G_{20} : Category one of complete but complementary information, implies that (eb) for a single observer; the complete Hilbert space describes (eb) one side of the horizon

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of notion of observer complementarity, which states that each observer

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

notion of observer complementarity, which states that each observer has complete but complementary information, implies that (eb) for a single observer; the complete Hilbert space describes (eb) one side of the horizon

G_{24} : Category one of notion of observer complementarity, which states that each observer has complete but complementary information

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of complete Hilbert space describes (eb) one side of the horizon for a single observer

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

notion of observer complementarity, which states that each observer has complete but complementary information, implies that for a single observer; the complete Hilbert space describes (eb) one side of the horizon

G_{28} : Category one of notion of observer complementarity, which states that each observer has complete but complementary information, implies that for a single observer; the complete Hilbert space

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of one side of the horizon

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Observer complementarity is implemented by (e) identifying antipodal states with (=) outgoing states

G_{32} : Category one of identifying antipodal states with (=) outgoing states

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of Observer complementarity

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Observer complementarity is implemented by identifying antipodal states with (=) outgoing states

G_{36} : Category one of Observer complementarity is implemented by identifying antipodal states

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of outgoing states

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

The de Sitter group acts on (e&eb) S-matrix elements

G_{40} : Category one of de Sitter group; S-matrix elements

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of S-matrix elements; de Sitter group

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Despite the fact that the de Sitter group has (e) no nontrivial finite-dimensional unitary representations, author shows that (eb) it is possible to construct (eb) an S-matrix that is finite-dimensional, unitary, and de Sitter-invariant.

G_{44} : Category one of no nontrivial finite-dimensional unitary representations

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of de Sitter group

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION TWENTY SIX A

S-Matrix That Is Finite-Dimensional, Unitary, And De Sitter-Invariant

INTRODUCTION—VARIABLES USED

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(1) Despite the fact that the de Sitter group has (e) no nontrivial finite-dimensional unitary representations, author show that (eb) it is possible to construct (eb) an S-matrix that is finite-dimensional, unitary, and de Sitter-invariant

(2) **Maulik Parikh and Erik Verlinde** present classes of examples that realize (eb) this idea holographically in terms of (e&eb) spinor fields on the boundary sphere.

(3) The finite dimensionality is due to (e) Fermi statistics and an ``exclusion principle" that truncates (e) the orthonormal basis in (eb) which the spinor fields can be expanded.

(4) The finite dimensionality is due to (e) Fermi statistics and an ``exclusion principle" that truncates (e) the orthonormal basis in (eb) which the spinor fields can be expanded

NOTATION

Module One

Despite the fact that the de Sitter group has (e) no nontrivial finite-dimensional unitary representations, author shows that (eb) it is possible to construct (eb) an S-matrix that is finite-dimensional, unitary, and de Sitter-invariant.

G_{13} : Category one of de Sitter group has no nontrivial finite-dimensional unitary representations nevertheless

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of an S-matrix that is finite-dimensional, unitary, and de Sitter-invariant

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Maulik Parikh and Erik Verlinde present

classes of examples that realize (eb) this idea holographically in terms of (e&eb) spinor fields on the boundary sphere

G_{16} : Category one of classes of examples

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of idea holographically in terms of (e&eb) spinor fields on the boundary sphere

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

classes of examples that realize this idea holographically in terms of (e&eb) spinor fields on the boundary sphere

G_{20} : Category one of classes of examples that realize this idea holographically; spinor fields on the boundary sphere

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of spinor fields on the boundary sphere; classes of examples that realize this idea holographically

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

The finite dimensionality is due to (e) Fermi statistics and an ``exclusion principle" that truncates (e) the orthonormal basis in (eb) which the spinor fields can be expanded

G_{24} : Category one of Fermi statistics and an ``exclusion principle" that truncates (e) the orthonormal basis in (eb) which the spinor fields can be expanded

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of finite dimensionality

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

finite dimensionality is due to Fermi statistics and an ``exclusion principle" that truncates (e) the orthonormal basis in (eb) which the spinor fields can be expanded

G_{28} : Category one of orthonormal basis in (eb) which the spinor fields can be expanded

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of finite dimensionality is due to Fermi statistics and an ``exclusion principle"

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

finite dimensionality is due to Fermi statistics and an ``exclusion principle" that truncates the orthonormal basis in (eb) which the spinor fields can be expanded

G_{32} : Category one of finite dimensionality is due to Fermi statistics and an ``exclusion principle" that truncates the orthonormal basis

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of spinor fields that an be expanded

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Multiple Modules

Exploring de Sitter Space and Holography Vijay Balasubramanian, Jan de Boer, Djordje Minic

Vijay Balasubramanian, Jan de Boer, Djordje Minic explore aspects of the

Physics of de Sitter (dS) spaces that are relevant to holography with (e&eb) a positive cosmological constant.

(1) First they display a nonlocal map that commutes with (e&eb) the de Sitter isometries, transforms the bulk-boundary propagator and solutions of free wave equations in de Sitter onto (e&eb) the same quantities in Euclidean anti-de Sitter (EAdS), and takes the two boundaries of dS to the single EAdS boundary via (e&eb) an antipodal identification.
(2) Second authors compute the action of scalar fields on dS as (=) a functional of boundary data.
(3) Third, they display a family of solutions to 3d gravity with a positive cosmological constant in which the equal time sections are (=) arbitrary genus Riemann surfaces, and compute the action of these spaces as (=) a functional of boundary data from the Einstein gravity and Chern-Simons gravity points of view.
(4) These studies suggest that if de Sitter space is dual to a Euclidean conformal field theory (CFT), this theory should involve (e&eb) two disjoint, but possibly entangled factors.
(5) They argue that these CFTs would be of (e) a novel form, with unusual hermiticity conditions relating left movers and (e&eb) right movers.
(6) After exploring these conditions in a toy model, they combine observations to propose (eb) that a holographic dual description of de Sitter space would involve (e&eb) a pure entangled state in a

product of two of our unconventional CFTs associated with (e&eb) the de Sitter boundaries.

(7) This state can be constructed to preserve (eb) the de Sitter symmetries and its decomposition in a basis appropriate to antipodal inertial observers would lead to (eb) the thermal properties of static patch. Journal reference: Class.Quant.Grav.19:5655-5700,2002; Ann.Phys.303:59-116,2003 DOI: 10.1016/S0003-4916(02)00020-9 Report number: VPI-IPPAP-02-05, UPR-1008-T, ITFA-2002-26 Cite as: arXiv:hep-th/0207245

Schwarzchild metric:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

$$f(r) = 1 - \frac{2a}{r} - br^2$$

Spatial Frequency:

$$f=(\sin\theta_2-\sin\theta_1)/\lambda$$

G_{36} : Category one of physics of de Sitter (dS) space that are relevant to holography; positive cosmological constant; LHS of above equations

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of positive cosmological constant ;physics of de Sitter (dS) space that are relevant to holography; RHS of above equations

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

First they display a nonlocal map that commutes with (e&eb) the de Sitter isometries, transforms the bulk-boundary propagator and solutions of free wave equations in de Sitter onto (e&eb) the same quantities in Euclidean anti-de Sitter (EAdS), and takes the two boundaries of dS to the single EAdS boundary via (e&eb) an antipodal identification

G_{40} : Category one of nonlocal map; de Sitter isometries, transforms the bulk-boundary propagator and solutions of free wave equations in de Sitter onto (e&eb) the same quantities in Euclidean anti-de Sitter (EAdS), and takes the two boundaries of dS to the single EAdS boundary via (e&eb) an antipodal identification

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of de Sitter isometries, transforms the bulk-boundary propagator and solutions of free wave equations in de Sitter onto (e&eb) the same quantities in Euclidean anti-de Sitter (EAdS), and takes the two boundaries of dS to the single EAdS boundary via (e&eb) an antipodal identification; nonlocal map

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

nonlocal map that commutes with the de Sitter isometries, transforms the bulk-boundary propagator and solutions of free wave equations in de Sitter onto (e&eb) the same quantities in Euclidean anti-de Sitter (EAdS), and takes the two boundaries of dS to the single EAdS boundary via (e&eb) an antipodal identification

G_{44} : Category one of nonlocal map that commutes with the de Sitter isometries, transforms the bulk-boundary propagator and solutions of free wave equations in de Sitter; same quantities in Euclidean anti-de Sitter (EAdS), and takes the two boundaries of dS to the single EAdS boundary via (e&eb) an antipodal identification

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of same quantities in Euclidean anti-de Sitter (EAdS), and takes the two boundaries of dS to the single EAdS boundary via (e&eb) an antipodal identification ;nonlocal map that commutes with the de Sitter isometries, transforms the bulk-boundary propagator and solutions of free wave equations in de Sitter

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION TWENTY SEVEN

Holographic Quantum Foam

INTRODUCTION—VARIABLES USED

Holographic Quantum Foam Y. Jack Ng

(1) Due to quantum fluctuations, probed at small scales, spacetime is very complicated -- something akin in complexity to (=) a turbulent froth which the late John Wheeler dubbed quantum foam, aka spacetime foam.
(2) Our recent work suggests that (1) we may be close to being able to detect (e&eb) quantum foam with extragalactic sources once the Very Large Telescope Interferometers (VLTI) are fully operational;
(3) (2) dark energy is arguably a cosmological manifestation of (e) quantum foam, the constituents of which obey (eb) infinite statistics;

(4) (3) in the gravitational context, turbulence is closely related to (e&eb) holographic quantum foam, partly validating (eb) Wheeler's picture of a turbulent spacetime arXiv: 1001.0411 [gr-qc]

The Kaluza Hypothesis

(5) In his 1921 paper, [2] Kaluza established all the elements of the classical 5-dimensional theory: the metric, the field equations, the equations of motion, the stress-energy tensor, and the cylinder condition. The theory has (e) no free parameters; it merely extends general relativity to (eb) five dimensions.

(6) One starts by hypothesizing a form of (e) the 5-dimensional metric $\widetilde{g}_{ab}$, where roman indices span 5 dimensions.

(7) Let us also introduce the 4-dimensional spacetime metric $g_{\mu\nu}$, where Greek indices span (e&eb) the usual 4 dimensions of (e) space and time; a 4-vector A^{μ} which will be identified with (e&eb) the electromagnetic vector potential; and a scalar field ϕ.

(8) Then decompose the 5D metric so that the 4D metric is framed by (e) the electromagnetic vector potential, with the scalar field at the fifth diagonal

NOTATION

Module One

Exploring de Sitter Space and Holography Vijay Balasubramanian, Jan de Boer, Djordje Minic

Third, they display a family of solutions to 3d gravity with a positive cosmological constant in which the equal time sections are (=) arbitrary genus Riemann surfaces, and compute the action of these spaces as (=) a functional of boundary data from the Einstein gravity and Chern-Simons gravity points of view.

(1) These studies suggest that if de Sitter space is dual to a Euclidean conformal field theory (CFT), this theory should involve (e&eb) two disjoint, but possibly entangled factors.

(2) They argue that these CFTs would be of (e) a novel form, with unusual hermiticity conditions relating left movers and (e&eb) right movers.

(3) After exploring these conditions in a toy model, they combine observations to propose (eb) that a holographic dual description of de Sitter space would involve (e&eb) a pure entangled state in a product of two of our unconventional CFTs associated with (e&eb) the de Sitter boundaries.

(4) This state can be constructed to preserve (eb) the de Sitter symmetries and its decomposition in a basis appropriate to antipodal inertial observers would lead to (eb) the thermal properties of static patch. Journal reference: Class.Quant.Grav.19:5655-5700,2002; Ann.Phys.303:59-116,2003 DOI: 10.1016/S0003-4916(02)00020-9 Report number: VPI-IPPAP-02-05, UPR-1008-T, ITFA-2002-26 Cite as: arXiv:hep-th/0207245

G_{13} : Category one of family of solutions to 3d gravity with a positive cosmological constant in which the equal time sections

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of arbitrary genus Riemann surfaces, and compute the action of these spaces as (=) a functional of boundary data from the Einstein gravity and Chern-Simons gravity points of view

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Due to quantum fluctuations, probed at small scales, spacetime is very complicated -- something akin in complexity to (=) a turbulent froth which the late John Wheeler dubbed quantum foam, aka spacetime foam.

G_{16} : Category one of spacetime Due to quantum fluctuations, probed at small scales

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of turbulent froth

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Authors' recent work suggests that (1) we may be close to being able to detect (e&eb) quantum foam with extragalactic sources once the Very Large Telescope Interferometers (VLTI) are fully operational

G_{20} : Category one of quantum foam; extragalactic sources

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of extragalactic sources ;quantum foam

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

These studies suggest that if de Sitter space is dual to a Euclidean conformal field theory (CFT), this theory should involve (e&eb) two disjoint, but possibly entangled factors

G_{24} : Category one of de Sitter space is dual to a Euclidean conformal field theory (CFT); two disjoint, but possibly entangled factors

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of two disjoint, but possibly entangled factors ;de Sitter space is dual to a Euclidean conformal field theory (CFT)

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

dark energy is arguably a cosmological manifestation of (e) quantum foam, the constituents of which obey (eb) infinite statistics

G_{28} : Category one of quantum foam, the constituents of which obey infinite statistics

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of dark energy is arguably a cosmological manifestation

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

turbulence is closely related to (e&eb) holographic quantum foam, partly validating (eb) Wheeler's picture of a turbulent spacetime arXiv: 1001.0411 [gr-qc]

G_{32} : Category one of turbulence; holographic quantum foam

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of holographic quantum foam; turbulence

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

4-dimensional spacetime metric $g_{\mu\nu}$, where Greek indices span (e&eb) the usual 4 dimensions of (e) space and time; a 4-vector A^{μ} which will be identified with (e&eb) the electromagnetic vector potential; and a scalar field ϕ.

G_{36} : Category one of**4**-dimensional spacetime metric $g_{\mu\nu}$, where Greek indices; usual 4 dimensions of (e) space and time; a 4-vector A^{μ} which will be identified with (e&eb) the electromagnetic vector potential; and a scalar field ϕ.

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of usual 4 dimensions of (e) space and time; a 4-vector A^{μ} which will be identified with (e&eb) the electromagnetic vector potential; and a scalar field ϕ. ;**4**-dimensional spacetime metric $g_{\mu\nu}$, where Greek indices

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

4-dimensional spacetime metric $g_{\mu\nu}$, where Greek indices span the usual 4 dimensions of (e) space and time; a 4-vector A^{μ} which will be identified with (e&eb) the electromagnetic vector potential; and a scalar field ϕ.

G_{40} : Category one of**4**-dimensional spacetime metric $g_{\mu\nu}$, where Greek indices span the usual 4 dimensions of (e) space and time; a 4-vector A^{μ} ; electromagnetic vector potential; and a scalar field ϕ.

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one o electromagnetic vector potential; and a scalar field ϕ.;f**4**-dimensional spacetime metric $g_{\mu\nu}$, where Greek indices span the usual 4 dimensions of (e) space and time; a 4-vector A^{μ}

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Exploring de Sitter Space and Holography Vijay Balasubramanian, Jan de Boer, Djordje Minic

They argue that these CFTs would be of a novel form, with unusual hermiticity conditions relating left movers and (e&eb) right movers

G_{44} : Category one of CFTs would be of a novel form, with unusual hermiticity conditions relating left movers; right movers

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of right movers ;CFTs would be of a novel form, with unusual hermiticity conditions relating left movers

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients

$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)} \ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,

are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) & \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) & \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for

category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad (60)$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt}=(a_{16})^{(2)}G_{17}-\left[\begin{matrix}(a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{44})^{(9,9)}(T_{45},t)}\end{matrix}\right]G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt}=(a_{17})^{(2)}G_{16}-\left[\begin{matrix}(a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9)}(T_{45},t)}\end{matrix}\right]G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt}=(a_{18})^{(2)}G_{17}-\left[\begin{matrix}(a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}\\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9)}(T_{45},t)}\end{matrix}\right]G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt}=(b_{16})^{(2)}T_{17}-\left[\begin{matrix}(b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}\ \boxed{-(b''_{13})^{(1,1,)}(G,t)}\ \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}\\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}\\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}\ \boxed{-(b''_{44})^{(9,9)}(G_{47},t)}\end{matrix}\right]T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients

for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)}&\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}&\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}&\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}&\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}&\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}&\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)}&\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}&\boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}&\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}&\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}&\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}&\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)}&\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}&\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}&\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}&\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}&\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}&\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{26} \quad (75)$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
are sixth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
are seventh augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
are eighth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)}&\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}&\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}&\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}&\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}&\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}&\boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)}&\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}&\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}&\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}&\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}&\boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}&\boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad (78)$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
are fourth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad (81)$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation

coefficients for category 1,2, and 3
$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3
$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3
$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3
$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3
$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3
$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3
$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3
$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3
$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3
$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3
$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3
$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3
$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \quad 85$$

$$- \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}, \boxed{(a''_{37})^{(7)}(T_{37},t)}, \boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\left[-(b''_{40})^{(8)}(G_{43},t)\right] & -(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\left[-(b''_{41})^{(8)}(G_{43},t)\right] & -(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\left[-(b''_{42})^{(8)}(G_{43},t)\right] & -(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq \ (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = \ (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \ , \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\,(a_i)^{(1)} + (a_i')^{(1)} + \ (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}\,(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i')^{(1)} + \ (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \leq (p_i)^{(2)} \leq \left(\hat{A}_{16}\right)^{(2)} \qquad 102$$

$$(b_i'')^{(2)}(G_{19},t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\big((G_{19}),t\big) = \ (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\big((G_{19}),t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\big[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\big]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}} \ , \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\,(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + \ (\hat{P}_{16})^{(2)}\,(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + \ (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \le (p_i)^{(3)} \le (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \le (r_i)^{(3)} \le (b_i')^{(3)} \le (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\,(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and \ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\,(a_i)^{(6)} + (a_i')^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)}\,(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(QQQQQ) $\quad (a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(RRRRR) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$(a_i'')^{(7)}(T_{37},t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$

$(b_i'')^{(7)}(G_{39},t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$ 132

(SSSSS) $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

(TTTTT)

$\lim_{G\to\infty}(b_i'')^{(7)}((G_{39}),t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \leq (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$

$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}((G_{39}),t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$. (T_{37}',t) and (T_{37},t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(UUUUU) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(VVVVV) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$

$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$

Where we suppose

$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$ 138

$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$ 139

$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$ 140

$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t}$ 142

$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t}$ 143

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ 144

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}$, $(r_i)^{(8)}, i = 40,41,42$,
Satisfy the inequalities

$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 145

$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 146

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad 146\ A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\hat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)} ||(G_{47}) - (G_{47})'|| e^{-(\hat{M}_{44})^{(9)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}\left[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)} (\hat{k}_{44})^{(9)}\right] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t} \ , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t} \ , \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \ , \ T_i(0) = T_i^0 \ , \ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} , T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\overline{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{16})^{(2)}, T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{20})^{(3)}, T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad (169)$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that (170)

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that (171)

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that (172)

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that (173)

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that (174)

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \leq G_{32}^{0}+\int_0^t\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)} =$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(u) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \leq G_{36}^{0}+\int_0^t\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)} =$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \leq G_{40}^{0}+\int_0^t\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)} = \quad 180$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t} \leq \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \leq G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \leq \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $\left(\tilde{G}, \tilde{T}\right) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)}\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) - (a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right| \; e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \qquad (186)$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13{,}14{,}15$ depend only on T_{14} and respectively on G(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: (187)

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. (188)

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}} \leq$
$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{\mathrm{A}}_{16})^{(2)} + (\widehat{\mathrm{P}}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)\mathrm{d}\left(\left((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)}\right)\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $\mathrm{G}_i\,(\mathrm{t}) = 0$ and $\mathrm{T}_i\,(\mathrm{t}) = 0$ 200

it results

$\mathrm{G}_i\,(\mathrm{t}) \geq \mathrm{G}_i^0\mathrm{e}^{\left[-\int_0^{\mathrm{t}}\{(a_i')^{(2)} - (a_i'')^{(2)}(\mathrm{T}_{17}(s_{(16)}), s_{(16)})\}\mathrm{d}s_{(16)}\right]} \geq 0$

$\mathrm{T}_i\,(\mathrm{t}) \geq \mathrm{T}_i^0\mathrm{e}^{(-(b_i')^{(2)}\mathrm{t})} > 0 \quad \text{for t} > 0$

Definition of $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1$, $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2$ and $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$\mathrm{G}_{16} < (\widehat{\mathrm{M}}_{16})^{(2)}$ it follows $\frac{\mathrm{dG}_{17}}{\mathrm{dt}} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)}\mathrm{G}_{17}$ and by integrating

$\mathrm{G}_{17} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 = \mathrm{G}_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$

In the same way , one can obtain

$\mathrm{G}_{18} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3 = \mathrm{G}_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{\mathrm{t}\to\infty}((b_i'')^{(2)}\,((G_{19})(\mathrm{t}), \mathrm{t})) = (b_{17}')^{(2)}$ then $\mathrm{T}_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$: $\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\ and\ (\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on $(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+\ (a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right) d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ (228)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}),s_{(24)})\}ds_{(24)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(4)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)}G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)}, (T_{35})^{(1)}), ((G_{35})^{(2)}, (T_{35})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})}, \widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})}, \widetilde{(T_{35})}\right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right)\left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)};\ (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ *and* $(\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ (250)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$
$sup_i\{max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}, max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)} \left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)}; (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t} \; and \; (\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)} (T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2 \; and \; \left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37} \; and \; G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1,\left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right),\quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$

From 99 to 44 it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(9)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)}, , (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad (333)$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad (334)$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)} t} - e^{-(a'_{26})^{(4)} t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)} t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad (335)$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad (336)$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G^0_{28} e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G^0_{28}}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G^0_{30} e^{-(S_2)^{(5)} t} \le G_{30}(t) \le \right. \quad 344$$

$$\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_2)^{(5)}((S_1)^{(5)}-(a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t}-e^{-(a'_{30})^{(5)}t}\right]+\ G_{30}^{0}e^{-(a'_{30})^{(5)}t}\Big)$$

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \qquad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \qquad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\le T_{30}(t)\le \qquad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$:

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)}\le-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\le-(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)}\le-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\le-(\tau_1)^{(6)}$$

Definition of $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$: 350

By $(v_1)^{(6)}>0$, $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$, $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\left(v^{(6)}\right)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\left(u^{(6)}\right)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

Definition of $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)}>0$, $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$, $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\left(v^{(6)}\right)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\left(u^{(6)}\right)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

Definition of $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$:-

If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)}=(v_0)^{(6)},(m_1)^{(6)}=(v_1)^{(6)},$ ***if*** $(v_0)^{(6)}<(v_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}$ $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}$ $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}$ $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}$ $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}$ $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})} \left[e^{(S_1)^{(6)} t} - e^{-(a'_{34})^{(6)} t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)} t} \right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^{0}e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}}G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}}G_{36}^{0}e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t}-e^{-(S_2)^{(7)}t}\right]+G_{38}^{0}e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left.\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_2)^{(7)}((S_1)^{(7)}-(a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t}-e^{-(a'_{38})^{(7)}t}\right]+\ G_{38}^{0}e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}}T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}}T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right) - (b''_{41})^{(8)}\left((G_{43}), t\right) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(v_1)^{(8)} > 0$, $(v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$, $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{v}_1)^{(8)}$,, $(\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{v}_1)^{(8)} > 0$, $(\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$, $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$:-

If we define $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$, $\boldsymbol{if}$ $(v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$, $\boldsymbol{if}$ $(v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$,
and $\boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}$, $\boldsymbol{if}$ $(\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, $\boldsymbol{if}$ $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, $\boldsymbol{if}$ $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}$ $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t})$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \qquad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq\frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \qquad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t}\leq T_{42}(t)\leq \qquad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)},(S_2)^{(8)},(R_1)^{(8)},(R_2)^{(8)}$:- 381

Where $(S_1)^{(8)}=(a_{40})^{(8)}(m_2)^{(8)}-(a'_{40})^{(8)}$

$(S_2)^{(8)}=(a_{42})^{(8)}-(p_{42})^{(8)}$

$(R_1)^{(8)}=(b_{40})^{(8)}(\mu_2)^{(8)}-(b'_{40})^{(8)}$

$(R_2)^{(8)}=(b'_{42})^{(8)}-(r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$:

$(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)}\leq-(a'_{44})^{(9)}+(a'_{45})^{(9)}-(a''_{44})^{(9)}(T_{45},t)+(a''_{45})^{(9)}(T_{45},t)\leq-(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)}\leq-(b'_{44})^{(9)}+(b'_{45})^{(9)}-(b''_{44})^{(9)}\big((G_{47}),t\big)-(b''_{45})^{(9)}\big((G_{47}),t\big)\leq-(\tau_1)^{(9)}$

Definition of $(v_1)^{(9)},(v_2)^{(9)},(u_1)^{(9)},(u_2)^{(9)},v^{(9)},u^{(9)}$:

By $(v_1)^{(9)}>0$, $(v_2)^{(9)}<0$ and respectively $(u_1)^{(9)}>0$, $(u_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_1)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)}>0$, $(\bar{v}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$, $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

Definition of $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$:-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)},\ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$
and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \le G_{46}(t) \le \right.$$
$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (v_1)^{(1)} \le (\bar{v}_1)^{(1)} \le \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$, we obtain 386

$$(v_1)^{(1)} \le v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $v^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{\mathrm{d}v^{(2)}}{\mathrm{dt}} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\right) - (a''_{17})^{(2)}(\mathrm{T}_{17}, \mathrm{t})v^{(2)} - (a_{17})^{(2)}v^{(2)}$$

Definition of $v^{(2)}$:- $\boxed{v^{(2)} = \frac{\mathrm{G}_{16}}{\mathrm{G}_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(v^{(2)}\right)^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)}\right) \leq \frac{\mathrm{d}v^{(2)}}{\mathrm{dt}} \leq -\left((a_{17})^{(2)}\left(v^{(2)}\right)^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$v^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (\mathrm{C})^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (\mathrm{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq v^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)}\right) \le \frac{dv^{(3)}}{dt} \le -\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \ge \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}}$$

it follows $(v_0)^{(3)} \le v^{(3)}(t) \le (v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \le v^{(3)}(t) \le (\bar{v}_1)^{(3)}$

If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)} \le \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}} \le v^{(3)}(t) \le$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (\bar{v}_1)^{(3)}$$

If $0 < (v_1)^{(3)} \le (\bar{v}_1)^{(3)} \le (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$, we obtain 403

$$(v_1)^{(3)} \le v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

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$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right) - (a''_{41})^{(8)}(T_{41},t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(8)} and\ (b''_i)^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\,]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\,]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\,]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0$ 479

$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0$ 480

$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0$ 481

$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0$ 482

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(u) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first

equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T^*_{33} for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$$

Definition and uniqueness of T^*_{37} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T^*_{37} for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$$

Definition and uniqueness of T^*_{41} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{25} such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507
$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{29} such that $\varphi((G_{31})^*) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{33} such that $\varphi({G_{35}}^*) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{37} such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{41} such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right]+(b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{45} such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)} G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)} G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)} T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)} T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)} G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)} G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)} T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)} T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)} G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)} G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)} T_{21}^*}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T_{22}^* = \frac{(b_{22})^{(3)} T_{21}^*}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]} \quad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \qquad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \qquad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \qquad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \qquad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \qquad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \qquad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i \qquad 532$$

$$\frac{\partial (a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial (b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \qquad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \qquad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \qquad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \qquad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \qquad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}\mathrm{T}^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}\mathrm{T}^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}\left((G_{27})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^{*}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^{*}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^{*}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^{*}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^{*}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^{*}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^{*}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^{*}\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

ASYMPTOTIC STABILITY ANALYSIS (586 A)

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\ B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\Big)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\Big]$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\Big)$$

$$+\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\Big)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\Big)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\Big]$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\Big)$$

$$+\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\Big)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\Big)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\Big]$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\Big)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\Big)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\Big)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\Big)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\Big)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\Big)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION TWENTY EIGHT

The Wightman Axioms

INTRODUCTION—VARIABLES USED

The Wightman axioms

The Millenium problem requires the proposed Yang-Mill theory to satisfy the Wightman axioms or similarly stringent axioms.[1] There are four axioms:

W0 (assumptions of relativistic quantum mechanics)

(1) Quantum mechanics is described according to von Neumann; in particular, the pure states are given by (eb) the rays, i.e. the one-dimensional subspaces, of someseparable complex Hilbert space.
(2) The Wightman axioms require that the Poincaré group acts (e&eb) unitarily on the Hilbert space.
(3) In other words, they have position dependent operators called quantum fields which form (eb) covariant representations of the Poincaré group.
(4) The group of space-time translations is (=) commutative, and so the operators can be (=) simultaneously diagonalized.
(5) The generators of these groups give us (eb) four self-adjoint operators, P_0, P_j, j = 1, 2, 3, which transform under (e&eb) the homogeneous group as a four-vector, called the energy-momentum four-vector.
(6) The second part of the zeroth axiom of Wightman is that the representation U (a, A) fulfills (eb)the spectral condition - that the simultaneous spectrum of energy-momentum is contained in (eb) the forward cone:

$$P_0 \geq 0 \text{.....} P_0^2 - P_j P_j \geq 0.$$

(7) The third part of the axiom is that there is a unique state, represented by (e) a ray in the Hilbert space, which is (=) invariant under the action of (e&eb) the Poincaré group. It is called a vacuum.
(8) W1 (assumptions on the domain and continuity of the field) For each test function f, there exists (eb) a set of operators $A_1(f), \ldots, A_n(f)$ which, together with (e&eb) their adjoints, are defined on (eb) a dense subset of (e) the Hilbert state space, containing (e) the vacuum.
(9) The fields A are (=) operator-valued tempered distributions.
(10) The Hilbert state space is spanned by (e&eb) the field polynomials acting on (e&eb) the vacuum (cyclicity condition).

NOTATION

Module One

Quantum mechanics is described according to von Neumann; in particular, the pure states are given by (eb) the rays, i.e. the one-dimensional subspaces, of someseparable complex Hilbert space

G_{13} : Category one of rays, i.e. the one-dimensional subspaces, of someseparable complex Hilbert space

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of Quantum mechanics is described according to von Neumann; in particular, the pure states

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Wightman axioms require that the Poincaré group acts (e&eb) unitarily on the Hilbert space

G_{16} : Category one of Wightman axioms require that the Poincaré group ; unitarily on the Hilbert space

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of unitarily on the Hilbert space; Wightman axioms require that the Poincaré group

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

they have position dependent operators called quantum fields which form (eb) covariant representations of the Poincaré group

G_{20} : Category one of they have position dependent operators called quantum fields

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of covariant representations of the Poincaré group

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

group of space-time translations is (=) commutative, and so the operators can be (=) simultaneously diagonalized

G_{24} : Category one of group of space-time translations

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of commutative, and so the operators can be simultaneously diagonalized

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

generators of these groups give us (eb) four self-adjoint operators, P_0, P_j, j = 1, 2, 3, which transform under (e&eb) the homogeneous group as a four-vector, called the energy-momentum four-vector

G_{28} : Category one of generators of these groups give us (eb) four self-adjoint operators, P_0, P_j, j = 1, 2, 3, which transform; homogeneous group as a four-vector, called the energy-momentum four-vector

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of homogeneous group as a four-vector, called the energy-momentum four-vector;

generators of these groups give us (eb) four self-adjoint operators, P_0, P_j, j = 1, 2, 3, which transform

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

The second part of the zeroth axiom of Wightman is that the representation U (a, A) fulfills (eb)the spectral condition - that the simultaneous spectrum of energy-momentum is contained in (eb) the forward cone:

$$P_0 \geq 0 \text{.....} P_0^2 - P_j P_j \geq 0.$$

G_{32} : Category one of second part of the zeroth axiom of Wightman is that the representation U (a, A) fulfills

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of spectral condition - that the simultaneous spectrum of energy-momentum is contained in the forward cone: $P_0 \geq 0$..... $P_0^2 - P_j P_j \geq 0.$

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Third part of the axiom is that there is a unique state, represented by (e) a ray in the Hilbert space, which is (=) invariant under the action of (e&eb) the Poincaré group. It is called a vacuum

G_{36} : Category one of ray in the Hilbert space, which is (=) invariant under the action of (e&eb) the Poincaré group. It is called a vacuum

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of third part of the axiom is that there is a unique state

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

Third part of the axiom is that there is a unique state, represented by a ray in the Hilbert space, which is invariant under the action of (e&eb) the Poincaré group. It is called a vacuum

G_{40} : Category one of Third part of the axiom is that there is a unique state, represented by a ray in the Hilbert space, which is invariant ; Poincaré group

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of Poincaré group; Third part of the axiom is that there is a unique state, represented by a ray in the Hilbert space, which is invariant

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

The fields A are (=) operator-valued tempered distributions

G_{44} : Category one of fields A

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of operator-valued tempered distributions

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}), t\right)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}), t\right)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\left((G_{39}), t\right)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & \boxed{+(a''_{13})^{(1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & \boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & \boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\,-(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\,-(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\,-(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21},t)\;\; +(a''_{16})^{(2,2,2)}(T_{17},t)\;\; +(a''_{13})^{(1,1,1,)}(T_{14},t)\\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)\;\; +(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)\;\; +(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a''_{36})^{(7,7,7,7)}(T_{37},t)\;\; +(a''_{40})^{(8,8,8,8)}(T_{41},t)\;\; +(a''_{44})^{(9,9,9)}(T_{45},t)\end{array}\right]G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c}(a'_{21})^{(3)}+(a''_{21})^{(3)}(T_{21},t)\;\; +(a''_{17})^{(2,2,2)}(T_{17},t)\;\; +(a''_{14})^{(1,1,1,)}(T_{14},t)\\ +(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)\;\; +(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)\;\; +(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a''_{37})^{(7,7,7,7)}(T_{37},t)\;\; +(a''_{41})^{(8,8,8,8)}(T_{41},t)\;\; +(a''_{45})^{(9,9,9)}(T_{45},t)\end{array}\right]G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21},t)\;\; +(a''_{18})^{(2,2,2)}(T_{17},t)\;\; +(a''_{15})^{(1,1,1,)}(T_{14},t)\\ +(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)\;\; +(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)\;\; +(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a''_{38})^{(7,7,7,7)}(T_{37},t)\;\; +(a''_{42})^{(8,8,8,8)}(T_{41},t)\;\; +(a''_{46})^{(9,9,9)}(T_{45},t)\end{array}\right]G_{22} \quad (69)$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\,-(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\,-(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\,-(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$ $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad 75$$

$(a''_{24})^{(4)}(T_{25},t)$, $(a''_{25})^{(4)}(T_{25},t)$, $(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$
are fourth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$
are fifth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$
are sixth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$
are seventh augmentation coefficients for category 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$
are eighth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

$Where\ \boxed{-(b_{24}'')^{(4)}(G_{27},t)}, \boxed{-(b_{25}'')^{(4)}(G_{27},t)}, \boxed{-(b_{26}'')^{(4)}(G_{27},t)}\ are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{28}'')^{(5,5,)}(G_{31},t)}, \boxed{-(b_{29}'')^{(5,5,)}(G_{31},t)}, \boxed{-(b_{30}'')^{(5,5,)}(G_{31},t)}\ are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{32}'')^{(6,6,)}(G_{35},t)}, \boxed{-(b_{33}'')^{(6,6,)}(G_{35},t)}, \boxed{-(b_{34}'')^{(6,6,)}(G_{35},t)}\ are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{13}'')^{(1,1,1,1)}(G,t)}, \boxed{-(b_{14}'')^{(1,1,1,1)}(G,t)}, \boxed{-(b_{15}'')^{(1,1,1,1)}(G,t)}$

$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{16}'')^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b_{17}'')^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b_{18}'')^{(2,2,2,2)}(G_{19},t)}$

$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{20}'')^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b_{21}'')^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b_{22}'')^{(3,3,3,3)}(G_{23},t)}$

$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{36}'')^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b_{37}'')^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b_{38}'')^{(7,7,7,7,7)}(G_{39},t)}$

$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{40}'')^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b_{41}'')^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b_{42}'')^{(8,8,8,8,8)}(G_{43},t)}$

$are\ eighth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{46}'')^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b_{45}'')^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b_{44}'')^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{c} (a_{28}')^{(5)}\boxed{+(a_{28}'')^{(5)}(T_{29},t)}\ \boxed{+(a_{24}'')^{(4,4,)}(T_{25},t)}\ \boxed{+(a_{32}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{36}'')^{(7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a_{40}'')^{(8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a_{44}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{c} (a_{29}')^{(5)}\boxed{+(a_{29}'')^{(5)}(T_{29},t)}\ \boxed{+(a_{25}'')^{(4,4,)}(T_{25},t)}\ \boxed{+(a_{33}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a_{45}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{c} (a_{30}')^{(5)}\boxed{+(a_{30}'')^{(5)}(T_{29},t)}\ \boxed{+(a_{26}'')^{(4,4,)}(T_{25},t)}\ \boxed{+(a_{34}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a_{46}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{30} \quad (81)$$

$Where\ \boxed{+(a_{28}'')^{(5)}(T_{29},t)}, \boxed{+(a_{29}'')^{(5)}(T_{29},t)}, \boxed{+(a_{30}'')^{(5)}(T_{29},t)}\ are\ first\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$And\ \boxed{+(a_{24}'')^{(4,4,)}(T_{25},t)}, \boxed{+(a_{25}'')^{(4,4,)}(T_{25},t)}, \boxed{+(a_{26}'')^{(4,4,)}(T_{25},t)}\ are\ second\ augmentation\ coefficient\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a_{32}'')^{(6,6,6)}(T_{33},t)}, \boxed{+(a_{33}'')^{(6,6,6)}(T_{33},t)}, \boxed{+(a_{34}'')^{(6,6,6)}(T_{33},t)}\ are\ third\ \ augmentation\ coefficient\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8\ ,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \quad 85$$

$$- \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & \boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} & \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} & \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)} & \boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} & \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)} & \boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} & \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)} & \boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} & \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq \ (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = \ (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \ , \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\,(a_i)^{(1)} + (a_i')^{(1)} + \ (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}\,(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i')^{(1)} + \ (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \leq (p_i)^{(2)} \leq \left(\hat{A}_{16}\right)^{(2)} \qquad 102$$

$$(b_i'')^{(2)}(G_{19},t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\big((G_{19}),t\big) = \ (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - \ T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\big((G_{19}),t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\big[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\big]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}} \ , \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\,(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + \ (\hat{P}_{16})^{(2)}\,(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + \ (\hat{Q}_{16})^{(2)}\ \ (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \le (p_i)^{(3)} \le (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \le (r_i)^{(3)} \le (b_i')^{(3)} \le (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\widehat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\widehat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\widehat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\,(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and \ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\,(a_i)^{(6)} + (a_i')^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)}\,(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(WWWWW) $\quad (a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(XXXXX) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$(a_i'')^{(7)}(T_{37},t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$

$(b_i'')^{(7)}(G_{39},t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$

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(YYYYY) $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

(ZZZZZ)

$\lim_{G\to\infty}(b_i'')^{(7)}((G_{39}),t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \leq (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$

$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}((G_{39}),t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$. (T_{37}',t) and (T_{37},t) are points belonging to the interval $\left[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\right]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(AAAAAA) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(BBBBBB) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$

$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$

Where we suppose

$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$ 138

$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$ 139

$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$ 140

$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t}$ 142

$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t}$ 143

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ 144

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}$, $(r_i)^{(8)}, i = 40,41,42$,
Satisfy the inequalities

$\frac{1}{(\hat{M}_{40})^{(8)}} [(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 145

$\frac{1}{(\hat{M}_{40})^{(8)}} [(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 146

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\hat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)} ||(G_{47}) - (G_{47})'|| e^{-(\hat{M}_{44})^{(9)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)} (\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \ , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \ , \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \ , \ T_i(0) = T_i^0 \ , \ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} , T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\overline{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that 174

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \leq G_{32}^{0}+\int_0^t\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(v) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \leq G_{36}^{0}+\int_0^t\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$
$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \leq G_{40}^{0}+\int_0^t\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}= \quad 180$$
$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t} \leq \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for G_{41} , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \leq G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \leq \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for G_{45} , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \left\{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\right\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)}\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) - (a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right| \; e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} \} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \qquad (186)$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13{,}14{,}15$ depend only on T_{14} and respectively on G(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: (187)

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. (188)

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $(\widetilde{G_{19}}, \widetilde{T_{19}}) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t (a_{16})^{(2)}|G_{17}^{(1)} - G_{17}^{(2)}|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}|G_{16}^{(1)} - G_{16}^{(2)}|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}\left|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right| \; e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t} \leq$
$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)d\left(\left((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)}\right)\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\}ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)}G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$: $\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0$ *and* $T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$G_{24}^{(2)} \left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)}\right) d\left(((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on T_{25} and respectively on (G_{27})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (228)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_2 / (a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)}, (T_{35})^{(1)}), ((G_{35})^{(2)}, (T_{35})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}), (\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}), (\widetilde{T_{35}}) \right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right| \; e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} \} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)} (\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)}; \; (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t} \; and \; (\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$ (250)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2 \; and \; \left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33} \; and \; G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$
$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t}\}$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)} \left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)} (T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b'_i)^{(7)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1,\left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right),\quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$

From 99 to 44 it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(9)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)},, (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)}, , (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)} t} - e^{-(a'_{26})^{(4)} t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)} t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G^0_{28} e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G^0_{28}}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G^0_{30} e^{-(S_2)^{(5)} t} \le G_{30}(t) \le \right. \quad 344$$

$$\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_2)^{(5)}((S_1)^{(5)}-(a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t}-e^{-(a'_{30})^{(5)}t}\right]+\ G_{30}^{0}e^{-(a'_{30})^{(5)}t}\Big)$$

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \qquad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \qquad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\le T_{30}(t)\le \qquad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$:

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)}\le-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\le-(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)}\le-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\le-(\tau_1)^{(6)}$$

Definition of $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$: 350

By $(v_1)^{(6)}>0$, $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$, $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

Definition of $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)}>0$, $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$, $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

Definition of $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$:-

If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)}=(v_0)^{(6)},(m_1)^{(6)}=(v_1)^{(6)},$ ***if*** $(v_0)^{(6)}<(v_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}$ $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}$ $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}$ $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}$ $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}$ $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})} \left[e^{(S_1)^{(6)} t} - e^{-(a'_{34})^{(6)} t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)} t} \right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^{0}e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}}G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}}G_{36}^{0}e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t}-e^{-(S_2)^{(7)}t}\right]+G_{38}^{0}e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left.\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_2)^{(7)}((S_1)^{(7)}-(a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t}-e^{-(a'_{38})^{(7)}t}\right]+\ G_{38}^{0}e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}}T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}}T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right) - (b''_{41})^{(8)}\left((G_{43}), t\right) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(v_1)^{(8)} > 0$, $(v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$, $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{v}_1)^{(8)}$,, $(\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{v}_1)^{(8)} > 0$, $(\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$, $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$:-

If we define $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$, $\boldsymbol{if}$ $(v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$, $\boldsymbol{if}$ $(v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$,
and $\boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}$, $\boldsymbol{if}$ $(\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, $\boldsymbol{if}$ $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, $\boldsymbol{if}$ $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}$ $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t})$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \qquad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq\frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \qquad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t}\leq T_{42}(t)\leq \qquad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)},(S_2)^{(8)},(R_1)^{(8)},(R_2)^{(8)}$:- 381

Where $(S_1)^{(8)}=(a_{40})^{(8)}(m_2)^{(8)}-(a'_{40})^{(8)}$

$(S_2)^{(8)}=(a_{42})^{(8)}-(p_{42})^{(8)}$

$(R_1)^{(8)}=(b_{40})^{(8)}(\mu_2)^{(8)}-(b'_{40})^{(8)}$

$(R_2)^{(8)}=(b'_{42})^{(8)}-(r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$:

$(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)}\leq-(a'_{44})^{(9)}+(a'_{45})^{(9)}-(a''_{44})^{(9)}(T_{45},t)+(a''_{45})^{(9)}(T_{45},t)\leq-(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)}\leq-(b'_{44})^{(9)}+(b'_{45})^{(9)}-(b''_{44})^{(9)}\big((G_{47}),t\big)-(b''_{45})^{(9)}\big((G_{47}),t\big)\leq-(\tau_1)^{(9)}$

Definition of $(v_1)^{(9)},(v_2)^{(9)},(u_1)^{(9)},(u_2)^{(9)},v^{(9)},u^{(9)}$:

By $(v_1)^{(9)}>0$, $(v_2)^{(9)}<0$ and respectively $(u_1)^{(9)}>0$, $(u_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_1)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)}>0$, $(\bar{v}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$, $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

Definition of $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$:-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)},\ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$
and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \le G_{46}(t) \le \right.$$
$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (v_1)^{(1)} \le (\bar{v}_1)^{(1)} \le \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$, we obtain 386

$$(v_1)^{(1)} \le v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $v^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\right) - (a''_{17})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{\mathrm{G}_{16}}{\mathrm{G}_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (\mathrm{C})^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (\mathrm{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)}\right) \le \frac{dv^{(3)}}{dt} \le -\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \ge \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}}$$

it follows $(v_0)^{(3)} \le v^{(3)}(t) \le (v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \le v^{(3)}(t) \le (\bar{v}_1)^{(3)}$

If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)} \le \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}} \le v^{(3)}(t) \le$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (\bar{v}_1)^{(3)}$$

If $0 < (v_1)^{(3)} \le (\bar{v}_1)^{(3)} \le (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$, we obtain 403

$$(v_1)^{(3)} \le v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

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$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(8)} and\ (b''_i)^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\,]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\,]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\,]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0$ 479

$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0$ 480

$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0$ 481

$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0$ 482

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(v) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first

equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T^*_{33} for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$$

Definition and uniqueness of T^*_{37} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T^*_{37} for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$$

Definition and uniqueness of T^*_{41} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{25} such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507
$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{29} such that $\varphi((G_{31})^*) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{33} such that $\varphi({G_{35}}^*) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{37} such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{41} such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right]+(b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{45} such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)} G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)} G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \qquad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \quad 530$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}\mathrm{T}^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}\mathrm{T}^*_{18}\mathbb{G}_j\right) \quad 539$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}\left((G_{27})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\ B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\Big)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\Big]$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\Big)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\Big)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\Big]$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\Big)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\Big)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\Big]$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\Big)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\Big)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\Big)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\Big)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION TWENTY NINE

W2 (Transformation Law Of The Field), W3 (Local Commutativity Or Microscopic Causality), Mass Gap

INTRODUCTION—VARIABLES USED

The Wightman axioms

W2 (transformation law of the field)

(1) The fields are covariant under the action (e&eb) of Poincaré group, and they transform (e&eb)

according to some representation S of (e) the Lorentz group, or SL(2,C) if the spin is not integer:

$$U(a,L)^{\dagger}A(x)U(a,L) = S(L)A(L^{-1}(x-a)).$$

W3 (local commutativity or microscopic causality)

(2) If the supports of two fields are (=) space-like separated, then (eb) the fields either commute or anticommute.
(3) Cyclicity of a vacuum and (e&eb) uniqueness of a vacuum are sometimes considered separately.
(4) Also, there is property of asymptotic completeness - that Hilbert state space is spanned by (e&eb) the asymptotic spaces H^{in} and H^{out}, appearing in (eb) the collision S matrix.
(5) The other important property of field theory is (=) mass gap which is not (e) required by the axioms - that energy-momentum spectrum has (e) a gap between zero and (e&eb) some positive number.

Main article: Mass gap

(6) In quantum field theory, the mass gap is (=) the difference in energy between the vacuum and the next lowest energy state.
(7) The energy of the vacuum is zero by definition, and assuming that all energy states can be thought of as (=) particles in plane-waves, (eb) the mass gap is the mass of the lightest particle.
(8) For a given real field $\phi(x)$, we can say that the theory has (e) a mass gap if the two-point function has the property

$$\langle \phi(0,t)\phi(0,0)\rangle \sim \sum_n A_n \exp\left(-\Delta_n t\right)$$

with $\Delta_0 > 0$ being the lowest energy value in the spectrum of the Hamiltonian and thus the mass gap. This quantity, easy to generalize to other fields, is what is generally measured in lattice computations. It was proved in this way that Yang–Mills theory develops a mass gap on a lattice.

Importance of Yang-Mills theory

(9) Most known and nontrivial (i.e. interacting) quantum field theories in 4 dimensions are effective field theories with a cutoff scale. Since the beta-function is positive for most models, it appears that most such models have (e) a Landau pole as it is not at all clear whether or not they have nontrivial UV fixed points.
(10) This means that if such a QFT is (=) well-defined at all scales, as it has to be to satisfy (eb) the axioms of axiomatic quantum field theory, (eb) it would have to be trivial (i.e. a free field theory).
(11) Quantum Yang-Mills theory with (e&eb) a non-abelian gauge group and no quarks is (=) an exception, because (e) asymptotic freedom characterizes this theory, meaning (eb) that it has (e) a trivial UV fixed point.
(12) Hence it is the simplest nontrivial constructive QFT in (eb) 4 dimensions. (QCD is a more complicated theory because it involves quarks.)

Quark confinement

(1) It has already been well proven—at least at the level of rigor of theoretical physics but not that of mathematical physics—that the quantum Yang–Mills theory for a non-abelian Lie group exhibits (eb) a property known as confinement.
(2) A consequence of this property is that beyond a certain scale, known as the QCD scale (more

properly, the confinement scale, as this theory is devoid of quarks), the color charges are connected by (e&eb) chromodynamic flux tubes leading to (eb) a linear potential between the charges.

(3) Hence free color charge and free gluons cannot (e) exist.

(4) In the absence of confinement, we would expect to see (eb) massless gluons, but since they are confined, all we would see are (=) color-neutral bound states of gluons, called glueballs.

(5) If glueballs exist, they are (=) massive, which is why we expect a mass gap.

NOTATION

Module One

The fields are covariant under the action of Poincaré group, and they transform (e&eb) according to some representation S of (e) the Lorentz group, or SL(2,C) if the spin is not integer:

$$U(a,L)^{\dagger}A(x)U(a,L) = S(L)A(L^{-1}(x-a)).$$

G_{13} : Category one of fields are covariant; action of Poincaré group, and they transform (e&eb) according to some representation S of (e) the Lorentz group, or SL(2,C) if the spin is not integer

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of action of Poincaré group, and they transform (e&eb) according to some representation S of (e) the Lorentz group, or SL(2,C) if the spin is not integer; fields are covariant

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

fields are covariant under the action of Poincaré group, and they transform according to some representation S of the Lorentz group, or SL(2,C) if the spin is not integer:

$$U(a,L)^{\dagger}A(x)U(a,L) = S(L)A(L^{-1}(x-a)).$$

G_{16} : Category one of Lorentz group, or SL(2,C) if the spin is not integer

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of fields are covariant under the action of Poincaré group, and they transform according to some representation S

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

If the supports of two fields are space-like separated, then (eb) the fields either commute or anticommute

G_{20} : Category one of supports of two fields are space-like separated

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of fields either commute or anticommute

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Cyclicity of a vacuum and (e&eb) uniqueness of a vacuum are sometimes considered separately

G_{24} : Category one of Cyclicity of a vacuum; uniqueness of a vacuum

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of uniqueness of a vacuum; Cyclicity of a vacuum

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Also, there is property of asymptotic completeness - that Hilbert state space is spanned by (e&eb) the asymptotic spaces H^{in} and H^{out}, appearing in (eb) the collision S matrix

G_{28} : Category one of Hilbert state space; asymptotic spaces H^{in} and H^{out}, appearing in the collision S matrix

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of asymptotic spaces H^{in} and H^{out}, appearing in (eb) the collision S matrix ;Hilbert state space

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

The other important property of field theory is (=) mass gap which is not (e) required by the axioms - that energy-momentum spectrum has (e) a gap between zero and (e&eb) some positive number

G_{32} : Category one of property of field theory

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of mass gap which is not required by the axioms

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

energy-momentum spectrum has (e) a gap between zero and some positive number

G_{36} : Category one of gap between zero and some positive number

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of energy-momentum spectrum

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

In quantum field theory, the mass gap is (=) the difference in energy between the vacuum and the next lowest energy state

G_{40} : Category one of mass gap

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of difference in energy between the vacuum and the next lowest energy state

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

The energy of the vacuum is zero by definition, and assuming that all energy states can be thought of as (=) particles in plane-waves, (eb) the mass gap is the mass of the lightest particle

G_{44} : Category one of energy of the vacuum is zero by definition, and assuming that all energy states

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of particles in plane-waves, (eb) the mass gap is the mass of the lightest particle

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION TWENTY NINE A

Importance Of Yang-Mills Theory

INTRODUCTION—VARIABLES USED

Wikipedia:

(1) For a given real field $\phi(x)$, we can say that the theory has (e) a mass gap if the two-point function has the property

$$\langle \phi(0,t)\phi(0,0)\rangle \sim \sum_n A_n \exp\left(-\Delta_n t\right)$$

with $\Delta_0 > 0$ being the lowest energy value in the spectrum of the Hamiltonian and thus the mass gap. This quantity, easy to generalize to other fields, is what is generally measured in lattice computations. It was proved in this way that Yang–Mills theory develops a mass gap on a lattice.

Importance of Yang-Mills theory

(2) Most known and nontrivial (i.e. interacting) quantum field theories in 4 dimensions are effective field theories with a cutoff scale. Since the beta-function is positive for most models, it appears that most such models have (e) a Landau pole as it is not at all clear whether or not they have nontrivial UV fixed points.
(3) This means that if such a QFT is (=) well-defined at all scales, as it has to be to satisfy (eb) the axioms of axiomatic quantum field theory, (eb) it would have to be trivial (i.e. a free field theory).
(4) Quantum Yang-Mills theory with (e&eb) a non-abelian gauge group and no quarks is (=) an exception, because (e) asymptotic freedom characterizes this theory, meaning (eb) that it has (e) a trivial UV fixed point.
(5) Hence it is the simplest nontrivial constructive QFT in (eb) 4 dimensions. (QCD is a more complicated theory because it involves quarks.)

Quark confinement

(6) It has already been well proven—at least at the level of rigor of theoretical physics but not that of mathematical physics—that the quantum Yang–Mills theory for a non-abelian Lie group exhibits (eb) a property known as confinement.
(7) A consequence of this property is that beyond a certain scale, known as the QCD scale (more properly, the confinement scale, as this theory is devoid of quarks), the color charges are connected by (e&eb) chromodynamic flux tubes leading to (eb) a linear potential between the charges.
(8) Hence free color charge and free gluons cannot (e) exist.
(9) In the absence of confinement, we would expect to see (eb) massless gluons, but since they are confined, all we would see are (=) color-neutral bound states of gluons, called glueballs.
(10) If glueballs exist, they are (=) massive, which is why we expect a mass gap.

NOTATION

Module One

energy of the vacuum is zero by definition, and assuming that all energy states can be thought of as particles in plane-waves, (eb) the mass gap is the mass of the lightest particle

G_{13} : Category one of energy of the vacuum is zero by definition, and assuming that all energy states can be thought of as particles in plane-waves

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of mass gap is the mass of the lightest particle

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

For a given real field $\phi(x)$, we can say that the theory has (e) a mass gap if the two-point function has the property

$$\langle \phi(0,t)\phi(0,0) \rangle \sim \sum_n A_n \exp\left(-\Delta_n t\right)$$

G_{16} : Category one of LHS of the above equation

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of RHS of the above equation

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Most known and nontrivial (i.e. interacting) quantum field theories in 4 dimensions are effective field theories with a cutoff scale.

Since the beta-function is positive for most models, it appears that most such models have (e) a Landau pole as it is not at all clear whether or not they have nontrivial UV fixed points

G_{20} : Category one of Landau pole as it is not at all clear whether or not they have nontrivial UV fixed points

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of beta-function is positive for most models, it appears that most such models

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

This means that if such a QFT is well-defined at all scales, as it has to be to satisfy (eb) the axioms of axiomatic quantum field theory, (eb) it would have to be trivial (i.e. a free field theory)

G_{24} : Category one of beta-function is positive for most models, it appears that most such models have a Landau pole as it is not at all clear whether or not they have nontrivial UV fixed points

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of QFT is well-defined at all scales, as it has to be to satisfy (eb) the axioms of axiomatic quantum field theory, (eb) it would have to be trivial (i.e. a free field theory)

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

QFT is well-defined at all scales, as it has to be to satisfy the axioms of axiomatic quantum field theory, (eb) it would have to be trivial (i.e. a free field theory)

G_{28} : Category one of QFT is well-defined at all scales, as it has to be to satisfy the axioms of axiomatic quantum field theory

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of it would have to be trivial (i.e. a free field theory)

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Quantum Yang-Mills theory with a non-abelian gauge group and no quarks is an exception, because (e) asymptotic freedom characterizes this theory, meaning (eb) that it has (e) a trivial UV fixed point

G_{32} : Category one of asymptotic freedom characterizes this theory, meaning) that it has a trivial UV fixed point

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of Quantum Yang-Mills theory with a non-abelian gauge group and no quarks is an

exception

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

It has already been well proven—at least at the level of rigor of theoretical physics but not that of mathematical physics—that the quantum Yang–Mills theory for a non-abelian Lie group exhibits (eb) a property known as confinement

G_{36} : Category one of quantum Yang–Mills theory for a non-abelian Lie group

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of confinement

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

A consequence of this property is that beyond a certain scale, known as the QCD scale (more properly, the confinement scale, as this theory is devoid of quarks), the color charges are connected by (e&eb) chromodynamic flux tubes leading to (eb) a linear potential between the charges.

G_{40} : Category one of confinement

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of color charges are connected by (e&eb) chromodynamic flux tubes leading to (eb) a linear potential between the charges.

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Multiple modules : Care

Color charges are connected by chromodynamic flux tubes leading to (eb) a linear potential between the charges.

G_{44} : Category one of Color charges; chromodynamic flux tubes leading to (eb) a linear potential between the charges.; Color charges are connected by chromodynamic flux tubes

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of chromodynamic flux tubes leading to (eb) a linear potential between the charges.; Color charges; linear potential between the charges.

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION THIRTY

Anti De Sitter Space And Holography

INTRODUCTION—VARIABLES USED

Anti De Sitter Space and Holography Edward Witten

(1) Recently, it has been proposed by Maldacena that large N limits of (e) certain conformal field theories in (eb) d dimensions can be described in terms of (e&eb) supergravity (and string theory) on the product of d+1-dimensional AdS space with (e&eb) a compact manifold.
(2) Here authors elaborate on this idea and propose a precise correspondence between conformal field theory observables and (e&eb) those of supergravity
(3) Correlation functions in conformal field theory are given by (eb) the dependence of the supergravity action on (eb) the asymptotic behavior at infinity.
(4) In particular, dimensions of operators in conformal field theory are given by (eb) masses of particles in supergravity.
(5) As quantitative confirmation of this correspondence, authors note that the Kaluza-Klein modes of Type IIB supergravity on (eb) AdS5×S5 match with (e&eb) the chiral operators of $\N=4$ super Yang-Mills theory in four dimensions.
(6) With some further assumptions, one can deduce a Hamiltonian version of the correspondence and show that (eb) the $\N=4$ theory has a large N phase transition related to (e&eb) the thermodynamics of AdS black holes. Journal reference: Adv.Theor.Math.Phys.2:253-291, 1998 Cite as: arXiv: hep-th/9802150

Chaos, Solitons & Fractals Volume 10, Issues 2–3, February–March 1999, Pages 213–223 Chaos in M (atrix) theory I.Ya. Aref'eva†, P.B. Medvedev‡, O.A. Rytchkov, I.V. Volovich† Available online 29 March 2003 doi: 10.1016/S0960-0779(98)00159-3

(7) **I.Ya. Aref'eva†, P.B. Medvedev‡, O.A. Rytchkov, I.V. Volovich†** consider the classical and quantum dynamics in (eb) M (atrix) theory.
(8) Using a simple ansatz they show that a classical trajectory exhibits (eb) a chaotic motion.
(9) They argue that the holographic feature of M (atrix) theory is related with (e&eb) the repulsive feature of energy eigenvalues in (eb) quantum chaotic system. Chaotic dynamics in (e) N = 2 supersymmetric Yang—Mills theory is also discussed.
(10) They demonstrate that after the separation of “slow” and “fast” modes there is (=) a singular contribution from the “slow” modes to (e&eb) the Hamiltonian of the “fast” modes. Copyright © 1998 Elsevier Science Ltd. All rights reserved.

Journal of High Energy Physics April 2013, 2013:83 Date: 12 Apr 2013 A string theory explanation

for quantum chaos in the hadronic spectrum Leopoldo A. Pando Zayas, Dori Reichmann

(1) In the 1950's Wigner and collaborators provided an explanation for the spectrum of (e) hadronic excitations in terms of (e&eb) Random Matrix Theory.
(2) In the 1980's it was understood that some hadronic spectral properties were generic to (e) systems whose classical limit is chaotic.
(3) They use string theory to demonstrate explicitly how, under very general conditions, recent holographic models of strong interactions have (e) a spectrum compatible with Wigner's conjecture. ArXiv ePrint: 1209.5902

NOTATION

Module One

The action for a SUGRA theory with chiral superfields X, is given by

$$S = \int d^4x d^2\Theta 2\mathcal{E}\left[\frac{3}{8}\left(\overline{D}^2 - 8R\right)e^{-K(\overline{X},X)/3} + W(X)\right] + c.c.$$

Heisenberg's reasoning

Before matrix mechanics, the old quantum theory described the motion of a particle by a classical orbit, with well defined position and momentum X(t), P(t), with the restriction that the time integral over one period T of the momentum times the velocity must be a positive integer multiple of Planck's constant

$$\int_0^T P\, dX = nh$$

.

While this restriction correctly selects orbits with more or less the right energy values En, the old quantum mechanical formalism did not describe time dependent processes, such as the emission or absorption of radiation.

When a classical particle is weakly coupled to a radiation field, so that the radiative damping can be neglected, it will emit radiation in a pattern which repeats itself every orbital period. The frequencies which make up the outgoing wave are then integer multiples of the orbital frequency, and this is a reflection of the fact that X(t) is periodic, so that its Fourier representation has frequencies 2πn/T only.

$$X(t) = \sum_{n=-\infty}^{\infty} e^{2\pi int/T} X_n$$

.

The coefficients Xn are complex numbers. The ones with negative frequencies must be the complex conjugates of the ones with positive frequencies, so that X(t) will always be real,

$$X_n = X^*_{-n}\,.$$

A quantum mechanical particle, on the other hand, can't emit radiation continuously, it can only emit photons. Assuming that the quantum particle started in orbit number n, emitted a photon, then ended up in orbit number m, the energy of the photon is En−Em, which means that its frequency is (En−Em)/h.

For large n and m, but with n−m relatively small, these are the classical frequencies by Bohr's correspondence principle

$$E_n - E_m \approx h(n-m)/T \,.$$

In the formula above, T is the classical period of either orbit n or orbit m, since the difference between them is higher order in h. But for n and m small, or if n − m is large, the frequencies are not integer multiples of any single frequency.

Since the frequencies which the particle emits are the same as the frequencies in the fourier description of its motion, this suggests that something in the time-dependent description of the particle is oscillating with frequency (En−Em)/h. Heisenberg called this quantity Xnm, and demanded that it should reduce to the classical Fourier coefficients in the classical limit. For large values of n, m but with n − m relatively small, Xnm is the (n−m)th Fourier coefficient of the classical motion at orbit n. Since Xnm has opposite frequency to Xmn, the condition that X is real becomes

$$X_{nm} = X^*_{mn} \,.$$

By definition, Xnm only has the frequency (En−Em)/h, so its time evolution is simple:

$$X_{nm}(t) = e^{2\pi i (E_n - E_m)t/h} X_{nm}(0) \,.$$

This is the original form of Heisenberg's equation of motion.

Given two arrays Xnm and Pnm describing two physical quantities, Heisenberg could form a new array of the same type by combining the terms XnkPkm, which also oscillate with the right frequency. Since the Fourier coefficients of the product of two quantities is the convolution of the Fourier coefficients of each one separately, the correspondence with Fourier series allowed Heisenberg to deduce the rule by which the arrays should be multiplied,

$$(XP)_{mn} = \sum_{k=0}^{\infty} X_{mk} P_{kn} \,.$$

Born pointed out that this is the law of matrix multiplication, so that the positions, the momentum, the energy, all the observable quantities in the theory, are interpreted as matrices. Under this multiplication rule, the product depends on the order: XP is different from PX.

The X matrix is a complete description of the motion of a quantum mechanical particle. Because the frequencies in the quantum motion are not multiples of a common frequency, the matrix elements cannot be interpreted as the Fourier coefficients of a sharp classical trajectory. Nevertheless, as matrices, X(t) and P(t) satisfy the classical equations of motion; also see Ehrenfest's theorem, below.

Matrix basics

When it was introduced by Werner Heisenberg, Max Born and Pascual Jordan in 1925, matrix mechanics was not immediately accepted and was a source of controversy, at first. Schrödinger's later introduction of wave mechanics was greatly favored.

Part of the reason was that Heisenberg's formulation was in an odd mathematical language, for the time, while Schrödinger's formulation was based on familiar wave equations. But there was also a deeper sociological reason. Quantum mechanics had been developing by two paths, one under the direction of Einstein and the other under the direction of Bohr. Einstein emphasized wave–particle duality, while Bohr

emphasized the discrete energy states and quantum jumps. DeBroglie had shown how to reproduce the discrete energy states in Einstein's framework--- the quantum condition is the standing wave condition, and this gave hope to those in the Einstein school that all the discrete aspects of quantum mechanics would be subsumed into a continuous wave mechanics.

Matrix mechanics, on the other hand, came from the Bohr school, which was concerned with discrete energy states and quantum jumps. Bohr's followers did not appreciate physical models which pictured electrons as waves, or as anything at all. They preferred to focus on the quantities which were directly connected to experiments.

In atomic physics, spectroscopy gave observational data on atomic transitions arising from the interactions of atoms with light quanta. The Bohr school required that only those quantities which were in principle measurable by spectroscopy should appear in the theory. These quantities include the energy levels and their intensities but they do not include the exact location of a particle in its Bohr orbit. It is very hard to imagine an experiment which could determine whether an electron in the ground state of a hydrogen atom is to the right or to the left of the nucleus. It was a deep conviction that such questions did not have an answer.

The matrix formulation was built on the premise that all physical observables are represented by matrices, whose elements are indexed by two different energy levels. The set of eigenvalues of the matrix were eventually understood to be the set of all possible values that the observable can have. Since Heisenberg's matrices are Hermitian, the eigenvalues are real.

If an observable is measured and the result is a certain eigenvalue, the corresponding eigenvector is the state of the system immediately after the measurement. The act of measurement in matrix mechanics 'collapses' the state of the system. If one measures two observables simultaneously, the state of the system collapses to a common eigenvector of the two observables. Since most matrices don't have any eigenvectors in common, most observables can never be measured precisely at the same time. This is the uncertainty principle.

If two matrices share their eigenvectors, they can be simultaneously diagonalized. In the basis where they are both diagonal, it is clear that their product does not depend on their order because multiplication of diagonal matrices is just multiplication of numbers. The uncertainty principle, by contrast, is an expression of the fact that often two matrices A and B do not always commute, i.e., that AB − BA does not necessarily equal 0. The fundamental commutation relation of matrix mechanics,

$$\sum_k (X_{nk}P_{km} - P_{nk}X_{km}) = \frac{ih}{2\pi}\,\delta_{nm}$$

Implies then that there are no states which simultaneously have a definite position and momentum. This principle of uncertainty holds for many other pairs of observables as well. For example, the energy does not commute with the position either, so it is impossible to precisely determine the position and energy of an electron in an atom.

G_{13} : Category one of LHS of the above equations

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of RHS of the above equations

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Recently, it has been proposed by Maldacena that

large N limits of (e) certain conformal field theories in (eb) d dimensions can be described in terms of (e&eb) supergravity (and string theory) on the product of d+1-dimensional AdS space with (e&eb) a compact manifold

G_{16} : Category one of certain conformal field theories in (eb) d dimensions can be described in terms of (e&eb) supergravity (and string theory) on the product of d+1-dimensional AdS space with (e&eb) a compact manifold

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of large N limits

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

large N limits of certain conformal field theories in (eb) d dimensions can be described in terms of (e&eb) supergravity (and string theory) on the product of d+1-dimensional AdS space with (e&eb) a compact manifold

G_{20} : Category one of large N limits of certain conformal field theories

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of d dimensions can be described in terms of (e&eb) supergravity (and string theory) on the product of d+1-dimensional AdS space with (e&eb) a compact manifold

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

large N limits of certain conformal field theories in d dimensions can be described in terms of (e&eb) supergravity (and string theory) on the product of d+1-dimensional AdS space with a compact manifold

G_{24} : Category one of large N limits of certain conformal field theories in d dimensions; supergravity (and string theory) on the product of d+1-dimensional AdS space with (e&eb) a compact manifold

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of supergravity (and string theory) on the product of d+1-dimensional AdS space with a compact manifold; large N limits of certain conformal field theories in d dimensions

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

large N limits of certain conformal field theories in d dimensions can be described in terms of supergravity (and string theory) on the product of d+1-dimensional AdS space with a compact manifold

G_{28} : Category one of large N limits of certain conformal field theories in d dimensions can be described in terms of supergravity (and string theory) on the product of d+1-dimensional AdS space; compact manifolds

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of compact manifold(s); large N limits of certain conformal field theories in d dimensions can be described in terms of supergravity (and string theory) on the product of d+1-dimensional AdS space

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

In particular, dimensions of operators in conformal field theory are given by (eb) masses of particles in supergravity

G_{32} : Category one of dimensions of operators in conformal field theory

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of masses of particles in supergravity

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

As quantitative confirmation of this correspondence, authors note that the Kaluza-Klein modes of Type IIB supergravity on AdS5×S5 match with (e&eb) the chiral operators of $\N=4$ super Yang-Mills theory in four dimensions

G_{36} : Category one of Kaluza-Klein modes of Type IIB supergravity on AdS5×S5; chiral operators of $\N=4$ super Yang-Mills theory in four dimensions

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of chiral operators of $\N=4$ super Yang-Mills theory in four dimensions; Kaluza-Klein modes of Type IIB supergravity on AdS5×S5

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

With some further assumptions, one can deduce a Hamiltonian version of the correspondence and show that (eb) the

$\N=4$ theory has a large N phase transition related to (e&eb) the thermodynamics of AdS black holes.

Journal reference: Adv.Theor.Math.Phys.2:253-291, 1998 Cite as: arXiv: hep-th/9802150

G_{40} : Category one of$\N=4$ theory has a large N phase transition; thermodynamics of AdS black holes.

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of thermodynamics of AdS black holes. ;$\N=4$ theory has a large N phase transition

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Using a simple ansatz they show that a

classical trajectory exhibits (eb) a chaotic motion

G_{44} : Category one of classical trajectory

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of chaotic motion

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},$
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},\ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}\ ,$
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}), t\big)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}), t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\big((G_{19}), t\big) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}),t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}),t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}),t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}),t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}),t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$
$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$
$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$
$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**
$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\begin{array}{ccc} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right]G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b_{20}'')^{(3,3,)}(G_{23},t)$, $-(b_{21}'')^{(3,3,)}(G_{23},t)$, $-(b_{22}'')^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b_{24}'')^{(4,4,4,4,)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b_{28}'')^{(5,5,5,5,)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b_{32}'')^{(6,6,6,6,)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b_{37}'')^{(7,7,)}(G_{39},t)$, $-(b_{36}'')^{(7,7,)}(G_{39},t)$, $-(b_{38}'')^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b_{40}'')^{(8,8)}(G_{43},t)$ $-(b_{41}'')^{(8,8)}(G_{43},t)$ $-(b_{42}'')^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{c} (a_{16}')^{(2)} + (a_{16}'')^{(2)}(T_{17},t) + (a_{13}'')^{(1,1,)}(T_{14},t) + (a_{20}'')^{(3,3,3)}(T_{21},t) \\ + (a_{24}'')^{(4,4,4,4,4)}(T_{25},t) + (a_{28}'')^{(5,5,5,5,5)}(T_{29},t) + (a_{32}'')^{(6,6,6,6,6)}(T_{33},t) \\ + (a_{36}'')^{(7,7,7)}(T_{37},t) + (a_{40}'')^{(8,8,8)}(T_{41},t) + (a_{44}'')^{(9,9)}(T_{45},t) \end{array}\right] G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{c} (a_{17}')^{(2)} + (a_{17}'')^{(2)}(T_{17},t) + (a_{14}'')^{(1,1,)}(T_{14},t) + (a_{21}'')^{(3,3,3)}(T_{21},t) \\ + (a_{25}'')^{(4,4,4,4,4)}(T_{25},t) + (a_{29}'')^{(5,5,5,5,5)}(T_{29},t) + (a_{33}'')^{(6,6,6,6,6)}(T_{33},t) \\ + (a_{37}'')^{(7,7,7)}(T_{37},t) + (a_{41}'')^{(8,8,8)}(T_{41},t) + (a_{45}'')^{(9,9)}(T_{45},t) \end{array}\right] G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{c} (a_{18}')^{(2)} + (a_{18}'')^{(2)}(T_{17},t) + (a_{15}'')^{(1,1,)}(T_{14},t) + (a_{22}'')^{(3,3,3)}(T_{21},t) \\ + (a_{26}'')^{(4,4,4,4,4)}(T_{25},t) + (a_{30}'')^{(5,5,5,5,5)}(T_{29},t) + (a_{34}'')^{(6,6,6,6,6)}(T_{33},t) \\ + (a_{38}'')^{(7,7,7)}(T_{37},t) + (a_{42}'')^{(8,8,8)}(T_{41},t) + (a_{46}'')^{(9,9)}(T_{45},t) \end{array}\right] G_{18} \quad 63$$

Where $+(a_{16}'')^{(2)}(T_{17},t)$, $+(a_{17}'')^{(2)}(T_{17},t)$, $+(a_{18}'')^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{13}'')^{(1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) \end{bmatrix} T_{18} \quad (66)$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)} & +(a''_{20})^{(3)}(T_{21},t) & +(a''_{16})^{(2,2,2)}(T_{17},t) & +(a''_{13})^{(1,1,1,)}(T_{14},t) \\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9)}(T_{45},t) \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)}\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)}\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad (69)$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)}$ are eight augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\boxed{-(b''_{20})^{(3)}(G_{23},t)}\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{13})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\boxed{-(b''_{21})^{(3)}(G_{23},t)}\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)}\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad (72)$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for*

category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & \boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & \boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & \boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* $1, 2\ 3$

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category $1, 2$ *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category $1, 2$ *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
are sixth augmentation coefficients for category $1, 2$ *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
are seventh augmentation coefficients for category $1, 2$ *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$

are eighth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$

are fourth detrition coefficients for category 1, 2 *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$

are fifth detrition coefficients for category 1, 2 *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$

are sixth detrition coefficients for category 1, 2 *and* 3

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$

are seventh detrition coefficients for category 1, 2 *and* 3

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$

are eighth detrition coefficients for category 1, 2 *and* 3

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{30} \quad 81$$

Where $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for

category 1,2, and 3

$-(b_{16}'')^{(2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b_{20}'')^{(3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b_{36}'')^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b_{42}'')^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b_{40}'')^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b_{46}'')^{(9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a_{32}')^{(6)} & +(a_{32}'')^{(6)}(T_{33},t) & +(a_{28}'')^{(5,5,5)}(T_{29},t) & +(a_{24}'')^{(4,4,4,)}(T_{25},t) \\ +(a_{13}'')^{(1,1,1,1,1,1)}(T_{14},t) & & +(a_{16}'')^{(2,2,2,2,2,2)}(T_{17},t) & +(a_{20}'')^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a_{36}'')^{(7,7,7,7,7,7,7)}(T_{37},t) & & +(a_{40}'')^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a_{44}'')^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a_{33}')^{(6)} & +(a_{33}'')^{(6)}(T_{33},t) & +(a_{29}'')^{(5,5,5)}(T_{29},t) & +(a_{25}'')^{(4,4,4,)}(T_{25},t) \\ +(a_{14}'')^{(1,1,1,1,1,1)}(T_{14},t) & & +(a_{17}'')^{(2,2,2,2,2,2)}(T_{17},t) & +(a_{21}'')^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a_{37}'')^{(7,7,7,7,7,7,7)}(T_{37},t) & & +(a_{41}'')^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a_{34}')^{(6)} & +(a_{34}'')^{(6)}(T_{33},t) & +(a_{30}'')^{(5,5,5)}(T_{29},t) & +(a_{26}'')^{(4,4,4,)}(T_{25},t) \\ +(a_{15}'')^{(1,1,1,1,1,1)}(T_{14},t) & & +(a_{18}'')^{(2,2,2,2,2,2)}(T_{17},t) & +(a_{22}'')^{(3,3,3,3,3,3)}(T_{21},t) \\ +(a_{38}'')^{(7,7,7,7,7,7,7)}(T_{37},t) & & +(a_{42}'')^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{34} \quad (87)$$

$+(a_{32}'')^{(6)}(T_{33},t)$, $+(a_{33}'')^{(6)}(T_{33},t)$, $+(a_{34}'')^{(6)}(T_{33},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

$+(a_{28}'')^{(5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5)}(T_{29},t)$ *are second augmentation coefficients for category* 1, 2 *and* 3

$+(a_{24}'')^{(4,4,4,)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,)}(T_{25},t)$ *are third augmentation coefficients for category* 1, 2 *and* 3

$+(a_{13}'')^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1,1)}(T_{14},t)$ - are fourth augmentation coefficients

$+(a_{16}'')^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2,2,2)}(T_{17},t)$ - fifth augmentation coefficients

$+(a_{20}'')^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a_{36}'')^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$+(a_{40}'')^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8,8,8)}(T_{41},t)$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \qquad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt}$$
$$= (a_{42})^{(8)} G_{41}$$
$$- \begin{bmatrix} (a'_{42})^{(8)} \boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$
$$(b_{40})^{(8)} T_{41} - \begin{bmatrix} (b'_{40})^{(8)} \boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$
$$(b_{41})^{(8)} T_{40} - \begin{bmatrix} (b'_{41})^{(8)} \boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$
$$(b_{42})^{(8)} T_{41} - \begin{bmatrix} (b'_{42})^{(8)} \boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{38}'')^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

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$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a_{44}')^{(9)}\boxed{+(a_{44}'')^{(9)}(T_{45},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a_{45}')^{(9)}\boxed{+(a_{45}'')^{(9)}(T_{45},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a_{46}')^{(9)}\boxed{+(a_{46}'')^{(9)}(T_{37},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{44}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{46}'')^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second

augmentation coefficient for category 1, 2 and 3
$+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3
$+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3
$+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3
$+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$
$$(b_{44})^{(9)}T_{45} -$$
$$\begin{bmatrix} (b_{44}')^{(9)}\left[-(b_{44}'')^{(9)}(G_{47},t)\right] & \left[-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{13}$$
$$\frac{dT_{45}}{dt} =$$
$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}\left[-(b_{45}'')^{(9)}(G_{47},t)\right] & \left[-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{14}$$
$$\frac{dT_{46}}{dt} =$$
$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}\left[-(b_{46}'')^{(9)}(G_{47},t)\right] & \left[-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{15}$$

Where $-(b_{44}'')^{(9)}(G_{47},t)$, $-(b_{45}'')^{(9)}(G_{47},t)$, $-(b_{46}'')^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3
$-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\ ,\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)},\ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} +\ (\hat{A}_{20})^{(3)} +\ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} +\ (\hat{B}_{20})^{(3)} +\ (\hat{Q}_{20})^{(3)}\ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)},\ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}), t\big) \le\ (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}\,(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\,\big((G_{27}), t\big) =\ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)},\ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30$ 122

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)}$$ 123

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}\left((G_{35}), t\right) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(CCCCCC) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(DDDDDD) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

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(EEEEEE) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(FFFFFF)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(GGGGGG) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(HHHHHH) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)} \qquad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be

noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\ ,\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}<1 \qquad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+\ (\hat{A}_{40})^{(8)}+(\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \qquad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[\ (b_i)^{(8)}+(b_i')^{(8)}+\ (\hat{B}_{40})^{(8)}+(\hat{Q}_{40})^{(8)}\ (\hat{k}_{40})^{(8)}]<1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \qquad 146A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t)-(a_i'')^{(9)}(T_{45},t)|\le(\hat{k}_{44})^{(9)}|T_{45}-\ T_{45}'|e^{-(\widehat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t)-(b_i'')^{(9)}\big((G_{47}),t\big)|<(\hat{k}_{44})^{(9)}||(G_{47})-(G_{47})'||e^{-(\widehat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$

and $(a_i'')^{(9)}(T_{45},t)$. (T_{45}',t) and (T_{45},t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)},(\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)}=1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},$ are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\ ,\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}<1$$

Definition of $(\hat{P}_{44})^{(9)},(\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},(\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)},(a_i')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(p_i)^{(9)},\ (r_i)^{(9)},i=44,45,46,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)}+(a_i')^{(9)}+\ (\hat{A}_{44})^{(9)}+\ (\hat{P}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)}+(b_i')^{(9)}+\ (\hat{B}_{44})^{(9)}+\ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\ ,T_i(0)$:

$$G_i(t)\le\ \left(\hat{P}_{13}\right)^{(1)}e^{(\hat{M}_{13})^{(1)}t}\ ,\ \boxed{G_i(0)=G_i^0>0}$$

$$T_i(t)\le\ (\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}\ ,\ \boxed{T_i(0)=T_i^0>0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\ ,T_i(0)$

$$G_i(t)\le\ (\hat{P}_{16})^{(2)}e^{(\hat{M}_{16})^{(2)}t}\ ,\ \ G_i(0)=G_i^0>0$$

$$T_i(t)\le\ (\hat{Q}_{16})^{(2)}e^{(\hat{M}_{16})^{(2)}t}\ ,\ \ T_i(0)=T_i^0>0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t)\le\ (\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}t}\ ,\ \ G_i(0)=G_i^0>0$$

$$T_i(t)\le\ (\hat{Q}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}t}\ ,\ \ T_i(0)=T_i^0>0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0)\ ,T_i(0)$:

$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{\left(\hat{M}_{24}\right)^{(4)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{\left(\hat{M}_{24}\right)^{(4)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0)$, $T_i(0)$:

$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{\left(\hat{M}_{28}\right)^{(5)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{\left(\hat{M}_{28}\right)^{(5)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)$, $T_i(0)$:

$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{\left(\hat{M}_{32}\right)^{(6)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{\left(\hat{M}_{32}\right)^{(6)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)$, $T_i(0)$:

$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{\left(\hat{M}_{36}\right)^{(7)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{\left(\hat{M}_{36}\right)^{(7)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)$, $T_i(0)$:

$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{\left(\hat{M}_{40}\right)^{(8)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{\left(\hat{M}_{40}\right)^{(8)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)$, $T_i(0)$:

$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{\left(\hat{M}_{44}\right)^{(9)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{\left(\hat{M}_{44}\right)^{(9)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{13})^{(1)}, T_i^0 \le (\hat{Q}_{13})^{(1)}$, 155

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} t}$ 156

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}, T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\tilde{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\tilde{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{28})^{(5)}$, $T_i^0 \le (\hat{Q}_{28})^{(5)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0,t)$

Proof: 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(w) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left(e^{(\tilde{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\tilde{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left(e^{(\tilde{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\tilde{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0\right) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\tilde{M}_{44})^{(9)}} \left(e^{(\tilde{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\tilde{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\tilde{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\tilde{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\tilde{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[(\widehat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on

$G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 19 to 24 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(1)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If $G_{13}\ \ is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}\,(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on

(G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)} t)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

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$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)}$$

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$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)}$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),\left((G_{23})^{(2)},(T_{23})^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

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$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\;(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on T_{21} and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \quad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 227

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24{,}25{,}26$ depend only on T_{25} and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(4)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25}\ \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t\ > t_4$

$$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\widehat{P}_{28})^{(5)}$ and $(\widehat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\widehat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \tag{234}$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \tag{235}$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric (236)

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{31}}),(\widetilde{T_{31}})$: $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\leq \tag{237}$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\ (G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(5)} t)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)}+\left((\widehat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\widehat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\widehat{Q}_{32})^{(6)}\right]\leq(\widehat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(6)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose

$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{39}}),(\widetilde{T_{39}}):\left((\widetilde{G_{39}}),(\widetilde{T_{39}})\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$

It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\leq \quad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\ (G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\widehat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{40})^{(8)}\right] \leq (\widehat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})}, \widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by 274

$(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$. The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right)) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)}, , (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}, \ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}, \ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13} e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13} e^{(S_1)^{(1)} t}$

where $(p_i)^{(1)}$ is defined by equation

$\frac{1}{(m_1)^{(1)}} G^0_{13} e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G^0_{13} e^{(S_1)^{(1)} t}$

$$\left(\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t}-e^{-(S_2)^{(1)}t}\right]+G_{15}^{0}e^{-(S_2)^{(1)}t}\le G_{15}(t)\le\right. \quad (286)$$

$$\left.\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}\right]+G_{15}^{0}e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation (291)

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: (292)

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(T_{17},t)+(a''_{17})^{(2)}(T_{17},t)\le-(\sigma_1)^{(2)} \quad (293)$$

$$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)} \quad (294)$$

Definition of $(\nu_1)^{(2)},(\nu_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: (295)

By $(\nu_1)^{(2)}>0$, $(\nu_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0$, $(u_2)^{(2)}<0$ the roots (296)

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_1)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ (297)

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and (298)

Definition of $(\bar{\nu}_1)^{(2)},,(\bar{\nu}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: (299)

By $(\bar{\nu}_1)^{(2)}>0$, $(\bar{\nu}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0$, $(\bar{u}_2)^{(2)}<0$ the (300)

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_2)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ (301)

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t}\right] + T_{18}^0 e^{-(R_2)^{(2)}t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations
$(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$
and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by
$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, ***if*** $(\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, ***if*** $(\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, ***if*** $(\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$$

$$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(v^{(4)}\big)^2 + (\sigma_1)^{(4)} v^{(4)} - (a_{24})^{(4)} = 0$

and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)},,(\bar{v}_2)^{(4)},(\bar{u}_1)^{(4)},(\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$ and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}$, $(\mu_2)^{(4)}$, $(v_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}$, $(\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, ***if*** $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)}t} - e^{-(a'_{26})^{(4)}t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)}t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^{0}}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t}-e^{-(R_2)^{(4)}t}\right]+T_{26}^{0}e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)},(S_2)^{(4)},(R_1)^{(4)},(R_2)^{(4)}$:- 337

Where $(S_1)^{(4)}=(a_{24})^{(4)}(m_2)^{(4)}-(a'_{24})^{(4)}$

$(S_2)^{(4)}=(a_{26})^{(4)}-(p_{26})^{(4)}$

$(R_1)^{(4)}=(b_{24})^{(4)}(\mu_2)^{(4)}-(b'_{24})^{(4)}$

$(R_2)^{(4)}=(b'_{26})^{(4)}-(r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)},(\sigma_2)^{(5)},(\tau_1)^{(5)},(\tau_2)^{(5)}$:

$(\sigma_1)^{(5)},(\sigma_2)^{(5)},(\tau_1)^{(5)},(\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)}\le-(a'_{28})^{(5)}+(a'_{29})^{(5)}-(a''_{28})^{(5)}(T_{29},t)+(a''_{29})^{(5)}(T_{29},t)\le-(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)}\le-(b'_{28})^{(5)}+(b'_{29})^{(5)}-(b''_{28})^{(5)}\big((G_{31}),t\big)-(b''_{29})^{(5)}\big((G_{31}),t\big)\le-(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)},(\nu_2)^{(5)},(u_1)^{(5)},(u_2)^{(5)},\nu^{(5)},u^{(5)}$: 339

By $(\nu_1)^{(5)}>0$, $(\nu_2)^{(5)}<0$ and respectively $(u_1)^{(5)}>0$, $(u_2)^{(5)}<0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2+(\sigma_1)^{(5)}\nu^{(5)}-(a_{28})^{(5)}=0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2+(\tau_1)^{(5)}u^{(5)}-(b_{28})^{(5)}=0$ and

Definition of $(\bar{\nu}_1)^{(5)},,(\bar{\nu}_2)^{(5)},(\bar{u}_1)^{(5)},(\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)}>0$, $(\bar{\nu}_2)^{(5)}<0$ and respectively $(\bar{u}_1)^{(5)}>0$, $(\bar{u}_2)^{(5)}<0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2+(\sigma_2)^{(5)}\nu^{(5)}-(a_{28})^{(5)}=0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2+(\tau_2)^{(5)}u^{(5)}-(b_{28})^{(5)}=0$

Definition of $(m_1)^{(5)},(m_2)^{(5)},(\mu_1)^{(5)},(\mu_2)^{(5)},(\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)},(m_2)^{(5)},(\mu_1)^{(5)},(\mu_2)^{(5)}$ by

$(m_2)^{(5)}=(\nu_0)^{(5)},(m_1)^{(5)}=(\nu_1)^{(5)}$, $\boldsymbol{if}\ (\nu_0)^{(5)}<(\nu_1)^{(5)}$

$(m_2)^{(5)}=(\nu_1)^{(5)},(m_1)^{(5)}=(\bar{\nu}_1)^{(5)}$, $\boldsymbol{if}\ (\nu_1)^{(5)}<(\nu_0)^{(5)}<(\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)}=\frac{G_{28}^{0}}{G_{29}^{0}}}$

$(m_2)^{(5)}=(\nu_1)^{(5)},(m_1)^{(5)}=(\nu_0)^{(5)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(5)}<(\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)}=(u_0)^{(5)},(\mu_1)^{(5)}=(u_1)^{(5)}$, $\boldsymbol{if}\ (u_0)^{(5)}<(u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, $\boldsymbol{if}$ $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}$ $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{28}(t) \leq G_{28}^0 e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{29}(t) \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \leq G_{30}(t) \leq \right. \quad 344$$

$$\left. \frac{(a_{30})^{(5)} G_{28}^0}{(m_2)^{(5)}((S_1)^{(5)} - (a'_{30})^{(5)})}\left[e^{(S_1)^{(5)} t} - e^{-(a'_{30})^{(5)} t}\right] + G_{30}^0 e^{-(a'_{30})^{(5)} t}\right)$$

$$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \leq T_{28}(t) \leq T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \leq T_{28}(t) \leq \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \leq T_{30}(t) \leq \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \leq -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \leq -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \leq -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \leq -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}$, $(m_2)^{(6)}$, $(\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}$, $(m_2)^{(6)}$, $(\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, $\boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad (354)$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad (355)$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t} - e^{-(a'_{34})^{(6)}t}\right] + G_{34}^0 e^{-(a'_{34})^{(6)}t}\right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad (356)$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}}T_{32}^0 e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(v_1)^{(7)}, (v_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, v^{(7)}, u^{(7)}$: 361

By $(v_1)^{(7)} > 0, (v_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{v}_1)^{(7)}, , (\bar{v}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)} > 0, (\bar{v}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)},\ \boldsymbol{if}\ (\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)},\ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}),t\big) - (b''_{45})^{(9)}\big((G_{47}),t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)},, (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, ***if*** $(\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, ***if*** $(\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, ***if*** $(\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,
and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44}e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44}e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G^0_{44}e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G^0_{44}e^{(S_1)^{(9)}t}$$

$$\Big(\frac{(a_{46})^{(9)}G^0_{44}}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})}\Big[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t}\Big] + G^0_{46}e^{-(S_2)^{(9)}t} \le G_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}G^0_{44}}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})}[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G^0_{46}e^{-(a'_{46})^{(9)}t}\Big)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{dv^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)v^{(2)} - (a_{17})^{(2)}v^{(2)}$$

Definition of $v^{(2)}$:- $\boxed{v^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(v^{(2)}\right)^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)}\right) \leq \frac{dv^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(v^{(2)}\right)^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{v}_1)^{(2)}, (v_0)^{(2)}$:-

For $0 < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (v_1)^{(2)} < (\bar{v}_1)^{(2)}$

$$v^{(2)}(t) \geq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(v_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (v_2)^{(2)}}}$$

it follows $(v_0)^{(2)} \leq v^{(2)}(t) \leq (v_1)^{(2)}$

In the same manner , we get 391

$$v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{v}_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (\bar{v}_2)^{(2)}}}$$

From which we deduce $(v_0)^{(2)} \leq v^{(2)}(t) \leq (\bar{v}_1)^{(2)}$ 392

If $0 < (v_1)^{(2)} < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{v}_1)^{(2)}$ we find like in the previous case, 393

$$(v_1)^{(2)} \leq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_2)^{(2)}\right)t\right]}} \leq v^{(2)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \leq (\bar{v}_1)^{(2)}$$

If $0 < (v_1)^{(2)} \leq (\bar{v}_1)^{(2)} \leq (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(v_1)^{(2)} \leq v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \leq (v_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(2)}(t)$:- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then
$(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

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If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\quad \boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$v^{(6)}(t) \le \frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)}=\frac{(\bar{v}_1)^{(6)}-(v_0)^{(6)}}{(v_0)^{(6)}-(\bar{v}_2)^{(6)}}}$$

From which we deduce $(v_0)^{(6)} \le v^{(6)}(t) \le (\bar{v}_1)^{(6)}$

If $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{v}_1)^{(6)}$ we find like in the previous case, 414

$$(v_1)^{(6)} \le \frac{(v_1)^{(6)}+(C)^{(6)}(v_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)}-(v_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)}-(v_2)^{(6)}\right)t\right]}} \le v^{(6)}(t) \le$$

$$\frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \le (\bar{v}_1)^{(6)}$$

If $0 < (v_1)^{(6)} \le (\bar{v}_1)^{(6)} \le \boxed{(v_0)^{(6)}=\frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(v_1)^{(6)} \le v^{(6)}(t) \le \frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \le (v_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(6)}(t)$:-

$$(m_2)^{(6)} \le v^{(6)}(t) \le (m_1)^{(6)}, \quad \boxed{v^{(6)}(t)=\frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \le u^{(6)}(t) \le (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t)=\frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(v_1)^{(6)} = (\bar{v}_1)^{(6)}$ if in addition $(v_0)^{(6)} = (v_1)^{(6)}$ then $v^{(6)}(t) = (v_0)^{(6)}$ and as a consequence $G_{32}(t) = (v_0)^{(6)}G_{33}(t)$ **this also defines** $(v_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(v_1)^{(6)}$ and $(\bar{v}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)}=\frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}\right)\leq\frac{dv^{(7)}}{dt}\leq-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0<\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}<(\nu_1)^{(7)}<(\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t)\geq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}\quad,\quad\boxed{(C)^{(7)}=\frac{(\nu_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)}\leq\nu^{(7)}(t)\leq(\nu_1)^{(7)}$

In the same manner , we get 417

$$\nu^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(7)}=\frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)}\leq\nu^{(7)}(t)\leq(\bar{\nu}_1)^{(7)}$

If $0<(\nu_1)^{(7)}<(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}<(\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)}\leq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}\leq\nu^{(7)}(t)\leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\bar{\nu}_1)^{(7)}$$

If $0<(\nu_1)^{(7)}\leq(\bar{\nu}_1)^{(7)}\leq\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)}\leq\nu^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)}\leq\nu^{(7)}(t)\leq(m_1)^{(7)},\quad\boxed{\nu^{(7)}(t)=\frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{40}'')^{(8)} = (a_{41}'')^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b_{40}'')^{(8)} = (b_{41}'')^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$ if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a_{44}')^{(9)} - (a_{45}')^{(9)} + (a_{44}'')^{(9)}(T_{45}, t)\right) - (a_{45}'')^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\quad \boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \leq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \leq \nu^{(9)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \leq (\bar{\nu}_1)^{(9)} \leq \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \leq \nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \leq \nu^{(9)}(t) \leq (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for**

the special case .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(v_1)^{(9)}$ and $(\bar{v}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a_i'')^{(1)} and\ (b_i'')^{(1)}$ are independent on t , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0$ 447

$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0$ 448

has a unique positive solution , which is an equilibrium solution

$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0$ 449

$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0$ 450

$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0$ 451

$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0$ 452

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(w) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T_{33}^{*} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^{*} for which $f(T_{33}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

Definition and uniqueness of T_{37}^{*} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^{*} for which $f(T_{37}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^{*})]}$$

Definition and uniqueness of T_{41}^{*} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^{*} for which $f(T_{41}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^{*})]}$$

Definition and uniqueness of T_{45}^{*} :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^{*} for which $f(T_{45}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^{*})]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi(G^{*}) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that

there exists a unique G_{37}^{*} such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)}G_{17}^{*}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)}G_{17}^{*}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T_{17}^{*})]} \qquad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)}T_{17}^{*}}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)}T_{17}^{*}}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)}G_{21}^{*}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)}G_{21}^{*}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$$T_{28}^* = \frac{(b_{28})^{(5)} T_{29}^*}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^*)]} \quad , \quad T_{30}^* = \frac{(b_{30})^{(5)} T_{29}^*}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^*)]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^* given by $\varphi((G_{35})^*) = 0$, T_{33}^* given by $f(T_{33}^*) = 0$ and

$$G_{32}^* = \frac{(a_{32})^{(6)} G_{33}^*}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34}^* = \frac{(a_{34})^{(6)} G_{33}^*}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

$$T_{32}^* = \frac{(b_{32})^{(6)} T_{33}^*}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^*)]} \quad , \quad T_{34}^* = \frac{(b_{34})^{(6)} T_{33}^*}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^*)]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^* given by $\varphi((G_{39})^*) = 0$, T_{37}^* given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

G_{41}^* given by $\varphi((G_{43})^*) = 0$, T_{41}^* given by $f(T_{41}^*) = 0$ and

$$G_{40}^{*}=\frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}\quad,\quad G_{42}^{*}=\frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

$$T_{40}^{*}=\frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}\quad,\quad T_{42}^{*}=\frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$$

Finally we obtain the unique solution of 89 to 99 523
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G_{45}^{*} given by $\varphi((G_{47})^{*})=0$, T_{45}^{*} given by $f(T_{45}^{*})=0$ and

$$G_{44}^{*}=\frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}\quad,\quad G_{46}^{*}=\frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$$

$$T_{44}^{*}=\frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}\quad,\quad T_{46}^{*}=\frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}\ and\ (b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i=G_i^{*}+\mathbb{G}_i\qquad,T_i=T_i^{*}+\mathbb{T}_i$$

$$\frac{\partial(a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^{*})=(q_{14})^{(1)}\quad,\ \frac{\partial(b_i'')^{(1)}}{\partial G_j}(\,G^{*}\,)=s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt}=-\big((a'_{13})^{(1)}+(p_{13})^{(1)}\big)\mathbb{G}_{13}+(a_{13})^{(1)}\mathbb{G}_{14}-(q_{13})^{(1)}G_{13}^{*}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt}=-\big((a'_{14})^{(1)}+(p_{14})^{(1)}\big)\mathbb{G}_{14}+(a_{14})^{(1)}\mathbb{G}_{13}-(q_{14})^{(1)}G_{14}^{*}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt}=-\big((a'_{15})^{(1)}+(p_{15})^{(1)}\big)\mathbb{G}_{15}+(a_{15})^{(1)}\mathbb{G}_{14}-(q_{15})^{(1)}G_{15}^{*}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt}=-\big((b'_{13})^{(1)}-(r_{13})^{(1)}\big)\mathbb{T}_{13}+(b_{13})^{(1)}\mathbb{T}_{14}+\textstyle\sum_{j=13}^{15}\big(s_{(13)(j)}T_{13}^{*}\mathbb{G}_j\big) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt}=-\big((b'_{14})^{(1)}-(r_{14})^{(1)}\big)\mathbb{T}_{14}+(b_{14})^{(1)}\mathbb{T}_{13}+\textstyle\sum_{j=13}^{15}\big(s_{(14)(j)}T_{14}^{*}\mathbb{G}_j\big) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt}=-\big((b'_{15})^{(1)}-(r_{15})^{(1)}\big)\mathbb{T}_{15}+(b_{15})^{(1)}\mathbb{T}_{14}+\textstyle\sum_{j=13}^{15}\big(s_{(15)(j)}T_{15}^{*}\mathbb{G}_j\big) \quad 530$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions

$(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \tag{532}$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \ , \ \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \tag{533}$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \tag{534}$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \tag{535}$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \tag{536}$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \tag{537}$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}')^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T_{17}^*\mathbb{G}_j\right) \tag{538}$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}')^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T_{18}^*\mathbb{G}_j\right) \tag{539}$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \ , \ \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}')^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \tag{541}$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}')^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \tag{542}$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}')^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \tag{543}$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}')^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T_{20}^*\mathbb{G}_j\right) \tag{544}$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^{*}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T_{22}^{*}\mathbb{G}_j\right) \quad 546$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^{*}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^{*}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^{*}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^{*}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^{*}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^{*}\mathbb{G}_j\right) \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}(\,(G_{31})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^{*}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}(\,(G_{35})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^{*}) = (q_{41})^{(8)} \quad , \frac{\partial(b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^{*}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^{*}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^{*}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^{*}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \ , \ \frac{\partial(b''_i)^{(9)}}{\partial G_j}(\ (G_{47})^*\ ) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\ B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\ C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\ D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\ E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\ F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\ G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14}+(b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\Big]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION THIRTY ONE

Mass-Matrix Ansatz And Flavor Nonconservation In Models

INTRODUCTION—VARIABLES USED

Mass-matrix ansatz and flavor nonconservation in models with multiple Higgs doublets Phys. Rev. D 35, 3484 – Published 1 June 1987 T. P. Cheng and Marc Sher

(1) Motivated by the existence of three fermion generations, authors consider models with three Higgs doublets (for each hypercharge).
(2) It is shown that, if the basic Yukawa coupling matrices are of the Fritzsch form or some generalizations thereof (two such examples are presented), then (eb) the neutral flavor-changing couplings have a hierarchical structure given by Δij √mimj , where Δij is a combination of mixing angles and the mi's are the relevant fermion masses.
(3) These couplings give (eb) a lower bound of the order of Δds times 1 TeV on (eb) the mass of the exchanged scalar from (e) the neutral-kaon mass difference.
(4) If the Higgs-boson mass is not significantly greater than this limit, (eb) one expects $D\bar{D}$ mixing comparable to (e&eb) the present experimental bound. DOI: http://dx.doi.org/10.1103/PhysRevD.35.3484

Radiative corrections to Higgs boson masses in the next-to-minimal supersymmetric standard model Phys. Rev. D 49, 2435 – Published 1 March 1994 T. Elliott, S. F. King, and P. L. White

(5) **T. Elliott, S. F. King, and P. L. White** perform a systematic study of radiative corrections to (e) the masses of the Higgs bosons in (eb) the minimal supersymmetric standard model (MSSM) augmented by (e) a single gauge singlet, the so-called next-to-minimal supersymmetric standard model (NMSSM).
(6) Method is based on (e) the one-loop effective potential and includes effects of (e&eb) top quark, squark, Higgs boson, and Higgsino loops.
(7) Authors discuss the renormalization group flows of (e) Yukawa couplings and the upper bound on (eb) the lightest CP-even neutral Higgs boson mass as (=) a function of the heavier top squark mass and top quark mass.
(8) They then give a general discussion of Higgs boson phenomenology including (e&eb) radiative corrections.
(9) Authors survey as much of the parameter space of (e) the Higgs sector of the NMSSM as is practical, and analyze the full spectrum of Higgs boson masses and couplings in (eb) these regions of parameter space.

NOTATION

Module One

It is shown that, if the basic Yukawa coupling matrices are of the Fritzsch form or some generalizations thereof (two such examples are presented), then (eb) the neutral flavor-changing couplings have a hierarchical structure given by Δij √mimj , where Δij is a combination of mixing angles and the mi's are the relevant fermion masses

G_{13} : Category one of basic Yukawa coupling matrices are of the Fritzsch form or some generalizations thereof (two such examples are presented)

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of neutral flavor-changing couplings have a hierarchical structure given by Δij √mimj , where Δij is a combination of mixing angles and the mi's are the relevant fermion masses

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

These couplings give (eb) a lower bound of the order of Δds times 1 TeV on (eb) the mass of the exchanged scalar from (e) the neutral-kaon mass difference

G_{16} : Category one of neutral flavor-changing couplings

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of lower bound of the order of Δds times 1 TeV on (eb) the mass of the exchanged scalar from (e) the neutral-kaon mass difference

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

These neutral flavor-changing couplings give (eb) a lower bound of the order of Δds times 1 TeV on (eb) the mass of the exchanged scalar from (e) the neutral-kaon mass difference

G_{20} : Category one of neutral flavor-changing couplings

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of lower bound of the order of Δds times 1 TeV on the mass of the exchanged scalar from (e) the neutral-kaon mass difference

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

neutral flavor-changing couplings give a lower bound of the order of Δds times 1 TeV on the mass of the exchanged scalar from (e) the neutral-kaon mass difference

G_{24} : Category one of neutral-kaon mass difference

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of neutral flavor-changing couplings give a lower bound of the order of Δds times 1 TeV on the mass of the exchanged scalar

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

If the Higgs-boson mass is not significantly greater than this limit, (eb) one expects DD^{-} mixing comparable to (e&eb) the present experimental bound.

DOI: http://dx.doi.org/10.1103/PhysRevD.35.3484

G_{28} : Category one of Higgs-boson mass is not significantly greater than this limit

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of DD^{-} mixing comparable to the present experimental bound.

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

T. Elliott, S. F. King, and P. L. White perform a systematic study of

radiative corrections to (e) the masses of the Higgs bosons in (eb) the minimal supersymmetric standard model (MSSM) augmented by (e) a single gauge singlet, the so-called next-to-minimal supersymmetric standard model (NMSSM)

G_{32} : Category one of masses of the Higgs bosons in (eb) the minimal supersymmetric standard model (MSSM) augmented by (e) a single gauge singlet, the so-called next-to-minimal supersymmetric standard model (NMSSM)

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of radiative corrections

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

radiative corrections to the masses of the Higgs bosons in (eb) the minimal supersymmetric standard model (MSSM) augmented by (e) a single gauge singlet, the so-called next-to-minimal supersymmetric standard model (NMSSM)

G_{36} : Category one of radiative corrections to the masses of the Higgs bosons

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of minimal supersymmetric standard model (MSSM) augmented by (e) a single gauge singlet, the so-called next-to-minimal supersymmetric standard model (NMSSM)

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

radiative corrections to the masses of the Higgs bosons in the minimal supersymmetric standard model (MSSM) augmented by (e) a single gauge singlet, the so-called next-to-minimal supersymmetric standard model (NMSSM)

G_{40} : Category one of single gauge singlet, the so-called next-to-minimal supersymmetric standard model (NMSSM)

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of radiative corrections to the masses of the Higgs bosons in the minimal supersymmetric standard model (MSSM) augmented

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Authors discuss the

renormalization group flows of (e) Yukawa couplings and the upper bound on (eb) the lightest CP-even neutral Higgs boson mass as (=) a function of the heavier top squark mass and top quark mass

G_{44} : Category one of Yukawa couplings and the upper bound on (eb) the lightest CP-even neutral Higgs boson mass as (=) a function of the heavier top squark mass and top quark mass

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of renormalization group flows

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}), t\right)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}), t\right)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\left((G_{39}), t\right)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}), t\right)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}), t\right)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}, t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}, t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}), t\right)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)}\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad (60)$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t) \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & +(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) & +(a''_{21})^{(3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t) \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & +(a''_{18})^{(2)}(T_{17},t) & +(a''_{15})^{(1,1,)}(T_{14},t) & +(a''_{22})^{(3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{38})^{(7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9)}(T_{45},t) \end{bmatrix} G_{18} \quad (63)$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)} & -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9)}(G_{47},t) \end{bmatrix} T_{18} \quad (66)$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{20})^{(3)}\,+(a''_{20})^{(3)}(T_{21},t)\,+(a''_{16})^{(2,2,2)}(T_{17},t)\,+(a''_{13})^{(1,1,1,)}(T_{14},t) \\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)\,+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)\,+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7,7)}(T_{37},t)\,+(a''_{40})^{(8,8,8,8)}(T_{41},t)\,+(a''_{44})^{(9,9,9)}(T_{45},t) \end{array}\right] G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c} (a'_{21})^{(3)}\,+(a''_{21})^{(3)}(T_{21},t)\,+(a''_{17})^{(2,2,2)}(T_{17},t)\,+(a''_{14})^{(1,1,1,)}(T_{14},t) \\ +(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)\,+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)\,+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{37})^{(7,7,7,7)}(T_{37},t)\,+(a''_{41})^{(8,8,8,8)}(T_{41},t)\,+(a''_{45})^{(9,9,9)}(T_{45},t) \end{array}\right] G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c} (a'_{22})^{(3)}\,+(a''_{22})^{(3)}(T_{21},t)\,+(a''_{18})^{(2,2,2)}(T_{17},t)\,+(a''_{15})^{(1,1,1,)}(T_{14},t) \\ +(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)\,+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)\,+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t) \\ +(a''_{38})^{(7,7,7,7)}(T_{37},t)\,+(a''_{42})^{(8,8,8,8)}(T_{41},t)\,+(a''_{46})^{(9,9,9)}(T_{45},t) \end{array}\right] G_{22} \quad (69)$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[\begin{array}{c} (b'_{20})^{(3)}\,-(b''_{20})^{(3)}(G_{23},t)\,-(b''_{16})^{(2,2,2)}(G_{19},t)\,-(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)\,-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)\,-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t)\,-(b''_{40})^{(8,8,8,8)}(G_{43},t)\,-(b''_{44})^{(9,9,9)}(G_{47},t) \end{array}\right] T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[\begin{array}{c} (b'_{21})^{(3)}\,-(b''_{21})^{(3)}(G_{23},t)\,-(b''_{17})^{(2,2,2)}(G_{19},t)\,-(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)\,-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)\,-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t)\,-(b''_{41})^{(8,8,8,8)}(G_{43},t)\,-(b''_{45})^{(9,9,9)}(G_{47},t) \end{array}\right] T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* $1, 2\ 3$

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$

$are\ fourth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$
$are\ fifth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$
$are\ sixth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$
$are\ seventh\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$
$are\ eighth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t)\\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t)\\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t)\end{bmatrix}T_{24} \quad 76$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t)\\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t)\\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t)\end{bmatrix}T_{25} \quad 77$$

$$\frac{dT_{26}}{dt}=(b_{26})^{(4)}T_{25}-\begin{bmatrix}(b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t)\\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t)\\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t)\end{bmatrix}T_{26} \quad 78$$

$Where\ -(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ $are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ $are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ $are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
$are\ eighth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad (81)$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \quad (84)$$

$where\ \boxed{-(b''_{28})^{(5)}(G_{31},t)}\ ,\boxed{-(b''_{29})^{(5)}(G_{31},t)}\ ,\boxed{-(b''_{30})^{(5)}(G_{31},t)}\ \ are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}\ ,\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}\ ,\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}\ are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}\ ,\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}\ ,\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}\ are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}\ ,\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}\ ,\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}\ ,\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}\ ,\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}\ ,\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}\ ,\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}\ ,\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}\ ,\boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}\ ,\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}\ ,\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}\ ,\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}\ ,\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32} \quad 85$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}\ ,\boxed{+(a''_{33})^{(6)}(T_{33},t)}\ ,\boxed{+(a''_{34})^{(6)}(T_{33},t)}\ are\ first\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\ ,\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\ ,\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\ are\ second\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}\ ,\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}\ ,\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}\ \ are\ third\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\ ,\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\ ,\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\ ,\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\ ,\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation

coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$ Eighth augmentation coefficients

$+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)} & -(b''_{32})^{(6)}(G_{35},t) & -(b''_{28})^{(5,5,5)}(G_{31},t) & -(b''_{24})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{13})^{(1,1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)} & -(b''_{33})^{(6)}(G_{35},t) & -(b''_{29})^{(5,5,5)}(G_{31},t) & -(b''_{25})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{14})^{(1,1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & -(b''_{34})^{(6)}(G_{35},t) & -(b''_{30})^{(5,5,5)}(G_{31},t) & -(b''_{26})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{15})^{(1,1,1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{34} \quad 90$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,)}(G_{27},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3

$-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3

$-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2, and 3

$-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \quad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt}$$ *95*

$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt}$$

$$= (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt}$$
$$= (a_{42})^{(8)} G_{41}$$
$$- \left[\begin{array}{ccc} (a'_{42})^{(8)} \boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array} \right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$
$$(b_{40})^{(8)} T_{41} - \left[\begin{array}{ccc} (b'_{40})^{(8)} \boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{13}$$

$$\frac{dT_{41}}{dt} =$$
$$(b_{41})^{(8)} T_{40} - \left[\begin{array}{ccc} (b'_{41})^{(8)} \boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{14}$$

$$\frac{dT_{42}}{dt} =$$
$$(b_{42})^{(8)} T_{41} - \left[\begin{array}{ccc} (b'_{42})^{(8)} \boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{38}'')^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second

augmentation coefficient for category 1, 2 and 3
$+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3
$+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3
$+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3
$+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$
$$(b_{44})^{(9)}T_{45} -$$
$$\begin{bmatrix} (b_{44}')^{(9)}\left[-(b_{44}'')^{(9)}(G_{47},t)\right] & \left[-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{13}$$
$$\frac{dT_{45}}{dt} =$$
$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}\left[-(b_{45}'')^{(9)}(G_{47},t)\right] & \left[-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{14}$$
$$\frac{dT_{46}}{dt} =$$
$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}\left[-(b_{46}'')^{(9)}(G_{47},t)\right] & \left[-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{15}$$

Where $-(b_{44}'')^{(9)}(G_{47},t)$, $-(b_{45}'')^{(9)}(G_{47},t)$, $-(b_{46}'')^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3
$-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}'| e^{-(\hat{M}_{20})^{(3)} t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} ||G_{23} - G_{23}'|| e^{-(\hat{M}_{20})^{(3)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and$(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\ ,\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)},\ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)},\ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}), t\big) \le \ (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}\,(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\,\big((G_{27}), t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)},\ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - \ T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30$ 122

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \le (p_i)^{(5)} \le (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \le (r_i)^{(5)} \le (b_i')^{(5)} \le (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)}$$ 123

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \le (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and $(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}\left((G_{35}), t\right) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(IIIIII) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(JJJJJJ) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

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(KKKKKK) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(LLLLLL)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(MMMMMM) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(NNNNNN) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)} \qquad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be

noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\ ,\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}<1 \qquad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+\ (\hat{A}_{40})^{(8)}+(\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \qquad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[\ (b_i)^{(8)}+(b_i')^{(8)}+\ (\hat{B}_{40})^{(8)}+(\hat{Q}_{40})^{(8)}\ (\hat{k}_{40})^{(8)}]<1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \qquad 146A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t)-(a_i'')^{(9)}(T_{45},t)|\le(\hat{k}_{44})^{(9)}|T_{45}-\ T_{45}'|e^{-(\widehat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t)-(b_i'')^{(9)}\big((G_{47}),t\big)|<(\hat{k}_{44})^{(9)}||(G_{47})-(G_{47})'||e^{-(\widehat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$

and $(a_i'')^{(9)}(T_{45},t)$. (T_{45}',t) and (T_{45},t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)},(\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)}=1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},$ are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\ ,\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}<1$$

Definition of $(\hat{P}_{44})^{(9)},(\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},(\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)},(a_i')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(p_i)^{(9)},\ (r_i)^{(9)},i=44,45,46,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)}+(a_i')^{(9)}+\ (\hat{A}_{44})^{(9)}+\ (\hat{P}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)}+(b_i')^{(9)}+\ (\hat{B}_{44})^{(9)}+\ (\hat{Q}_{44})^{(9)}\ \ (\hat{k}_{44})^{(9)}]<1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\ ,T_i(0)$:

$$G_i(t)\leq\ \left(\hat{P}_{13}\right)^{(1)}e^{(\hat{M}_{13})^{(1)}t}\ ,\ \boxed{G_i(0)=G_i^0>0}$$

$$T_i(t)\leq\ (\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}\quad ,\quad \boxed{T_i(0)=T_i^0>0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\ ,T_i(0)$

$$G_i(t)\leq\ (\hat{P}_{16})^{(2)}e^{(\hat{M}_{16})^{(2)}t}\quad ,\quad G_i(0)=G_i^0>0$$

$$T_i(t)\leq\ (\hat{Q}_{16})^{(2)}e^{(\hat{M}_{16})^{(2)}t}\quad ,\quad T_i(0)=T_i^0>0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t)\leq\ (\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}t}\quad ,\quad G_i(0)=G_i^0>0$$

$$T_i(t)\leq\ (\hat{Q}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}t}\quad ,\quad T_i(0)=T_i^0>0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0)\ ,T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0)$, $T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)$, $T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)$, $T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)$, $T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)$, $T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{13})^{(1)}$, $T_i^0 \le (\hat{Q}_{13})^{(1)}$, 155

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 156

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^{0} + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^{0} + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{20})^{(3)}, T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By 161

$$\bar{G}_{20}(t) = G_{20}^{0} + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^{0} + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^{0} + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^{0} + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^{0} + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^{0} + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G_{23}(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^{0} + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{28})^{(5)}$, $T_i^0 \le (\hat{Q}_{28})^{(5)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0,t)$

Proof: 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(x) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left(e^{(\tilde{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\tilde{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left(e^{(\tilde{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\tilde{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0\right) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\tilde{M}_{44})^{(9)}} \left(e^{(\tilde{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\tilde{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\tilde{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\tilde{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\tilde{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[(\widehat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on

$G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 19 to 24 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(1)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If $G_{13}\ \ is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}\,(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on

(G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)} t)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18}. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16}, G_{18} and G_{16}, G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18}. The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2:

Indeed let t_2 be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

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$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)}$$

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$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)}$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

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$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\,(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on T_{21} and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$

In the same way , one can obtain

$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right)) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \quad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 227

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24{,}25{,}26$ depend only on T_{25} and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(4)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\,((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\widehat{P}_{28})^{(5)}$ and $(\widehat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\widehat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \tag{234}$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \tag{235}$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric (236)

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})},\widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})},\widetilde{(T_{31})}\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\leq \tag{237}$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\ (G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(5)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)}+\left((\widehat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\widehat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\widehat{Q}_{32})^{(6)}\right]\leq(\widehat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(6)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose

$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{39}}),(\widetilde{T_{39}}):\left((\widetilde{G_{39}}),(\widetilde{T_{39}})\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$

It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\leq \quad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\ (G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\widehat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{40})^{(8)}\right] \leq (\widehat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})}, \widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by 274

$(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right), \quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$. The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right)) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)}, , (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}, \ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}, \ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13} e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13} e^{(S_1)^{(1)} t}$

where $(p_i)^{(1)}$ is defined by equation

$\frac{1}{(m_1)^{(1)}} G^0_{13} e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G^0_{13} e^{(S_1)^{(1)} t}$

$$\left(\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t}-e^{-(S_2)^{(1)}t}\right]+G_{15}^{0}e^{-(S_2)^{(1)}t}\le G_{15}(t)\le\right. \quad (286)$$

$$\left.\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}\right]+G_{15}^{0}e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation (291)

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: (292)

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(T_{17},t)+(a''_{17})^{(2)}(T_{17},t)\le-(\sigma_1)^{(2)} \quad (293)$$

$$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)} \quad (294)$$

Definition of $(\nu_1)^{(2)},(\nu_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: (295)

By $(\nu_1)^{(2)}>0$, $(\nu_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0$, $(u_2)^{(2)}<0$ the roots (296)

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_1)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ (297)

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and (298)

Definition of $(\bar{\nu}_1)^{(2)},,(\bar{\nu}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: (299)

By $(\bar{\nu}_1)^{(2)}>0$, $(\bar{\nu}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0$, $(\bar{u}_2)^{(2)}<0$ the (300)

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_2)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ (301)

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t}\right] + T_{18}^0 e^{-(R_2)^{(2)}t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$$ 318

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations
$(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$
and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(v^{(3)}\big)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by
$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$$

$$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}(v^{(4)})^2 + (\sigma_1)^{(4)} v^{(4)} - (a_{24})^{(4)} = 0$

and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)},,(\bar{v}_2)^{(4)},(\bar{u}_1)^{(4)},(\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$ and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}$, $(\mu_2)^{(4)}$, $(v_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}$, $(\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, ***if*** $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)}t} - e^{-(a'_{26})^{(4)}t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)}t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^{0}}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t}-e^{-(R_2)^{(4)}t}\right]+T_{26}^{0}e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)},(S_2)^{(4)},(R_1)^{(4)},(R_2)^{(4)}$:- 337

Where $(S_1)^{(4)}=(a_{24})^{(4)}(m_2)^{(4)}-(a'_{24})^{(4)}$

$(S_2)^{(4)}=(a_{26})^{(4)}-(p_{26})^{(4)}$

$(R_1)^{(4)}=(b_{24})^{(4)}(\mu_2)^{(4)}-(b'_{24})^{(4)}$

$(R_2)^{(4)}=(b'_{26})^{(4)}-(r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)},(\sigma_2)^{(5)},(\tau_1)^{(5)},(\tau_2)^{(5)}$:

$(\sigma_1)^{(5)},(\sigma_2)^{(5)},(\tau_1)^{(5)},(\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)}\le-(a'_{28})^{(5)}+(a'_{29})^{(5)}-(a''_{28})^{(5)}(T_{29},t)+(a''_{29})^{(5)}(T_{29},t)\le-(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)}\le-(b'_{28})^{(5)}+(b'_{29})^{(5)}-(b''_{28})^{(5)}\big((G_{31}),t\big)-(b''_{29})^{(5)}\big((G_{31}),t\big)\le-(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)},(\nu_2)^{(5)},(u_1)^{(5)},(u_2)^{(5)},\nu^{(5)},u^{(5)}$: 339

By $(\nu_1)^{(5)}>0$, $(\nu_2)^{(5)}<0$ and respectively $(u_1)^{(5)}>0$, $(u_2)^{(5)}<0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2+(\sigma_1)^{(5)}\nu^{(5)}-(a_{28})^{(5)}=0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2+(\tau_1)^{(5)}u^{(5)}-(b_{28})^{(5)}=0$ and

Definition of $(\bar{\nu}_1)^{(5)},,(\bar{\nu}_2)^{(5)},(\bar{u}_1)^{(5)},(\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)}>0$, $(\bar{\nu}_2)^{(5)}<0$ and respectively $(\bar{u}_1)^{(5)}>0$, $(\bar{u}_2)^{(5)}<0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2+(\sigma_2)^{(5)}\nu^{(5)}-(a_{28})^{(5)}=0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2+(\tau_2)^{(5)}u^{(5)}-(b_{28})^{(5)}=0$

Definition of $(m_1)^{(5)},(m_2)^{(5)},(\mu_1)^{(5)},(\mu_2)^{(5)},(\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)},(m_2)^{(5)},(\mu_1)^{(5)},(\mu_2)^{(5)}$ by

$(m_2)^{(5)}=(\nu_0)^{(5)},(m_1)^{(5)}=(\nu_1)^{(5)}$, $\boldsymbol{if}\ (\nu_0)^{(5)}<(\nu_1)^{(5)}$

$(m_2)^{(5)}=(\nu_1)^{(5)},(m_1)^{(5)}=(\bar{\nu}_1)^{(5)}$, $\boldsymbol{if}\ (\nu_1)^{(5)}<(\nu_0)^{(5)}<(\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)}=\frac{G_{28}^{0}}{G_{29}^{0}}}$

$(m_2)^{(5)}=(\nu_1)^{(5)},(m_1)^{(5)}=(\nu_0)^{(5)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(5)}<(\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)}=(u_0)^{(5)},(\mu_1)^{(5)}=(u_1)^{(5)}$, $\boldsymbol{if}\ (u_0)^{(5)}<(u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, $\boldsymbol{if}$ $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}$ $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{28}(t) \leq G_{28}^0 e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{29}(t) \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \leq G_{30}(t) \leq \right. \quad 344$$

$$\left. \frac{(a_{30})^{(5)} G_{28}^0}{(m_2)^{(5)}((S_1)^{(5)} - (a'_{30})^{(5)})}\left[e^{(S_1)^{(5)} t} - e^{-(a'_{30})^{(5)} t}\right] + G_{30}^0 e^{-(a'_{30})^{(5)} t}\right)$$

$$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \leq T_{28}(t) \leq T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \leq T_{28}(t) \leq \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \leq T_{30}(t) \leq \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \leq -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \leq -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \leq -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \leq -(\tau_1)^{(6)}$$

Definition of $(v_1)^{(6)}, (v_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, v^{(6)}, u^{(6)}$: 350

By $(v_1)^{(6)} > 0$, $(v_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{v}_1)^{(6)},, (\bar{v}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)} > 0$, $(\bar{v}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}$, $(m_2)^{(6)}$, $(\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}$, $(m_2)^{(6)}$, $(\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t} - e^{-(a'_{34})^{(6)}t}\right] + G_{34}^0 e^{-(a'_{34})^{(6)}t}\right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(v_1)^{(7)}, (v_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, v^{(7)}, u^{(7)}$: 361

By $(v_1)^{(7)} > 0, (v_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{v}_1)^{(7)}, , (\bar{v}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)} > 0, (\bar{v}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)},\ \boldsymbol{if}\ (\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)},\ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}),t\big) - (b''_{45})^{(9)}\big((G_{47}),t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)},, (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, ***if*** $(\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, ***if*** $(\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, ***if*** $(\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,
and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44}e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44}e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G^0_{44}e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G^0_{44}e^{(S_1)^{(9)}t}$$

$$\Big(\frac{(a_{46})^{(9)}G^0_{44}}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})}\Big[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t}\Big] + G^0_{46}e^{-(S_2)^{(9)}t} \le G_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}G^0_{44}}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})}[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G^0_{46}e^{-(a'_{46})^{(9)}t}\Big)$$

$$\boxed{T_{44}^{0}e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^{0}e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^{0}e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^{0}e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^{0}}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t}-e^{-(b'_{46})^{(9)}t}\right]+T_{46}^{0}e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^{0}}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}-e^{-(R_2)^{(9)}t}\right]+T_{46}^{0}e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^{0}}{G_{14}^{0}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \le \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \le \nu^{(1)}(t) \le (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \le \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \le \nu^{(1)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \le (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \le (\bar{\nu}_1)^{(1)} \le \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \le \nu^{(1)}(t) \le \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \le (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \le \nu^{(1)}(t) \le (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \le u^{(1)}(t) \le (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{dv^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)v^{(2)} - (a_{17})^{(2)}v^{(2)}$$

Definition of $v^{(2)}$:- $\boxed{v^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(v^{(2)}\right)^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)}\right) \leq \frac{dv^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(v^{(2)}\right)^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{v}_1)^{(2)}, (v_0)^{(2)}$:-

For $0 < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (v_1)^{(2)} < (\bar{v}_1)^{(2)}$

$$v^{(2)}(t) \geq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(v_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (v_2)^{(2)}}}$$

it follows $(v_0)^{(2)} \leq v^{(2)}(t) \leq (v_1)^{(2)}$

In the same manner , we get 391

$$v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{v}_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (\bar{v}_2)^{(2)}}}$$

From which we deduce $(v_0)^{(2)} \leq v^{(2)}(t) \leq (\bar{v}_1)^{(2)}$ 392

If $0 < (v_1)^{(2)} < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{v}_1)^{(2)}$ we find like in the previous case, 393

$$(v_1)^{(2)} \leq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_2)^{(2)}\right)t\right]}} \leq v^{(2)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \leq (\bar{v}_1)^{(2)}$$

If $0 < (v_1)^{(2)} \leq (\bar{v}_1)^{(2)} \leq (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(v_1)^{(2)} \leq v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \leq (v_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(2)}(t)$:- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then
$(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

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If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\quad \boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$v^{(6)}(t) \le \frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{v}_1)^{(6)}-(v_0)^{(6)}}{(v_0)^{(6)}-(\bar{v}_2)^{(6)}}}$$

From which we deduce $(v_0)^{(6)} \le v^{(6)}(t) \le (\bar{v}_1)^{(6)}$

If $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{v}_1)^{(6)}$ we find like in the previous case, 414

$$(v_1)^{(6)} \le \frac{(v_1)^{(6)}+(C)^{(6)}(v_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)}-(v_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)}-(v_2)^{(6)}\right)t\right]}} \le v^{(6)}(t) \le$$

$$\frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \le (\bar{v}_1)^{(6)}$$

If $0 < (v_1)^{(6)} \le (\bar{v}_1)^{(6)} \le \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(v_1)^{(6)} \le v^{(6)}(t) \le \frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \le (v_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(6)}(t)$:-

$$(m_2)^{(6)} \le v^{(6)}(t) \le (m_1)^{(6)}, \quad \boxed{v^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \le u^{(6)}(t) \le (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(v_1)^{(6)} = (\bar{v}_1)^{(6)}$ if in addition $(v_0)^{(6)} = (v_1)^{(6)}$ then $v^{(6)}(t) = (v_0)^{(6)}$ and as a consequence $G_{32}(t) = (v_0)^{(6)}G_{33}(t)$ **this also defines** $(v_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(v_1)^{(6)}$ and $(\bar{v}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}\right)\leq\frac{dv^{(7)}}{dt}\leq-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)},(\nu_0)^{(7)}$:-

$$\text{For } 0<\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}<(\nu_1)^{(7)}<(\bar{\nu}_1)^{(7)}$$

$$v^{(7)}(t)\geq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}\quad,\quad\boxed{(C)^{(7)}=\frac{(\nu_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)}\leq v^{(7)}(t)\leq(\nu_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(7)}=\frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)}\leq v^{(7)}(t)\leq(\bar{\nu}_1)^{(7)}$

If $0<(\nu_1)^{(7)}<(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}<(\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)}\leq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}\leq v^{(7)}(t)\leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\bar{\nu}_1)^{(7)}$$

If $0<(\nu_1)^{(7)}\leq(\bar{\nu}_1)^{(7)}\leq\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)}\leq v^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(7)}(t)$:-

$$(m_2)^{(7)}\leq v^{(7)}(t)\leq(m_1)^{(7)},\quad\boxed{v^{(7)}(t)=\frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)}+(C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)}-(\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)}+(\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)}-(\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{40}'')^{(8)} = (a_{41}'')^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)}G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b_{40}'')^{(8)} = (b_{41}'')^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$ if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a_{44}')^{(9)} - (a_{45}')^{(9)} + (a_{44}'')^{(9)}(T_{45}, t)\right) - (a_{45}'')^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\quad \boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \leq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \leq \nu^{(9)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \leq (\bar{\nu}_1)^{(9)} \leq \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \leq \nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \leq \nu^{(9)}(t) \leq (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for**

the special case .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(v_1)^{(9)}$ and $(\bar{v}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a_i'')^{(1)} and\ (b_i'')^{(1)}$ are independent on t , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0$ 447

$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0$ 448

has a unique positive solution , which is an equilibrium solution

$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0$ 449

$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0$ 450

$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0$ 451

$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0$ 452

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(x) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T_{33}^* :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

Definition and uniqueness of T_{37}^* :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^* for which $f(T_{37}^*) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

Definition and uniqueness of T_{41}^* :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that

there exists a unique G_{37}^{*} such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)}G_{17}^{*}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)}G_{17}^{*}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T_{17}^{*})]} \quad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)}T_{17}^{*}}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)}T_{17}^{*}}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^{*})]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)}G_{21}^{*}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)}G_{21}^{*}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$$T_{28}^* = \frac{(b_{28})^{(5)} T_{29}^*}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^*)]} \quad , \quad T_{30}^* = \frac{(b_{30})^{(5)} T_{29}^*}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^*)]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^* given by $\varphi((G_{35})^*) = 0$, T_{33}^* given by $f(T_{33}^*) = 0$ and

$$G_{32}^* = \frac{(a_{32})^{(6)} G_{33}^*}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34}^* = \frac{(a_{34})^{(6)} G_{33}^*}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

$$T_{32}^* = \frac{(b_{32})^{(6)} T_{33}^*}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^*)]} \quad , \quad T_{34}^* = \frac{(b_{34})^{(6)} T_{33}^*}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^*)]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^* given by $\varphi((G_{39})^*) = 0$, T_{37}^* given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

G_{41}^* given by $\varphi((G_{43})^*) = 0$, T_{41}^* given by $f(T_{41}^*) = 0$ and

$$G_{40}^{*}=\frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}\quad,\quad G_{42}^{*}=\frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

$$T_{40}^{*}=\frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}\quad,\quad T_{42}^{*}=\frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*})=0$, T_{45}^{*} given by $f(T_{45}^{*})=0$ and

$$G_{44}^{*}=\frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}\quad,\quad G_{46}^{*}=\frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$$

$$T_{44}^{*}=\frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}\quad,\quad T_{46}^{*}=\frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}\ and\ (b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i=G_i^{*}+\mathbb{G}_i\qquad,T_i=T_i^{*}+\mathbb{T}_i$$

$$\frac{\partial(a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^{*})=(q_{14})^{(1)}\quad,\ \frac{\partial(b_i'')^{(1)}}{\partial G_j}(\,G^{*}\,)=s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt}=-\big((a'_{13})^{(1)}+(p_{13})^{(1)}\big)\mathbb{G}_{13}+(a_{13})^{(1)}\mathbb{G}_{14}-(q_{13})^{(1)}G_{13}^{*}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt}=-\big((a'_{14})^{(1)}+(p_{14})^{(1)}\big)\mathbb{G}_{14}+(a_{14})^{(1)}\mathbb{G}_{13}-(q_{14})^{(1)}G_{14}^{*}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt}=-\big((a'_{15})^{(1)}+(p_{15})^{(1)}\big)\mathbb{G}_{15}+(a_{15})^{(1)}\mathbb{G}_{14}-(q_{15})^{(1)}G_{15}^{*}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt}=-\big((b'_{13})^{(1)}-(r_{13})^{(1)}\big)\mathbb{T}_{13}+(b_{13})^{(1)}\mathbb{T}_{14}+\textstyle\sum_{j=13}^{15}\big(s_{(13)(j)}T_{13}^{*}\mathbb{G}_j\big) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt}=-\big((b'_{14})^{(1)}-(r_{14})^{(1)}\big)\mathbb{T}_{14}+(b_{14})^{(1)}\mathbb{T}_{13}+\textstyle\sum_{j=13}^{15}\big(s_{(14)(j)}T_{14}^{*}\mathbb{G}_j\big) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt}=-\big((b'_{15})^{(1)}-(r_{15})^{(1)}\big)\mathbb{T}_{15}+(b_{15})^{(1)}\mathbb{T}_{14}+\textstyle\sum_{j=13}^{15}\big(s_{(15)(j)}T_{15}^{*}\mathbb{G}_j\big) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions

$(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \tag{532}$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \ , \ \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \tag{533}$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \tag{534}$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \tag{535}$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \tag{536}$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \tag{537}$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}')^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T_{17}^*\mathbb{G}_j\right) \tag{538}$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}')^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T_{18}^*\mathbb{G}_j\right) \tag{539}$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \ , \ \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}')^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \tag{541}$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}')^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \tag{542}$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}')^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \tag{543}$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}')^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T_{20}^*\mathbb{G}_j\right) \tag{544}$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^{*}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T_{22}^{*}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^{*}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^{*}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^{*}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^{*}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^{*}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^{*}\mathbb{G}_j\right) \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}(\,(G_{31})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^{*}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^{*}) = (q_{41})^{(8)} \quad , \frac{\partial(b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^{*}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^{*}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^{*}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^{*}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \ , \ \frac{\partial(b_i'')^{(9)}}{\partial G_j}(\ (G_{47})^*\ ) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\ B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\ C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\ D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\ E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\ F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\ G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14}+(b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION THIRTY TWO

Radiative Corrections To Higgs Boson Masses

INTRODUCTION—VARIABLES USED

Radiative corrections to Higgs boson masses in the next-to-minimal supersymmetric standard model Phys. Rev. D 49, 2435 – Published 1 March 1994 T. Elliott, S. F. King, and P. L. White

(1) Characteristic signatures of (e) the NMSSM such as light charged bosons and (e&eb) weakly coupled neutral scalars are discussed, as are the relative sizes of the various radiative corrections.
(2) The MSSM is also discussed as (=) a limiting case of the NMSSM for comparison. DOI: http://dx.doi.org/10.1103/PhysRevD.49.2435

International Journal of Modern Physics A Particles and Fields; Gravitation; Cosmology Volume 04, Issue 14, August 20, 1989 MANUEL DREES, Int. J. Mod. Phys. A, 04, 3635 (1989). DOI: 10.1142/S0217751X89001448 SUPERSYMMETRIC MODELS WITH EXTENDED HIGGS SECTOR

(3) The Higgs sector of supersymmetric models with (e&eb) an additional SU (2) ×U (1) Y singlet or an additional pair of Higgs doublet superfields is investigated.
(4) It is shown that in the model with a Higgs singlet bounds on (eb) Higgs boson masses are considerably weaker than in (eb) the minimal supersymmetric model;
(5) In particular, all neutral scalars can be heavier (e) than about 150 GeV or lighter than, whereas the mass of the charged Higgs boson is (=) essentially unconstrained.
(6) The four doublet model resembles the minimal model more closely; in particular, the bounds for (e) the heaviest and lightest neutral scalar and (e&eb) the heaviest charged Higgs bosons are unaltered.
(7) In this model, many relations between masses of different Higgs bosons can be found; 44 of the 66 possible decays of one Higgs boson into (e&eb) another Higgs boson plus a real W or Z boson are kinematically forbidden.
(8) It is also shown that in both models, the lightest neutral scalar becomes (=) indistinguishable from (e) the standard model Higgs boson if (e) all other Higgs bosons of the model are much heavier than Mw.

NOTATION

Module One

renormalization group flows of Yukawa couplings and the upper bound on (eb) the lightest CP-even neutral Higgs boson mass as (=) a function of the heavier top squark mass and top quark mass

G_{13} : Category one of renormalization group flows of Yukawa couplings and the upper bound

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

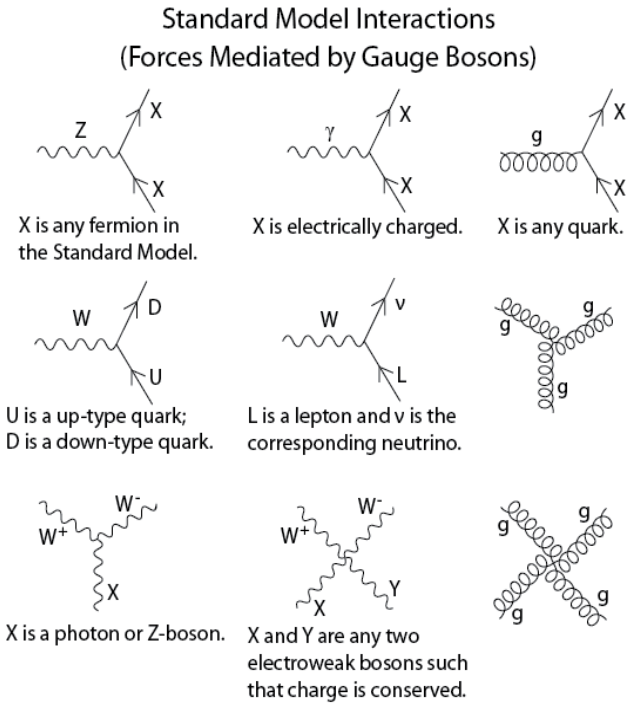
T_{13} : Category one of lightest CP-even neutral Higgs boson mass as (=) a function of the heavier top squark mass and top quark mass

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

lightest CP-even neutral Higgs boson mass as (=) a function of the heavier top squark mass and top quark mass



$$\beta_{y_t} = k\, y_t \left[-\frac{9}{4}g^2 - \frac{17}{12}g'^2 - 8g_s^2 + \frac{9}{2}s(t)y_t^2 \right]$$
$$+ k^2 y_t \left[-\frac{23}{4}g^4 - \frac{3}{4}g^2 g'^2 + \frac{1187}{216}g'^4 + 9g^2 g_s^2 \right.$$
$$+ \frac{19}{9}g'^2 g_s^2 - 108 g_s^4 + \left(\frac{225}{16}g^2 + \frac{131}{16}g'^2 + 36 g_s^2\right) s(t) y_t^2$$
$$\left. + 6\left(-2s^2(t)y_t^4 - 2s^3(t)y_t^2\lambda + s^2(t)\lambda^2\right) \right]. \qquad \text{(A3)}$$

$$\partial_t h^2 = (d - 4 + 2\eta_\psi + \eta_\sigma)h^2 + \frac{1}{N_f} 8 v_d h^4 l_{1,1}^{(FB)d}(2h^2\kappa, [u'(\kappa) + 2\kappa u''(\kappa)]; \eta_\psi, \eta_\sigma)$$
$$- \frac{1}{N_f}(48\kappa u''(\kappa) + 32\kappa^2 u'''(\kappa)) v_d h^4 l_{1,2}^{(FB)d}(2h^2\kappa, [u'(\kappa) + 2\kappa u''(\kappa)]; \eta_\psi, \eta_\sigma)$$
$$- \frac{1}{N_f} 32 v_d h^6 \kappa l_{2,1}^{(FB)d}(2h^2\kappa, [u'(\kappa) + 2\kappa u''(\kappa)]; \eta_\psi, \eta_\sigma), \qquad (23)$$

$$\mathcal{C}_L \cong \left[-g_2\mu_2^* B_3 m_2 M_W \frac{M_2 \sin\beta + \mu_0 \cos\beta}{(\mu_0 M_2 - M_W^2 \sin 2\beta)^2 M_s^2} - \frac{y_{e_2}}{\sqrt{2}} \frac{\mu_3 B_2^* M_2}{(\mu_0 M_2 - M_W^2 \sin 2\beta) M_s^2} \right] (\tan\beta \sin\alpha - \cos\alpha),$$

$$\mathcal{C}_R \cong \left[-g_2\mu_3 B_2^* m_3 M_W \frac{M_2^* \sin\beta + \mu_0^* \cos\beta}{(\mu_0 M_2 - M_W^2 \sin 2\beta)^2 M_s^2} - \frac{y_{e_3}}{\sqrt{2}} \frac{\mu_2^* B_3 M_2^*}{(\mu_0 M_2 - M_W^2 \sin 2\beta) M_s^2} \right] (\tan\beta \sin\alpha - \cos\alpha). \quad (5)$$

$$(-i\kappa_0)^3 \int \frac{d^4q}{(2\pi)^4} \frac{i^2}{(q^2 - M_0^2)(-q + \lambda m_0)^2} = \frac{\kappa_0^3}{16\pi^2} \int d(q_E^2) \frac{q_E^2 [q_E^2 - (\lambda m_0)^2]}{(q_E^2 + M_0^2)[q_E^2 + (\lambda m_0)^2]^2}$$

$$= \frac{\kappa_0^3}{16\pi^2} \left\{ M_0^2 \frac{(\lambda m_0)^2 + M_0^2}{[(\lambda m_0)^2 - M_0^2]^2} \ln\left(1 + \frac{\Lambda^2}{M_0^2}\right) + (\lambda m_0)^2 \frac{(\lambda m_0)^2 - 3M_0^2}{[(\lambda m_0)^2 - M_0^2]^2} \right.$$

$$\left. \times \ln\left(1 + \frac{\Lambda^2}{(\lambda m_0)^2}\right) - \frac{2(\lambda m_0)^2 \Lambda^2}{[(\lambda m_0)^2 - M_0^2][\Lambda^2 + (\lambda m_0)^2]} \right\}, \quad (56)$$

$$S_{\text{vis}} = \int d^4x \sqrt{-g_{\text{vis}}} \hat{Y}_6 \bar{L}_{0L}(x) H(x, \pi, 0) \psi_R(x, \pi, 0) + \text{h.c.},$$

$$S_{\text{vis}'} = \int d^4x \sqrt{-g_{\text{vis}'}} \hat{Y}_6' \bar{L}_{0L}(x) H(x, \pi, \pi) \psi_R(x, \pi, \pi) + \text{h.c.}. \quad (6)$$

$$\frac{\partial n_{\mathcal{L}_i}}{\partial t} = \epsilon^{\mathrm{W}i}_{N_1}(t)\langle\Gamma_{N_1}\rangle\left(n_{N_1} - n^{\mathrm{eq}}_{N_1}\right),$$

$$\begin{aligned}\epsilon^{\mathrm{W}i}_{N_1}(t) = &-\frac{4}{\langle\Gamma_{N_1}\rangle}\sum_{j=1}^{3}\mathrm{Im}\left(\lambda_{1i}\lambda_{1j}\lambda^{\dagger}_{j2}\lambda^{\dagger}_{i2}\right)\\ &\times\int_0^t \mathrm{d}t_z\int_0^{t_z}\mathrm{d}t_2\int_0^{t_2}\mathrm{d}t_1 \mathrm{e}^{-\Gamma_{N_2}(t_z-t_2)}\mathrm{e}^{-\left(\Gamma_{\ell_j}+\Gamma_H\right)(t_2-t_1)}\\ &\times\int\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\frac{f^{\mathrm{eq}}_{N_1}}{n^{\mathrm{eq}}_{N_1}}\int\frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\frac{1-f^{\mathrm{eq}}_{\ell_j}(\mathbf{p})+f^{\mathrm{eq}}_{H}(\mathbf{k}-\mathbf{p})}{2\omega_{\ell_j}(\mathbf{p})2\omega_H(\mathbf{k}-\mathbf{p})\omega_{N_1}(\mathbf{k})}\\ &\times\int\frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3}\frac{1-f^{\mathrm{eq}}_{\ell_i}(\mathbf{q})+f^{\mathrm{eq}}_{H}(\mathbf{k}-\mathbf{q})}{2\bar{\omega}_{\ell_i}(\mathbf{q})2\bar{\omega}_H(\mathbf{k}-\mathbf{q})\omega_{N_2}(\mathbf{k})}\\ &\times\sin\left(\omega_{N_1}(t-t_1)+\left(\omega_{\ell_j}+\omega_H\right)(t_1-t_2)+\omega_{N_2}(t_2-t_z)\right.\\ &\left.+\left(\bar{\omega}_{\ell_i}+\bar{\omega}_H\right)(t_z-t)\right)\mathrm{Tr}\left(M_1P_{\mathrm{L}}pM_2\not{q}\right),\end{aligned} \tag{33}$$

$$\begin{aligned}S_{scalar} = &-\int \mathrm{d}^4x\left(h^{\dagger}_a h^{\dagger}_b\right)\begin{pmatrix}0 & m^2_c\\ m^2_c & 0\end{pmatrix}\begin{pmatrix}h_a\\ h_b\end{pmatrix}\\ &-\int \mathrm{d}^4x\left(\frac{\lambda_a}{2}(h^{\dagger}_a h_a)^2+\frac{\lambda_b}{2}(h^{\dagger}_b h_b)^2\right.\\ &+\lambda_c(h^{\dagger}_a h_b)(h^{\dagger}_a h_b)+\frac{\lambda_d}{2}(h^{\dagger}_a h_a)(h^{\dagger}_b h_b)\\ &+\frac{\lambda_e}{2}(h^{\dagger}_a h_b)(h^{\dagger}_b h_a)+\lambda_f(h^{\dagger}_a h_a)(h^{\dagger}_a h_b)\\ &\left.+\lambda_g(h^{\dagger}_b h_b)(h^{\dagger}_b h_a)+\mathrm{H.c.}\right),\end{aligned} \tag{13}$$

Quantum Boltzmann equations and leptogenesis Andrea De Simone1 and Antonio Riotto2,3 JCAP08(2007)002. doi:10.1088/1475-7516/2007/08/002 IOP Publishing Ltd

The closed time-path formalism is a powerful Green's function formulation to describe non-equilibrium phenomena in field theory and it leads to a complete non-equilibrium quantum kinetic theory. We make use of this formalism to write down the set of quantum Boltzmann equations relevant for leptogenesis. They manifest memory effects and off-shell corrections. In particular, memory effects lead to a time-dependent CP asymmetry whose value at a given instant of time depends upon the previous history of the system. This result is particularly relevant when the asymmetry is generated by the decays of nearly mass-degenerate heavy states, as in resonant or soft leptogenesis.

The flow equation for the effective action

In quantum field theory, the effective action Γ is an analogue of the classical action functional S and depends on the fields of a given theory. It includes all quantum and thermal fluctuations. Variation of Γ yields exact quantum field equations, for example for cosmology or the electrodynamics of superconductors. Mathematically, Γ is the generating functional of the one-particle irreducible Feynman diagrams. Interesting physics, as propagators and effective couplings for interactions, can be straightforwardly extracted from it. In a generic interacting field theory the effective action Γ, however, is difficult to obtain. FRG provides a practical tool to calculate Γ employing the renormalization

group concept.

The central object in FRG is a scale-dependent effective action functional Γ_k often called average action or flowing action. The dependence on the RG sliding scale k is introduced by adding a regulator (infrared cutoff) R_k to the full inverse propagator $\Gamma_k^{(2)}$. Roughly speaking, the regulator R_k decouples slow modes with momenta $q \lesssim k$ by giving them a large mass, while high momentum modes are not affected. Thus, Γ_k includes all quantum and statistical fluctuations with momenta $q \gtrsim k$. The flowing action Γ_k obeys the exact functional flow equation

$$\partial_k \Gamma_k = \frac{1}{2} \mathrm{STr}\, \partial_k R_k \left(\Gamma_k^{(2)} + R_k\right)^{-1},$$

derived by Christof Wetterich in 1993 and Tim R. Morris in 1994 Here ∂_k denotes a derivative with respect to the RG scale k at fixed values of the fields. The functional differential equation for Γ_k must be supplemented with the initial condition $\Gamma_{k\to\Lambda} = S$, where the "classical action" S describes the physics at the microscopic ultraviolet scale $k = \Lambda$. Importantly, in the infrared limit $k \to 0$ the full effective action $\Gamma = \Gamma_{k\to 0}$ is obtained. In the Wetterich equation STr denotes a super trace which sums over momenta, frequencies, internal indices, and fields (taking bosons with a plus and fermions with a minus sign). The exact flow equation for Γ_k has a one-loop structure. This is an important simplification compared to perturbation theory, where multi-loop diagrams must be included. The second functional derivative $\Gamma_k^{(2)}$ is the full inverse field propagator modified by the presence of the regulator R_k.

The renormalization group evolution of Γ_k can be illustrated in the theory space, which is a multi-dimensional space of all possible running couplings $\{c_n\}$ allowed by the symmetries of the problem. As schematically shown in the figure, at the microscopic ultraviolet scale $k = \Lambda$ one starts with the initial condition $\Gamma_{k=\Lambda} = S$.

Functional renormalization-group for Wick-ordered effective interaction

Contrary to the flow equation for the effective action, this scheme is formulated for the effective interaction

$$\mathcal{V}[\eta, \eta^+] = -\ln Z[G_0^{-1}\eta, G_0^{-1}\eta^+] - \eta G_0^{-1}\eta^+$$

which generates n-particle interaction vertices, amputated by the bare propagators G_0; $Z[\eta, \eta^+]$ is the "standard" generating functional for the n-particle Green functions.

The Wick ordering of effective interaction with respect to Green function D can be defined by

$$\mathcal{W}[\eta, \eta^+] = \exp(-\Delta_D)\mathcal{V}[\eta, \eta^+].$$

where $\Delta = D\delta^2/(\delta\eta\delta\eta^+)$ is the Laplacian in the field space. This operation is similar to Normal order and excludes from the interaction all possible terms, formed by a convolution of source fields with respective Green function D. Introducing some cutoff Λ the Polchinskii equation

$$\frac{\partial}{\partial\Lambda}V_\Lambda(\psi) = -\dot{\Delta}_{G_{0,\Lambda}}V_\Lambda(\psi) + \Delta^{12}_{\dot{G}_{0,\Lambda}}\mathcal{V}^{(1)}_\Lambda\mathcal{V}^{(2)}_\Lambda$$

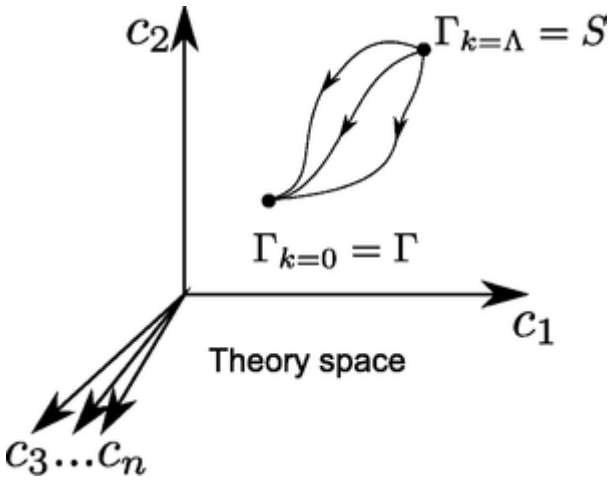
takes the form of the Wick-ordered equation

$$\partial_\Lambda\mathcal{W}_\Lambda = -\Delta_{\dot{D}_\Lambda+\dot{G}_{0,\Lambda}}\mathcal{W}_\Lambda + e^{-\Delta^{12}_{D_\Lambda}}\Delta^{12}_{\dot{G}_{0,\Lambda}}\mathcal{W}^{(1)}_\Lambda\mathcal{W}^{(2)}_\Lambda$$

where

$$\Delta^{12}_{\dot{G}_{0,\Lambda}}\mathcal{V}^{(1)}_\Lambda\mathcal{V}^{(2)}_\Lambda = \frac{1}{2}\left(\frac{\delta V_\Lambda(\psi)}{\delta\psi}, \dot{G}_{0,\Lambda}\frac{\delta V_\Lambda(\psi)}{\delta\psi}\right)$$

Applications



As the sliding scale k is lowered, the flowing action Γ_k evolves in the theory space according to the functional flow equation. The choice of the regulator R_k is not unique, which introduces some scheme dependence into the renormalization group flow. For this reason, different choices of the regulator R_k correspond to the different paths in the figure. At the infrared scale $k = 0$, however, the full effective action $\Gamma_{k=0} = \Gamma$ is recovered for every choice of the cut-off R_k, and all trajectories meet at the same point in the theory space.

G_{16} : Category one of lightest CP-even neutral Higgs boson mass; LHS of above equations

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of function of the heavier top squark mass and top quark mass ; RHS of above equations

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

The Higgs sector of supersymmetric models with (e&eb) an additional SU (2) ×U (1) Y singlet or an additional pair of Higgs doublet superfields is investigated

G_{20} : Category one of Higgs sector of supersymmetric models; additional SU (2) ×U (1) Y singlet or an additional pair of Higgs doublet superfields

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of additional SU (2) ×U (1) Y singlet or an additional pair of Higgs doublet superfields; Higgs sector of supersymmetric models

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

It is shown that in the model with a Higgs singlet bounds on (eb) Higgs boson masses are considerably weaker than in (eb) the minimal supersymmetric model

G_{24} : Category one of model with a Higgs singlet bounds

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of Higgs boson masses are considerably weaker than in (eb) the minimal supersymmetric model

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

model with a Higgs singlet bounds on Higgs boson masses are considerably weaker than in (eb) the minimal supersymmetric model

G_{28} : Category one of model with a Higgs singlet bounds on Higgs boson masses are considerably weaker

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of minimal supersymmetric model

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

In particular, all neutral scalars can be heavier (e) than about 150 GeV or lighter than, whereas the

mass of the charged Higgs boson is (=) essentially unconstrained

G_{32} : Category one of mass of the charged Higgs boson

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of unconstrained

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

The four doublet model resembles the minimal model more closely; in particular, the

bounds for the heaviest and lightest neutral scalar and (e&eb) the heaviest charged Higgs bosons are unaltered

G_{36} : Category one of bounds for the heaviest and lightest neutral scalar; heaviest charged Higgs bosons are unaltered

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of heaviest charged Higgs bosons are unaltered; bounds for the heaviest and lightest neutral scalar

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

It is also shown that in both models, the

lightest neutral scalar becomes indistinguishable from (e) the standard model Higgs boson if (e) all other Higgs bosons of the model are much heavier than Mw.

G_{40} : Category one of standard model Higgs boson if (e) all other Higgs bosons of the model are much heavier than Mw.

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of lightest neutral scalar becomes indistinguishable

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

lightest neutral scalar becomes indistinguishable from the standard model Higgs boson if (e) all other Higgs bosons of the model are much heavier than Mw

G_{44} : Category one of all other Higgs bosons of the model are much heavier than Mw

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of lightest neutral scalar becomes indistinguishable from the standard model Higgs boson

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \qquad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**
$-(b''_{24})^{(4)}((G_{27}),t) =$ **First detritions factor**
Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$
$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$
$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$
$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28} \quad 28$$
$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$
$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**
$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**
Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**
Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**
Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}),t\right)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}),t\right)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}),t\right)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}),t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}),t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) & +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) & +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) & +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t) \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & +(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) & +(a''_{21})^{(3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t) \end{bmatrix} G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad 63$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition

coefficients for category 1,2 and 3

$-(b_{28}'')^{(5,5,5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b_{32}'')^{(6,6,6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b_{36}'')^{(7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b_{40}'')^{(8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b_{44}'')^{(9,9)}(G_{47},t)$, $-(b_{46}'')^{(9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c}(a_{20}')^{(3)}\,+(a_{20}'')^{(3)}(T_{21},t)\;+(a_{16}'')^{(2,2,2)}(T_{17},t)\;+(a_{13}'')^{(1,1,1,)}(T_{14},t)\\ +(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)\;+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)\;+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a_{36}'')^{(7,7,7,7)}(T_{37},t)\;+(a_{40}'')^{(8,8,8,8)}(T_{41},t)\;+(a_{44}'')^{(9,9,9)}(T_{45},t)\end{array}\right]G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c}(a_{21}')^{(3)}\,+(a_{21}'')^{(3)}(T_{21},t)\;+(a_{17}'')^{(2,2,2)}(T_{17},t)\;+(a_{14}'')^{(1,1,1,)}(T_{14},t)\\ +(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)\;+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)\;+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a_{37}'')^{(7,7,7,7)}(T_{37},t)\;+(a_{41}'')^{(8,8,8,8)}(T_{41},t)\;+(a_{45}'')^{(9,9,9)}(T_{45},t)\end{array}\right]G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c}(a_{22}')^{(3)}\,+(a_{22}'')^{(3)}(T_{21},t)\;+(a_{18}'')^{(2,2,2)}(T_{17},t)\;+(a_{15}'')^{(1,1,1,)}(T_{14},t)\\ +(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)\;+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)\;+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a_{38}'')^{(7,7,7,7)}(T_{37},t)\;+(a_{42}'')^{(8,8,8,8)}(T_{41},t)\;+(a_{46}'')^{(9,9,9)}(T_{45},t)\end{array}\right]G_{22} \quad 69$$

$+(a_{20}'')^{(3)}(T_{21},t)$, $+(a_{21}'')^{(3)}(T_{21},t)$, $+(a_{22}'')^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a_{40}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a_{44}'')^{(9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for

category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$ $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{25} \quad (74)$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}, \boxed{(a''_{25})^{(4)}(T_{25},t)}, \boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
are sixth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
are seventh augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
are eighth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad 78$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$

are fourth detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b_{16}'')^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b_{17}'')^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b_{18}'')^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b_{20}'')^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b_{21}'')^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b_{22}'')^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b_{36}'')^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b_{37}'')^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b_{38}'')^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b_{40}'')^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b_{41}'')^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b_{42}'')^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category 1, 2 *and* 3
$\boxed{-(b_{46}'')^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b_{45}'')^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b_{44}'')^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{ccc} (a_{28}')^{(5)} & \boxed{+(a_{28}'')^{(5)}(T_{29},t)} & \boxed{+(a_{24}'')^{(4,4,)}(T_{25},t)} \quad \boxed{+(a_{32}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{36}'')^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{ccc} (a_{29}')^{(5)} & \boxed{+(a_{29}'')^{(5)}(T_{29},t)} & \boxed{+(a_{25}'')^{(4,4,)}(T_{25},t)} \quad \boxed{+(a_{33}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{ccc} (a_{30}')^{(5)} & \boxed{+(a_{30}'')^{(5)}(T_{29},t)} & \boxed{+(a_{26}'')^{(4,4,)}(T_{25},t)} \quad \boxed{+(a_{34}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{30} \quad (81)$$

Where $\boxed{+(a_{28}'')^{(5)}(T_{29},t)}, \boxed{+(a_{29}'')^{(5)}(T_{29},t)}, \boxed{+(a_{30}'')^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3
And $\boxed{+(a_{24}'')^{(4,4,)}(T_{25},t)}, \boxed{+(a_{25}'')^{(4,4,)}(T_{25},t)}, \boxed{+(a_{26}'')^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3
$\boxed{+(a_{32}'')^{(6,6,6)}(T_{33},t)}, \boxed{+(a_{33}'')^{(6,6,6)}(T_{33},t)}, \boxed{+(a_{34}'')^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3
$\boxed{+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3
$\boxed{+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3
$\boxed{+(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)}, \boxed{+(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)}, \boxed{+(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3
$\boxed{+(a_{36}'')^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3
$\boxed{+(a_{40}'')^{(8,8\,,8,8,8,8)}(T_{41},t)}, \boxed{+(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3
$\boxed{+(a_{46}'')^{(9,9,9,9,9)}(T_{45},t)}, \boxed{+(a_{45}'')^{(9,9,9,9,9)}(T_{45},t)}, \boxed{+(a_{44}'')^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation

coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)} & +(a''_{32})^{(6)}(T_{33},t) & +(a''_{28})^{(5,5,5)}(T_{29},t) & +(a''_{24})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)} & +(a''_{33})^{(6)}(T_{33},t) & +(a''_{29})^{(5,5,5)}(T_{29},t) & +(a''_{25})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{33} \quad (86)$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[\begin{array}{ccc} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{array}\right] T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* $1,2$ *and* 3

$-(b_{13}'')^{(1,1,1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1,1,1)}(G,t)$, $-(b_{15}'')^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b_{16}'')^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b_{20}'')^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3

$-(b_{36}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3

$-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2, and 3

$-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a_{36}')^{(7)} & +(a_{36}'')^{(7)}(T_{37},t) & +(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t) & +(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a_{37}')^{(7)} & +(a_{37}'')^{(7)}(T_{37},t) & +(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a_{38}')^{(7)} & +(a_{38}'')^{(7)}(T_{37},t) & +(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t) & +(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t) \\ & +(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t) & +(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t) & +(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t) \\ & +(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t) & +(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{15}$$

Where $(a_{36}'')^{(7)}(T_{37},t)$, $(a_{37}'')^{(7)}(T_{37},t)$, $(a_{38}'')^{(7)}(T_{37},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\,-(b''_{36})^{(7)}(G_{39},t) & -(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1)}(G,t) & -(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\,-(b''_{37})^{(7)}(G_{39},t) & -(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1)}(G,t) & -(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\,-(b''_{38})^{(7)}(G_{39},t) & -(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1)}(G,t) & -(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$\frac{dT_{40}}{dt} =$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt}$$

$$= (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt}$$
$$= (a_{45})^{(9)} G_{44}$$
$$- \begin{bmatrix} (a'_{45})^{(9)} \boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt}$$
$$= (a_{46})^{(9)} G_{45}$$
$$- \begin{bmatrix} (a'_{46})^{(9)} \boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$
$$(b_{44})^{(9)} T_{45} -$$
$$\begin{bmatrix} (b'_{44})^{(9)} \boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$
$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G,t) \le (r_i)^{(1)} \le (b'_i)^{(1)} \le (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a''_i)^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b''_i)^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}', t) - (a_i'')^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}', t)$ and $(a_i'')^{(1)}(T_{14}, t)$. (T_{14}', t) and (T_{14}, t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}\big((G_{19}), t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, \ (\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq \ (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 113$$

$\lim_{G\to\infty}(b_i'')^{(3)}\,(G_{23},t) = \ (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$

$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)},$ are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, \ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$

$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$

$(b_i'')^{(4)}\big((G_{27}),t\big) \le \ (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$

$lim_{T_2\to\infty}(a_i'')^{(4)}\,(T_{25},t) = (p_i)^{(4)}$ 118

$\lim_{G\to\infty}(b_i'')^{(4)}\,\big((G_{27}),t\big) = \ (r_i)^{(4)}$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}\,(T_{29}, t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G \to \infty}(b_i'')^{(5)}\,(G_{31}, t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\,(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ \ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}\,(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}\,\big((G_{35}), t\big) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\big((G_{35}), t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and \ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\,(a_i)^{(6)} + (a_i')^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)}\,(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(OOOOOO) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(PPPPPP) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \leq \ (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$$

132

(QQQQQQ) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(RRRRRR)

$\lim_{G\to\infty}(b_i'')^{(7)}\left((G_{39}),t\right) = \ (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, \ (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}\left((G_{39}),t\right)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $\left[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\right]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(SSSSSS) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(TTTTTT) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, \ (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\,(a_i)^{(7)} + (a_i')^{(7)} + \ (\hat{A}_{36})^{(7)} + \ (\hat{P}_{36})^{(7)}\,(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[\ (b_i)^{(7)} + (b_i')^{(7)} + \ (\hat{B}_{36})^{(7)} + \ (\hat{Q}_{36})^{(7)}\ \ (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}, \ (r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41},t) \le (p_i)^{(8)} \le (\hat{A}_{40})^{(8)} \quad 138$$

$$(b_i'')^{(8)}((G_{43}),t) \le (r_i)^{(8)} \le (b_i')^{(8)} \le (\hat{B}_{40})^{(8)} \quad 139$$

$$\lim_{T_2\to\infty}(a_i'')^{(8)}(T_{41},t) = (p_i)^{(8)} \quad 140$$

$$\lim_{G\to\infty}(b_i'')^{(8)}((G_{43}),t) = (r_i)^{(8)} \quad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}',t) - (a_i'')^{(8)}(T_{41},t)| \le (\hat{k}_{40})^{(8)}|T_{41} - T_{41}'|e^{-(\hat{M}_{40})^{(8)}t} \quad 142$$

$$|(b_i'')^{(8)}((G_{43})',t) - (b_i'')^{(8)}((G_{43}),t)| < (\hat{k}_{40})^{(8)}||(G_{43}) - (G_{43})'||e^{-(\hat{M}_{40})^{(8)}t} \quad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}',t)$ and $(a_i'')^{(8)}(T_{41},t)$. (T_{41}',t) and (T_{41},t) are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \quad 144$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad 145$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)}(\hat{k}_{40})^{(8)}] < 1 \quad 146$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \quad 146A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t) \le (p_i)^{(9)} \le (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t) \le (r_i)^{(9)} \le (b_i')^{(9)} \le (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty} (a_i'')^{(9)}(T_{45},t) = (p_i)^{(9)}$$

$$\lim_{G\to\infty} (b_i'')^{(9)}(G_{47},t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t) - (a_i'')^{(9)}(T_{45},t)| \le (\hat{k}_{44})^{(9)}|T_{45} - T_{45}'|e^{-(\hat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t) - (b_i'')^{(9)}\big((G_{47}),t\big)| < (\hat{k}_{44})^{(9)}||(G_{47}) - (G_{47})'||e^{-(\hat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$ and$(a_i'')^{(9)}(T_{45},t)$. (T_{45}',t) and (T_{45},t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}$, $(r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(b_i)^{(9)} + (b_i')^{(9)} + (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}(\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\,, T_i(0)$

$$G_i(t) \le \left(\hat{P}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t} \;,\quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le \left(\hat{Q}_{16}\right)^{(2)} e^{(\hat{M}_{16})^{(2)}t} \;,\quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \le \left(\hat{P}_{20}\right)^{(3)} e^{(\hat{M}_{20})^{(3)}t} \;,\quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \le \left(\hat{Q}_{20}\right)^{(3)} e^{(\hat{M}_{20})^{(3)}t} \;,\quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)\,, T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t} \;,\quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)\,, T_i(0)$:

$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)\,, T_i(0)$:

$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)\,, T_i(0)$:

$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le \left(\hat{P}_{13}\right)^{(1)}$, $T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}$, 155

$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 156

$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{16})^{(2)}$, $T_i^0 \leq (\hat{Q}_{16})^{(2)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''^{(2)}_{16}\right)\left(T_{17}(s_{(16)}), s_{(16)}\right)\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}\left(T_{17}(s_{(16)}), s_{(17)}\right)\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}\left(T_{17}(s_{(16)}), s_{(16)}\right)\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}\left(G_{19}(s_{(16)}), s_{(16)}\right)\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{20})^{(3)}$, $T_i^0 \leq (\hat{Q}_{20})^{(3)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''^{(3)}_{20}\right)\left(T_{21}(s_{(20)}), s_{(20)}\right)\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}\left(T_{21}(s_{(20)}), s_{(20)}\right)\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}\left(T_{21}(s_{(20)}), s_{(20)}\right)\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)},\ T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)},\ T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

By

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad (169)$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that (170)

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that (171)

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that (172)

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that (173)

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that (174)

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^0)e^{-(\hat{M}_{28})^{(5)}t} \le \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^0)e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^0}{G_{29}^0}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \le G_{32}^0 + \int_0^t\left[(a_{32})^{(6)}\left(G_{33}^0+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)} =$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^0+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^0)e^{-(\hat{M}_{32})^{(6)}t} \le \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^0)e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^0}{G_{33}^0}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(y) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t\left[(a_{36})^{(7)}\left(G_{37}^0+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)} =$$
$$\left(1+(a_{36})^{(7)}t\right)G_{37}^0+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^0)e^{-(\hat{M}_{36})^{(7)}t} \le \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^0)e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^0}{G_{37}^0}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t\left[(a_{40})^{(8)}\left(G_{41}^0+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)} = \quad 180$$
$$\left(1+(a_{40})^{(8)}t\right)G_{41}^0+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^0)e^{-(\hat{M}_{40})^{(8)}t} \le \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^0)e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^0}{G_{41}^0}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \leq G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \leq \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $\left(\tilde{G}, \tilde{T}\right) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)}\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) - (a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right| \; e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} \} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \qquad (186)$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: (187)

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. (188)

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}\left|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right| \; e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\} ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}t} \leq$
$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right) d\left(\left((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)}\right)\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$: $\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0$ *and* $T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$G_{24}^{(2)} \left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)}\right) d\left(((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on T_{25} and respectively on (G_{27})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (228)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_2 / (a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\widehat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{32})^{(6)}\right] \leq (\widehat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})}, \widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})}, \widetilde{(T_{35})}\right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right)\left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)};\ (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ *and* $(\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ (250)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)} \left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)} (T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1,\left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right),\quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$

From 99 to 44 it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(9)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)}, , (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)} t} - e^{-(a'_{26})^{(4)} t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)} t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28}e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G^0_{28}e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}}G^0_{28}e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}}G^0_{28}e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)}G^0_{28}}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G^0_{30}e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

$$\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_2)^{(5)}((S_1)^{(5)}-(a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t}-e^{-(a'_{30})^{(5)}t}\right]+\ G_{30}^{0}e^{-(a'_{30})^{(5)}t}\Big)$$

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\le T_{30}(t)\le \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$:

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$-(\sigma_2)^{(6)}\le-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\le-(\sigma_1)^{(6)}$

$-(\tau_2)^{(6)}\le-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\le-(\tau_1)^{(6)}$

Definition of $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$: 350

By $(v_1)^{(6)}>0$, $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$, $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

Definition of $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)}>0$, $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$, $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

Definition of $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$:-

If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)}=(v_0)^{(6)},(m_1)^{(6)}=(v_1)^{(6)},$ ***if*** $(v_0)^{(6)}<(v_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}$ $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}$ $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}$ $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}$ $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}$ $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})} \left[e^{(S_1)^{(6)} t} - e^{-(a'_{34})^{(6)} t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)} t} \right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^{0}e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}}G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}}G_{36}^{0}e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t}-e^{-(S_2)^{(7)}t}\right]+G_{38}^{0}e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left.\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_2)^{(7)}((S_1)^{(7)}-(a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t}-e^{-(a'_{38})^{(7)}t}\right]+\ G_{38}^{0}e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}}T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}}T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right) - (b''_{41})^{(8)}\left((G_{43}), t\right) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(v_1)^{(8)} > 0$, $(v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$, $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{v}_1)^{(8)}$,, $(\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{v}_1)^{(8)} > 0$, $(\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$, $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$:-

If we define $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$, $\boldsymbol{if}$ $(v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$, $\boldsymbol{if}$ $(v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$,
and $\boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}$, $\boldsymbol{if}$ $(\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, $\boldsymbol{if}$ $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, $\boldsymbol{if}$ $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}$ $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t})$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \qquad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \qquad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t}\leq T_{42}(t)\leq \qquad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)},(S_2)^{(8)},(R_1)^{(8)},(R_2)^{(8)}$:- 381

Where $(S_1)^{(8)}=(a_{40})^{(8)}(m_2)^{(8)}-(a'_{40})^{(8)}$

$(S_2)^{(8)}=(a_{42})^{(8)}-(p_{42})^{(8)}$

$(R_1)^{(8)}=(b_{40})^{(8)}(\mu_2)^{(8)}-(b'_{40})^{(8)}$

$(R_2)^{(8)}=(b'_{42})^{(8)}-(r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$:

$(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)}\leq -(a'_{44})^{(9)}+(a'_{45})^{(9)}-(a''_{44})^{(9)}(T_{45},t)+(a''_{45})^{(9)}(T_{45},t)\leq -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)}\leq -(b'_{44})^{(9)}+(b'_{45})^{(9)}-(b''_{44})^{(9)}\big((G_{47}),t\big)-(b''_{45})^{(9)}\big((G_{47}),t\big)\leq -(\tau_1)^{(9)}$

Definition of $(\nu_1)^{(9)},(\nu_2)^{(9)},(u_1)^{(9)},(u_2)^{(9)},\nu^{(9)},u^{(9)}$:

By $(\nu_1)^{(9)}>0$, $(\nu_2)^{(9)}<0$ and respectively $(u_1)^{(9)}>0$, $(u_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2+(\sigma_1)^{(9)}\nu^{(9)}-(a_{44})^{(9)}=0$ and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

Definition of $(\bar{\nu}_1)^{(9)},,(\bar{\nu}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)}>0$, $(\bar{\nu}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$, $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2+(\sigma_2)^{(9)}\nu^{(9)}-(a_{44})^{(9)}=0$ and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

Definition of $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)},\ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$
and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \le G_{46}(t) \le \right.$$
$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (v_1)^{(1)} \le (\bar{v}_1)^{(1)} \le \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$, we obtain 386

$$(v_1)^{(1)} \le v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $v^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\right) - (a''_{17})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{\mathrm{G}_{16}}{\mathrm{G}_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (\mathrm{C})^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (\mathrm{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \le \nu^{(2)}(t) \le (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \le \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \le \nu^{(2)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \le (\bar{\nu}_1)^{(2)} \le (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \le \nu^{(2)}(t) \le \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \le (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \le \nu^{(2)}(t) \le (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \le u^{(2)}(t) \le (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)}\right) \le \frac{dv^{(3)}}{dt} \le -\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \ge \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}}$$

it follows $(v_0)^{(3)} \le v^{(3)}(t) \le (v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \le v^{(3)}(t) \le (\bar{v}_1)^{(3)}$

If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)} \le \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}} \le v^{(3)}(t) \le$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (\bar{v}_1)^{(3)}$$

If $0 < (v_1)^{(3)} \le (\bar{v}_1)^{(3)} \le (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$, we obtain 403

$$(v_1)^{(3)} \le v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

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$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(8)} and\ (b''_i)^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\ ]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\ ]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\ ]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0$ 479

$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0$ 480

$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0$ 481

$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0$ 482

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(y) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first

equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T^*_{33} for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$$

Definition and uniqueness of T^*_{37} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T^*_{37} for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$$

Definition and uniqueness of T^*_{41} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{25} such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507
$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{29} such that $\varphi((G_{31})^*) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{33} such that $\varphi({G_{35}}^*) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{37} such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{41} such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right]+(b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{45} such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)} G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)} G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)} T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)} T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)} G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)} G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)} T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)} T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)} G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)} G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)} T_{21}^*}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T_{22}^* = \frac{(b_{22})^{(3)} T_{21}^*}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]} \quad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \qquad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \qquad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \qquad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \qquad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \qquad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \qquad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i \qquad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \qquad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \qquad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \qquad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \qquad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \qquad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}\mathrm{T}^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}\mathrm{T}^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}\left((G_{27})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)}$, $\frac{\partial (b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\big((a_{32}')^{(6)} + (p_{32})^{(6)}\big)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\big((a_{33}')^{(6)} + (p_{33})^{(6)}\big)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\big((a_{34}')^{(6)} + (p_{34})^{(6)}\big)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\big((b_{32}')^{(6)} - (r_{32})^{(6)}\big)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\big(s_{(32)(j)}T_{32}^*\mathbb{G}_j\big) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\big((b_{33}')^{(6)} - (r_{33})^{(6)}\big)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\big(s_{(33)(j)}T_{33}^*\mathbb{G}_j\big) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\big((b_{34}')^{(6)} - (r_{34})^{(6)}\big)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\big(s_{(34)(j)}T_{34}^*\mathbb{G}_j\big) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial (b_i'')^{(7)}}{\partial G_j}\big((G_{39})^{**} \big) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\big((a_{36}')^{(7)} + (p_{36})^{(7)}\big)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\big((a_{37}')^{(7)} + (p_{37})^{(7)}\big)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\big((a_{38}')^{(7)} + (p_{38})^{(7)}\big)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\big((b_{36}')^{(7)} - (r_{36})^{(7)}\big)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\big(s_{(36)(j)}T_{36}^*\mathbb{G}_j\big) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\big((b_{37}')^{(7)} - (r_{37})^{(7)}\big)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\big(s_{(37)(j)}T_{37}^*\mathbb{G}_j\big) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\big((b_{38}')^{(7)} - (r_{38})^{(7)}\big)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\big(s_{(38)(j)}T_{38}^*\mathbb{G}_j\big) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

(586 A)

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\ B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\Big)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\Big]$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\Big)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\Big)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\Big]$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\Big)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\Big)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\Big]$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\Big)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\Big)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\Big)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\Big)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\Big)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\Big)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION THIRTY THREE

Quantum Transport

INTRODUCTION—VARIABLES USED

Dephasing enhanced transport in non-equilibrium strongly-correlated quantum systems J. J. Mendoza-Arenas, T. Grujic, D. Jaksch, S. R. Clark

(1) A key insight from recent studies is that noise, such as dephasing, can improve (eb) the efficiency of quantum transport by suppressing (e) coherent single-particle interference effects.
(2) However, it is not yet clear whether dephasing can enhance (eb) transport in (eb) an interacting many-body system.

(3) Here authors address this question by analysing the transport properties of (e) a boundary driven spinless fermion chain with (e&eb) nearest-neighbour interactions subject to (e&eb) bulk dephasing.
(4) The many-body non-equilibrium stationary state is determined using (e) large scale matrix product simulations of the corresponding quantum master equation.
(5) Authors find dephasing enhanced transport only in (eb) the strongly interacting regime, where it is shown to induce (eb) incoherent transitions bridging the gap between bound dark-states and (e&eb) bands of mobile eigenstates.
(6) The generic nature of the effect is illustrated and shown not (e) to depend on the integrability of the model considered.
(7) As a result dephasing enhanced transport is expected to persist in (eb) more realistic driven systems of strongly-correlated particles. Cite as: arXiv: 1302.5629 [quant-ph]

NOTATION

Module One

Introduction

A master equation is a phenomenological set of first-order differential equations describing the time evolution of (usually) the probability of a system to occupy each one of a discrete set of states with regard to a continuous time variable t. The most familiar form of a master equation is a matrix form:

$$\frac{d\vec{P}}{dt} = \mathbf{A}\vec{P},$$

where $\vec{P}$ is a column vector (where element i represents state i), and $\mathbf{A}$ is the matrix of connections. The way connections among states are made determines the dimension of the problem; it is either

a d-dimensional system (where d is 1,2,3,...), where any state is connected with exactly its 2d nearest neighbors, or a network, where every pair of states may have a connection (depending on the network's properties).

When the connections are time-independent rate constants, the master equation represents a kinetic scheme, and the process is Markovian (any jumping time probability density function for state i is an exponential, with a rate equal to the value of the connection). When the connections depend on the actual time (i.e. matrix $\mathbf{A}$ depends on the time, $\mathbf{A} \to \mathbf{A}(t)$), the process is not stationary and the master equation reads

$$\frac{d\vec{P}}{dt} = \mathbf{A}(t)\vec{P}.$$

When the connections represent multi exponential jumping time probability density functions, the process is semi-Markovian, and the equation of motion is anintegro-differential equation termed the generalized master equation:

$$\frac{d\vec{P}}{dt} = \int_0^t \mathbf{A}(t-\tau)\vec{P}(\tau)d\tau.$$

The matrix $\mathbf{A}$ can also represent birth and death, meaning that probability is injected (birth) or taken from (death) the system, where then, the process is not in equilibrium.

Detailed description of the matrix $\mathbf{A}$, and properties of the system

Let $\mathbf{A}$ be the matrix describing the transition rates (also known as kinetic rates or reaction rates). As always, the first subscript represents the row, the second subscript the column. That is, the source is given by the second subscript, and the destination by the first subscript. This is the opposite of what one might expect, but it is technically convenient.

For each state k, the increase in occupation probability depends on the contribution from all other states to k, and is given by:

$$\sum_{\ell} A_{k\ell} P_{\ell},$$

where P_{ℓ}, is the probability for the system to be in the state ℓ, while the matrix $\mathbf{A}$ is filled with a grid of transition-rate constants. Similarly, P_k contributes to the occupation of all other states P_{ℓ},

$$\sum_{\ell} A_{\ell k} P_k,$$

In probability theory, this identifies the evolution as a continuous-time Markov process, with the integrated master equation obeying a Chapman–Kolmogorov equation.

The master equation can be simplified so that the terms with $\ell = k$ do not appear in the summation. This allows calculations even if the main diagonal of the $\mathbf{A}$ is not defined or has been assigned an arbitrary value.

$$\frac{dP_k}{dt} = \sum_{\ell} (A_{k\ell} P_{\ell}) = \sum_{\ell \neq k} (A_{k\ell} P_{\ell}) + A_{kk} P_k = \sum_{\ell \neq k} (A_{k\ell} P_{\ell} - A_{\ell k} P_k).$$

The final equality arises from the fact that

$$\sum_{\ell} (A_{\ell k}) = 0$$

and so

$$A_{kk} = -\sum_{\ell \neq k} (A_{\ell k}) \Rightarrow A_{kk} P_k = -\sum_{\ell \neq k} (A_{\ell k} P_k)$$

.

The master equation exhibits detailed balance if each of the terms of the summation disappears separately at equilibrium — i.e. if, for all states k and ℓ having equilibrium probabilities π_k and π_{ℓ},

$$A_{k\ell} \pi_{\ell} = A_{\ell k} \pi_k.$$

These symmetry relations were proved on the basis of the time reversibility of microscopic dynamics

(microscopic reversibility) as Onsager reciprocal relations.

G_{13} : Category one of LHS of above equations

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of RHS of above equations

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

A key insight from recent studies is that noise, such as dephasing, can improve (eb) the efficiency of quantum transport by suppressing (e) coherent single-particle interference effects

G_{16} : Category one of noise, such as dephasing

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of efficiency of quantum transport by suppressing (e) coherent single-particle interference effects

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

noise, such as dephasing, can improve the efficiency of quantum transport by suppressing (e) coherent single-particle interference effects

G_{20} : Category one of coherent single-particle interference effects

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of noise, such as dephasing, can improve the efficiency of quantum transport

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

However, it is not yet clear whether

dephasing can enhance (eb) transport in (eb) an interacting many-body system.

We assume the proposition and give a model

G_{24} : Category one of dephasing

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of transport in (eb) an interacting many-body system

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

dephasing can enhance transport in (eb) an interacting many-body system.

G_{28} : Category one of dephasing can enhance transport

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of interacting many-body system

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Here authors address this question by analysing the

transport properties of (e) a boundary driven spinless fermion chain with (e&eb) nearest-neighbour interactions subject to (e&eb) bulk dephasing

G_{32} : Category one of boundary driven spinless fermion chain with (e&eb) nearest-neighbour interactions subject to (e&eb) bulk dephasing

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of transport properties

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

transport properties of a boundary driven spinless fermion chain with (e&eb) nearest-neighbour interactions subject to (e&eb) bulk dephasing

G_{36} : Category one of transport properties of a boundary driven spinless fermion chain; nearest-neighbour

interactions subject to bulk dephasing

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of nearest-neighbour interactions subject to bulk dephasing; transport properties of a boundary driven spinless fermion chain

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

transport properties of a boundary driven spinless fermion chain with nearest-neighbour interactions subject to (e&eb) bulk dephasing

G_{40} : Category one of transport properties of a boundary driven spinless fermion chain with nearest-neighbour interactions; bulk dephasing

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of bulk dephasing; transport properties of a boundary driven spinless fermion chain with nearest-neighbour interactions

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

The many-body non-equilibrium stationary state is determined using (e) large scale matrix product simulations of the corresponding quantum master equation

Non-equilibrium thermodynamics From Wikipedia, the free encyclopedia

The Onsager relations

$$I_i = \partial S / \partial E_i .$$

$$k_b M = S - \sum_i (I_i E_i),$$

$$k_b \, dM = \sum_i (E_i \, dI_i).$$

Following Section III of Rayleigh (1873), Onsager (1931, I) showed that in the regime where both the flows (J_i) are small and the thermodynamic forces (F_i) vary slowly, the rate of creation of entropy (σ)

is linearly related to the flows:

$$\sigma = \sum_i J_i \frac{\partial F_i}{\partial x_i}$$

and the flows are related to the gradient of the forces, parametrized by a matrix of coefficients conventionally denoted L:

$$J_i = \sum_j L_{ij} \frac{\partial F_j}{\partial x_j}$$

from which it follows that:

$$\sigma = \sum_{i,j} L_{ij} \frac{\partial F_i}{\partial x_i} \frac{\partial F_j}{\partial x_j}$$

The second law of thermodynamics requires that the matrix L be positive definite. Statistical mechanics considerations involving microscopic reversibility of dynamics imply that the matrix L is symmetric. This fact is called the Onsager reciprocal relations.

G_{44} : Category one of large scale matrix product simulations of the corresponding quantum master equation; LHS of above equations

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of many-body non-equilibrium stationary state; RHS of above equations

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}$ $(a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$ $(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$

$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$
$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor
$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}),t)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}),t)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}),t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}),t) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$
$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$
$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad 60$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}$ $\boxed{-(b''_{42})^{(8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,

2 and 3

$[-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)]$, $[-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)]$, $[-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)]$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}[+(a''_{16})^{(2)}(T_{17},t)] & [+(a''_{13})^{(1,1,)}(T_{14},t)] & [+(a''_{20})^{(3,3,3)}(T_{21},t)] \\ [+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)] & [+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)] & [+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)] \\ [+(a''_{36})^{(7,7,7)}(T_{37},t)] & [+(a''_{40})^{(8,8,8)}(T_{41},t)] & [+(a''_{44})^{(9,9)}(T_{45},t)] \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}[+(a''_{17})^{(2)}(T_{17},t)] & [+(a''_{14})^{(1,1,)}(T_{14},t)] & [+(a''_{21})^{(3,3,3)}(T_{21},t)] \\ [+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)] & [+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)] & [+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)] \\ [+(a''_{37})^{(7,7,7)}(T_{37},t)] & [+(a''_{41})^{(8,8,8)}(T_{41},t)] & [+(a''_{45})^{(9,9)}(T_{45},t)] \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}[+(a''_{18})^{(2)}(T_{17},t)] & [+(a''_{15})^{(1,1,)}(T_{14},t)] & [+(a''_{22})^{(3,3,3)}(T_{21},t)] \\ [+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)] & [+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)] & [+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)] \\ [+(a''_{38})^{(7,7,7)}(T_{37},t)] & [+(a''_{42})^{(8,8,8)}(T_{41},t)] & [+(a''_{46})^{(9,9)}(T_{45},t)] \end{bmatrix} G_{18} \quad (63)$$

Where $[+(a''_{16})^{(2)}(T_{17},t)]$, $[+(a''_{17})^{(2)}(T_{17},t)]$, $[+(a''_{18})^{(2)}(T_{17},t)]$ are first augmentation coefficients for category 1, 2 and 3

$[+(a''_{13})^{(1,1,)}(T_{14},t)]$, $[+(a''_{14})^{(1,1,)}(T_{14},t)]$, $[+(a''_{15})^{(1,1,)}(T_{14},t)]$ are second augmentation coefficient for category 1, 2 and 3

$[+(a''_{20})^{(3,3,3)}(T_{21},t)]$, $[+(a''_{21})^{(3,3,3)}(T_{21},t)]$, $[+(a''_{22})^{(3,3,3)}(T_{21},t)]$ are third augmentation coefficient for category 1, 2 and 3

$[+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)]$, $[+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)]$, $[+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)]$ are fourth augmentation coefficient for category 1, 2 and 3

$[+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)]$, $[+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)]$, $[+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)]$ are fifth augmentation coefficient for category 1, 2 and 3

$[+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)]$, $[+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)]$, $[+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)]$ are sixth augmentation coefficient for category 1, 2 and 3

$[+(a''_{36})^{(7,7,7)}(T_{37},t)]$, $[+(a''_{37})^{(7,7,7)}(T_{37},t)]$, $[+(a''_{38})^{(7,7,7)}(T_{37},t)]$ are seventh augmentation coefficient for category 1, 2 and 3

$[+(a''_{40})^{(8,8,8)}(T_{41},t)]$, $[+(a''_{41})^{(8,8,8)}(T_{41},t)]$, $[+(a''_{42})^{(8,8,8)}(T_{41},t)]$ are eight augmentation coefficient for category 1, 2 and 3

$[+(a''_{44})^{(9,9)}(T_{45},t)]$, $[+(a''_{45})^{(9,9)}(T_{45},t)]$, $[+(a''_{46})^{(9,9)}(T_{45},t)]$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}[-(b''_{16})^{(2)}(G_{19},t)] & [-(b''_{13})^{(1,1,)}(G,t)] & [-(b''_{20})^{(3,3,3,)}(G_{23},t)] \\ [-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)] & [-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)] & [-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)] \\ [-(b''_{36})^{(7,7,7)}(G_{39},t)] & [-(b''_{40})^{(8,8,8)}(G_{43},t)] & [-(b''_{44})^{(9,9)}(G_{47},t)] \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}[-(b''_{17})^{(2)}(G_{19},t)] & [-(b''_{14})^{(1,1,)}(G,t)] & [-(b''_{21})^{(3,3,3,)}(G_{23},t)] \\ [-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)] & [-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)] & [-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)] \\ [-(b''_{37})^{(7,7,7)}(G_{39},t)] & [-(b''_{41})^{(8,8,8)}(G_{43},t)] & [-(b''_{45})^{(9,9)}(G_{47},t)] \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)} & \boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{24} \quad 73$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)} & \boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{25} \quad 74$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)} & \boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
are sixth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
are seventh augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
are eighth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)} & \boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad 76$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)} & \boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad (78)$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
are fourth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad (81)$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation

coefficients for category 1,2, and 3
$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3
$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3
$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3
$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3
$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3
$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3
$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3
$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3
$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3
$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3
$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3
$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3
$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \quad 85$$

$$- \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & \boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} & \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} & \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)} & \boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} & \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)} & \boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} & \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)} & \boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} & \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b_{36}')^{(7)}\boxed{-(b_{36}'')^{(7)}(G_{39},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b_{37}')^{(7)}\boxed{-(b_{37}'')^{(7)}(G_{39},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b_{38}')^{(7)}\boxed{-(b_{38}'')^{(7)}(G_{39},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\,-(b''_{40})^{(8)}(G_{43},t) & -(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\,-(b''_{41})^{(8)}(G_{43},t) & -(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\,-(b''_{42})^{(8)}(G_{43},t) & -(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq \ (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = \ (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \ , \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\,(a_i)^{(1)} + (a_i')^{(1)} + \ (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}\,(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i')^{(1)} + \ (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \leq (p_i)^{(2)} \leq \left(\hat{A}_{16}\right)^{(2)} \qquad 102$$

$$(b_i'')^{(2)}(G_{19},t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\big((G_{19}),t\big) = \ (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - \ T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\big((G_{19}),t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\big[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\big]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}} \ , \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\,(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + \ (\hat{P}_{16})^{(2)}\,(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + \ (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \le (p_i)^{(3)} \le (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \le (r_i)^{(3)} \le (b_i')^{(3)} \le (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\widehat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\widehat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\widehat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - \ T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\,(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and \ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\,(a_i)^{(6)} + (a_i')^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)}\,(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(UUUUUU) $\quad (a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(VVVVVV) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$(a_i'')^{(7)}(T_{37},t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$

$(b_i'')^{(7)}(G_{39},t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$ 132

(WWWWWW) $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

(XXXXXX)

$\lim_{G\to\infty}(b_i'')^{(7)}\left((G_{39}),t\right) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \leq (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$

$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}\left((G_{39}),t\right)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$. (T_{37}',t) and (T_{37},t) are points belonging to the interval $\left[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\right]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(YYYYYY) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(ZZZZZZ) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$

$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$

Where we suppose

$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$ 138

$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$ 139

$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$ 140

$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t}$ 142

$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t}$ 143

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ 144

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}$, $(r_i)^{(8)}, i = 40,41,42$,
Satisfy the inequalities

$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 145

$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 146

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad 146\ A$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\hat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)} ||(G_{47}) - (G_{47})'|| e^{-(\hat{M}_{44})^{(9)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}\left[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)} (\hat{k}_{44})^{(9)}\right] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t} \ , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t} \ , \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \ , \ T_i(0) = T_i^0 \ , \ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} , T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\overline{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^{0} + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^{0} + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^{0} + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^{0} + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^{0} + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^{0} + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By

$$\bar{G}_{32}(t) = G_{32}^{0} + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^{0} + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^{0} + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^{0} + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^{0} + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^{0} + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that 174

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)} =$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(z) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \leq G_{36}^{0}+\int_{0}^{t}\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)} =$$
$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \leq G_{40}^{0}+\int_{0}^{t}\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)} = \quad 180$$
$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t} \leq \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for G_{41} , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \leq G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \leq \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for G_{45} , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \left\{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\right\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $\left(\tilde{G}, \tilde{T}\right) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)}\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) - (a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right| \; e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \qquad (186)$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13{,}14{,}15$ depend only on T_{14} and respectively on G(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: (187)

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. (188)

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}} \leq$
$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{\mathrm{A}}_{16})^{(2)} + (\widehat{\mathrm{P}}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)\mathrm{d}\left(\left((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)}\right)\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where G_i (t) $= 0$ and T_i (t) $= 0$ 200

it results

G_i (t) $\geq \mathrm{G}_i^0\mathrm{e}^{\left[-\int_0^{\mathrm{t}}\{(a_i')^{(2)} - (a_i'')^{(2)}(\mathrm{T}_{17}(s_{(16)}), s_{(16)})\}\mathrm{d}s_{(16)}\right]} \geq 0$

T_i (t) $\geq \mathrm{T}_i^0\mathrm{e}^{(-(b_i')^{(2)}\mathrm{t})} > 0$ for t > 0

Definition of $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1$, $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2$ and $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$\mathrm{G}_{16} < (\widehat{\mathrm{M}}_{16})^{(2)}$ it follows $\frac{\mathrm{dG}_{17}}{\mathrm{dt}} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)}\mathrm{G}_{17}$ and by integrating

$\mathrm{G}_{17} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 = \mathrm{G}_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$

In the same way , one can obtain

$\mathrm{G}_{18} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3 = \mathrm{G}_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{\mathrm{t}\to\infty}((b_i'')^{(2)}((G_{19})(\mathrm{t}), \mathrm{t})) = (b_{17}')^{(2)}$ then $\mathrm{T}_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$: $\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0$ *and* $T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$G_{24}^{(2)} \left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)}\right) d\left(((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ (228)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_2 / (a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\widehat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{32})^{(6)}\right] \leq (\widehat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})}, \widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})}, \widetilde{(T_{35})}\right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right)\left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)};\ (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ *and* $(\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ (250)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)} \left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)} (T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b'_i)^{(7)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\,((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1,\left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right),\quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$

From 99 to 44 it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(9)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)}, , (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)} t} - e^{-(a'_{26})^{(4)} t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)} t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28}e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G^0_{28}e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}}G^0_{28}e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}}G^0_{28}e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)}G^0_{28}}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G^0_{30}e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

$$\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_2)^{(5)}((S_1)^{(5)}-(a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t}-e^{-(a'_{30})^{(5)}t}\right]+\ G_{30}^{0}e^{-(a'_{30})^{(5)}t}\Big)$$

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\le T_{30}(t)\le \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$:

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)}\le-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\le-(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)}\le-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\le-(\tau_1)^{(6)}$$

Definition of $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$: 350

By $(v_1)^{(6)}>0$, $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$, $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

Definition of $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)}>0$, $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$, $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

Definition of $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$:-

If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)}=(v_0)^{(6)},(m_1)^{(6)}=(v_1)^{(6)},$ ***if*** $(v_0)^{(6)}<(v_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}$ $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}$ $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}$ $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}$ $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}$ $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})} \left[e^{(S_1)^{(6)} t} - e^{-(a'_{34})^{(6)} t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)} t} \right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^{0}e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}}G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}}G_{36}^{0}e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t}-e^{-(S_2)^{(7)}t}\right]+G_{38}^{0}e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left.\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_2)^{(7)}((S_1)^{(7)}-(a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t}-e^{-(a'_{38})^{(7)}t}\right]+\ G_{38}^{0}e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}}T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}}T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right) - (b''_{41})^{(8)}\left((G_{43}), t\right) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(v_1)^{(8)} > 0$, $(v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$, $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{v}_1)^{(8)}$,, $(\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{v}_1)^{(8)} > 0$, $(\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$, $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$:-

If we define $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$, $\boldsymbol{if}$ $(v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$, $\boldsymbol{if}$ $(v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$,
and $\boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}$, $\boldsymbol{if}$ $(\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, $\boldsymbol{if}$ $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, $\boldsymbol{if}$ $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}$ $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t})$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t}\leq T_{42}(t)\leq \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)},(S_2)^{(8)},(R_1)^{(8)},(R_2)^{(8)}$:- 381

Where $(S_1)^{(8)}=(a_{40})^{(8)}(m_2)^{(8)}-(a'_{40})^{(8)}$

$(S_2)^{(8)}=(a_{42})^{(8)}-(p_{42})^{(8)}$

$(R_1)^{(8)}=(b_{40})^{(8)}(\mu_2)^{(8)}-(b'_{40})^{(8)}$

$(R_2)^{(8)}=(b'_{42})^{(8)}-(r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$:

$(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)}\leq -(a'_{44})^{(9)}+(a'_{45})^{(9)}-(a''_{44})^{(9)}(T_{45},t)+(a''_{45})^{(9)}(T_{45},t)\leq -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)}\leq -(b'_{44})^{(9)}+(b'_{45})^{(9)}-(b''_{44})^{(9)}\big((G_{47}),t\big)-(b''_{45})^{(9)}\big((G_{47}),t\big)\leq -(\tau_1)^{(9)}$

Definition of $(\nu_1)^{(9)},(\nu_2)^{(9)},(u_1)^{(9)},(u_2)^{(9)},\nu^{(9)},u^{(9)}$:

By $(\nu_1)^{(9)}>0$, $(\nu_2)^{(9)}<0$ and respectively $(u_1)^{(9)}>0$, $(u_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2+(\sigma_1)^{(9)}\nu^{(9)}-(a_{44})^{(9)}=0$ and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

Definition of $(\bar{\nu}_1)^{(9)},,(\bar{\nu}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)}>0$, $(\bar{\nu}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$, $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2+(\sigma_2)^{(9)}\nu^{(9)}-(a_{44})^{(9)}=0$ and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

Definition of $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)},\ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$
and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \le G_{46}(t) \le \right.$$
$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (v_1)^{(1)} \le (\bar{v}_1)^{(1)} \le \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$, we obtain 386

$$(v_1)^{(1)} \le v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $v^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\right) - (a''_{17})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{\mathrm{G}_{16}}{\mathrm{G}_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (\mathrm{C})^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (\mathrm{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)}\right) \le \frac{dv^{(3)}}{dt} \le -\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \ge \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}}$$

it follows $(v_0)^{(3)} \le v^{(3)}(t) \le (v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \le v^{(3)}(t) \le (\bar{v}_1)^{(3)}$

If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)} \le \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}} \le v^{(3)}(t) \le$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (\bar{v}_1)^{(3)}$$

If $0 < (v_1)^{(3)} \le (\bar{v}_1)^{(3)} \le (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$, we obtain 403

$$(v_1)^{(3)} \le v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

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$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{36}'')^{(7)} = (a_{37}'')^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b_{36}'')^{(7)} = (b_{37}'')^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a_{40}')^{(8)} - (a_{41}')^{(8)} + (a_{40}'')^{(8)}(T_{41}, t)\right) - (a_{41}'')^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(8)} and\ (b''_i)^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\ ]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\ ]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\ ]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0$ 479

$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0$ 480

$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0$ 481

$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0$ 482

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(z) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first

equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T^*_{33} for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$$

Definition and uniqueness of T^*_{37} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T^*_{37} for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$$

Definition and uniqueness of T^*_{41} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{25} such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507
$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{29} such that $\varphi((G_{31})^*) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{33} such that $\varphi({G_{35}}^*) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{37} such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{41} such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right]+(b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{45} such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)} G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)} G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \qquad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \qquad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \qquad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \qquad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \qquad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \qquad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \qquad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i \qquad 532$$

$$\frac{\partial (a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial (b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \qquad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \qquad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \qquad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \qquad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \qquad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}\mathrm{T}^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}\mathrm{T}^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}\left((G_{27})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^{*}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^{*}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^{*}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^{*}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^{*}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^{*}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^{*}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^{*}\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

(586 A)

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\ B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\Big)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\Big]$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\Big)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\Big)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\Big]$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\Big)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\Big)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\Big]$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\Big)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\Big)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\Big)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\Big)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION THIRTY FOUR

Dynamical Mean-Field Theory Of Strongly Correlated Fermion Systems

INTRODUCTION—VARIABLES USED

Dynamical mean-field theory of strongly correlated fermion systems and the limit of infinite dimensions Antoine Georges, Gabriel Kotliar, Werner Krauth, and Marcelo J. Rozenberg Rev. Mod. Phys. 68, 13 – Published 1 January 1996

(1) Authors review the dynamical mean-field theory of (e) strongly correlated electron systems which is based on (e) a mapping of lattice models onto (e&eb) quantum impurity models subject to a self-consistency condition.

(2) This mapping is exact for (e) models of correlated electrons in (eb) the limit of large lattice coordination (or infinite spatial dimensions).
(3) It extends the standard mean-field construction from classical statistical mechanics to (e&eb) quantum problems.
(4) Authors also discuss the physical ideas underlying this theory and its mathematical derivation. Various analytic and numerical techniques that have been developed recently in order to analyze and solve the dynamical mean-field equations are reviewed and compared to each other. The method can be used for the determination of (eb) phase diagrams (by comparing the stability of various types of long-range order), and the calculation of thermodynamic properties, one-particle Green's functions, and response functions.
(5) They review in detail the recent progress in understanding the Hubbard model and (e&eb) the Mott metal-insulator transition within this approach, including some comparison to experiments on three-dimensional transition-metal oxides.
(6) They present an overview of the rapidly developing field of applications of this method to other systems. The present limitations of the approach and possible extensions of the formalism are finally discussed. Computer programs for the numerical implementation of this method are also provided with this article. DOI: http://dx.doi.org/10.1103/RevModPhys.68.13

Quantum mechanical framework (Wikipedia):.

Pure states

(7) Consider two noninteracting systems A and B, with respective Hilbert spaces HA and HB. The Hilbert space of the composite system is the tensor product

$$H_A \otimes H_B.$$

(8) If the first system is in state $|\psi\rangle_A$ and the second in state $|\phi\rangle_B$, the state of the composite system is

$$|\psi\rangle_A \otimes |\phi\rangle_B.$$

(9) States of the composite system which can be represented in this form are called separable states, or (in the simplest case) product states.
(10) Not all states are separable states (and thus product states). Fix a basis $\{|i\rangle_A\}$ for HA and a basis $\{|j\rangle_B\}$ for HB. The most general state in HA ⊗ HB is of the form

$$|\psi\rangle_{AB} = \sum_{i,j} c_{ij}|i\rangle_A \otimes |j\rangle_B .$$

(11) This state is separable if (e) there exist c_i^A, c_j^B so that $c_{ij}=c_i^A c_j^B$, yielding $|\psi\rangle_A=\sum_i c_i^A|i\rangle_A$ and $|\phi\rangle_B=\sum_j c_j^B|j\rangle_B$. It is inseparable if for all c_i^A, c_j^B we have $c_{ij}\neq c_i^A c_j^B$. If a state is inseparable, it is called an entangled state.
(12) For example, given two basis vectors $\{|0\rangle_A,|1\rangle_A\}$ of HA and two basis vectors $\{|0\rangle_B,|1\rangle_B\}$ of HB, the following is an entangled state:

$$\frac{1}{\sqrt{2}}\left(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B\right).$$

(13) If the composite system is in this state, (eb) it is impossible to attribute to either system A or

system B a definite pure state.

(14) Another way to say this is that while the von Neumann entropy of the whole state is zero (as it is for any pure state), (eb)the entropy of the subsystems is greater than zero.

(15) In this sense, the systems are "entangled". This has specific empirical ramifications for interferometry.[39] It is worthwhile to note that the above example is one of four Bell states, which are (maximally) (=) entangled pure states (pure states of the HA ⊗ HB space, but which cannot be separated into pure states of each HA and HB).

(16) Now suppose Alice is an observer for system A, and Bob is an observer for system B. If in the entangled state given above Alice makes a measurement in the $\{|0\rangle,|1\rangle\}$ eigenbasis of A, there are two possible outcomes, occurring with equal probability:

(i) Alice measures 0, and the state of the system collapses to $|0\rangle_A|1\rangle_B$.

(ii) Alice measures 1, and the state of the system collapses to $|1\rangle_A|0\rangle_B$.

(17) If the former occurs, then (eb) any subsequent measurement performed by Bob, in the same basis, will always (eb) return 1.

(18) If the latter occurs, (Alice measures 1) then Bob's measurement (eb) will return 0 with certainty.

(19) Thus, system B has been altered by (e&eb) Alice performing a local measurement on system A.

(20) This remains true even if (e) the systems A and B are spatially separated. This is the foundation of the EPR paradox.

(21) The outcome of Alice's measurement is random. Alice cannot decide which state to (eb) collapse the composite system into, and therefore cannot transmit (e&eb) information to Bob by acting on her system. Causality is thus preserved, in this particular scheme. For the general argument, see no-communication theorem.

NOTATION

Module One

Anderson impurity model from Wikipedia, the free encyclopedia

The Anderson impurity model is a Hamiltonian that is used to describe magnetic impurities embedded in metallic hosts. It is often applied to the description ofKondo-type of problems, such as heavy fermion systems and Kondo insulators. In its simplest form, the model contains a term describing the kinetic energy of the conduction electrons, a two-level term with an on-site Coulomb repulsion that models the impurity energy levels, and a hybridization term that couples conduction and impurity orbitals. For a single impurity, the Hamiltonian takes the form

$$H = \sum_{\sigma} \epsilon_f f_\sigma^\dagger f_\sigma + \sum_{<j,j'>\sigma} t_{jj'} c_{j\sigma}^\dagger c_{j'\sigma} + \sum_{j,\sigma} (V_j f_\sigma^\dagger c_{j\sigma} + V_j^* c_{j\sigma}^\dagger f_\sigma) + U f_\uparrow^\dagger f_\uparrow f_\downarrow^\dagger f_\downarrow$$

where the f operator corresponds to the annihilation operator of an impurity, and c corresponds to a conduction electron annihilation operator, and σ labels the spin. The onsite Coulomb repulsion is U, which is usually the dominant energy scale, and $t_{jj'}$ is the hopping strength from site j to site j'. A significant feature of this model is the hybridization term V, which allows the f electrons in heavy fermion systems to become mobile, despite the fact they are separated by a distance greater than the Hill limit.

In heavy-fermion systems, we find we have a lattice of impurities. The relevant model is then the periodic Anderson model.

$$H = \sum_{j\sigma} \epsilon_f f^{\dagger}_{j\sigma} f_{j\sigma} + \sum_{<j,j'>\sigma} t_{jj'} c^{\dagger}_{j\sigma} c_{j'\sigma} + \sum_{j,\sigma} (V_j f^{\dagger}_{\sigma} c_{j\sigma} + V^*_j c^{\dagger}_{\sigma} f_{j\sigma}) + U \sum_{j} f^{\dagger}_{j\uparrow} f_{j\uparrow} f^{\dagger}_{j\downarrow} f_{j\downarrow}$$

There are other variants of the Anderson model, for instance the SU(4) Anderson model, which is used to describe impurities which have an orbital, as well as a spin, degree of freedom. This is relevant in carbon nanotube quantum dot systems. The SU(4) Anderson model Hamiltonian is

$$H = \sum_{i\sigma} \epsilon_f f^{\dagger}_{i\sigma} f_{i\sigma} + \sum_{<j,j'>\sigma} t_{ijj'} c^{\dagger}_{ij\sigma} c_{ij'\sigma} + \sum_{ij,\sigma} (V_j f^{\dagger}_{i\sigma} c_{ij\sigma} + V^*_j c^{\dagger}_{ij\sigma} f_{i\sigma}) + \sum_{i\sigma,i'\sigma'} \frac{U}{2} n_{i\sigma} n_{i'\sigma'}$$

where i and i' label the orbital degree of freedom (which can take one of two values), and n represents a number operator

G_{13} : Category one of LHS of above equations

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of RHS of the above equations

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Authors review the

dynamical mean-field theory of (e) strongly correlated electron systems which is based on (e) a mapping of lattice models onto (e&eb) quantum impurity models subject to a self-consistency condition

G_{16} : Category one of strongly correlated electron systems which is based on (e) a mapping of lattice models onto (e&eb) quantum impurity models subject to a self-consistency condition

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of dynamical mean-field theory

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

dynamical mean-field theory of strongly correlated electron systems which is based on (e) a mapping of lattice models onto (e&eb) quantum impurity models subject to a self-consistency condition

G_{20} : Category one of mapping of lattice models onto (e&eb) quantum impurity models subject to a self-consistency condition

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of dynamical mean-field theory of strongly correlated electron systems

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

dynamical mean-field theory of strongly correlated electron systems which is based on a mapping of lattice models onto (e&eb) quantum impurity models subject to a self-consistency condition

G_{24} : Category one of dynamical mean-field theory of strongly correlated electron systems which is based on a mapping of lattice models; quantum impurity models subject to a self-consistency condition

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of quantum impurity models subject to a self-consistency condition; dynamical mean-field theory of strongly correlated electron systems which is based on a mapping of lattice models

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

This mapping is exact for models of correlated electrons in (eb) the limit of large lattice coordination (or infinite spatial dimensions)

G_{28} : Category one of mapping is exact for models of correlated electrons

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of limit of large lattice coordination (or infinite spatial dimensions)

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

They review in detail the recent progress in understanding the Hubbard model and (e&eb) the Mott metal-insulator transition within this approach, including some comparison to experiments on three-dimensional transition-metal oxides

Hubbard model from Wikipedia, the free encyclopedia

(1) The Hubbard model is an approximate model used, especially in solid state physics, to describe (eb) the transition between conducting and insulating systems.
(2) The Hubbard model, named after John Hubbard, is the simplest model of (e)interacting particles in

a lattice, with only two terms in the Hamiltonian (see example below): a kinetic term allowing for (eb) tunneling ('hopping') of particles between sites of the lattice and a potential term consisting of an on-site interaction.

(3) The particles can either be fermions, as in Hubbard's original work, or bosons, when the model is referred to as either the 'Bose–Hubbard model' or the 'boson Hubbard model'.

(4) The Hubbard model is a good approximation for (e) particles in a periodic potential at sufficiently low temperatures that all the particles are in the lowest Bloch band, as long as any long-range interactions between the particles can be ignored.

(5) If interactions between particles on different sites of the lattice are included, the model is often referred to as the 'extended Hubbard model'.

Theory (Narrow energy band theory)

(6) The Hubbard model is based on (eb) the tight-binding approximation from solid state physics.

(7) In the tight-binding approximation, electrons are viewed as (=) occupying the standard orbitals of their constituent atoms, and then 'hopping' between atoms during conduction.

(8) Mathematically, this is represented as (=) a 'hopping integral' or 'transfer integral' between (e&eb) neighboring atoms, which can be viewed as the physical principle that creates electron bands in crystalline materials, due to (e) overlapping between atomic orbitals.

(9) The width of the band depends upon (=) the overlapping amplitude.

(10) However, the more general band theories do not (e) consider interactions between electrons explicitly.

(11) They consider the interaction of a single electron with (e&eb) the potential of nuclei and other electrons in (eb) an average way only.

(12) By formulating conduction in terms of the hopping integral, however, the Hubbard model is able to include the so-called 'onsite repulsion', which stems from the Coulomb repulsion between electrons at the same atomic orbitals. This sets up a competition between the hopping integral, which is a function of the distance and angles between neighboring atoms, and the on-site Coulomb repulsion, which is not considered in the usual band theories. The Hubbard model can therefore explain the transition from metal to insulator in certain metal oxides as they are heated by the increase in nearest neighbor spacing, which reduces the 'hopping integral' to the point where the onsite potential is dominant. Similarly, this can explain the transition from conductor to insulator in systems such as rare-earth pyrochlores as the atomic number of the rare-earth metal increases, because the lattice parameter increases (or the angle between atoms can also change — see Crystal structure) as the rare-earth element atomic number increases, thus changing the relative importance of the hopping integral compared to the onsite repulsion.

Example: 1D chain of hydrogen atoms

The hydrogen atom has only one electron, in the so-called s orbital, which can either be spin up (↑) or spin down (↓). This orbital can be occupied by at most two electrons, one with spin up and one down (see Pauli exclusion principle).

Now, consider a 1D chain of hydrogen atoms. Under band theory, we would expect the 1s orbital to form a continuous band, which would be exactly half-full. The 1-D chain of hydrogen atoms is thus predicted to be a conductor under conventional band theory.

But now consider the case where the spacing between the hydrogen atoms is gradually increased. At some point we expect that the chain must become an insulator.

Expressed in terms of the Hubbard model, on the other hand, the Hamiltonian is now made up of two components. The first component is the hopping integral. The hopping integral is typically represented by the letter t because it represents the kinetic energy of electrons hopping between atoms. The second term in the Hubbard model is then the on-site repulsion, typically represented by the letter U because it represents the potential energy arising from the charges on the electrons. Written out in second quantization notation, the Hubbard Hamiltonian then takes the form:

$$H = -t \sum_{\langle i,j\rangle,\sigma} (c^{\dagger}_{i,\sigma} c_{j,\sigma} + c^{\dagger}_{j,\sigma} c_{i,\sigma}) + U \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow},$$

where $\langle i, j \rangle$ represents nearest-neighbor interaction on the lattice.

If we consider the Hamiltonian without the contribution of the second term, we are simply left with the tight binding formula from regular band theory.

When the second term is included, however, we end up with a more realistic model that also predicts a transition from conductor to insulator as the inter-atomic spacing is increased. In the limit where the spacing is infinite (or if we ignore the first term), the chain simply resolves into a set of isolated magnetic moments. Additionally, when there are some contributions from the first term, but the material remains an insulator, the overlap integral provides for exchange interactions between neighboring magnetic moments which may lead to a variety of interesting magnetic correlations, such as ferromagnetic, antiferromagnetic, etc. depending on the exact solutions of the model. The one-dimensional Hubbard model was solved by Lieb and Wu using the Bethe ansatz. Essential progress has been achieved in the 1990s: a hidden symmetry was discovered, the scattering matrix, correlation functions, thermodynamic and quantum entanglement were evaluated,

More complex systems

Although the Hubbard model is useful in describing systems such as a 1-D chain of hydrogen atoms, it is important to note that in more complex systems there may be other effects that the Hubbard model does not consider. In general, insulators can be divided into Mott-Hubbard type insulators (see Mott insulator) and charge transfer insulators.

Consider the following description of a Mott–Hubbard insulator:

(Ni2+O2−)2 --> Ni3+O2− + Ni1+O2−

This can be seen as analogous to the Hubbard model for hydrogen chains, where conduction between unit cells can be described by a transfer integral.

However, it is possible for the electrons to exhibit another kind of behavior:

Ni2+O2− --> Ni1+O1−

This is known as charge transfer, and results in charge transfer insulators. Note that this is quite different from the Mott–Hubbard insulator model because there is no electron transfer between unit cells, only within a unit cell.

Both of these effects may be present and competing in complex ionic systems.

G_{32} : Category one of first term on the Hamiltonian and reactions of Mott–Hubbard insulator

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of second term of the Hamiltonian and reactions of Mott–Hubbard insulator

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Consider two noninteracting systems A and B, with respective Hilbert spaces HA and HB. The Hilbert space of the composite system is the tensor product

$$H_A \otimes H_B.$$

(1) If the first system is in state $|\psi\rangle_A$ and the second in state $|\phi\rangle_B$, the state of the composite system is

$$|\psi\rangle_A \otimes |\phi\rangle_B.$$

G_{36} : Category one of first system is in state $|\psi\rangle_A$ and the second in state $|\phi\rangle_B$,

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of state of the composite system is $|\psi\rangle_A \otimes |\phi\rangle_B$.

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

Not all states are separable states (and thus product states). Fix a basis $\{|i\rangle_A\}$ for HA and a basis $\{|j\rangle_B\}$ for HB. The most general state in HA ⊗ HB is of the form

$$|\psi\rangle_{AB} = \sum_{i,j} c_{ij}|i\rangle_A \otimes |j\rangle_B .$$

This state is separable if (e) there exist c_i^A , c_j^B so that $c_{ij}=c_i^A c_j^B$, yielding $|\psi\rangle_A=\sum_i c_i^A|i\rangle_A$ and $|\phi\rangle_B=\sum_j c_j^B|j\rangle_B$. It is inseparable if for all c_i^A , c_j^B we have $c_{ij}\neq c_i^A c_j^B$. If a state is inseparable, it is called an entangled state

G_{40} : Category one of exist c_i^A , c_j^B so that $c_{ij}=c_i^A c_j^B$, yielding $|\psi\rangle_A=\sum_i c_i^A|i\rangle_A$ and $|\phi\rangle_B=\sum_j c_j^B|j\rangle_B$. It is inseparable if for all c_i^A , c_j^B we have $c_{ij}\neq c_i^A c_j^B$. If a state is inseparable, it is called an entangled state

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of state is separable

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Now suppose Alice is an observer for system A, and Bob is an observer for system B. If in the entangled state given above Alice makes a measurement in the $\{|0\rangle,|1\rangle\}$ eigenbasis of A, there are two possible outcomes, occurring with equal probability:

(iii) Alice measures 0, and the state of the system collapses to $|0\rangle_A|1\rangle_B$.
(iv) Alice measures 1, and the state of the system collapses to $|1\rangle_A|0\rangle_B$.

G_{44} : Category one of Alice measures 0,

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of state of the system collapses to $|0\rangle_A|1\rangle_B$.

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION THIRTY FIVE

Holographic Matrix Models And Quantum Foam

INTRODUCTION—VARIABLES USED

Ben Craps et al JHEP10 (2005)005 doi:10.1088/1126-6708/2005/10/005 A matrix big bang Ben Craps1, Savdeep Sethi2 and Erik Verlinde1

(1) The light-like linear dilaton background represents (eb) a particularly simple time-dependent 1/2 BPS solution of (e) critical type-IIA superstring theory in (eb) ten dimensions.
(2) Its lift to M-theory, as well as its Einstein frame metric, are singular in the sense that the geometry

is (=) geodesically incomplete and the Riemann tensor diverges along (e&eb) a light-like subspace of codimension one.

(3) Authors study this background as a model for (e) a big bang type singularity in (eb) string theory/M-theory.

(4) They construct the dual Matrix theory description in terms of (e&eb) a (1+1)-d supersymmetric Yang-Mills theory on (eb) a time-dependent world-sheet given by (eb) the Milne orbifold of (1+1)-d Minkowski space.

(5) Model provides a framework in which the physics of (e) the singularity appears to be under control.

S. CECOTTI, S. FERRARA, and L. GIRARDELLO, Int. J. Mod. Phys. A 04, 2475 (1989). DOI: 10.1142/S0217751X89000972 GEOMETRY OF TYPE II SUPERSTRINGS AND THE MODULI OF SUPERCONFORMAL FIELD THEORIES: This work is supported in part by The United States Department of Energy under contract No. DE-AA03–76SF00034.

We study general properties of the low-energy effective theory for 4D type II superstrings obtained by the compactification on an abstract (2,2) superconformal system. This is the basic step towards the construction of their moduli space. We give an explicit and general algorithm to convert the effective Lagrangian for the type IIA into that of type IIB superstring defined by the same (2,2) superconformal system (and vice versa). This map converts Kahler manifolds into quaternionic ones (and quaternionic into Kahlerian ones) and has a deep geometrical meaning. The relationship with the theory of normal quaternionic manifolds (and algebras), as well as with Jordan algebras, is outlined. It turns out that only a restricted class of quarternionic geometries is allowed in the string case. We derive a general and explicit formula for the (fully nonlinear) couplings of the vector-multiplets (IIA case) in terms of the basic three-point functions of the underlying superconformal theory. A number of illustrative examples are also presented.

Miguel S. Costa and Michael Gutperle JHEP03 (2001)027 doi:10.1088/1126-6708/2001/03/027 The Kaluza Klein Melvin solution in M-theory

We study some aspects of the Kaluza-Klein Melvin solution in M-theory. The associated magnetic field has a maximal critical value B = ±1/R where R is the radius of the compactification circle. It is argued that the Melvin background of type IIA with magnetic field B and of type 0A with magnetic field B' = B−1/R are equivalent. Evidence for this conjecture is provided using a further circle compactification and a `9-11' flip. We show that partition functions of nine-dimensional type-IIA strings and of a (−1)Fσ1/2 type-IIA orbifold both with NS-NS Melvin flux tubes are related by such shift of the magnetic field. Then the instabilities of both IIA and 0A Melvin solutions are analyzed. For each theory there is an instanton associated to the decay of spacetime. In the IIA case the decay mode is associated to the nucleation of D6/Dbar 6-brane pairs, while in the 0A case spacetime decays through Witten's bubble production.

NOTATION

Module One

The light-like linear dilaton background represents (eb) a particularly simple time-dependent 1/2 BPS solution of (e) critical type-IIA superstring theory in (eb) ten dimensions

G_{13} : Category one of light-like linear dilaton background

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of particularly simple time-dependent 1/2 BPS solution of (e) critical type-IIA superstring theory in (eb) ten dimensions

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

light-like linear dilaton background represents a particularly simple time-dependent 1/2 BPS solution of (e) critical type-IIA superstring theory in ten dimensions

$$T_{\mathrm{H}} = \frac{\hbar c^3}{8\pi G M k_B} \quad (\approx \frac{1.227 \times 10^{23}\ kg}{M}\ K)$$

$$S_{\mathrm{B}} = \frac{k_B 4\pi G}{\hbar c} M^2.$$

$$S_{\mathrm{B}} = \frac{A k_B}{4 l_{\mathrm{P}}^2},$$

Bogomol'nyi–Prasad–Sommerfield bound from Wikipedia, the free encyclopedia

In a theory of U(1) Yang-Mills-Higgs, the energy at a given time t is given by

$$E = \int d^3x \ \left[\frac{1}{2}\overrightarrow{D\varphi}^T \cdot \overrightarrow{D\varphi} + \frac{1}{2}\pi^T\pi + V(\varphi) + \frac{1}{2g^2}\,\mathrm{Tr}\left[\vec{E}\cdot\vec{E} + \vec{B}\cdot\vec{B}\right]\right]$$

where D is the covariant derivative and V is the potential. If we assume that V is nonnegative and is zero only for the Higgs vacuum and that the Higgs field is in thé adjoint representation, then

$$\begin{aligned} E &\geq \int d^3x \ \left[\frac{1}{2}\,\mathrm{Tr}\left[\overrightarrow{D\varphi}\cdot\overrightarrow{D\varphi}\right] + \frac{1}{2g^2}\,\mathrm{Tr}\left[\vec{B}\cdot\vec{B}\right]\right] \\ &\geq \int d^3x \ \mathrm{Tr}\left[\frac{1}{2}\left(\overrightarrow{D\varphi} \mp \frac{1}{g}\vec{B}\right)^2 \pm \frac{1}{g}\overrightarrow{D\varphi}\cdot\vec{B}\right] \\ &\geq \pm\frac{1}{g}\int d^3x \ \mathrm{Tr}\left[\overrightarrow{D\varphi}\cdot\vec{B}\right] \\ &= \pm\frac{1}{g}\int_{S^2\ \text{boundary}} \mathrm{Tr}\left[\varphi\vec{B}\cdot d\vec{S}\right]. \end{aligned}$$

Therefore,

$$E \geq \left\| \int_{S^2} \mathrm{Tr}\left[\varphi\vec{B}\cdot d\vec{S}\right] \right\|.$$

Saturation happens when $\pi = 0$ and

$$\overrightarrow{D\varphi} \mp \frac{1}{g}\vec{B} = 0$$

The Bogomol'nyi equation. The other condition for saturation is the Higgs mass and self-interaction are zero, which is the case in N=2 supersymmetric theories.

This quantity is the absolute value of the magnetic flux.

A slight generalization applying to dyons also exists. For that, the Higgs field needs to be a complex adjoint, not a real adjoint.

Supersymmetry

In supersymmetry, the BPS bound is saturated when half (or a quarter or an eighth) of the SUSY generators are unbroken. This happens when the mass is equal to the central extension, which is typically a topological charge.

In fact, most bosonic BPS bounds actually come from the bosonic sector of a supersymmetric theory and this explains their origin.

G_{16} : Category one of critical type-IIA superstring theory in ten dimensions; LHS of above equations

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of light-like linear dilaton background represents a particularly simple time-dependent 1/2 BPS solution; RHS of above equations

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Its lift to M-theory, as well as its

Einstein frame metric, are singular in the sense that the geometry is (=) geodesically incomplete and the Riemann tensor diverges along (e&eb) a light-like subspace of codimension one

G_{20} : Category one of Einstein frame metric, are singular in the sense that the geometry

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of geodesically incomplete and the Riemann tensor diverges along (e&eb) a light-like subspace of codimension one

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Einstein frame metric, are singular in the sense that the geometry is geodesically incomplete and the

Riemann tensor diverges along (e&eb) a light-like subspace of codimension one

G_{24} : Category one of Einstein frame metric, are singular in the sense that the geometry is geodesically incomplete and the Riemann tensor diverges; light-like subspace of codimension one

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of light-like subspace of codimension one ;Einstein frame metric, are singular in the sense that the geometry is geodesically incomplete and the Riemann tensor diverges

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Authors study this background as a model for (e) a big bang type singularity in (eb) string theory/M-theory

G_{28} : Category one of big bang type singularity

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of string theory/M-theory

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

They construct the

dual Matrix theory description in terms of (e&eb) a (1+1)-d supersymmetric Yang-Mills theory on (eb) a time-dependent world-sheet given by (eb) the Milne orbifold of (1+1)-d Minkowski space

G_{32} : Category one of dual Matrix theory description; (1+1)-d supersymmetric Yang-Mills theory on (eb) a time-dependent world-sheet given by (eb) the Milne orbifold of (1+1)-d Minkowski space

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of (1+1)-d supersymmetric Yang-Mills theory on (eb) a time-dependent world-sheet given by (eb) the Milne orbifold of (1+1)-d Minkowski space ;dual Matrix theory description

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

dual Matrix theory description in terms of a (1+1)-d supersymmetric Yang-Mills theory on (eb) a time-dependent world-sheet given by (eb) the Milne orbifold of (1+1)-d Minkowski space

G_{36} : Category one of dual Matrix theory description in terms of a (1+1)-d supersymmetric Yang-Mills theory

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of time-dependent world-sheet given by (eb) the Milne orbifold of (1+1)-d Minkowski space

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

dual Matrix theory description in terms of a (1+1)-d supersymmetric Yang-Mills theory on a time-dependent world-sheet given by (eb) the Milne orbifold of (1+1)-d Minkowski space

G_{40} : Category one of dual Matrix theory description in terms of a (1+1)-d supersymmetric Yang-Mills theory on a time-dependent world-sheet

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of Milne orbifold of (1+1)-d Minkowski space

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Field Equations from the Kaluza Hypothesis (Wikipedia) (for references kindly see the Wikipedia article)

The field equations of the 5-dimensional theory were never adequately provided by Kaluza or Klein, mainly regarding the scalar field. The full Kaluza field equations are generally attributed to Thiry,[7] who most famously obtained vacuum field equations, although Kaluza [2] originally provided a stress-energy tensor for his theory and Thiry included a stress-energy tensor in his thesis. But as described by Gonner,[5] several independent groups worked on the field equations in the 1940s and earlier. Thiry is perhaps best known only because an English translation was provided by Applequist, Chodos, & Freund in their review book.[19] Applequist et al. also provided an English translation of Kaluza's paper. There are no English translations of the Jordan papers.[9][10][12]

To obtain the 5D field equations, the 5D connections $\widetilde{\Gamma}^{a}_{bc}$ are calculated from the 5D metric $\widetilde{g}_{ab}$, and the 5D Ricci tensor $\widetilde{R}_{ab}$ is calculated from the 5D connections.

The classic results of Thiry and other authors presume the cylinder condition:

$$\frac{\partial \tilde{g}_{ab}}{\partial x^5} = 0$$

.

Without this assumption, the field equations become much more complex, providing many more degrees of freedom that can be identified with various new fields. Paul Wesson and colleagues have pursued relaxation of the cylinder condition to gain extra terms that can be identified with the matter fields,[20] for which Kaluza [2]otherwise inserted a stress-energy tensor by hand.

It has been an objection to the original Kaluza hypothesis to invoke the fifth dimension only to negate its dynamics. But Thiry argued [5] that the interpretation of the Lorentz force law in terms of a 5-dimensional geodesic mitigates strongly for a fifth dimension irrespective of the cylinder condition. Most authors have therefore employed the cylinder condition in deriving the field equations. Furthermore, vacuum equations are typically assumed for which

$$\tilde{R}_{ab} = 0$$

where

$$\tilde{R}_{ab} \equiv \partial_c \tilde{\Gamma}^c_{ab} - \partial_b \tilde{\Gamma}^c_{ca} + \tilde{\Gamma}^c_{cd} \tilde{\Gamma}^d_{ab} - \tilde{\Gamma}^c_{bd} \tilde{\Gamma}^d_{ac}$$

and

$$\tilde{\Gamma}^a_{bc} \equiv \frac{1}{2} \tilde{g}^{ad} (\partial_b \tilde{g}_{dc} + \partial_c \tilde{g}_{db} - \partial_d \tilde{g}_{bc})$$

The vacuum field equations obtained in this way by Thiry [7] and Jordan's group [9][10][12] are as follows.

The field equation for ϕ is obtained from

$$\tilde{R}_{55} = 0 \Rightarrow \Box \phi = \frac{1}{4} \phi^3 F^{\alpha\beta} F_{\alpha\beta}$$

where $F_{\alpha\beta} \equiv \partial_\alpha A_\beta - \partial_\beta A_\alpha$, where $\Box \equiv g^{\mu\nu} \nabla_\mu \nabla_\nu$, and where ∇_μ is a standard, 4D covariant derivative. It shows that the electromagnetic field is a source for the scalar field. Note that the scalar field cannot be set to a constant without constraining the electromagnetic field. The earlier treatments by Kaluza and Klein did not have an adequate description of the scalar field, and did not realize the implied constraint on the electromagnetic field by assuming the scalar field to be constant.

The field equation for A^ν is obtained from

$$\tilde{R}_{5\alpha} = 0 = \frac{1}{2} g^{\beta\mu} \nabla_\mu (\phi^3 F_{\alpha\beta})$$

It has the form of the vacuum Maxwell equations if the scalar field is constant.

The field equation for the 4D Ricci tensor $R_{\mu\nu}$ is obtained from

$$\tilde{R}_{\mu\nu}-\frac{1}{2}\tilde{g}_{\mu\nu}\tilde{R}=0\Rightarrow R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=\frac{1}{2}\phi^2\left(g^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta}-\frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right)+\frac{1}{\phi}\left(\nabla_\mu\nabla_\nu\phi-g_{\mu\nu}\Box\phi\right)$$

where R is the standard 4D Ricci scalar.

This equation shows the remarkable result, called the "Kaluza miracle", that the precise form for the electromagnetic stress-energy tensor emerges from the 5D vacuum equations as a source in the 4D equations: field from the vacuum. This relation allows the definitive identification of A^μ with the electromagnetic vector potential. Therefore the field needs to be rescaled with a conversion constant k such that $A^\mu \rightarrow kA^\mu$.

The relation above shows that we must have

$$\frac{k^2}{2}=\frac{8\pi G}{c^4}\frac{1}{\mu_0}$$

where G is the gravitational constant and μ_0 is the permeability of free space. In the Kaluza theory, the gravitational constant can be understood as an electromagnetic coupling constant in the metric. There is also a stress-energy tensor for the scalar field. The scalar field behaves like a variable gravitational constant, in terms of modulating the coupling of electromagnetic stress energy to spacetime curvature. The sign of ϕ^2 in the metric is fixed by correspondence with 4D theory so that electromagnetic energy densities are positive. This turns out to imply that the 5th coordinate is spacelike in its signature in the metric.

In the presence of matter, the 5D vacuum condition can not be assumed. Indeed, Kaluza did not assume it. The full field equations require evaluation of the 5D Einstein tensor

$$\tilde{G}_{ab}\equiv\tilde{R}_{ab}-\frac{1}{2}\tilde{g}_{ab}\tilde{R}$$

as seen in the recovery of the electromagnetic stress-energy tensor above. The 5D curvature tensors are complex, and most English-language reviews contain errors in either $\tilde{G}_{ab}$ or $\tilde{R}_{ab}$, as does the English translation of.[7] See [18] for a complete set of 5D curvature tensors under the cylinder condition, evaluated using tensor algebra software.

Equations of Motion from the Kaluza Hypothesis

The equations of motion are obtained from the 5-dimensional geodesic hypothesis in terms of a 5-velocity $\tilde{U}^a\equiv dx^a/ds$:

$$\tilde{U}^b\tilde{\nabla}_b\tilde{U}^a=\frac{d\tilde{U}^a}{ds}+\tilde{\Gamma}^a_{bc}\tilde{U}^b\tilde{U}^c=0$$

This equation can be recast in several ways, and it has been studied in various forms by authors including Kaluza, Pauli, Gross & Perry Gegenberg & Kunstatter, and Wesson & Ponce de Leon, but it is instructive to

convert it back to the usual 4-dimensional length element $c^2 d\tau^2 \equiv g_{\mu\nu} dx^\mu dx^\nu$, which is related to the 5-dimensional length element ds as given above:

$$ds^2 = c^2 d\tau^2 + \phi^2 (k A_\nu dx^\nu + dx^5)^2$$

Then the 5D geodesic equation can be written for the spacetime components of the 4velocity, $U^\nu \equiv dx^\nu / d\tau$

$$\frac{dU^\nu}{d\tau} + \tilde{\Gamma}^\mu_{\alpha\beta} U^\alpha U^\beta + 2\tilde{\Gamma}^\mu_{5\alpha} U^\alpha U^5 + \tilde{\Gamma}^\mu_{55} (U^5)^2 + U^\mu \frac{d}{d\tau} \ln\left(\frac{c d\tau}{ds}\right) = 0$$

:

The term quadratic in U^ν provides the 4D geodesic equation plus some electromagnetic terms:

$$\tilde{\Gamma}^\mu_{\alpha\beta} = \Gamma^\mu_{\alpha\beta} + \frac{1}{2} g^{\mu\nu} k^2 \phi^2 (A_\alpha F_{\beta\nu} + A_\beta F_{\alpha\nu} + A_\alpha A_\beta \partial_\nu \ln \phi^2)$$

The term linear in U^ν provides the Lorentz force law:

$$\tilde{\Gamma}^\mu_{5\alpha} = \frac{1}{2} g^{\mu\nu} k \phi^2 (F_{\alpha\nu} - A_\alpha \partial_\nu \ln \phi^2)$$

This is another expression of the "Kaluza miracle". The same hypothesis for the 5D metric that provides electromagnetic stress-energy in the Einstein equations also provides the Lorentz force law in the equation of motions along with the 4D geodesic equation. Yet correspondence with the Lorentz force law requires that we identify the component of 5-velocity along the 5th dimension with electric charge:

$$k U^5 = k \frac{dx^5}{d\tau} \rightarrow \frac{q}{mc}$$

where m is particle mass and q is particle electric charge. Thus, electric charge is understood as motion along the 5th dimension. The fact that the Lorentz force law could be understood as a geodesic in 5 dimensions was to Kaluza a primary motivation for considering the 5-dimensional hypothesis, even in the presence of the aesthetically-unpleasing cylinder condition.

Yet there is a problem: the term quadratic in U^5.

$$\tilde{\Gamma}^\mu_{55} = -\frac{1}{2} g^{\mu\alpha} \partial_\alpha \phi^2$$

If there is no gradient in the scalar field, the term quadratic in U^5 vanishes. But otherwise the expression above implies

$$U^5 \sim c \frac{q/m}{G^{1/2}}$$

For elementary particles, $U^5 > 10^{20} c$. The term quadratic in U^5 should dominate the equation,

perhaps in contradiction to experience. This was the main shortfall of the 5-dimensional theory as Kaluza saw it,[2] and he gives it some discussion in his original article.

The equation of motion for U^5 is particularly simple under the cylinder condition. Start with the alternate form of the geodesic equation, written for the covariant 5-velocity:

$$\frac{d\tilde{U}_a}{ds} = \frac{1}{2}\tilde{U}^b\tilde{U}^c\frac{\partial\tilde{g}_{bc}}{\partial x^a}$$

This means that under the cylinder condition, $\tilde{U}_5$ is a constant of the 5-dimensional motion:

$$\tilde{U}_5 = \tilde{g}_{5a}\tilde{U}^a = \phi^2\frac{cd\tau}{ds}(kA_\nu U^\nu + U^5) = \text{constant}$$

Kaluza's Hypothesis for the Matter Stress-Energy Tensor

Kaluza [2] proposed a 5D matter stress tensor $\tilde{T}^{ab}_M$ of the form

$$\tilde{T}^{ab}_M = \rho\frac{dx^a}{ds}\frac{dx^b}{ds}$$

Here ρ is a density and the length element ds is as defined above.

Then, the spacetime component gives a typical "dust" stress energy tensor:

$$\tilde{T}^{\mu\nu}_M = \rho\frac{dx^\mu}{ds}\frac{dx^\nu}{ds}$$

The mixed component provides a 4-current source for the Maxwell equations:

$$\tilde{T}^{5\mu}_M = \rho\frac{dx^\mu}{ds}\frac{dx^5}{ds} = \rho U^\mu\frac{q}{kmc}$$

Just as the 5-dimensional metric comprises the 4-D metric framed by the electromagnetic vector potential, the 5-dimensional stress-energy tensor comprises the 4-D stress-energy tensor framed by the vector 4-current.

G_{44} : Category one of LHS of above equations

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of RHS of above equations

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION THIRTY SIX

Spacetime Foam

INTRODUCTION—VARIABLES USED

Entropy 2008, 10(4), 441-461; doi: 10.3390/e10040441 Spacetime Foam: From Entropy and Holography to Infinite Statistics and Nonlocality Y. Jack Ng

(1) Due to quantum fluctuations, spacetime is (=) foamy on small scales.
(2) The degree of foaminess is found to be (=) consistent with holography, a principle prefigured in (eb) the physics of black hole entropy.
(3) It has bearing on the ultimate accuracies of clocks and measurements and the physics of quantum computation. Consistent with existing archived data on active galactic nuclei from (e) the Hubble Space Telescope, the application of the holographic spacetime foam model to cosmology requires (e) the existence of dark energy which, authors argue, is composed of (e) an enormous number of inert "particles" of extremely long wavelength.
(4) Authors suggest that these "particles" obey (eb) infinite statistics in which all representations of (e) the particle permutation group can occur, and that the nonlocality present in (eb) systems obeying infinite statistics may be related to (e&eb)the nonlocality present in holographic theories.
(5) They also propose to detect spacetime foam by looking for (e) halos in the images of distant quasars, and argue that it does not (e) modify the GZK cutoff in the ultra-high energy cosmic ray spectrum and its contributions to (e&eb) time-offlight differences of high energy gamma rays from (e) distant GRB are too small to be detectable.

NOTATION

Module One

Quantum foam From Wikipedia, the free encyclopedia

Quantum foam (also referred to as space-time foam) is a concept in quantum mechanics devised by John Wheeler in 1955. The foam is supposed to be conceptualized as the foundation of the fabric of the universe.[1] Additionally, quantum foam can be used as a qualitative description of subatomic space-time turbulence at extremely small distances (on the order of the Planck length). At such small scales of time and space, the Heisenberg uncertainty principle allows energy to briefly decay into particles and antiparticles and then annihilate without violating physical conservation laws. As the scale of time and space being discussed shrinks, the energy of the virtual particles increases. According to Einstein's theory of general relativity, energy curves space-time. This suggests that—at sufficiently small scales—the energy of these fluctuations would be large enough to cause significant departures from the smooth space-time seen at larger scales, giving space-time a "foamy" character. With an incomplete theory of quantum gravity, it is impossible to be certain what space-time would look like at these small scales, because existing theories of gravity do not give accurate predictions in that realm. Therefore, any of the developing theories of quantum gravity may improve our understanding of quantum foam as they are tested. However, observations of radiation from nearby quasars by Floyd Stecker of NASA's Goddard Space Flight Center have placed strong experimental limits on the possible violations of Einstein's special theory of relativity implied by the existence of quantum foam.[2] Thus experimental evidence so far has given a range of values in which scientists can test for quantum foam.

Spin foam in loop quantum gravity

Loop Quantum Gravity has a covariant formulation that, at present, provides the best formulation of the dynamics of the theory of Quantum Gravity. This is a Quantum Field Theory where the invariance under diffeomorphisms of general relativity is implemented. The resulting path integral represents a sum over all the possible configuration of the geometry, coded in the spinfoam. A spin network is defined as a diagram (like the Feynman diagram) that makes a basis of connections between the elements of a differentiable manifold for the Hilbert spaces defined over them. Spin networks provide a representation for computations of amplitudes between two different hypersurfaces of the manifold. Any evolution of spin network provides spin foam over a manifold of one dimension higher than the dimensions of the corresponding spin network. Spin foam may be viewed as a quantum history

$$Z := \sum_{\Gamma} w(\Gamma) \left[\sum_{j_f, i_e} \prod_{f} A_f(j_f) \prod_{e} A_e(j_f, i_e) \prod_{v} A_v(j_f, i_e) \right]$$

he Dirac sea is a theoretical model of the vacuum as an infinite sea of particles with negative energy. It was first postulated by the British physicist Paul Dirac in 1930 to explain the anomalous negative-energy quantum states predicted by the Dirac equation for relativistic electrons. The positron, the antimatter counterpart of the electron, was originally conceived of as ahole in the Dirac Sea, well before its experimental discovery in 1932.

The equation relating energy, mass and momentum in special relativity is:

$$E^2 = p^2c^2 + m^2c^4,$$

In the special case of a particle at rest (i.e. p = 0), the above equation reduces to $E^2 = m^2c^4$, which is usually quoted as the familiar $E = mc^2$. However, this is a simplification because, while $x \cdot x = x^2$, we can also see that $(-x) \cdot (-x) = x^2$. Therefore, the correct equation to use to relate energy and mass in the Hamiltonian of the Dirac equation is:

$$E = \pm mc^2.$$

Here the negative solution was used to predict the existence of antimatter, discovered by Carl Anderson as the positron. The interpretation of this result requires a Dirac sea, showing that the Dirac equation is not merely a combination of special relativity and quantum field theory, but it also implies that the number of particles cannot be conserved.

The Dirac sea interpretation and the modern QFT interpretation are related by what may be thought of as a very simple Bogoliubov transformation, an identification between the creation and annihilation operators of two different free field theories. In the modern interpretation, the field operator for a Dirac spinor is a sum of creation operators and annihilation operators, in a schematic notation:

$$\psi(x) = \sum a^\dagger(k) e^{ikx} + a(k) e^{-ikx}$$

An operator with negative frequency lowers the energy of any state by an amount proportional to the frequency, while operators with positive frequency raise the energy of any state.

Quantum Fluctuation:

$$\Delta E \Delta t \approx \frac{h}{4\pi}$$

Quantum fluctuations of a field

A quantum fluctuation is the temporary appearance of energetic particles out of empty space, as allowed by the uncertainty principle. The uncertainty principle states that for a pair of conjugate variables such as position/momentum or energy/time, it is impossible to have a precisely determined value of each member of the pair at the same time. For example, a particle pair can pop out of the vacuum during a very short time interval.

An extension is applicable to the "uncertainty in time" and "uncertainty in energy" (including the rest mass energy mc^2). When the mass is very large like amacroscopic object, the uncertainties and thus the quantum effect become very small, and classical physics is applicable. This was proposed by scientist Adam Jonathon Davis' study in 1916 at Harvard's Laboratory 1996a. Davis' theory was later proven in the 1920s by Louis de Broglie and became a law of quantum physics.

In quantum field theory, fields undergo quantum fluctuations. A reasonably clear distinction can be made between quantum fluctuations and thermal fluctuations of a quantum field (at least for a free field; for interacting fields, renormalization substantially complicates matters). For the quantized Klein–Gordon field in thevacuum state, we can calculate the probability density that we would observe a configuration $\varphi_t(x)$ at a time t in terms of its Fourier transform $\tilde{\varphi}_t(k)$ to be

$$\rho_0[\varphi_t] = \exp\left[-\frac{1}{\hbar}\int \frac{d^3k}{(2\pi)^3}\tilde{\varphi}_t^*(k)\sqrt{|k|^2+m^2}\;\tilde{\varphi}_t(k)\right].$$

In contrast, for the classical Klein–Gordon field at non-zero temperature, the Gibbs probability density that we would observe a configuration $\varphi_t(x)$ at a time t is

$$\rho_E[\varphi_t] = \exp\left[-H[\varphi_t]/k_\mathrm{B}T\right] = \exp\left[-\frac{1}{k_\mathrm{B}T}\int \frac{d^3k}{(2\pi)^3}\tilde{\varphi}_t^*(k)\tfrac{1}{2}(|k|^2+m^2)\;\tilde{\varphi}_t(k)\right].$$

The amplitude of quantum fluctuations is controlled by the amplitude of Planck's constant $\hbar$, just as the amplitude of thermal fluctuations is controlled by $k_\mathrm{B}T$, where k_B is Boltzmann's constant. Note that the following three points are closely related:

Planck's constant has units of action instead of units of energy,

the quantum kernel is $\sqrt{|k|^2+m^2}$ instead of $\frac{1}{2}(|k|^2+m^2)$ (the quantum kernel is nonlocal from a classical heat kernel viewpoint, but it is local in the sense that it does not allow signals to be transmitted),

the quantum vacuum state is Lorentz invariant (although not manifestly in the above), whereas the classical thermal state is not (the classical dynamics is Lorentz invariant, but the Gibbs probability density is not a Lorentz invariant initial condition).

We can construct a classical continuous random field that has the same probability density as the quantum vacuum state, so that the principal difference from quantum field theory is the measurement theory (measurement in quantum theory is different from measurement for a classical continuous random field, in that classical measurements are always mutually compatible — in quantum mechanical terms they always commute). Quantum effects that are consequences only of quantum fluctuations, not of subtleties of measurement incompatibility, can alternatively be models of classical continuous random fields.

Derivation of Casimir effect assuming zeta-regularization

In the original calculation done by Casimir, he considered the space between a pair of conducting metal plates at distance a apart. In this case, the standing waves are particularly easy to calculate, because the transverse component of the electric field and the normal component of the magnetic field must vanish on the surface of a conductor. Assuming the parallel plates lie in the xy-plane, the standing waves are

$$\psi_n(x,y,z;t) = e^{-i\omega_n t} e^{ik_x x + ik_y y} \sin(k_n z)$$

where ψ stands for the electric component of the electromagnetic field, and, for brevity, the polarization and the magnetic components are ignored here. Here, k_x and k_y are the wave vectors in directions parallel to the plates, and

$$k_n = \frac{n\pi}{a}$$

is the wave-vector perpendicular to the plates. Here, n is an integer, resulting from the requirement that ψ vanish on the metal plates. The frequency of this wave is

$$\omega_n = c\sqrt{{k_x}^2 + {k_y}^2 + \frac{n^2\pi^2}{a^2}}$$

where c is the speed of light. The vacuum energy is then the sum over all possible excitation modes. Since the area of the plates is large, we may sum by integrating over two of the dimensions in k-space. The assumption of periodic boundary conditions yields,

$$\langle E \rangle = \frac{\hbar}{2} \cdot 2 \int \frac{A dk_x dk_y}{(2\pi)^2} \sum_{n=1}^{\infty} \omega_n$$

where A is the area of the metal plates, and a factor of 2 is introduced for the two possible polarizations of the wave. This expression is clearly infinite, and to proceed with the calculation, it is convenient to introduce a regulator (discussed in greater detail below). The regulator will serve to make the expression finite, and in the end will be removed. The zeta-regulated version of the energy per unit-area of the plate is

$$\frac{\langle E(s) \rangle}{A} = \hbar \int \frac{dk_x dk_y}{(2\pi)^2} \sum_{n=1}^{\infty} \omega_n |\omega_n|^{-s}.$$

In the end, the limit $s \to 0$ is to be taken. Here s is just a complex number, not to be confused with the shape discussed previously. This integral/sum is finite for sreal and larger than 3. The sum has a pole at s = 3, but may be analytically continued to s = 0, where the expression is finite. The above expression simplifies

to:

$$\frac{\langle E(s)\rangle}{A} = \frac{\hbar c^{1-s}}{4\pi^2}\sum_n \int_0^\infty 2\pi q dq \left| q^2 + \frac{\pi^2 n^2}{a^2}\right|^{(1-s)/2},$$

where polar coordinates $q^2 = k_x^2 + k_y^2$ were introduced to turn the double integral into a single integral. The q in front is the Jacobian, and the 2π comes from the angular integration. The integral converges if Re[s] > 3, resulting in

$$\frac{\langle E(s)\rangle}{A} = -\frac{\hbar c^{1-s}\pi^{2-s}}{2a^{3-s}}\frac{1}{3-s}\sum_n |n|^{3-s}.$$

The sum diverges at s in the neighborhood of zero, but if the damping of large-frequency excitations corresponding to analytic continuation of the Riemann zeta function to s = 0 is assumed to make sense physically in some way, then one has

$$\frac{\langle E\rangle}{A} = \lim_{s\to 0}\frac{\langle E(s)\rangle}{A} = -\frac{\hbar c\pi^2}{6a^3}\zeta(-3).$$

But $\zeta(-3) = 1/120$ and so one obtains

$$\frac{\langle E\rangle}{A} = \frac{-\hbar c\pi^2}{3\cdot 240 a^3}.$$

The analytic continuation has evidently lost an additive positive infinity, somehow exactly accounting for the zero-point energy (not included above) outside the slot between the plates, but which changes upon plate movement within a closed system. The Casimir force per unit area F_c/A for idealized, perfectly conducting plates with vacuum between them is

$$\frac{F_c}{A} = -\frac{d}{da}\frac{\langle E\rangle}{A} = -\frac{\hbar c\pi^2}{240a^4}$$

G_{13} : Category one of LHS of finalized equations in the above expatiation

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of RHS of finalized equations in the above expatiation

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

The degree of foaminess is found to be (=) consistent with holography, a principle prefigured in (eb) the

physics of black hole entropy

G_{16} : Category one of degree of foaminess; holography, a principle prefigured in (eb) the physics of black hole entropy

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of holography, a principle prefigured in (eb) the physics of black hole entropy; degree of foaminess

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Due to quantum fluctuations, spacetime is (=) foamy on small scales

G_{20} : Category one of quantum fluctuations

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of spacetime is foamy on small scales

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

degree of foaminess is found to be consistent with holography, a principle prefigured in (eb) the physics of black hole entropy

Source: Wikipedia

$$S_{\mathrm{BH}} = \frac{kA}{4\ell_{\mathrm{P}}^2}$$

$$dE = \frac{\kappa}{8\pi}\, dA + \Omega\, dJ + \Phi\, dQ,$$

$$T_{\mathrm{H}} = \frac{\kappa}{2\pi}.$$

$$S_{\mathrm{BH}} = \frac{A}{4}.$$

The holographic principle is a property of string theories and a supposed property of quantum gravity that states that the description of a volume of space can be thought of as encoded on a boundary to the region—preferably a light-like boundary like a gravitational horizon. First proposed by Gerard't Hooft, it was given a precise string-theory interpretation by Leonard Susskind [1] who combined his ideas with previous ones of 't

Hooft and Charles Thorn.[1][2] As pointed out by Raphael Bousso,[3] Thorn observed in 1978 that string theory admits a lower-dimensional description in which gravity emerges from it in what would now be called a holographic way. In a larger sense, the theory suggests that the entire universe can be seen as a two-dimensional information structure "painted" on the cosmological horizon[clarification needed], such that the three dimensions we observe are an effective description only at macroscopic scales and at low energies. Cosmological holography has not been made mathematically precise, partly because the particle horizon has a finite area and grows with time.[4][5] The holographic principle was inspired by black hole thermodynamics, which conjectures that the maximal entropy in any region scales with the radius squared, and not cubed as might be expected. In the case of a black hole, the insight was that the informational content of all the objects that have fallen into the hole might be entirely contained in surface fluctuations of the event horizon. The holographic principle resolves the black hole information paradox within the framework of string theory.[6] However, there exist classical solutions to the Einstein equations that allow values of the entropy larger than those allowed by an area law, hence in principle larger than those of a black hole. These are the so-called "Wheeler's bags of gold". The existence of such solutions conflicts with the holographic interpretation, and their effects in a quantum theory of gravity including the holographic principle are not yet fully understood.[7] (Wikipedia**). Note the principle of holography is embedded in the physics of black hole entropy**

G_{24} : Category one of degree of foaminess is found to be consistent with holography; LHS of above equations

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of black hole entropy; RHS of above equations

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

It has bearing on the ultimate accuracies of clocks and measurements and the physics of quantum computation. Consistent with existing archived data on active galactic nuclei from (e) the Hubble Space Telescope, the

application of the holographic spacetime foam model to cosmology requires (e) the existence of dark energy which, authors argue, is composed of (e) an enormous number of inert “particles” of extremely long wavelength

G_{28} : Category one of existence of dark energy which, authors argue, is composed of (e) an enormous number of inert “particles” of extremely long wavelength

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of application of the holographic spacetime foam model to cosmology

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

application of the holographic spacetime foam model to cosmology requires the existence of dark energy which, authors argue, is composed of (e) an enormous number of inert "particles" of extremely long wavelength

G_{32} : Category one of enormous number of inert "particles" of extremely long wavelength

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of application of the holographic spacetime foam model to cosmology requires the existence of dark energy

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

May 2000, Volume 30, Issue 5, pp 795-805: Measuring the Foaminess of Space-Time with Gravity-Wave Interferometers Y. Jack Ng, H. van Dam Foundations of Physics

(1) By analyzing a gedanken experiment designed to measure the distance l between two spatially separated points, authors find that this distance cannot (e) be measured with uncertainty less than (ll 2 P) 1/3 , considerably larger than the Planck scale lP (or the string scale in string theories), the conventional-wisdom uncertainty in distance measurements.
(2) This limitation to space-time measurements is interpreted as resulting from (e) quantum fluctuations of space-time itself.
(3) Thus, at very short distance scales, space-time is "foamy."
(4) This intrinsic foaminess of space-time provides (eb) another source of noise in the interferometers.
(5) The LIGO/VIRGO and LISA generations of gravity-wave interferometers, through future refinements, are expected to reach (eb) displacement noise levels low enough to test our proposed degree of foaminess in the structure of space-time.

Source: Wikipedia:

String theory as currently understood makes a series of predictions for the structure of the Universe at the largest scales. Many phases in string theory have very large, positive vacuum energy.[23] Regions of the Universe that are in such a phase will inflate exponentially rapidly in a process known as eternal inflation. As such, the theory predicts that most of the Universe is very rapidly expanding. However, these expanding phases are not stable, and can decay via the nucleation of bubbles of lower vacuum energy. Since our local region of the Universe is not very rapidly expanding, string theory predicts we are inside such a bubble. The spatial curvature of the "universe" inside the bubbles that form by this process is negative, a testable prediction.[24] Moreover, other bubbles will eventually form in the parent vacuum outside the bubble and collide with it. These collisions lead to potentially observable imprints on cosmology.[25] However, it is possible that neither of these will be observed if the spatial curvature is too small and the collisions are too rare. Under certain circumstances, fundamental strings produced at or near the end of inflation can be "stretched" to astronomical proportions. These cosmic strings could be observed in various ways, for instance by their gravitational lensing effects. However, certain field theories also predict cosmic strings

arising from topological defects in the field configuration.[26] (Wikipedia)

limitation to space-time measurements is interpreted as resulting from (e) quantum fluctuations of space-time itself

G_{36} : Category one of quantum fluctuations of space-time itself

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of limitation to space-time measurements is interpreted as resulting

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

Authors suggest that these "particles" obey (eb) infinite statistics in which all representations of (e) the particle permutation group can occur, and that the nonlocality present in (eb) systems obeying infinite statistics may be related to (e&eb)the nonlocality present in holographic theories.

They also propose to detect spacetime foam by looking for (e) halos in the images of distant quasars, and argue that it does not (e) modify the GZK cutoff in the ultra-high energy cosmic ray spectrum and its contributions to (e&eb) time-offlight differences of high energy gamma rays from (e) distant GRB are too small to be detectable.

G_{40} : Category one of these "particles"

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of infinite statistics in which all representations of (e) the particle permutation group can occur, and that the nonlocality present in (eb) systems obeying infinite statistics may be related to (e&eb)the nonlocality present in holographic theories

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

these "particles" obey infinite statistics in which all representations of (e) the particle permutation group can occur, and that the nonlocality present in (eb) systems obeying infinite statistics may be related to (e&eb)the nonlocality present in holographic theories

G_{44} : Category one of particle permutation group can occur, and that the nonlocality present in (eb) systems obeying infinite statistics may be related to (e&eb)the nonlocality present in holographic theories

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of these "particles" obey infinite statistics in which all representations

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}\ ,\ (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},\ \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**
$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**
Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$
$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$
$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$
$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$
$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$
$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**
Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$
$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$
$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$
$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$
$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$
$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**
Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$
$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$
$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}$ $\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}$ are eight augmentation coefficient for 1,2,3

$\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)}\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) & \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{15} \quad (60)$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t) & \\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t) & \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & +(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) & +(a''_{21})^{(3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t) & \\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t) & \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & +(a''_{18})^{(2)}(T_{17},t) & +(a''_{15})^{(1,1,)}(T_{14},t) & +(a''_{22})^{(3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33},t) & \\ +(a''_{38})^{(7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9)}(T_{45},t) & \end{bmatrix} G_{18} \quad (63)$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$-(b_{44}'')^{(9,9)}(G_{47},t)$, $-(b_{46}'')^{(9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a_{20}')^{(3)} & +(a_{20}'')^{(3)}(T_{21},t) & +(a_{16}'')^{(2,2,2)}(T_{17},t) & +(a_{13}'')^{(1,1,1,)}(T_{14},t) \\ +(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t) & +(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t) & +(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a_{36}'')^{(7,7,7,7)}(T_{37},t) & +(a_{40}'')^{(8,8,8,8)}(T_{41},t) & +(a_{44}'')^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a_{21}')^{(3)} & +(a_{21}'')^{(3)}(T_{21},t) & +(a_{17}'')^{(2,2,2)}(T_{17},t) & +(a_{14}'')^{(1,1,1,)}(T_{14},t) \\ +(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t) & +(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t) & +(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a_{37}'')^{(7,7,7,7)}(T_{37},t) & +(a_{41}'')^{(8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a_{22}')^{(3)} & +(a_{22}'')^{(3)}(T_{21},t) & +(a_{18}'')^{(2,2,2)}(T_{17},t) & +(a_{15}'')^{(1,1,1,)}(T_{14},t) \\ +(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t) & +(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t) & +(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a_{38}'')^{(7,7,7,7)}(T_{37},t) & +(a_{42}'')^{(8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{22} \quad (69)$$

$+(a_{20}'')^{(3)}(T_{21},t)$, $+(a_{21}'')^{(3)}(T_{21},t)$, $+(a_{22}'')^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a_{40}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a_{44}'')^{(9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b_{20}')^{(3)} & -(b_{20}'')^{(3)}(G_{23},t) & -(b_{16}'')^{(2,2,2)}(G_{19},t) & -(b_{13}'')^{(1,1,1,)}(G,t) \\ -(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b_{36}'')^{(7,7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8,8)}(G_{43},t) & -(b_{44}'')^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b_{21}')^{(3)} & -(b_{21}'')^{(3)}(G_{23},t) & -(b_{17}'')^{(2,2,2)}(G_{19},t) & -(b_{14}'')^{(1,1,1,)}(G,t) \\ -(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b_{37}'')^{(7,7,7,7)}(G_{39},t) & -(b_{41}'')^{(8,8,8,8)}(G_{43},t) & -(b_{45}'')^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* $1, 2\ 3$

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$

$are\ fourth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$
$are\ fifth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$
$are\ sixth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$
$are\ seventh\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$
$are\ eighth\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$
$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

$Where\ -(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ $are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ $are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ $are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
$are\ eighth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$
$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad (81)$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \quad (84)$$

$where\ \boxed{-(b''_{28})^{(5)}(G_{31},t)}\ ,\boxed{-(b''_{29})^{(5)}(G_{31},t)}\ ,\boxed{-(b''_{30})^{(5)}(G_{31},t)}\ \ are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}\ ,\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}\ ,\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}\ are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}\ ,\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}\ ,\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}\ are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}\ ,\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}\ ,\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}\ ,\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}\ ,\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}\ ,\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}\ ,\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}\ ,\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)}\ ,\boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)}\ ,\boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)}\ ,\boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)}\ ,\boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)}\ ,\boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt}=(a_{32})^{(6)}G_{33} \qquad 85$$

$$-\begin{bmatrix}(a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{32}$$

$$\frac{dG_{33}}{dt}=(a_{33})^{(6)}G_{32}-\begin{bmatrix}(a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{33} \qquad 86$$

$$\frac{dG_{34}}{dt}=(a_{34})^{(6)}G_{33}-\begin{bmatrix}(a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{34} \qquad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}\ ,\boxed{+(a''_{33})^{(6)}(T_{33},t)}\ ,\boxed{+(a''_{34})^{(6)}(T_{33},t)}\ are\ first\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\ ,\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\ ,\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\ are\ second\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}\ ,\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}\ ,\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}\ \ are\ third\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\ ,\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\ ,\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\ ,\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\ ,\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation

coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$ Eighth augmentation coefficients

$+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)} & -(b''_{32})^{(6)}(G_{35},t) & -(b''_{28})^{(5,5,5)}(G_{31},t) & -(b''_{24})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{13})^{(1,1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)} & -(b''_{33})^{(6)}(G_{35},t) & -(b''_{29})^{(5,5,5)}(G_{31},t) & -(b''_{25})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{14})^{(1,1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & -(b''_{34})^{(6)}(G_{35},t) & -(b''_{30})^{(5,5,5)}(G_{31},t) & -(b''_{26})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{15})^{(1,1,1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{34} \quad 90$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,)}(G_{27},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3

$-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3

$-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2, and 3

$-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \qquad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt}$$

$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt}$$

$$= (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt}$$
$$= (a_{42})^{(8)} G_{41}$$
$$- \left[\begin{array}{c} (a'_{42})^{(8)} \boxed{+(a''_{42})^{(8)}(T_{41},t)} \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array} \right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$
$$(b_{40})^{(8)} T_{41} - \left[\begin{array}{c} (b'_{40})^{(8)} \boxed{-(b''_{40})^{(8)}(G_{43},t)} \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{13}$$

$$\frac{dT_{41}}{dt} =$$
$$(b_{41})^{(8)} T_{40} - \left[\begin{array}{c} (b'_{41})^{(8)} \boxed{-(b''_{41})^{(8)}(G_{43},t)} \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{14}$$

$$\frac{dT_{42}}{dt} =$$
$$(b_{42})^{(8)} T_{41} - \left[\begin{array}{c} (b'_{42})^{(8)} \boxed{-(b''_{42})^{(8)}(G_{43},t)} \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{38}'')^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

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$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a_{44}')^{(9)}\boxed{+(a_{44}'')^{(9)}(T_{45},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a_{45}')^{(9)}\boxed{+(a_{45}'')^{(9)}(T_{45},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a_{46}')^{(9)}\boxed{+(a_{46}'')^{(9)}(T_{37},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{44}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{46}'')^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second

augmentation coefficient for category 1, 2 and 3
$+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3
$+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3
$+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3
$+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3
$+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)$ are Seventh augmentation coefficient for category 1, 2 and 3
$+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ $+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3
$+(a_{40}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$
$$(b_{44})^{(9)}T_{45} -$$
$$\begin{bmatrix} (b_{44}')^{(9)}\left[-(b_{44}'')^{(9)}(G_{47},t)\right] & \left[-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{13}$$
$$\frac{dT_{45}}{dt} =$$
$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}\left[-(b_{45}'')^{(9)}(G_{47},t)\right] & \left[-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{14}$$
$$\frac{dT_{46}}{dt} =$$
$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}\left[-(b_{46}'')^{(9)}(G_{47},t)\right] & \left[-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)\right] \end{bmatrix} T_{15}$$

Where $-(b_{44}'')^{(9)}(G_{47},t)$, $-(b_{45}'')^{(9)}(G_{47},t)$, $-(b_{46}'')^{(9)}(G_{47},t)$ are first detrition coefficients for category 1, 2 and 3
$-(b_{16}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3
$-(b_{20}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3
$-(b_{24}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}'| e^{-(\hat{M}_{20})^{(3)} t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} ||G_{23} - G_{23}'|| e^{-(\hat{M}_{20})^{(3)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and $(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\ ,\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)},\ (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} + \ (\hat{A}_{20})^{(3)} + \ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} + \ (\hat{B}_{20})^{(3)} + \ (\hat{Q}_{20})^{(3)}\ \ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)},\ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}), t\big) \le \ (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}\,(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\,\big((G_{27}), t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)},\ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - \ T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30$ 122

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}((G_{31}), t) \leq (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)}$$ 123

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\ ,\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)},\ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ \ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)},\ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \le (p_i)^{(6)} \le (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \le\ \ (r_i)^{(6)} \le (b_i')^{(6)} \le (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}\,(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G\to\infty}(b_i'')^{(6)}\left((G_{35}), t\right) = \ \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)},\ \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \le (\hat{k}_{32})^{(6)}|T_{33} - \ T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(AAAAAAA) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(BBBBBBB) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

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(CCCCCCC) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(DDDDDDD)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(EEEEEEE) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(FFFFFFF) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)} \qquad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be

noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\ ,\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}<1 \qquad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+\ (\hat{A}_{40})^{(8)}+(\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \qquad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[\ (b_i)^{(8)}+(b_i')^{(8)}+\ (\hat{B}_{40})^{(8)}+(\hat{Q}_{40})^{(8)}\ (\hat{k}_{40})^{(8)}]<1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \qquad 146A$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t)-(a_i'')^{(9)}(T_{45},t)|\le(\hat{k}_{44})^{(9)}|T_{45}-\ T_{45}'|e^{-(\widehat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t)-(b_i'')^{(9)}\big((G_{47}),t\big)|<(\hat{k}_{44})^{(9)}||(G_{47})-(G_{47})'||e^{-(\widehat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$

and$(a_i'')^{(9)}(T_{45},t)$. (T_{45}',t) and (T_{45},t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)},(\widehat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{44})^{(9)}=1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$:

$(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},$ are positive constants

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\ ,\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}<1$$

Definition of $(\hat{P}_{44})^{(9)},(\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\widehat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},(\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)},(a_i')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(p_i)^{(9)},\ (r_i)^{(9)},i=44,45,46,$
satisfy the inequalities

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[(a_i)^{(9)}+(a_i')^{(9)}+\ (\hat{A}_{44})^{(9)}+\ (\hat{P}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)}+(b_i')^{(9)}+\ (\hat{B}_{44})^{(9)}+\ (\hat{Q}_{44})^{(9)}\ \ (\hat{k}_{44})^{(9)}]<1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\ ,T_i(0)$:

$$G_i(t)\leq\ \left(\hat{P}_{13}\right)^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ ,\ \boxed{G_i(0)=G_i^0>0}$$

$$T_i(t)\leq\ (\hat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\quad ,\quad \boxed{T_i(0)=T_i^0>0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\ ,T_i(0)$

$$G_i(t)\leq\ (\hat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\quad ,\quad G_i(0)=G_i^0>0$$

$$T_i(t)\leq\ (\hat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}\quad ,\quad T_i(0)=T_i^0>0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t)\leq\ (\hat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\quad ,\quad G_i(0)=G_i^0>0$$

$$T_i(t)\leq\ (\hat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}\quad ,\quad T_i(0)=T_i^0>0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0)\ ,T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{24}\right)^{(4)} e^{(\hat{M}_{24})^{(4)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0)$, $T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{28}\right)^{(5)} e^{(\hat{M}_{28})^{(5)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0)$, $T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{32}\right)^{(6)} e^{(\hat{M}_{32})^{(6)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)$, $T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)$, $T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)$, $T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{13})^{(1)}, T_i^0 \le (\hat{Q}_{13})^{(1)}$, 155

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} t}$ 156

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} t}$ 157

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}, T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\tilde{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\tilde{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)} (G_{19}(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{32})^{(6)}$, $T_i^0 \leq (\hat{Q}_{32})^{(6)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0,t)$

Proof: 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(aa) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left(e^{(\tilde{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\tilde{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left(e^{(\tilde{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\tilde{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0\right) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\tilde{M}_{44})^{(9)}} \left(e^{(\tilde{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\tilde{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\tilde{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\tilde{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\tilde{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[(\widehat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on

$G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 19 to 24 it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(1)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If $G_{13}\ \ is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}\,(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right),\quad t = log\,\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on

(G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\,(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on T_{21} and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \quad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 227

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24{,}25{,}26$ depend only on T_{25} and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(4)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\,((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\widehat{P}_{28})^{(5)}$ and $(\widehat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\widehat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \tag{234}$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \tag{235}$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric (236)

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{31}}),(\widetilde{T_{31}})$: $\left((\widetilde{G_{31}}),(\widetilde{T_{31}})\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\leq \tag{237}$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\ (G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(5)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)}+\left((\widehat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\widehat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\widehat{Q}_{32})^{(6)}\right]\leq(\widehat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(6)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose

$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)}+\left((\hat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{36})^{(7)}\right]\leq(\hat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{39}}),(\widetilde{T_{39}}):\left((\widetilde{G_{39}}),(\widetilde{T_{39}})\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$

It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}\left|(a''_{36})^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a''_{36})^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\ e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\leq \quad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\ (G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})}, \widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by 274

$(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ *and* $(\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$. The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right)) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0, (\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0, (u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0, (\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0, (\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}, \ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}, \ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13} e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13} e^{(S_1)^{(1)} t}$

where $(p_i)^{(1)}$ is defined by equation

$\frac{1}{(m_1)^{(1)}} G^0_{13} e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G^0_{13} e^{(S_1)^{(1)} t}$

$$\left(\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t}-e^{-(S_2)^{(1)}t}\right]+G_{15}^{0}e^{-(S_2)^{(1)}t}\le G_{15}(t)\le\right. \quad (286)$$

$$\left.\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}\right]+G_{15}^{0}e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation (291)

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: (292)

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(T_{17},t)+(a''_{17})^{(2)}(T_{17},t)\le-(\sigma_1)^{(2)} \quad (293)$$

$$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)} \quad (294)$$

Definition of $(\nu_1)^{(2)},(\nu_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: (295)

By $(\nu_1)^{(2)}>0$, $(\nu_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0$, $(u_2)^{(2)}<0$ the roots (296)

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_1)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ (297)

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and (298)

Definition of $(\bar{\nu}_1)^{(2)},,(\bar{\nu}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: (299)

By $(\bar{\nu}_1)^{(2)}>0$, $(\bar{\nu}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0$, $(\bar{u}_2)^{(2)}<0$ the (300)

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_2)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ (301)

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t}\right] + T_{18}^0 e^{-(R_2)^{(2)}t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations
$(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$
and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by
$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, ***if*** $(\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, ***if*** $(\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, ***if*** $(\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$$

$$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}(v^{(4)})^2 + (\sigma_1)^{(4)} v^{(4)} - (a_{24})^{(4)} = 0$

and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)},,(\bar{v}_2)^{(4)},(\bar{u}_1)^{(4)},(\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$ and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)},(\mu_2)^{(4)},(v_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)},(\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, ***if*** $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)}G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)}G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)}t} - e^{-(a'_{26})^{(4)}t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)}t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^{0}}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t}-e^{-(R_2)^{(4)}t}\right]+T_{26}^{0}e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)},(S_2)^{(4)},(R_1)^{(4)},(R_2)^{(4)}$:- 337

Where $(S_1)^{(4)}=(a_{24})^{(4)}(m_2)^{(4)}-(a'_{24})^{(4)}$

$(S_2)^{(4)}=(a_{26})^{(4)}-(p_{26})^{(4)}$

$(R_1)^{(4)}=(b_{24})^{(4)}(\mu_2)^{(4)}-(b'_{24})^{(4)}$

$(R_2)^{(4)}=(b'_{26})^{(4)}-(r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)},(\sigma_2)^{(5)},(\tau_1)^{(5)},(\tau_2)^{(5)}$:

$(\sigma_1)^{(5)},(\sigma_2)^{(5)},(\tau_1)^{(5)},(\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)}\le-(a'_{28})^{(5)}+(a'_{29})^{(5)}-(a''_{28})^{(5)}(T_{29},t)+(a''_{29})^{(5)}(T_{29},t)\le-(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)}\le-(b'_{28})^{(5)}+(b'_{29})^{(5)}-(b''_{28})^{(5)}\big((G_{31}),t\big)-(b''_{29})^{(5)}\big((G_{31}),t\big)\le-(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)},(\nu_2)^{(5)},(u_1)^{(5)},(u_2)^{(5)},\nu^{(5)},u^{(5)}$: 339

By $(\nu_1)^{(5)}>0$, $(\nu_2)^{(5)}<0$ and respectively $(u_1)^{(5)}>0$, $(u_2)^{(5)}<0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2+(\sigma_1)^{(5)}\nu^{(5)}-(a_{28})^{(5)}=0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2+(\tau_1)^{(5)}u^{(5)}-(b_{28})^{(5)}=0$ and

Definition of $(\bar{\nu}_1)^{(5)},,(\bar{\nu}_2)^{(5)},(\bar{u}_1)^{(5)},(\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)}>0$, $(\bar{\nu}_2)^{(5)}<0$ and respectively $(\bar{u}_1)^{(5)}>0$, $(\bar{u}_2)^{(5)}<0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2+(\sigma_2)^{(5)}\nu^{(5)}-(a_{28})^{(5)}=0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2+(\tau_2)^{(5)}u^{(5)}-(b_{28})^{(5)}=0$

Definition of $(m_1)^{(5)},(m_2)^{(5)},(\mu_1)^{(5)},(\mu_2)^{(5)},(\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)},(m_2)^{(5)},(\mu_1)^{(5)},(\mu_2)^{(5)}$ by

$(m_2)^{(5)}=(\nu_0)^{(5)},(m_1)^{(5)}=(\nu_1)^{(5)}$, $\boldsymbol{if}\ (\nu_0)^{(5)}<(\nu_1)^{(5)}$

$(m_2)^{(5)}=(\nu_1)^{(5)},(m_1)^{(5)}=(\bar{\nu}_1)^{(5)}$, $\boldsymbol{if}\ (\nu_1)^{(5)}<(\nu_0)^{(5)}<(\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)}=\frac{G_{28}^{0}}{G_{29}^{0}}}$

$(m_2)^{(5)}=(\nu_1)^{(5)},(m_1)^{(5)}=(\nu_0)^{(5)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(5)}<(\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)}=(u_0)^{(5)},(\mu_1)^{(5)}=(u_1)^{(5)}$, $\boldsymbol{if}\ (u_0)^{(5)}<(u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, $\boldsymbol{if}$ $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}$ $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{28}(t) \leq G_{28}^0 e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{29}(t) \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})} \left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t} \right] + G_{30}^0 e^{-(S_2)^{(5)} t} \leq G_{30}(t) \leq \right. \quad 344$$

$$\left. \frac{(a_{30})^{(5)} G_{28}^0}{(m_2)^{(5)}((S_1)^{(5)} - (a'_{30})^{(5)})} \left[e^{(S_1)^{(5)} t} - e^{-(a'_{30})^{(5)} t} \right] + G_{30}^0 e^{-(a'_{30})^{(5)} t} \right)$$

$$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \leq T_{28}(t) \leq T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \leq T_{28}(t) \leq \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})} \left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t} \right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \leq T_{30}(t) \leq \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})} \left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t} \right] + T_{30}^0 e^{-(R_2)^{(5)} t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \leq -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \leq -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \leq -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}((G_{35}), t) - (b''_{33})^{(6)}((G_{35}), t) \leq -(\tau_1)^{(6)}$$

Definition of $(v_1)^{(6)}, (v_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, v^{(6)}, u^{(6)}$: 350

By $(v_1)^{(6)} > 0$, $(v_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{v}_1)^{(6)},, (\bar{v}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)} > 0$, $(\bar{v}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(v^{(6)}\right)^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}$, $(m_2)^{(6)}$, $(\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}$, $(m_2)^{(6)}$, $(\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, ***if*** $(\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, ***if*** $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, ***if*** $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, ***if*** $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, ***if*** $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}$, ***if*** $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t} - e^{-(a'_{34})^{(6)}t}\right] + G_{34}^0 e^{-(a'_{34})^{(6)}t}\right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^{0}e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}}T_{32}^{0}e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^{0}}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t}-e^{-(b'_{34})^{(6)}t}\right]+T_{34}^{0}e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^{0}}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t}-e^{-(R_2)^{(6)}t}\right]+T_{34}^{0}e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(v_1)^{(7)}, (v_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, v^{(7)}, u^{(7)}$: 361

By $(v_1)^{(7)} > 0, (v_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{v}_1)^{(7)}, , (\bar{v}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)} > 0, (\bar{v}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)},\ \boldsymbol{if}\ (\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)},\ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}),t\big) - (b''_{45})^{(9)}\big((G_{47}),t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)},, (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, ***if*** $(\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, ***if*** $(\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, ***if*** $(\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, ***if*** $(u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, ***if*** $(u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,
and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}$, ***if*** $(\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44}e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44}e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G^0_{44}e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G^0_{44}e^{(S_1)^{(9)}t}$$

(

$$\frac{(a_{46})^{(9)}G^0_{44}}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})}\Big[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t}\Big] + G^0_{46}e^{-(S_2)^{(9)}t} \le G_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}G^0_{44}}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})}[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G^0_{46}e^{-(a'_{46})^{(9)}t})$$

$$\boxed{T_{44}^{0}e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^{0}e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}}$$

$$\frac{1}{(\mu_1)^{(9)}}T_{44}^{0}e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}}T_{44}^{0}e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}$$

$$\frac{(b_{46})^{(9)}T_{44}^{0}}{(\mu_1)^{(9)}\left((R_1)^{(9)}-(b'_{46})^{(9)}\right)}\left[e^{(R_1)^{(9)}t}-e^{-(b'_{46})^{(9)}t}\right]+T_{46}^{0}e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^{0}}{(\mu_2)^{(9)}\left((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)}\right)}\left[e^{\left((R_1)^{(9)}+(r_{44})^{(9)}\right)t}-e^{-(R_2)^{(9)}t}\right]+T_{46}^{0}e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^{0}}{G_{14}^{0}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)}+(C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{dv^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)v^{(2)} - (a_{17})^{(2)}v^{(2)}$$

Definition of $v^{(2)}$:- $\boxed{v^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(v^{(2)}\right)^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)}\right) \leq \frac{dv^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(v^{(2)}\right)^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{v}_1)^{(2)}, (v_0)^{(2)}$:-

For $0 < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (v_1)^{(2)} < (\bar{v}_1)^{(2)}$

$$v^{(2)}(t) \geq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(v_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (v_2)^{(2)}}}$$

it follows $(v_0)^{(2)} \leq v^{(2)}(t) \leq (v_1)^{(2)}$

In the same manner , we get 391

$$v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{v}_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (\bar{v}_2)^{(2)}}}$$

From which we deduce $(v_0)^{(2)} \leq v^{(2)}(t) \leq (\bar{v}_1)^{(2)}$ 392

If $0 < (v_1)^{(2)} < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{v}_1)^{(2)}$ we find like in the previous case, 393

$$(v_1)^{(2)} \leq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_2)^{(2)}\right)t\right]}} \leq v^{(2)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \leq (\bar{v}_1)^{(2)}$$

If $0 < (v_1)^{(2)} \leq (\bar{v}_1)^{(2)} \leq (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(v_1)^{(2)} \leq v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \leq (v_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(2)}(t)$:- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a_{20}')^{(3)} - (a_{21}')^{(3)} + (a_{20}'')^{(3)}(T_{21}, t)\right) - (a_{21}'')^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $v^{(4)}$:- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \leq \frac{dv^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(4)}, (v_0)^{(4)}$:-

For $0 < \boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (v_1)^{(4)} < (\bar{v}_1)^{(4)}$

$$v^{(4)}(t) \geq \frac{(v_1)^{(4)} + (C)^{(4)}(v_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((v_1)^{(4)} - (v_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((v_1)^{(4)} - (v_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(v_1)^{(4)} - (v_0)^{(4)}}{(v_0)^{(4)} - (v_2)^{(4)}}}$$

it follows $(v_0)^{(4)} \leq v^{(4)}(t) \leq (v_1)^{(4)}$

In the same manner , we get 405

$$v^{(4)}(t) \leq \frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)}(\bar{v}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{v}_1)^{(4)} - (v_0)^{(4)}}{(v_0)^{(4)} - (\bar{v}_2)^{(4)}}}$$

From which we deduce $(v_0)^{(4)} \leq v^{(4)}(t) \leq (\bar{v}_1)^{(4)}$

If $0 < (v_1)^{(4)} < (v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{v}_1)^{(4)}$ we find like in the previous case, 406

$$(v_1)^{(4)} \leq \frac{(v_1)^{(4)} + (C)^{(4)}(v_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((v_1)^{(4)} - (v_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((v_1)^{(4)} - (v_2)^{(4)}\right)t\right]}} \leq v^{(4)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)}(\bar{v}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}\right)t\right]}} \leq (\bar{v}_1)^{(4)}$$

If $0 < (v_1)^{(4)} \leq (\bar{v}_1)^{(4)} \leq \boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(v_1)^{(4)} \leq v^{(4)}(t) \leq \frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)}(\bar{v}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}\right)t\right]}} \leq (v_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(4)}(t)$:-

$$(m_2)^{(4)} \leq v^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{v^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then
$(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

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If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\quad \boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$v^{(6)}(t) \le \frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{v}_1)^{(6)}-(v_0)^{(6)}}{(v_0)^{(6)}-(\bar{v}_2)^{(6)}}}$$

From which we deduce $(v_0)^{(6)} \le v^{(6)}(t) \le (\bar{v}_1)^{(6)}$

If $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{v}_1)^{(6)}$ we find like in the previous case, 414

$$(v_1)^{(6)} \le \frac{(v_1)^{(6)}+(C)^{(6)}(v_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)}-(v_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)}-(v_2)^{(6)}\right)t\right]}} \le v^{(6)}(t) \le$$

$$\frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \le (\bar{v}_1)^{(6)}$$

If $0 < (v_1)^{(6)} \le (\bar{v}_1)^{(6)} \le \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(v_1)^{(6)} \le v^{(6)}(t) \le \frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \le (v_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(6)}(t)$:-

$$(m_2)^{(6)} \le v^{(6)}(t) \le (m_1)^{(6)}, \quad \boxed{v^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \le u^{(6)}(t) \le (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(v_1)^{(6)} = (\bar{v}_1)^{(6)}$ if in addition $(v_0)^{(6)} = (v_1)^{(6)}$ then $v^{(6)}(t) = (v_0)^{(6)}$ and as a consequence $G_{32}(t) = (v_0)^{(6)}G_{33}(t)$ **this also defines** $(v_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(v_1)^{(6)}$ and $(\bar{v}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}\right)\leq\frac{dv^{(7)}}{dt}\leq-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0<\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}<(\nu_1)^{(7)}<(\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t)\geq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}\quad,\quad\boxed{(C)^{(7)}=\frac{(\nu_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)}\leq\nu^{(7)}(t)\leq(\nu_1)^{(7)}$

In the same manner , we get 417

$$\nu^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(7)}=\frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)}\leq\nu^{(7)}(t)\leq(\bar{\nu}_1)^{(7)}$

If $0<(\nu_1)^{(7)}<(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}<(\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)}\leq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}\leq\nu^{(7)}(t)\leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\bar{\nu}_1)^{(7)}$$

If $0<(\nu_1)^{(7)}\leq(\bar{\nu}_1)^{(7)}\leq\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)}\leq\nu^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)}\leq\nu^{(7)}(t)\leq(m_1)^{(7)},\quad\boxed{\nu^{(7)}(t)=\frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (v_1)^{(8)} < (v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{v}_1)^{(8)}$ we find like in the previous case, 423

$$(v_1)^{(8)} \leq \frac{(v_1)^{(8)}+(C)^{(8)}(v_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((v_1)^{(8)}-(v_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((v_1)^{(8)}-(v_2)^{(8)}\right)t\right]}} \leq v^{(8)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(8)}+(\bar{C})^{(8)}(\bar{v}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}} \leq (\bar{v}_1)^{(8)}$$

If $0 < (v_1)^{(8)} \leq (\bar{v}_1)^{(8)} \leq \boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(v_1)^{(8)} \leq v^{(8)}(t) \leq \frac{(\bar{v}_1)^{(8)}+(\bar{C})^{(8)}(\bar{v}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}} \leq (v_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(8)}(t)$:-

$$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(v_1)^{(8)} = (\bar{v}_1)^{(8)}$ if in addition $(v_0)^{(8)} = (v_1)^{(8)}$ then $v^{(8)}(t) = (v_0)^{(8)}$ and as a consequence $G_{40}(t) = (v_0)^{(8)}G_{41}(t)$ **this also defines** $(v_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(v_1)^{(8)}$ and $(\bar{v}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \leq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \leq \nu^{(9)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \leq (\bar{\nu}_1)^{(9)} \leq \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \leq \nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \leq \nu^{(9)}(t) \leq (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for**

the special case .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(v_1)^{(9)}$ and $(\bar{v}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a_i'')^{(1)} and\ (b_i'')^{(1)}$ are independent on t , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0$ 447

$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0$ 448

has a unique positive solution , which is an equilibrium solution

$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0$ 449

$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0$ 450

$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0$ 451

$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0$ 452

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(aa) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T_{33}^{*} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^{*} for which $f(T_{33}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

Definition and uniqueness of T_{37}^{*} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^{*} for which $f(T_{37}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^{*})]}$$

Definition and uniqueness of T_{41}^{*} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^{*} for which $f(T_{41}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^{*})]}$$

Definition and uniqueness of T_{45}^{*} :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^{*} for which $f(T_{45}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^{*})]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi(G^{*}) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that

there exists a unique G_{37}^{*} such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)}G_{17}^{*}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)}G_{17}^{*}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T_{17}^{*})]} \quad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)}T_{17}^{*}}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)}T_{17}^{*}}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^{*})]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)}G_{21}^{*}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)}G_{21}^{*}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$$T_{28}^{*} = \frac{(b_{28})^{(5)} T_{29}^{*}}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^{*})]} \quad , \quad T_{30}^{*} = \frac{(b_{30})^{(5)} T_{29}^{*}}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^{*})]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)} G_{33}^{*}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)} G_{33}^{*}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

$$T_{32}^{*} = \frac{(b_{32})^{(6)} T_{33}^{*}}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)} T_{33}^{*}}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^{*})]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$$G_{36}^{*} = \frac{(a_{36})^{(7)} G_{37}^{*}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38}^{*} = \frac{(a_{38})^{(7)} G_{37}^{*}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^{*})]}$$

$$T_{36}^{*} = \frac{(b_{36})^{(7)} T_{37}^{*}}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^{*})]} \quad , \quad T_{38}^{*} = \frac{(b_{38})^{(7)} T_{37}^{*}}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^{*})]}$$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$$G_{40}^{*}=\frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}\quad,\quad G_{42}^{*}=\frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

$$T_{40}^{*}=\frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}\quad,\quad T_{42}^{*}=\frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*})=0$, T_{45}^{*} given by $f(T_{45}^{*})=0$ and

$$G_{44}^{*}=\frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}\quad,\quad G_{46}^{*}=\frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$$

$$T_{44}^{*}=\frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}\quad,\quad T_{46}^{*}=\frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}\ and\ (b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i=G_i^{*}+\mathbb{G}_i\qquad,T_i=T_i^{*}+\mathbb{T}_i$$

$$\frac{\partial(a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^{*})=(q_{14})^{(1)}\quad,\ \frac{\partial(b_i'')^{(1)}}{\partial G_j}(\,G^{*}\,)=s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt}=-\big((a'_{13})^{(1)}+(p_{13})^{(1)}\big)\mathbb{G}_{13}+(a_{13})^{(1)}\mathbb{G}_{14}-(q_{13})^{(1)}G_{13}^{*}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt}=-\big((a'_{14})^{(1)}+(p_{14})^{(1)}\big)\mathbb{G}_{14}+(a_{14})^{(1)}\mathbb{G}_{13}-(q_{14})^{(1)}G_{14}^{*}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt}=-\big((a'_{15})^{(1)}+(p_{15})^{(1)}\big)\mathbb{G}_{15}+(a_{15})^{(1)}\mathbb{G}_{14}-(q_{15})^{(1)}G_{15}^{*}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt}=-\big((b'_{13})^{(1)}-(r_{13})^{(1)}\big)\mathbb{T}_{13}+(b_{13})^{(1)}\mathbb{T}_{14}+\textstyle\sum_{j=13}^{15}\big(s_{(13)(j)}T_{13}^{*}\mathbb{G}_j\big) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt}=-\big((b'_{14})^{(1)}-(r_{14})^{(1)}\big)\mathbb{T}_{14}+(b_{14})^{(1)}\mathbb{T}_{13}+\textstyle\sum_{j=13}^{15}\big(s_{(14)(j)}T_{14}^{*}\mathbb{G}_j\big) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt}=-\big((b'_{15})^{(1)}-(r_{15})^{(1)}\big)\mathbb{T}_{15}+(b_{15})^{(1)}\mathbb{T}_{14}+\textstyle\sum_{j=13}^{15}\big(s_{(15)(j)}T_{15}^{*}\mathbb{G}_j\big) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions

$(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \tag{532}$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \ , \ \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \tag{533}$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \tag{534}$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \tag{535}$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \tag{536}$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \tag{537}$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}')^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T_{17}^*\mathbb{G}_j\right) \tag{538}$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}')^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T_{18}^*\mathbb{G}_j\right) \tag{539}$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \ , \ \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}')^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \tag{541}$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}')^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \tag{542}$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}')^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \tag{543}$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}')^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T_{20}^*\mathbb{G}_j\right) \tag{544}$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^{*}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T_{22}^{*}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^{*}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^{*}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^{*}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^{*}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^{*}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^{*}\mathbb{G}_j\right) \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}(\,(G_{31})^*\,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^{*}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^{*}) = (q_{41})^{(8)} \quad , \frac{\partial(b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^{*}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^{*}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^{*}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^{*}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \ , \ \frac{\partial(b_i'')^{(9)}}{\partial G_j}(\ (G_{47})^*\ ) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\ B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\ C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\ D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\ E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\ F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\ G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left(\ (a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)\ (\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left(\ (b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)\ (\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14}+(b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\Big]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION THIRTY SIX A

Measuring The Foaminess Of Space-Time

INTRODUCTION—VARIABLES USED

May 2000, Volume 30, Issue 5, pp 795-805: Measuring the Foaminess of Space-Time with Gravity-Wave Interferometers Y. Jack Ng, H. van Dam Foundations of Physics

(1) By analyzing a gedanken experiment designed to measure the distance l between two spatially separated points, authors find that this distance cannot (e) be measured with uncertainty less than (ll 2 P) 1/3 , considerably larger than the Planck scale lP (or the string scale in string theories), the conventional-wisdom uncertainty in distance measurements.
(2) This limitation to space-time measurements is interpreted as resulting from (e) quantum fluctuations of space-time itself.
(3) Thus, at very short distance scales, space-time is “foamy.”
(4) This intrinsic foaminess of space-time provides (eb) another source of noise in the interferometers.
(5) The LIGO/VIRGO and LISA generations of gravity-wave interferometers, through future refinements, are expected to reach (eb) displacement noise levels low enough to test our proposed degree of foaminess in the structure of space-time.

Source: Wikipedia:

String theory as currently understood makes a series of predictions for the structure of the Universe at the largest scales. Many phases in string theory have very large, positive vacuum energy.[23] Regions of the Universe that are in such a phase will inflate exponentially rapidly in a process known as eternal inflation. As such, the theory predicts that most of the Universe is very rapidly expanding. However, these expanding phases are not stable, and can decay via the nucleation of bubbles of lower vacuum energy. Since our local region of the Universe is not very rapidly expanding, string theory predicts we are inside such a bubble. The spatial curvature of the "universe" inside the bubbles that form by this process is negative, a testable prediction.[24] Moreover, other bubbles will eventually form in the parent vacuum outside the bubble and collide with it. These collisions lead to potentially observable imprints on cosmology.[25] However, it is possible that neither of these will be observed if the spatial curvature is too small and the collisions are too rare. Under certain circumstances, fundamental strings produced at or near the end of inflation can be "stretched" to astronomical proportions. These cosmic strings could be observed in various ways, for instance by their gravitational lensing effects. However, certain field theories also predict cosmic strings arising from topological defects in the field configuration.[26] (Wikipedia)

NOTATION

Module One

these "particles" obey infinite statistics in which all representations of the particle permutation group can occur, and that the nonlocality present in (eb) systems obeying infinite statistics may be related to (e&eb)the nonlocality present in holographic theories

G_{13} : Category one of these "particles" obey infinite statistics in which all representations of the particle permutation group can occur, and that the nonlocality present

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of systems obeying infinite statistics may be related to (e&eb)the nonlocality present in holographic theories

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

these "particles" obey infinite statistics in which all representations of the particle permutation group can occur, and that the nonlocality present in systems obeying infinite statistics may be related to (e&eb)the nonlocality present in holographic theories

G_{16} : Category one of these "particles" obey infinite statistics in which all representations of the particle permutation group can occur, and that the nonlocality present in systems obeying infinite statistics; nonlocality present in holographic theories

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of nonlocality present in holographic theories; these "particles" obey infinite statistics in which all representations of the particle permutation group can occur, and that the nonlocality present in systems obeying infinite statistics

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

They also propose to detect spacetime foam by looking for (e) halos in the images of distant quasars, and argue that it does not (e) modify the GZK cutoff in the ultra-high energy cosmic ray spectrum and its contributions to (e&eb) time-offlight differences of high energy gamma rays from (e) distant GRB are too small to be detectable

G_{20} : Category one of halos in the images of distant quasars, and argue that it does not (e) modify the GZK cutoff in the ultra-high energy cosmic ray spectrum and its contributions to (e&eb) time-offlight differences of high energy gamma rays from (e) distant GRB are too small to be detectable

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of detect spacetime foam by looking

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

detect spacetime foam by looking for halos in the images of distant quasars, and argue that it does not (e) modify the GZK cutoff in the ultra-high energy cosmic ray spectrum and its contributions to (e&eb) time-offlight differences of high energy gamma rays from (e) distant GRB are too small to be detectable

G_{24} : Category one of modification of the GZK cutoff in the ultra-high energy cosmic ray spectrum and its contributions to (e&eb) time-offlight differences of high energy gamma rays from (e) distant GRB are too small to be detectable

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of detect spacetime foam by looking for halos in the images of distant quasars, and argue that it

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

detect spacetime foam by looking for halos in the images of distant quasars, and argue that it does not modify the GZK cutoff in the ultra-high energy cosmic ray spectrum and its contributions to (e&eb) time-offlight differences of high energy gamma rays from (e) distant GRB are too small to be detectable

G_{28} : Category one of detect spacetime foam by looking for halos in the images of distant quasars, and argue that it does not modify the GZK cutoff in the ultra-high energy cosmic ray spectrum; time-offlight differences of high energy gamma rays from (e) distant GRB are too small to be detectable

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of time-offlight differences of high energy gamma rays from (e) distant GRB are too small to be detectable ;detect spacetime foam by looking for halos in the images of distant quasars, and argue that it does not modify the GZK cutoff in the ultra-high energy cosmic ray spectrum

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

detect spacetime foam by looking for halos in the images of distant quasars, and argue that it does not modify the GZK cutoff in the ultra-high energy cosmic ray spectrum and its contributions to time-offlight differences of high energy gamma rays from (e) distant GRB are too small to be detectable

G_{32} : Category one of distant GRB are too small to be detectable

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of detect spacetime foam by looking for halos in the images of distant quasars, and argue that it does not modify the GZK cutoff in the ultra-high energy cosmic ray spectrum and its contributions to time-offlight differences of high energy gamma rays

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Thus, at very short distance scales, space-time is "foamy."

G_{36} : Category one of space-time at very short distance scales

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of "foamy." (note the degree of foaminess change)

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

This intrinsic foaminess of space-time provides (eb) another source of noise in the interferometers

G_{40} : Category one of intrinsic foaminess of space-time

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of another source of noise in the interferometers

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

The LIGO/VIRGO and LISA generations of gravity-wave interferometers, through future refinements, are expected to reach (eb) displacement noise levels low enough to test our proposed degree of foaminess in the structure of space-time.

G_{44} : Category one of generations of gravity-wave interferometers(Interferometry makes use of the principle of superposition to combine waves in a way that will cause the result of their combination to have some

meaningful property that is diagnostic of the original state of the waves. This works because when two waves with the same frequency combine, the resulting pattern is determined by the phase difference between the two waves—waves that are in phase will undergo constructive interference while waves that are out of phase will undergo destructive interference. Most interferometers use light or some other form of electromagnetic wave.[2]:3–12 (Wikipedia: Note the refinement means better the phase difference and other parameters : these factors are to be taken in to consideration in the prediction model that is presently suggested)

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of displacement noise levels low enough to test proposed degree of foaminess in the structure of space-time

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION THIRTY SEVEN

Fuzzball (String Theory)

INTRODUCTION—VARIABLES USED

Fuzzball (string theory) From Wikipedia, the free encyclopedia

(1) Theorized fuzzballs, like classic black holes, distort (e&eb) spacetime and bend light
(2) Here, the edge of the central dark spot, the event horizon, delineates not only the threshold where its escape velocity equals the speed of light but also a fuzzball's physical surface. (Artist rendition)
(3) Fuzzballs are theorized by some superstring theory scientists to be (=) the true quantum description of black holes. The theory resolves two intractable problems that classic black holes pose for modern physics:
(4) The information paradox wherein the quantum information bound in in‑falling matter and energy entirely disappears into (eb) a singularity; that is, the black hole would undergo (e&eb) zero physical change in its composition regardless of the nature of what fell into it.
(5) The singularity at the heart of the black hole, where conventional black hole theory says there is infinite spacetime curvature due to an infinitely intense gravitational field from a region of zero volume. Modern physics breaks down when such parameters are infinite and zero.[1]
(6) Fuzzball theory replaces the singularity at the heart of a black hole by positing that the entire region within the black hole's event horizon is actually a ball of strings, which are advanced as the ultimate building blocks of matter and energy. Strings are thought to be bundles of energy vibrating in complex ways in both the three physical dimensions of space as well as in compact directions—extra dimensions interwoven in the quantum foam (also known as spacetime foam).

Physical characteristics

(7) In some types of superstring theory—the basis of fuzzball theory—the extra dimensions of

spacetime are thought to take the form of a 6-dimensional Calabi–Yau manifold. Samir Mathur of Ohio State University, with postdoctoral researcher Oleg Lunin, proposed via two papers in 2002 that black holes are actually spheres of strings with a definite volume; they are not a singularity, which the classic view holds to be a zero-dimensional, zero-volume point into which a black hole's entire mass is concentrated.[2]

(8) String theory holds that the fundamental constituents of subatomic particles, including the force carriers (e.g. Leptons, photons, and gluons), all composed of a one-dimensional string of energy that takes on its identity by vibrating in different modes and/or frequencies. Quite unlike the view of a black hole as a singularity, a small fuzzball can be thought of as an extra-dense neutron star where its neutrons have decomposed, or "melted," liberating the quarks (strings in string theory) composing them. Accordingly, fuzzballs can be regarded as the most extreme form of degenerate matter.

(9) Whereas the event horizon of a classic black hole is thought to be very well defined and distinct, Mathur and Lunin further calculated that the event horizon of a fuzzball would, at an extremely small scale (likely on the order of a few Planck lengths),[3] be very much like a mist: fuzzy, hence the name "fuzzball." They also found that the physical surface of the fuzzball would have a radius equal to that of the event horizon of a classic black hole; for both, the Schwarzschild radius for a median-size stellar-mass black hole of 6.8 solar masses (M⊙) is 20 kilometers.

(10) With classical black holes, objects passing through the event horizon on their way to the singularity are thought to enter a realm of curved spacetime where the escape velocity exceeds the speed of light. It is a realm that is devoid of all structure. Further, at the singularity—the heart of a classic black hole—spacetime is thought to have infinite curvature (that is, gravity is thought to have infinite intensity) since its mass is believed to have collapsed to zero (infinitely small) volume where it has infinite density. Such infinite conditions are problematic with known physics because key calculations utterly collapse. With a fuzzball, however, the strings comprising an object are believed to simply fall onto and absorb into the surface of the fuzzball, which corresponds to the event horizon—the threshold at which the escape velocity equals the speed of light.

(11) A fuzzball is a black hole; spacetime, photons, and all else that is not exquisitely close to the surface of a fuzzball are thought to be affected in precisely the same fashion as with a classic black hole featuring a singularity at its center. Classic black holes and fuzzballs differ only at the quantum level; that is, they differ only in their internal composition as well as how they affect virtual particles that form close to their event horizons (see Information paradox, below). Fuzzball theory is thought by its proponents to be the true quantum description of black holes.

(12) Cygnus X-1, an 8.7 M⊙ black hole only 6,000 light years away in our own Milky Way galaxy, belongs to a binary system along with a blue supergiant variable star. If Cygnus X-1 is actually a fuzzball, its surface has a diameter of 51 kilometers.[4] Credit: ESA (Artist rendition)

(13) Since the volume of fuzzballs is a function of the Schwarzschild radius (2,954 kilometers per M⊙), fuzzballs have a variable density that decreases as the inverse square of their mass (twice the mass is twice the diameter, which is eight times the volume, resulting in one‑quarter the density). A typical 6.8 M⊙ fuzzball would have a mean density of 4.0×1017 kg/m3.[5] A bit of such a fuzzball the size of a drop of water would have a mass of twenty million metric tons, which is the mass of a granite ball 240 meters in diameter.[6] Though such densities are almost unimaginably extreme, they are, mathematically speaking, infinitely far from infinite density. Although the densities of typical stellar-mass fuzzballs are quite great—about the same as neutron stars[7]—their densities are many orders of magnitude less than the Planck density (5.155×1096 kg/m3), which is equivalent to the mass of the universe packed into the volume of a single atomic nucleus.

(14) Fuzzballs become less dense as their mass increases due to fractional tension. When matter or energy (strings) fall onto a fuzzball, more strings aren't simply added to the fuzzball; strings fuse

together, and in doing so, all the quantum information of the in‑falling strings becomes part of larger, more complex strings. Due to fractional tension, string tension exponentially decreases as they become more complex with more modes of vibration, relaxing to considerable lengths. The "mathematical beauty" of the string theory formulas Mathur and Lunin employed lies in how the fractional tension values produce fuzzball radii that precisely equal Schwarzschild radii, which Karl Schwarzschild calculated using an entirely different mathematical technique 87 years earlier.

(15) Due to the mass-density inverse-square rule, all fuzzballs need not have unimaginable densities. There are also supermassive black holes, which are found at the center of virtually all galaxies. Sagittarius A*, the black hole at the center of our Milky Way galaxy, is 4.3 million M⊙. If it is actually a fuzzball, it has a mean density that is "only" 51 times that of gold. At 3.9 billion M⊙, a fuzzball would have a radius of 77 astronomical units—about the same size as the termination shock of our solar system's heliosphere—and a mean density equal to that of the Earth's atmosphere at sea level (1.2 kg/m3).

(16) Irrespective of a fuzzball's mass and resultant density, the determining factor establishing where its surface lies is the threshold at which the fuzzball's escape velocity precisely equals the speed of light.[8] Escape velocity, as its name suggests, is the velocity a body must achieve to escape from a massive object. For earth, this is 11.2 km/s. In the other direction, a massive object's escape velocity is equal to the impact velocity achieved by a falling body that has fallen from the edge of a massive object's sphere of gravitational influence. Thus, event horizons—for both classic black holes and fuzzballs—lie precisely at the point where spacetime has warped to such an extent that falling bodies just achieve the speed of light. According to Albert Einstein, via his special theory of relativity, the speed of light is the maximum permissible velocity in spacetime. At this velocity, infalling matter and energy impacts the surface of the fuzzball and its now-liberated, individual strings contribute to the fuzzball's makeup.

NOTATION

Module One

Theorized fuzzballs, like classic black holes, distort (e&eb) spacetime and bend light

G_{13} : Category one of Theorized fuzzballs, like classic black holes; spacetime and bend light

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of spacetime and bend light ;Theorized fuzzballs, like classic black holes

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Fuzzballs are theorized by some superstring theory scientists to be (=) the true quantum description of black holes.

The theory resolves two intractable problems that classic black holes pose for modern physics

G_{16} : Category one of Fuzzballs are theorized

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of true quantum description of black holes.

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

The information paradox wherein the quantum information bound in in‐falling matter and energy entirely disappears into (eb) a singularity; that is, the black hole would undergo (e&eb) zero physical change in its composition regardless of the nature of what fell into it.

G_{20} : Category one of quantum information bound in in‐falling matter and energy

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of singularity

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

black hole would undergo (e&eb) zero physical change in its composition regardless of the nature of what fell into it.

G_{24} : Category one of black hole; zero physical change in its composition regardless of the nature of what fell into it.

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of zero physical change in its composition regardless of the nature of what fell into it.; black hole

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Fuzzball theory replaces the singularity at the heart of a black hole by positing that the entire region within the black hole's event horizon is actually a ball of strings, which are advanced as the ultimate building blocks of matter and energy.

Strings are thought to be bundles of energy vibrating in complex ways in both the three physical dimensions of space as well as in compact directions—extra dimensions interwoven in the quantum foam (also known as spacetime foam)

G_{28} : Category one of Fuzzball theory; singularity at the heart of a black hole by positing that the entire region within the black hole's event horizon is actually a ball of strings, which are advanced as the ultimate building blocks of matter and energy.

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of singularity at the heart of a black hole by positing that the entire region within the black hole's event horizon is actually a ball of strings, which are advanced as the ultimate building blocks of matter and energy. ;Fuzzball theory

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Fuzzball theory replaces the singularity at the heart of a black hole by positing that the entire region within the black hole's event horizon is actually a ball of strings, which are advanced as the ultimate building blocks of matter and energy

G_{32} : Category one of Fuzzball theory replaces the singularity at the heart of a black hole by positing that the entire region within the black hole's event horizon

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of ball of strings, which are advanced as the ultimate building blocks of matter and energy

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Fuzzball theory replaces the singularity at the heart of a black hole by positing that the entire region within the black hole's event horizon is actually a ball of strings, which are advanced as the ultimate building blocks of matter and energy

G_{36} : Category one of Fuzzball theory replaces the singularity at the heart of a black hole by positing that the entire region within the black hole's event horizon is actually a ball of strings, which are advanced

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of ultimate building blocks of matter and energy

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

They also found that the physical surface of the fuzzball would have a radius equal to that of the event horizon of a classic black hole; for both, the Schwarzschild radius for a median-size stellar-mass black hole of 6.8 solar masses (M⊙) is 20 kilometers

G_{40} : Category one of physical surface of the fuzzball would have a radius

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of event horizon of a classic black hole; for both, the Schwarzschild radius for a median-size stellar-mass black hole of 6.8 solar masses (M⊙) is 20 kilometers

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Irrespective of a fuzzball's mass and resultant density, the determining factor establishing where its surface lies is the threshold at which the fuzzball's escape velocity precisely equals the speed of light.[8]

G_{44} : Category one of fuzzball's mass and resultant density, the determining factor establishing where its surface lies is the threshold at which the fuzzball's escape velocity

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of speed of light

T_{45} : Category two of SAS

T_{46} : Category three of SAS

ECTION THIRTY EIGHT

Black Hole Statistics And Holography

INTRODUCTION—VARIABLES USED

Peter G. Shepard JHEP10 (2005)072 doi:10.1088/1126-6708/2005/10/072 Black hole statistics from holography (Please see article for references)

(1) **Peter G. Shepard** study the microstates of the ``small" black hole in (eb) the 1/2-BPS sector of AdS5 × S5, the superstar [1], using (e) the powerful holographic description provided by LLM [2].
(2) The system demonstrates the inherently statistical nature of black holes, with the geometry of [1]

emerging (eb) only after averaging over an ensemble of geometries.

(3) The individual microstate geometries differ (e&eb) in the highly non-trivial topology of (e) quantum foam at their core, and the entropy can be understood as (=) a partition of N units of flux among 5-cycles, as required by (e) flux quantization.

(4) While the system offers confirmation of the most controversial aspect of Mathur and Lunin's recent ``fuzzball'' proposal [3, 4], one could see signs of a discrepancy in interpreting its details

NOTATION

Module One

Fuzzballs become less dense as their mass increases due to fractional tension.

When matter or energy (strings) fall onto a fuzzball, more strings aren't simply added to the fuzzball; strings fuse together, and in doing so, all the quantum information of the infallings strings becomes part of (e&eb) larger, more complex strings.

String tension Due to fractional tension exponentially decreases as (e&eb) they become more complex with more modes of vibration, relaxing to considerable lengths.

The "mathematical beauty" of the string theory formulas Mathur and Lunin employed lies in how the fractional tension values produce fuzzball radii that precisely equal Schwarzschild radii, which Karl Schwarzschild calculated using an entirely different mathematical technique 87 years earlier

G_{13} : Category one of Fuzzballs

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of less dense as their mass increases due to fractional tension

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

When matter or energy (strings) fall onto a fuzzball, more strings aren't simply added to the fuzzball; strings fuse together, and in doing so, all the quantum information of the infallings strings becomes part of (e&eb) larger, more complex strings

G_{16} : Category one of matter or energy (strings) fall onto a fuzzball, more strings aren't simply added to the fuzzball ,strings; all the quantum information of the infallings strings becomes part of (e&eb) larger, more complex strings

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of all the quantum information of the infallings strings becomes part of (e&eb) larger, more complex strings; matter or energy (strings) fall onto a fuzzball, more strings aren't simply added to the fuzzball; strings

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

matter or energy (strings) fall onto a fuzzball, more strings aren't simply added to the fuzzball; strings fuse together, and in doing so, all the quantum information of the infallings strings becomes part of (e&eb) larger, more complex strings

G_{20} : Category one of matter or energy (strings) fall onto a fuzzball, more strings aren't simply added to the fuzzball; strings fuse together, and in doing so, all the quantum information of the infallings strings; larger, more complex strings

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of larger, more complex strings; matter or energy (strings) fall onto a fuzzball, more strings aren't simply added to the fuzzball; strings fuse together, and in doing so, all the quantum information of the infallings strings

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

String tension Due to fractional tension exponentially decreases as (e&eb) they become more complex with more modes of vibration, relaxing to considerable lengths

G_{24} : Category one of String tension Due to fractional tension exponentially decreases; they become more complex with more modes of vibration, relaxing to considerable lengths

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of they become more complex with more modes of vibration, relaxing to considerable lengths ;String tension Due to fractional tension exponentially decreases

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

The "mathematical beauty" of the string theory formulas Mathur and Lunin employed lies in how the

fractional tension values produce fuzzball radii that precisely equal Schwarzschild radii,

which Karl Schwarzschild calculated using an entirely different mathematical technique 87 years earlier

G_{28} : Category one of fractional tension values

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of fuzzball radii that precisely equal Schwarzschild radii

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Peter G. Shepard study the

microstates of the ``small" black hole in (eb) the 1/2-BPS sector of AdS5 × S5, the superstar [1], using (e) the powerful holographic description provided by LLM [2]

G_{32} : Category one of microstates of the ``small" black hole

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of1/2-BPS sector of AdS5 × S5, the superstar [1], using (e) the powerful holographic description provided by LLM [2]

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

The system demonstrates the inherently statistical nature of black holes, with the geometry of [1] emerging (eb) only after averaging over an ensemble of geometries

G_{36} : Category one of system demonstrates the inherently statistical nature of black holes, with the geometry; averaging over an ensemble of geometries

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of averaging over an ensemble of geometries; system demonstrates the inherently statistical nature of black holes, with the geometry

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

The individual microstate geometries differ in the highly non-trivial topology of (e) quantum foam at their core, and the entropy can be understood as (=) a partition of N units of flux among 5-cycles, as required by flux quantization

G_{40} : Category one of individual microstate geometries; highly non-trivial topology of (e) quantum foam at their core, and the entropy can be understood as (=) a partition of N units of flux among 5-cycles, as required by (e) flux quantization

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of highly non-trivial topology of (e) quantum foam at their core, and the entropy can be understood as (=) a partition of N units of flux among 5-cycles, as required by (e) flux quantization; individual microstate geometries

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

individual microstate geometries differ in the highly non-trivial topology of quantum foam at their core, and the entropy can be understood as (=) a partition of N units of flux among 5-cycles, as required by flux quantization

G_{44} : Category one of individual microstate geometries differ in the highly non-trivial topology of quantum foam at their core, and the entropy

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of partition of N units of flux among 5-cycles, as required by flux quantization

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION THIRTY NINE

Gravitational Thermodynamics

INTRODUCTION—VARIABLES USED

Vijay Balasubramanian et al JHEP12 (2005)006 doi:10.1088/1126-6708/2005/12/006 The library of Babel: on the origin of gravitational thermodynamics Vijay Balasubramanian1, Jan de Boer2, Vishnu Jejjala3 and Joan Simón1

(1) **Vijay Balasubramanian1, Jan de Boer2, Vishnu Jejjala3 and Joan Simón1** show that heavy pure states of gravity can appear to be (=) mixed states to almost all probes.
(2) For AdS5 Schwarzschild black holes, arguments are made using (e) the field theory dual to string theory in such spacetimes.
(3) Results follow from applying information theoretic notions to (e&eb) field theory operators capable of describing very heavy states in gravity.
(4) For half-BPS states of the theory which are incipient black holes, our account is exact: typical microstates are described in gravity by (e) a spacetime ``foam", the precise details of which are

almost invisible to almost all probes.

(5) They show that universal low-energy effective description of a foam of given global charges is via (e&eb) certain singular spacetime geometries.

(6) When one of the specified charges is the number of D-branes, (eb) the effective singular geometry is the half-BPS ``superstar".

(7) They propose this as the general mechanism by which the effective thermodynamic character of gravity emerges

NOTATION

Module One

Vijay Balasubramanian1, Jan de Boer2, Vishnu Jejjala3 and Joan Simón1 show that

heavy pure states of gravity can appear to be (=) mixed states to almost all probes

G_{13} : Category one of heavy pure states of gravity

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of mixed states to almost all probes

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

For AdS5 Schwarzschild black holes, arguments are made using (e) the field theory dual to string theory in such spacetimes

G_{16} : Category one of field theory dual to string theory in such spacetimes; AdS5 Schwarzschild black holes

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of AdS5 Schwarzschild black holes; field theory dual to string theory in such spacetimes

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Results follow from applying

information theoretic notions to (e&eb) field theory operators capable of describing very heavy states in gravity

G_{20} : Category one of information theoretic notions; field theory operators capable of describing very heavy states in gravity

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of field theory operators capable of describing very heavy states in gravity; information theoretic notions

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

For half-BPS states of the theory which are incipient black holes, our account is exact:

typical microstates are described in gravity by (e) a spacetime ``foam", the precise details of which are almost invisible to almost all probes

G_{24} : Category one of spacetime ``foam"

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of typical microstates are described in gravity

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

They show that universal low-energy effective description of a foam of given global charges is via (e&eb) certain singular spacetime geometries

G_{28} : Category one of universal low-energy effective description of a foam of given global charges; certain singular spacetime geometries

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of certain singular spacetime geometries; universal low-energy effective description of a foam of given global charges

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

When one of the specified charges is the number of D-branes, (eb) the effective singular geometry is the half-BPS ``superstar"

G_{32} : Category one of one of the specified charges is the number of D-branes

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of effective singular geometry is the half-BPS ``superstar''

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Induced gravity From Wikipedia, the free encyclopedia

Induced gravity (or emergent gravity) is an idea in quantum gravity that space-time curvature and its dynamics emerge as (eb) a mean field approximation of underlying microscopic degrees of freedom, similar to the fluid mechanics approximation of Bose–Einstein condensates. The concept was originally proposed by Andrei Sakharov in 1967.

(1) Sakharov observed that many condensed matter systems give rise to (eb) emergent phenomena which are identical to general relativity.
(2) For example, crystal defects can look like curvature and torsion in (eb) an Einstein–Cartan spacetime.
(3) This allows one to create a theory of gravity with torsion from (e) a World Crystal model of spacetime [1] in which the lattice spacing is of the order of a Planck length. Sakharov's idea was to start with an arbitrary background pseudo-Riemannian manifold (in modern treatments, possibly with torsion) and introduce quantum fields (matter) on it but not introduce any gravitational dynamics explicitly.
(4) This gives rise to an effective action which to one-loop order contains (e) the Einstein–Hilbert action with a cosmological constant.
(5) In other words, general relativity arises as an emergent property of (e) matter fields and is not put in by hand. On the other hand, such models typically predict huge cosmological constants.
(6) Some argue that the particular models proposed by Sakharov and others have been proven impossible by the Weinberg–Witten theorem. But, models with emergent gravity are always possible as long as other things, such as spacetime dimensions, emerge together with (e&eb) gravity.
(7) Developments in AdS/CFT correspondence after 1997 suggest (eb) that the microphysical degrees of freedom in induced gravity might be radically different.
(8) The bulk space-time arises as (=) an emergent phenomenon of the quantum degrees of freedom that live in (eb) the boundary of the space-time.

G_{36} : Category one of Induced gravity (or emergent gravity) is an idea in quantum gravity that space-time curvature and its dynamics

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of mean field approximation of underlying microscopic degrees of freedom, similar to the fluid mechanics approximation of Bose–Einstein condensates

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

Sakharov observed that

many condensed matter systems give rise to (eb) emergent phenomena which are identical to general relativity

G_{40} : Category one of many condensed matter systems

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of emergent phenomena which are identical to general relativity

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

For example, crystal defects can look like curvature and torsion in (eb) an Einstein–Cartan spacetime

G_{44} : Category one of crystal defects can look like curvature and torsion

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of Einstein–Cartan spacetime

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION FORTY

Quantum Optical Coherence In Cytoskeletal Microtubules

INTRODUCTION—VARIABLES USED

Biosystems Volume 32, Issue 3, 1994, Pages 195–209 Quantum optical coherence in cytoskeletal microtubules: implications for brain function Mari Jibu, a, b, Scott Haganc, Stuart R. Hameroff, Karl H. Pribram, Kunio Yasue doi:10.1016/0303-2647(94)90043-4

(1) 'Laser-like,' long-range coherent quantum phenomena may occur biologically within (eb) cytoskeletal microtubules.
(2) This paper presents a theoretical prediction of (e) the occurrence in biological media of (e) the phenomena which we term 'superradiance' and 'self-induced transparency'.
(3) Interactions between the electric dipole field of water molecules confined within the hollow core of

microtubules and (e&eb) the quantized electromagnetic radiation field are considered, and microtubules are theorized to play the roles of (e) non-linear coherent optical devices.

(4) Superradiance is a specific quantum mechanical ordering phenomenon with (e&eb) characteristic times much shorter than those of thermal interaction.

(5) Consequently, optical signalling (and computation) in microtubules would be free from (e) both thermal noise and loss

(6) Superradiant optical computing in networks of microtubules and other cytoskeletal structures may provide (eb) a basis for biomolecular cognition and a substrate for consciousness.

Mathematics and Computers in Simulation Volume 40, Issues 3–4, April 1996, Pages 453–480 Orchestrated reduction of quantum coherence in brain microtubules: A model for consciousness Stuart Hameroff, a, , Roger Penrose b

(1) Features of consciousness difficult to understand in terms of conventional neuroscience have evoked application of quantum theory, which describes the fundamental behavior of matter and energy. In this paper authors propose that aspects of quantum theory (e.g. quantum coherence) and of a newly proposed physical phenomenon of (e) quantum wave function "self-collapse" (objective reduction: OR - Penrose, 1994) are essential for consciousness, and occur in (eb) cytoskeletal microtubules and other structures within each of the brain's neurons.

(2) The particular characteristics of microtubules suitable for quantum effects include (e) their crystal-like lattice structure, hollow inner core, organization of cell function and capacity for information processing.

(3) They envisage that conformational states of microtubule subunits (tubulins) are coupled to (e&eb) internal quantum events, and cooperatively interact (compute) with (e&eb) other tubulins.

(4) Authors further assume that macroscopic coherent superposition of quantum-coupled tubulin conformational states occur throughout (e&eb) significant brain volumes and provide (eb) the global binding essential to consciousness.

(5) Authors equate the emergence of the microtubule quantum coherence with (=) pre-conscious processing which grows (for up to 500 ms) until the mass-energy difference among the separated states of tubulins reaches a (eb) threshold related to quantum gravity.

(6) According to the arguments for OR put forth in Penrose (1994), superpositioned states each have (e) their own space-time geometries.

(7) When the degree of coherent mass-energy difference leads to (eb) sufficient separation of space-time geometry, the system must choose and decay (reduce, collapse) to (e&eb) a single universe state.

(8) In this way, a transient superposition of slightly differing space-time geometries persists until (eb) an abrupt quantum → classical reduction occurs.

(9) Unlike the random, "subjective reduction" (SR, or R) of standard quantum theory caused by (e) observation or environmental entanglement, the OR authors propose in microtubules is (=) a self-collapse and it results in particular patterns of microtubule-tubulin conformational states that regulate (e&eb) neuronal activities including synaptic functions.

(10) Possibilities and probabilities for post-reduction tubulin states are influenced by (e&eb) factors including attachments of microtubule-associated proteins (MAPs) acting as (=) "nodes" which tune and "orchestrate" (e&eb) the quantum oscillations.

(11) They thus term the self-tuning OR process in microtubules "orchestrated objective reduction" ("Orch OR"), and calculate an estimate for the number of tubulins (and neurons) whose coherence for relevant time periods (e.g. 500 ms) will elicit Orch OR

(12). In **providing a connection** among (1) pre-conscious to conscious transition, (2) fundamental space-time notions, (3) non-computability, and (4) binding of various (time scale and spatial) reductions into an instantaneous event ("conscious now"), authors believe Orch OR in brain

microtubules is (=) the most specific and plausible model for consciousness yet proposed.

NOTATION

Module One

‘Laser-like,’ long-range coherent quantum phenomena may occur biologically within (eb) cytoskeletal microtubules

G_{13} : Category one of ‘Laser-like,’ long-range coherent quantum phenomena may occur biologically

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of cytoskeletal microtubules

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

This paper presents a theoretical prediction of (e) the occurrence in biological media of (e) the phenomena which we term ‘superradiance’ and ‘self-induced transparency’

Interactions between the

electric dipole field of water molecules confined within the hollow core of microtubules and (e&eb) the quantized electromagnetic radiation field are considered, and microtubules are theorized to play the roles of (e) non-linear coherent optical devices

G_{16} : Category one of electric dipole field of water molecules confined within the hollow core of microtubules; quantized electromagnetic radiation field are considered, and microtubules are theorized to play the roles of (e) non-linear coherent optical devices

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of quantized electromagnetic radiation field are considered, and microtubules are theorized to play the roles of (e) non-linear coherent optical devices; electric dipole field of water molecules confined within the hollow core of microtubules

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

electric dipole field of water molecules confined within the hollow core of microtubules and the quantized electromagnetic radiation field are considered, and microtubules are theorized to play the roles of (e) non-linear coherent optical devices

G_{20} : Category one of non-linear coherent optical devices

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of electric dipole field of water molecules confined within the hollow core of microtubules and the quantized electromagnetic radiation field are considered, and microtubules are theorized to play the roles

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Superradiance is a specific quantum mechanical ordering phenomenon with (e&eb) characteristic times much shorter than those of thermal interaction

G_{24} : Category one of Superradiance is a specific quantum mechanical ordering phenomenon; characteristic times much shorter than those of thermal interaction

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of characteristic times much shorter than those of thermal interaction ;Superradiance is a specific quantum mechanical ordering phenomenon

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Consequently,
optical signalling (and computation) in microtubules would be free from (e) both thermal noise and loss
G_{28} : Category one of thermal noise and loss

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of optical signalling (and computation) in microtubules would be free

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Superradiant optical computing in networks of microtubules and other cytoskeletal structures may provide (eb) a basis for biomolecular cognition and a substrate for consciousness

G_{32} : Category one of Superradiant optical computing in networks of microtubules and other cytoskeletal structures

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of basis for biomolecular cognition and a substrate for consciousness

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Features of consciousness difficult to understand in terms of conventional neuroscience have evoked application of quantum theory, which describes the fundamental behavior of matter and energy. In this paper authors propose that

aspects of quantum theory (e.g. quantum coherence) and of a newly proposed physical phenomenon of (e) quantum wave function "self-collapse" (objective reduction: OR - Penrose, 1994) are essential for consciousness, and occur in (eb) cytoskeletal microtubules and other structures within each of the brain's neurons.

G_{36} : Category one of quantum wave function "self-collapse" (objective reduction: OR - Penrose, 1994) are essential for consciousness, and occur in (eb) cytoskeletal microtubules and other structures within each of the brain's neurons.

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of aspects of quantum theory (e.g. quantum coherence) and of a newly proposed physical phenomenon

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

aspects of quantum theory (e.g. quantum coherence) and of a newly proposed physical phenomenon of quantum wave function "self-collapse" (objective reduction: OR - Penrose, 1994) are essential for consciousness, and occur in (eb) cytoskeletal microtubules and other structures within each of the brain's neurons.

G_{40} : Category one of aspects of quantum theory (e.g. quantum coherence) and of a newly proposed physical phenomenon of quantum wave function "self-collapse" (objective reduction: OR - Penrose, 1994) are essential for consciousness

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of cytoskeletal microtubules and other structures within each of the brain's neurons.

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

aspects of quantum theory (e.g. quantum coherence) and of a newly proposed physical phenomenon of quantum wave function "self-collapse" (objective reduction: OR - Penrose, 1994) are essential for consciousness, and occur in) cytoskeletal microtubules and other structures within each of the brain's neurons.

G_{44} : Category one of aspects of quantum theory (e.g. quantum coherence) and of a newly proposed physical phenomenon of quantum wave function "self-collapse" (objective reduction: OR - Penrose, 1994) are essential for consciousness, and occur in) cytoskeletal microtubules and other structures

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of each of the brain's neurons.

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION FORTY A

Superpositioned States Each Have Their Own Space-Time Geometries

INTRODUCTION—VARIABLES USED

Mathematics and Computers in Simulation Volume 40, Issues 3–4, April 1996, Pages 453–480 Orchestrated reduction of quantum coherence in brain microtubules: A model for consciousness Stuart Hameroff, a, , Roger Penrose b

(1) The particular characteristics of microtubules suitable for quantum effects include (e) their crystal-like lattice structure, hollow inner core, organization of cell function and capacity for information processing.
(2) They envisage that conformational states of microtubule subunits (tubulins) are coupled to (e&eb) internal quantum events, and cooperatively interact (compute) with (e&eb) other tubulins.
(3) Authors further assume that macroscopic coherent superposition of quantum-coupled tubulin conformational states occur throughout (e&eb) significant brain volumes and provide (eb) the global binding essential to consciousness.
(4) Authors equate the emergence of the microtubule quantum coherence with (=) pre-conscious processing which grows (for up to 500 ms) until the mass-energy difference among the separated states of tubulins reaches a (eb) threshold related to quantum gravity.
(5) According to the arguments for OR put forth in Penrose (1994), **superpositioned states each have (e) their own space-time geometries.**
(6) When the degree of coherent mass-energy difference leads to (eb) sufficient separation of space-time geometry, the system must choose and decay (reduce, collapse) to (e&eb) a single universe

state.

(7) In this way, a transient superposition of slightly differing space-time geometries persists until (eb) an abrupt quantum → classical reduction occurs.

(8) Unlike the random, "subjective reduction" (SR, or R) of standard quantum theory caused by (e) observation or environmental entanglement, the OR authors propose in microtubules is (=) a self-collapse and it results in particular patterns of microtubule-tubulin conformational states that regulate (e&eb) neuronal activities including synaptic functions.

(9) Possibilities and probabilities for post-reduction tubulin states are influenced by (e&eb) factors including attachments of microtubule-associated proteins (MAPs) acting as (=) "nodes" which tune and "orchestrate" (e&eb) the quantum oscillations.

(10) They thus term the self-tuning OR process in microtubules "orchestrated objective reduction" ("Orch OR"), and calculate an estimate for the number of tubulins (and neurons) whose coherence for relevant time periods (e.g. 500 ms) will elicit Orch OR

(11). In **providing a connection** among (1) pre-conscious to conscious transition, (2) fundamental space-time notions, (3) non-computability, and (4) binding of various (time scale and spatial) reductions into an instantaneous event ("conscious now"), authors believe Orch OR in brain microtubules is (=) the most specific and plausible model for consciousness yet proposed

NOTATION

Module One

The particular characteristics of microtubules suitable for quantum effects include (e) their crystal-like lattice structure, hollow inner core, organization of cell function and capacity for information processing.

G_{13} : Category one of their crystal-like lattice structure, hollow inner core, organization of cell function and capacity for information processing

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of particular characteristics of microtubules suitable for quantum effects

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Information theory From Wikipedia, the free encyclopedia

Information theory is a branch of applied mathematics, electrical engineering, and computer science involving the quantification of information. Information theory was developed by Claude E. Shannon to find fundamental limits on signal processing operations such as compressing data and on reliably storing and communicating data. Since its inception it has broadened to find applications in many other areas, including statistical inference, natural language processing, cryptography, neurobiology,[1] the evolution[2] and function[3] of molecular codes, model selection in ecology,[4] thermal physics,[5] quantum computing, linguistics, plagiarism detection,[6] pattern recognition, anomaly detection and other forms of data analysis.[7]

The landmark event that established the discipline of information theory, and brought it to immediate worldwide attention, was the publication of Claude E. Shannon's classic paper "A Mathematical Theory of

Communication" in the Bell System Technical Journal in July and October 1948. Prior to this paper, limited information-theoretic ideas had been developed at Bell Labs, all implicitly assuming events of equal probability. Harry Nyquist's 1924 paper, Certain Factors Affecting Telegraph Speed, contains a theoretical section quantifying "intelligence" and the "line speed" at which it can be transmitted by a communication system, giving the relation $W = K \log m$ (recalling Boltzmann's constant), where W is the speed of transmission of intelligence, m is the number of different voltage levels to choose from at each time step, and K is a constant. Ralph Hartley's 1928 paper, Transmission of Information, uses the word information as a measurable quantity, reflecting the receiver's ability to distinguish one sequence of symbols from any other, thus quantifying information as $H = \log S^n = n \log S$, where S was the number of possible symbols, and n the number of symbols in a transmission. The unit of information was therefore the decimal digit, much later renamed the Hartley in his honour as a unit or scale or measure of information. Alan Turing in 1940 used similar ideas as part of the statistical analysis of the breaking of the German Second World War Enigma ciphers. Much of the mathematics behind information theory with events of different probabilities was developed for the field of thermodynamics by Ludwig Boltzmann and J. Willard Gibbs. Connections between information-theoretic entropy and thermodynamic entropy, including the important contributions by Rolf Landauer in the 1960s, are explored in Entropy in thermodynamics and information theory.

In Shannon's revolutionary and groundbreaking paper, the work for which had been substantially completed at Bell Labs by the end of 1944, Shannon for the first time introduced the qualitative and quantitative model of communication as a statistical process underlying information theory, opening with the assertion that

"The fundamental problem of communication is that of reproducing at one point, either exactly or approximately, a message selected at another point."

With it came the ideas of

the information entropy and redundancy of a source, and its relevance through the source coding theorem;

the mutual information, and the channel capacity of a noisy channel, including the promise of perfect loss-free communication given by the noisy-channel coding theorem;

the practical result of the Shannon–Hartley law for the channel capacity of a Gaussian channel; as well as

the bit—a new way of seeing the most fundamental unit of information.

Quantities of information

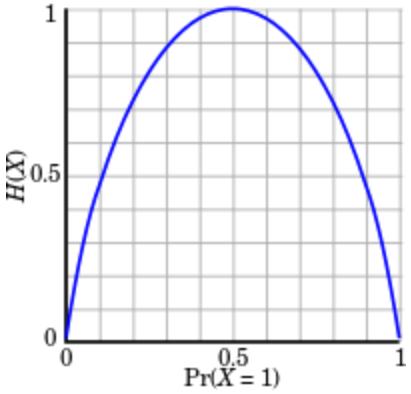
Information theory is based on probability theory and statistics. The most important quantities of information are entropy, the information in a random variable, andmutual information, the amount of information in common between two random variables. The former quantity gives the limit on how far message data can becompressed, while the latter can be used to find the communication rate across a channel.

The choice of logarithmic base in the following formulae determines the unit of information entropy that is used. The most common unit of information is the bit, based on the binary logarithm. Other units include the nat, which is based on the natural logarithm, and the Hartley, which is based on the common logarithm.

In what follows, an expression of the form $p \log p$ is considered by convention to be equal to zero

whenever $p = 0$. This is justified because $\lim_{p \to 0+} p \log p = 0$ for any logarithmic base.

Entropy



Entropy of a Bernoulli trial as a function of success probability, often called the binary entropy function, $H_{\mathrm{b}}(p)$. The entropy is maximized at 1 bit per trial when the two possible outcomes are equally probable, as in an unbiased coin toss.

The entropy, H, of a discrete random variable X is a measure of the amount of uncertainty associated with the value of X.

Suppose one transmits 1000 bits (0s and 1s). If the value of each these bits is known (has a specific value with certainty) ahead of transmission, it is clear that no information is transmitted. If, however, each bit is independently equally likely to be 0 or 1, 1000 shannons of information (also often called bits, in the information theoretic sense) have been transmitted. Between these two extremes, information can be quantified as follows. If $\mathbb{X}$ is the set of all messages $\{x_1, ..., x_n\}$ that X could be, and $p(x)$ is the probability of some $x \in \mathbb{X}$, then the entropy, H, of X is defined:

$$H(X) = \mathbb{E}_X[I(x)] = -\sum_{x \in \mathbb{X}} p(x) \log p(x).$$

(Here, $I(x)$ is the self-information, which is the entropy contribution of an individual message, and $\mathbb{E}_X$ is the expected value.) A property of entropy is that it is maximized when all the messages in the message space are equiprobable $p(x) = 1/n$,—i.e., most unpredictable—in which case $H(X) = \log n$.

The special case of information entropy for a random variable with two outcomes is the binary entropy function, usually taken to the logarithmic base 2, thus having the Shannon (Sh) as unit:

$$H_{\mathrm{b}}(p) = -p \log_2 p - (1-p) \log_2 (1-p).$$

Joint entropy

The joint entropy of two discrete random variables X and Y is merely the entropy of their pairing: (X, Y). This implies that if X and Y are independent, then their joint entropy is the sum of

their individual entropies.

For example, if (X, Y) represents the position of a chess piece — X the row and Y the column, then the joint entropy of the row of the piece and the column of the piece will be the entropy of the position of the piece.

$$H(X,Y) = \mathbb{E}_{X,Y}[-\log p(x,y)] = -\sum_{x,y} p(x,y) \log p(x,y)$$

Despite similar notation, joint entropy should not be confused with cross entropy.

Conditional entropy (equivocation)

The conditional entropy or conditional uncertainty of X given random variable Y (also called the equivocation of X about Y) is the average conditional entropy over Y

$$H(X|Y) = \mathbb{E}_Y[H(X|y)] = -\sum_{y \in Y} p(y) \sum_{x \in X} p(x|y) \log p(x|y) = -\sum_{x,y} p(x,y) \log \frac{p(x,y)}{p(y)}.$$

Because entropy can be conditioned on a random variable or on that random variable being a certain value, care should be taken not to confuse these two definitions of conditional entropy, the former of which is in more common use. A basic property of this form of conditional entropy is that:

$$H(X|Y) = H(X,Y) - H(Y).$$

Mutual information (transinformation)

Mutual information measures the amount of information that can be obtained about one random variable by observing another. It is important in communication where it can be used to maximize the amount of information shared between sent and received signals. The mutual information of X relative to Y is given by:

$$I(X;Y) = \mathbb{E}_{X,Y}[SI(x,y)] = \sum_{x,y} p(x,y) \log \frac{p(x,y)}{p(x)\,p(y)}$$

Here SI (Specific mutual Information) is the pointwise mutual information.

A basic property of the mutual information is that

$$I(X;Y) = H(X) - H(X|Y).$$

That is, knowing Y, we can save an average of $I(X;Y)$ bits in encoding X compared to not knowing Y.

Mutual information is symmetric:

$$I(X;Y) = I(Y;X) = H(X) + H(Y) - H(X,Y).$$

Mutual information can be expressed as the average Kullback–Leibler divergence (information gain) between the posterior probability distribution of X given the value of Y and the prior distribution on X:

$$I(X;Y) = \mathbb{E}_{p(y)}[D_{\mathrm{KL}}(p(X|Y=y)\|p(X))].$$

In other words, this is a measure of how much, on the average, the probability distribution on X will change if we are given the value of Y. This is often recalculated as the divergence from the product of the marginal distributions to the actual joint distribution:

$$I(X;Y) = D_{\mathrm{KL}}(p(X,Y)\|p(X)p(Y)).$$

Mutual information is closely related to the log-likelihood ratio test in the context of contingency tables and the multinomial distribution and to Pearson's $\chi 2$ test: mutual information can be considered a statistic for assessing independence between a pair of variables, and has a well-specified asymptotic distribution.

Pattern formation From Wikipedia, the free encyclopedia

Analysis

Further information: Gradient Pattern Analysis

The analysis of pattern-forming systems often consists of finding a Partial differential equation model of the system (the Swift-Hohenberg equation is one such model) of the form

$$\frac{\partial u}{\partial t} = F(u,t)$$

where F is generically a nonlinear differential operator, and postulating solutions of the form

$$u(\mathbf{x},t) = \sum_j z_j(t)e^{i\mathbf{k}_j\cdot\mathbf{x}} + z_j(t)^* e^{-i\mathbf{k}_j\cdot\mathbf{x}}$$

where the z_j are complex amplitudes associated to different modes in the solution and the $\mathbf{k}_j$ are the wave-vectors associated to a lattice, e.g. a square or hexagonal lattice in two dimensions. There is in general no rigorous justification for this restriction to a lattice.

Symmetry considerations can now be taken into account, and evolution equations obtained for the complex amplitudes governing the solution. This reduction puts the problem into the form of a system of first-order Ordinary differential equation, which can be analysed using standard methods (see dynamical systems). The same formalism can also be used to analyse bifurcations in pattern-forming systems, for example to analyse the formation of convection rolls in a Rayleigh-Bénardexperiment as the temperature is increased.

G_{16} : Category one of LHS of above equations

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of RHS of above equations

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Authors further assume that

macroscopic coherent superposition of quantum-coupled tubulin conformational states occur throughout (e&eb) significant brain volumes and provide (eb) the global binding essential to consciousness

G_{20} : Category one of macroscopic coherent superposition of quantum-coupled tubulin conformational states occur; significant brain volumes and provide (eb) the global binding essential to consciousness

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of significant brain volumes and provide (eb) the global binding essential to consciousness ;macroscopic coherent superposition of quantum-coupled tubulin conformational states occur

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

macroscopic coherent superposition of quantum-coupled tubulin conformational states occur throughout significant brain volumes and provide (eb) the global binding essential to consciousness

G_{24} : Category one of macroscopic coherent superposition of quantum-coupled tubulin conformational states occur throughout significant brain volumes

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of global binding essential to consciousness

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Authors equate the emergence of the microtubule quantum coherence with (=) pre-conscious processing which grows (for up to 500 ms) until the mass-energy difference among the separated states of tubulins reaches a (eb) threshold related to quantum gravity

G_{28} : Category one of emergence of the microtubule quantum coherence

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of pre-conscious processing which grows (for up to 500 ms) until the mass-energy

difference among the separated states of tubulins reaches a (eb) threshold related to quantum gravity

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Authors equate the emergence of the microtubule quantum coherence with pre-conscious processing which grows (for up to 500 ms) until the mass-energy difference among the separated states of tubulins reaches a (eb) threshold related to quantum gravity

G_{32} : Category one of emergence of the microtubule quantum coherence with pre-conscious processing which grows (for up to 500 ms) until the mass-energy difference among the separated states of tubulins

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of threshold related to quantum gravity

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

According to the arguments for OR put forth in Penrose (1994), **superpositioned states each have (e) their own space-time geometries**

G_{36} : Category one of **their own space-time geometries**

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of **superpositioned states**

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

When the degree of coherent mass-energy difference leads to (eb) sufficient separation of space-time geometry, the system must choose and decay (reduce, collapse) to (e&eb) a single universe state

G_{40} : Category one of degree of coherent mass-energy difference

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of sufficient separation of space-time geometry, the system must choose and decay (reduce, collapse) to (e&eb) a single universe state

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

When the degree of coherent mass-energy difference leads to sufficient separation of space-time geometry, the system must choose and decay (reduce, collapse) to (e&eb) a single universe state

G_{44} : Category one of degree of coherent mass-energy difference leads to sufficient separation of space-time geometry, the system must choose and decay (reduce, collapse); single universe state

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of single universe state; degree of coherent mass-energy difference leads to sufficient separation of space-time geometry, the system must choose and decay (reduce, collapse)

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}), t\right)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}), t\right)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\left((G_{39}), t\right)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}),t\right)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}),t\right)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}),t\right)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}),t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}),t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & \boxed{+(a''_{13})^{(1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & \boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & \boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\,-(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\,-(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\,-(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{20})^{(3)}\,+(a''_{20})^{(3)}(T_{21},t)\,+(a''_{16})^{(2,2,2)}(T_{17},t)\,+(a''_{13})^{(1,1,1,)}(T_{14},t)\\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)\,+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)\,+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a''_{36})^{(7,7,7,7)}(T_{37},t)\,+(a''_{40})^{(8,8,8,8)}(T_{41},t)\,+(a''_{44})^{(9,9,9)}(T_{45},t)\end{array}\right]G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c}(a'_{21})^{(3)}\,+(a''_{21})^{(3)}(T_{21},t)\,+(a''_{17})^{(2,2,2)}(T_{17},t)\,+(a''_{14})^{(1,1,1,)}(T_{14},t)\\ +(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)\,+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)\,+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a''_{37})^{(7,7,7,7)}(T_{37},t)\,+(a''_{41})^{(8,8,8,8)}(T_{41},t)\,+(a''_{45})^{(9,9,9)}(T_{45},t)\end{array}\right]G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{22})^{(3)}\,+(a''_{22})^{(3)}(T_{21},t)\,+(a''_{18})^{(2,2,2)}(T_{17},t)\,+(a''_{15})^{(1,1,1,)}(T_{14},t)\\ +(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)\,+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)\,+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a''_{38})^{(7,7,7,7)}(T_{37},t)\,+(a''_{42})^{(8,8,8,8)}(T_{41},t)\,+(a''_{46})^{(9,9,9)}(T_{45},t)\end{array}\right]G_{22} \quad (69)$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\left[-(b''_{20})^{(3)}(G_{23},t)\right] & \left[-(b''_{16})^{(2,2,2)}(G_{19},t)\right] & \left[-(b''_{13})^{(1,1,1,)}(G,t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{36})^{(7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{40})^{(8,8,8,8)}(G_{43},t)\right] & \left[-(b''_{44})^{(9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\left[-(b''_{21})^{(3)}(G_{23},t)\right] & \left[-(b''_{17})^{(2,2,2)}(G_{19},t)\right] & \left[-(b''_{14})^{(1,1,1,)}(G,t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{37})^{(7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{41})^{(8,8,8,8)}(G_{43},t)\right] & \left[-(b''_{45})^{(9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\left[-(b''_{22})^{(3)}(G_{23},t)\right] & \left[-(b''_{18})^{(2,2,2)}(G_{19},t)\right] & \left[-(b''_{15})^{(1,1,1,)}(G,t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{38})^{(7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{42})^{(8,8,8,8)}(G_{43},t)\right] & \left[-(b''_{46})^{(9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$ $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad 75$$

$(a''_{24})^{(4)}(T_{25},t)$, $(a''_{25})^{(4)}(T_{25},t)$, $(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$
are fourth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$
are fifth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$
are sixth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$
are seventh augmentation coefficients for category 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$
are eighth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

$Where\ \boxed{-(b_{24}'')^{(4)}(G_{27},t)}, \boxed{-(b_{25}'')^{(4)}(G_{27},t)}, \boxed{-(b_{26}'')^{(4)}(G_{27},t)}\ are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{28}'')^{(5,5,)}(G_{31},t)}, \boxed{-(b_{29}'')^{(5,5,)}(G_{31},t)}, \boxed{-(b_{30}'')^{(5,5,)}(G_{31},t)}\ are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{32}'')^{(6,6,)}(G_{35},t)}, \boxed{-(b_{33}'')^{(6,6,)}(G_{35},t)}, \boxed{-(b_{34}'')^{(6,6,)}(G_{35},t)}\ are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{13}'')^{(1,1,1,1)}(G,t)}, \boxed{-(b_{14}'')^{(1,1,1,1)}(G,t)}, \boxed{-(b_{15}'')^{(1,1,1,1)}(G,t)}$

$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{16}'')^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b_{17}'')^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b_{18}'')^{(2,2,2,2)}(G_{19},t)}$

$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{20}'')^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b_{21}'')^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b_{22}'')^{(3,3,3,3)}(G_{23},t)}$

$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{36}'')^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b_{37}'')^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b_{38}'')^{(7,7,7,7,7)}(G_{39},t)}$

$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{40}'')^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b_{41}'')^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b_{42}'')^{(8,8,8,8,8)}(G_{43},t)}$

$are\ eighth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{46}'')^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b_{45}'')^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b_{44}'')^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{c} (a_{28}')^{(5)}\boxed{+(a_{28}'')^{(5)}(T_{29},t)}\ \boxed{+(a_{24}'')^{(4,4,)}(T_{25},t)}\ \boxed{+(a_{32}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{36}'')^{(7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a_{40}'')^{(8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a_{44}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{c} (a_{29}')^{(5)}\boxed{+(a_{29}'')^{(5)}(T_{29},t)}\ \boxed{+(a_{25}'')^{(4,4,)}(T_{25},t)}\ \boxed{+(a_{33}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a_{45}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{c} (a_{30}')^{(5)}\boxed{+(a_{30}'')^{(5)}(T_{29},t)}\ \boxed{+(a_{26}'')^{(4,4,)}(T_{25},t)}\ \boxed{+(a_{34}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a_{46}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{30} \quad (81)$$

$Where\ \boxed{+(a_{28}'')^{(5)}(T_{29},t)}, \boxed{+(a_{29}'')^{(5)}(T_{29},t)}, \boxed{+(a_{30}'')^{(5)}(T_{29},t)}\ are\ first\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$And\ \boxed{+(a_{24}'')^{(4,4,)}(T_{25},t)}, \boxed{+(a_{25}'')^{(4,4,)}(T_{25},t)}, \boxed{+(a_{26}'')^{(4,4,)}(T_{25},t)}\ are\ second\ augmentation\ coefficient\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a_{32}'')^{(6,6,6)}(T_{33},t)}, \boxed{+(a_{33}'')^{(6,6,6)}(T_{33},t)}, \boxed{+(a_{34}'')^{(6,6,6)}(T_{33},t)}\ are\ third\ \ augmentation\ coefficient\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8\ ,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \quad 85$$

$$- \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}, \boxed{(a''_{37})^{(7)}(T_{37},t)}, \boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\left[-(b''_{40})^{(8)}(G_{43},t)\right] & \left[-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\left[-(b''_{41})^{(8)}(G_{43},t)\right] & \left[-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\left[-(b''_{42})^{(8)}(G_{43},t)\right] & \left[-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq \ (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = \ (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \ , \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\,(a_i)^{(1)} + (a_i')^{(1)} + \ (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}\,(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i')^{(1)} + \ (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \leq (p_i)^{(2)} \leq \left(\hat{A}_{16}\right)^{(2)} \qquad 102$$

$$(b_i'')^{(2)}(G_{19},t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\big((G_{19}),t\big) = \ (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - \ T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\big((G_{19}),t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\big[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\big]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}} \ , \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\,(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + \ (\hat{P}_{16})^{(2)}\,(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + \ (\hat{Q}_{16})^{(2)}\ \ (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \le (p_i)^{(3)} \le (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \le (r_i)^{(3)} \le (b_i')^{(3)} \le (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - \ T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32{,}33{,}34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and \ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32{,}33{,}34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\,(a_i)^{(6)} + (a_i')^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)}\,(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(GGGGGGG) $\quad (a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36{,}37{,}38$ 131

(HHHHHHH) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$(a_i'')^{(7)}(T_{37},t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$

$(b_i'')^{(7)}(G_{39},t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$ 132

(IIIIIII) $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

(JJJJJJJ)

$\lim_{G\to\infty}(b_i'')^{(7)}((G_{39}),t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \leq (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$

$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}((G_{39}),t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$. (T_{37}',t) and (T_{37},t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(KKKKKKK) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(LLLLLLL) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$

$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$

Where we suppose

$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$ 138

$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$ 139

$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$ 140

$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}$ are positive constants and $i = 40,41,42$

They satisfy Lipschitz condition:

$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t}$ 142

$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t}$ 143

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ 144

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}$, $(r_i)^{(8)}, i = 40,41,42$,
Satisfy the inequalities

$\frac{1}{(\hat{M}_{40})^{(8)}} [(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 145

$\frac{1}{(\hat{M}_{40})^{(8)}} [(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 146

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\hat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)} ||(G_{47}) - (G_{47})'|| e^{-(\hat{M}_{44})^{(9)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}\left[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)} (\hat{k}_{44})^{(9)}\right] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)} t}$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)} t}$, $T_i(0) = T_i^0 > 0$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$, $T_i(0) = T_i^0 > 0$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \ , \ T_i(0) = T_i^0 \ , \ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} , T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\overline{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^{0} + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^{0} + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^{0} + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^{0} + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^{0} + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^{0} + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By

$$\bar{G}_{32}(t) = G_{32}^{0} + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^{0} + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^{0} + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^{0} + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^{0} + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^{0} + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{\mathrm{T}}_{42}(\mathrm{t}) = \mathrm{T}_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that 174

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \leq G_{32}^{0}+\int_0^t\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(bb) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \leq G_{36}^{0}+\int_0^t\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \leq G_{40}^{0}+\int_0^t\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}= \quad 180$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t} \leq \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for G_{41} , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \leq G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \leq \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for G_{45} , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \left\{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\right\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)}\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) - (a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right| \; e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \qquad (186)$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13{,}14{,}15$ depend only on T_{14} and respectively on G(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: (187)

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. (188)

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$

$sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\hat{M}_{16})^{(2)}t}\}$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $(\widetilde{G_{19}}, \widetilde{T_{19}}) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t (a_{16})^{(2)}|G_{17}^{(1)} - G_{17}^{(2)}|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}ds_{(16)} +$

$\int_0^t\{(a_{16}')^{(2)}|G_{16}^{(1)} - G_{16}^{(2)}|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}\left|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t} \leq$
$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)d\left(\left((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)}\right)\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\}ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0$ for $t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)}G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$: $\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$G_{20}^{(2)} \left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ *and* $(\widehat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a'_{22})^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$G_{24}^{(2)} \left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)}\right) d\left(((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on T_{25} and respectively on (G_{27})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (228)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_2 / (a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28}, G_{30} and G_{28}, G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30}. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\tilde{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\tilde{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\tilde{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\tilde{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\tilde{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\tilde{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}), (\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}), (\widetilde{T_{35}}) \right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right| \; e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} \} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)} (\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)}; \; (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t} \; and \; (\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$ (250)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2 \; and \; \left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33} \; and \; G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$
$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t}\}$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)} \left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)} (T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}),(\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}),(\widetilde{T_{43}})\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1,\left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right),\quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{47}}),(\widetilde{T_{47}}):\left((\widetilde{G_{47}}),(\widetilde{T_{47}})\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$

From 99 to 44 it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(9)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)}, , (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)},\ \boldsymbol{if}\ (\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)} t} - e^{-(a'_{26})^{(4)} t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)} t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G^0_{28} e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G^0_{28}}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G^0_{30} e^{-(S_2)^{(5)} t} \le G_{30}(t) \le \right. \quad 344$$

$$\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_2)^{(5)}((S_1)^{(5)}-(a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t}-e^{-(a'_{30})^{(5)}t}\right]+\ G_{30}^{0}e^{-(a'_{30})^{(5)}t}\Big)$$

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\le T_{30}(t)\le \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$:

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$-(\sigma_2)^{(6)}\le-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\le-(\sigma_1)^{(6)}$

$-(\tau_2)^{(6)}\le-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\le-(\tau_1)^{(6)}$

Definition of $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$: 350

By $(v_1)^{(6)}>0$, $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$, $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

Definition of $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)}>0$, $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$, $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

Definition of $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$:-

If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)}=(v_0)^{(6)},(m_1)^{(6)}=(v_1)^{(6)},$ ***if*** $(v_0)^{(6)}<(v_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}$ $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}$ $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}$ $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}$ $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}$ $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})} \left[e^{(S_1)^{(6)} t} - e^{-(a'_{34})^{(6)} t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)} t} \right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^{0}e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}}G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}}G_{36}^{0}e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t}-e^{-(S_2)^{(7)}t}\right]+G_{38}^{0}e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left.\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_2)^{(7)}((S_1)^{(7)}-(a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t}-e^{-(a'_{38})^{(7)}t}\right]+\ G_{38}^{0}e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}}T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}}T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right) - (b''_{41})^{(8)}\left((G_{43}), t\right) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(v_1)^{(8)} > 0$, $(v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$, $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{v}_1)^{(8)}$,, $(\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{v}_1)^{(8)} > 0$, $(\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$, $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$:-

If we define $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$, $\boldsymbol{if}$ $(v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$, $\boldsymbol{if}$ $(v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$,
and $\boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}$, $\boldsymbol{if}$ $(\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, $\boldsymbol{if}$ $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, $\boldsymbol{if}$ $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}$ $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t})$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t}\leq T_{42}(t)\leq \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)},(S_2)^{(8)},(R_1)^{(8)},(R_2)^{(8)}$:- 381

Where $(S_1)^{(8)}=(a_{40})^{(8)}(m_2)^{(8)}-(a'_{40})^{(8)}$

$(S_2)^{(8)}=(a_{42})^{(8)}-(p_{42})^{(8)}$

$(R_1)^{(8)}=(b_{40})^{(8)}(\mu_2)^{(8)}-(b'_{40})^{(8)}$

$(R_2)^{(8)}=(b'_{42})^{(8)}-(r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$:

$(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)}\leq -(a'_{44})^{(9)}+(a'_{45})^{(9)}-(a''_{44})^{(9)}(T_{45},t)+(a''_{45})^{(9)}(T_{45},t)\leq -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)}\leq -(b'_{44})^{(9)}+(b'_{45})^{(9)}-(b''_{44})^{(9)}\big((G_{47}),t\big)-(b''_{45})^{(9)}\big((G_{47}),t\big)\leq -(\tau_1)^{(9)}$

Definition of $(v_1)^{(9)},(v_2)^{(9)},(u_1)^{(9)},(u_2)^{(9)},v^{(9)},u^{(9)}$:

By $(v_1)^{(9)}>0$, $(v_2)^{(9)}<0$ and respectively $(u_1)^{(9)}>0$, $(u_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_1)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)}>0$, $(\bar{v}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$, $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

Definition of $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$:-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)},\ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$
and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \le G_{46}(t) \le \right.$$
$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (v_1)^{(1)} \le (\bar{v}_1)^{(1)} \le \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$, we obtain 386

$$(v_1)^{(1)} \le v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $v^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\right) - (a''_{17})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{\mathrm{G}_{16}}{\mathrm{G}_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (\mathrm{C})^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (\mathrm{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)}\right) \le \frac{dv^{(3)}}{dt} \le -\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \ge \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}}$$

it follows $(v_0)^{(3)} \le v^{(3)}(t) \le (v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \le v^{(3)}(t) \le (\bar{v}_1)^{(3)}$

If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)} \le \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}} \le v^{(3)}(t) \le$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (\bar{v}_1)^{(3)}$$

If $0 < (v_1)^{(3)} \le (\bar{v}_1)^{(3)} \le (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$, we obtain 403

$$(v_1)^{(3)} \le v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

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$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{36}'')^{(7)} = (a_{37}'')^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b_{36}'')^{(7)} = (b_{37}'')^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a_{40}')^{(8)} - (a_{41}')^{(8)} + (a_{40}'')^{(8)}(T_{41}, t)\right) - (a_{41}'')^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(8)} and\ (b''_i)^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\,]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\,]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\,]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0$ 479

$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0$ 480

$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0$ 481

$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0$ 482

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(bb) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first

equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T^*_{33} for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$$

Definition and uniqueness of T^*_{37} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T^*_{37} for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$$

Definition and uniqueness of T^*_{41} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{25} such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507
$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{29} such that $\varphi((G_{31})^*) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{33} such that $\varphi({G_{35}}^*) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{37} such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{41} such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right]+(b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{45} such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)} G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)} G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \qquad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \qquad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \qquad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \qquad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \qquad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \qquad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \qquad 530$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i \qquad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \qquad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \qquad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \qquad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \qquad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \qquad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}\mathrm{T}^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}\mathrm{T}^*_{18}\mathbb{G}_j\right) \quad 539$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}\left((G_{27})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)}$, $\frac{\partial (b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\big((a_{32}')^{(6)} + (p_{32})^{(6)}\big)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\big((a_{33}')^{(6)} + (p_{33})^{(6)}\big)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\big((a_{34}')^{(6)} + (p_{34})^{(6)}\big)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\big((b_{32}')^{(6)} - (r_{32})^{(6)}\big)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\big(s_{(32)(j)}T_{32}^*\mathbb{G}_j\big) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\big((b_{33}')^{(6)} - (r_{33})^{(6)}\big)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\big(s_{(33)(j)}T_{33}^*\mathbb{G}_j\big) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\big((b_{34}')^{(6)} - (r_{34})^{(6)}\big)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\big(s_{(34)(j)}T_{34}^*\mathbb{G}_j\big) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial (b_i'')^{(7)}}{\partial G_j}\big((G_{39})^{**} \big) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\big((a_{36}')^{(7)} + (p_{36})^{(7)}\big)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\big((a_{37}')^{(7)} + (p_{37})^{(7)}\big)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\big((a_{38}')^{(7)} + (p_{38})^{(7)}\big)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\big((b_{36}')^{(7)} - (r_{36})^{(7)}\big)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\big(s_{(36)(j)}T_{36}^*\mathbb{G}_j\big) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\big((b_{37}')^{(7)} - (r_{37})^{(7)}\big)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\big(s_{(37)(j)}T_{37}^*\mathbb{G}_j\big) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\big((b_{38}')^{(7)} - (r_{38})^{(7)}\big)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\big(s_{(38)(j)}T_{38}^*\mathbb{G}_j\big) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\Big)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\Big]$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\Big)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\Big)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\Big]$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\Big)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\Big)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\Big]$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\Big)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\Big)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\Big)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\Big)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\Big)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\Big)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION FORTY ONE

Pre-Conscious To Conscious Transition

INTRODUCTION—VARIABLES USED

Science 19 November 2004: Vol. 306 no. 5700 pp. 1330-1336 DOI: 10.1126/science.1104149 Quantum-Enhanced Measurements: Beating the Standard Quantum Limit Vittorio Giovannetti1, Seth Lloyd2,*, Lorenzo Maccone3

(1) Quantum mechanics, through the Heisenberg uncertainty principle, imposes (e&eb) limits on the precision of measurement.
(2) Conventional measurement techniques typically fail (e) to reach these limits.

(3) Conventional bounds to the precision of measurements such as the shot noise limit or the standard quantum limit are not(e) as fundamental as the Heisenberg limits and can be beaten using (e) quantum strategies that employ (e) "quantum tricks" such as squeezing and entanglement.

Quantum (in) stability of dilatonic AdS backgrounds and the holographic renormalization group with gravity Phys. Rev. D 62, 064006 – Published 31 July 2000 Shin'ichi Nojiri, Sergei D. Odintsov, and Sergio Zerbini

(4) The stability of dilatonic AdS spaces due to (e) quantum effects of dilaton coupled conformal matter is considered.
(5) When such spaces do not exist on a classical level, (eb) their dynamical generation occurs.
(6) Explicit examples corresponding to the quantum creation of (e) a 4D dilatonic AdS universe and a 2D dilatonic AdS black hole (BH) are presented.
(7) Motivated by the holographic RG, in a similar approach, the (e&eb) complete 5D effective action is discussed.
(8) The intermediate region where it is (=) the sum of two parts, the bulk (classical gravity) and (e&eb) boundary quantum action, is investigated.

NOTATION

Module One

(1) In this way, a transient superposition of slightly differing space-time geometries persists until (eb) an abrupt quantum → classical reduction occurs.
(2) Unlike the random, "subjective reduction" (SR, or R) of standard quantum theory caused by (e) observation or environmental entanglement, the OR authors propose in microtubules is (=) a self-collapse and it results in particular patterns of microtubule-tubulin conformational states that regulate (e&eb) neuronal activities including synaptic functions.
(3) Possibilities and probabilities for post-reduction tubulin states are influenced by (e&eb) factors including attachments of microtubule-associated proteins (MAPs) acting as (=) "nodes" which tune and "orchestrate" (e&eb) the quantum oscillations.
(4) They thus term the self-tuning OR process in microtubules "orchestrated objective reduction" ("Orch OR"), and calculate an estimate for the number of tubulins (and neurons) whose coherence for relevant time periods (e.g. 500 ms) will elicit Orch OR
(5) In **providing a connection** among (1) pre-conscious to conscious transition, (2) fundamental space-time notions, (3) non-computability, and (4) binding of various (time scale and spatial) reductions into an instantaneous event ("conscious now"), authors believe Orch OR in brain microtubules is (=) the most specific and plausible model for consciousness yet propos

G_{13} : Category one of abrupt quantum → classical reduction occurs

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of transient superposition of slightly differing space-time geometries persists

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

abrupt quantum → classical reduction occurs

G_{16} : Category one of quantum

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of classical

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Unlike the random, "subjective reduction" (SR, or R) of standard quantum theory caused by (e) observation or environmental entanglement, the OR authors propose in microtubules is (=) a self-collapse and it results in particular patterns of microtubule-tubulin conformational states that regulate (e&eb) neuronal activities including synaptic functions

G_{20} : Category one of observation or environmental entanglement, the OR authors propose in microtubules is (=) a self-collapse and it results in particular patterns of microtubule-tubulin conformational states that regulate (e&eb) neuronal activities including synaptic functions

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of "subjective reduction" (SR, or R) of standard quantum theory

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

"subjective reduction" (SR, or R) of standard quantum theory caused by observation or environmental entanglement, the OR authors propose in microtubules is (=) a self-collapse and it results in particular patterns of microtubule-tubulin conformational states that regulate (e&eb) neuronal activities including synaptic functions

G_{24} : Category one of "subjective reduction" (SR, or R) of standard quantum theory caused by observation or environmental entanglement, the OR authors propose in microtubules

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of self-collapse and it results in particular patterns of microtubule-tubulin conformational states that regulate (e&eb) neuronal activities including synaptic functions

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Possibilities and probabilities for post-reduction tubulin states are influenced by (e&eb) factors including attachments of microtubule-associated proteins (MAPs) acting as (=) "nodes" which tune and "orchestrate"

(e&eb) the quantum oscillations

G_{28} : Category one of Possibilities and probabilities for post-reduction tubulin states; factors including attachments of microtubule-associated proteins (MAPs) acting as (=) "nodes" which tune and "orchestrate" (e&eb) the quantum oscillations

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of factors including attachments of microtubule-associated proteins (MAPs) acting as (=) "nodes" which tune and "orchestrate" (e&eb) the quantum oscillations ;Possibilities and probabilities for post-reduction tubulin states

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Possibilities and probabilities for post-reduction tubulin states are influenced by factors including attachments of microtubule-associated proteins (MAPs) acting as (=) "nodes" which tune and "orchestrate" (e&eb) the quantum oscillations

G_{32} : Category one of Possibilities and probabilities for post-reduction tubulin states are influenced by factors including attachments of microtubule-associated proteins (MAPs); "nodes" which tune and "orchestrate" (e&eb) the quantum oscillations

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of "nodes" which tune and "orchestrate" (e&eb) the quantum oscillations; Possibilities and probabilities for post-reduction tubulin states are influenced by factors including attachments of microtubule-associated proteins (MAPs)

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Possibilities and probabilities for post-reduction tubulin states are influenced by factors including attachments of microtubule-associated proteins (MAPs) acting as "nodes" which tune and "orchestrate" (e&eb) the quantum oscillations

G_{36} : Category one of Possibilities and probabilities for post-reduction tubulin states are influenced by factors including attachments of microtubule-associated proteins (MAPs) acting as "nodes" which tune and "orchestrate" ; quantum oscillations

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of quantum oscillations; Possibilities and probabilities for post-reduction tubulin states are influenced by factors including attachments of microtubule-associated proteins (MAPs) acting as "nodes" which tune and "orchestrate"

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

In **providing a connection** among (1) pre-conscious to conscious transition, (2) fundamental space-time notions, (3) non-computability, and (4) binding of various (time scale and spatial) reductions into an instantaneous event ("conscious now"), authors believe Orch OR in brain microtubules is (=) the most specific and plausible model for consciousness yet propos

G_{40} : Category one of pre-conscious to conscious transition; fundamental space-time notions,

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of fundamental space-time notions ; pre-conscious to conscious transition

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

G_{44} : Category one of non-computability; binding of various (time scale and spatial) reductions into an instantaneous event ("conscious now")

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of binding of various (time scale and spatial) reductions into an instantaneous event ("conscious now"); non-computability

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION FORTY TWO

Quantum-Enhanced Measurements

INTRODUCTION—VARIABLES USED

Science 19 November 2004: Vol. 306 no. 5700 pp. 1330-1336 DOI: 10.1126/science.1104149 Quantum-Enhanced Measurements: Beating the Standard Quantum Limit Vittorio Giovannetti1, Seth Lloyd2,*,

Lorenzo Maccone3

(1) Quantum mechanics, through the Heisenberg uncertainty principle, imposes (e&eb) limits on the precision of measurement.
(2) Conventional measurement techniques typically fail (e) to reach these limits.
(3) Conventional bounds to the precision of measurements such as the shot noise limit or the standard quantum limit are not(e) as fundamental as the Heisenberg limits and can be beaten using (e) quantum strategies that employ (e) "quantum tricks" such as squeezing and entanglement.

Quantum (in) stability of dilatonic AdS backgrounds and the holographic renormalization group with gravity Phys. Rev. D 62, 064006 – Published 31 July 2000 Shin'ichi Nojiri, Sergei D. Odintsov, and Sergio Zerbini

(4) The stability of dilatonic AdS spaces due to (e) quantum effects of dilaton coupled conformal matter is considered.
(5) When such spaces do not exist on a classical level, (eb) their dynamical generation occurs.
(6) Explicit examples corresponding to the quantum creation of (e) a 4D dilatonic AdS universe and a 2D dilatonic AdS black hole (BH) are presented.
(7) Motivated by the holographic RG, in a similar approach, the (e&eb) complete 5D effective action is discussed.
(8) The intermediate region where it is (=) the sum of two parts, the bulk (classical gravity) and (e&eb) boundary quantum action, is investigated

NOTATION

Module One

Quantum mechanics, through the Heisenberg uncertainty principle, imposes (e&eb) limits on the precision of measurement

G_{13} : Category one of Quantum mechanics, through the Heisenberg uncertainty principle; limits on the precision of measurement

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of limits on the precision of measurement; Quantum mechanics, through the Heisenberg uncertainty principle

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Conventional measurement techniques typically fail (e) to reach these limits

G_{16} : Category one of reach these limits; Conventional measurement techniques typically

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of Conventional measurement techniques typically; reach these limits

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Conventional bounds to the precision of measurements such as the shot noise limit or the standard quantum limit are not(e) as fundamental as the Heisenberg limits and can be beaten using (e) quantum strategies that employ (e) "quantum tricks" such as squeezing and entanglement.

G_{20} : Category one of fundamental as the Heisenberg limits and can be beaten using (e) quantum strategies that employ (e) "quantum tricks" such as squeezing and entanglement

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of Conventional bounds to the precision of measurements such as the shot noise limit or the standard quantum limit

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Conventional bounds to the precision of measurements such as the shot noise limit or the standard quantum limit are not as fundamental as the Heisenberg limits and can be beaten using (e) quantum strategies that employ (e) "quantum tricks" such as squeezing and entanglement

G_{24} : Category one of quantum strategies that employ (e) "quantum tricks" such as squeezing and entanglement

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of Conventional bounds to the precision of measurements such as the shot noise limit or the standard quantum limit are not as fundamental as the Heisenberg limits and can be beaten

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Conventional bounds to the precision of measurements such as the shot noise limit or the standard quantum limit are not as fundamental as the Heisenberg limits and can be beaten using quantum strategies that employ (e) "quantum tricks" such as squeezing and entanglement

G_{28} : Category one of "quantum tricks" such as squeezing and entanglement

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of Conventional bounds to the precision of measurements such as the shot noise limit or

the standard quantum limit are not as fundamental as the Heisenberg limits and can be beaten using quantum strategies

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Stability of dilatonic AdS spaces due to (e) quantum effects of dilaton coupled conformal matter is considered.

G_{32} : Category one of quantum effects of dilaton coupled conformal matter

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of Stability of dilatonic AdS spaces

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

When such spaces do not exist on a classical level, (eb) their dynamical generation occurs

G_{36} : Category one of such spaces do not exist on a classical level

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of their dynamical generation occurs

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

Explicit examples corresponding to the

quantum creation of (e) a 4D dilatonic AdS universe and a 2D dilatonic AdS black hole (BH)

are presented

G_{40} : Category one of4D dilatonic AdS universe and a 2D dilatonic AdS black hole (BH)

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of quantum creation

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Motivated by the holographic RG, in a similar approach, the (e&eb) complete 5D effective action is discussed

intermediate region where it is the sum of two parts, the bulk (classical gravity) and (e&eb) boundary quantum action, is investigated

G_{44} : Category one of intermediate region; intermediate region where it is the sum of two parts, the bulk (classical gravity); boundary quantum action

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of sum of two parts, the bulk (classical gravity) and (e&eb) boundary quantum action, is investigated; boundary quantum action; intermediate region where it is the sum of two parts, the bulk (classical gravity)

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION FORTY TWO A

Dilatonic Ads Backgrounds And The Holographic Renormalization Group

INTRODUCTION—VARIABLES USED

Quantum (in) stability of dilatonic AdS backgrounds and the holographic renormalization group with gravity Phys. Rev. D 62, 064006 – Published 31 July 2000 Shin'ichi Nojiri, Sergei D. Odintsov, and Sergio Zerbini

(1) The effective equation solution representing a (eb) 5D AdS universe with a (e&eb) warp scale factor is found.
(2) A four-dimensional de Sitter or AdS world is generated on (eb) the boundary of such a universe as a result of (e) quantum effects. DOI: http://dx.doi.org/10.1103/PhysRevD.62.064006

Physics Letters B Volume 575, Issues 3–4, 27 November 2003, Pages 318–324 A "general boundary" formulation for quantum mechanics and quantum gravity Robert Oeckl doi:10.1016/j.physletb.2003.08.043

(3) Author proposes to formalize quantum theories as (=) topological quantum field theories in (eb) a generalized sense, associating state spaces with (e&eb) boundaries of arbitrary (and possibly finite) regions of space–time.
(4) They further propose to obtain such "general boundary" quantum theories through (e&eb) a generalized path integral quantization.

(5) They show how both, non-relativistic quantum mechanics and (e&eb) quantum field theory can be given (eb) a "general boundary" formulation.

(6) Surprisingly, even in the non-relativistic case, features normally associated with quantum field theory emerge from (e) consistency conditions.

(7) This includes states with (e&eb) arbitrary particle number and pair creation.

(8) They also note how three-dimensional quantum gravity is (=) an example for a realization of both proposals and suggest application to (e&eb) four-dimensional quantum gravity

NOTATION

Module One

Effective equation solution representing a (eb)
5D AdS universe with a (e&eb) warp scale factor is found.

G_{13} : Category one of5D AdS universe; warp scale factor

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of warp scale factor ;5D AdS universe

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

A four-dimensional de Sitter or AdS world is generated on (eb) the boundary of such a universe as a result of (e) quantum effects.

DOI: http://dx.doi.org/10.1103/PhysRevD.62.064006

G_{16} : Category one of four-dimensional de Sitter or AdS world

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of boundary of such a universe as a result of (e) quantum effects

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

four-dimensional de Sitter or AdS world is generated on the boundary of such a universe as a result of (e) quantum effects

G_{20} : Category one of quantum effects

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of four-dimensional de Sitter or AdS world is generated on the boundary of such a universe

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Author proposes to formalize

quantum theories as (=) topological quantum field theories in (eb) a generalized sense, associating state spaces with (e&eb) boundaries of arbitrary (and possibly finite) regions of space–time

Mathematical formulations

The original Atiyah-Segal axioms

Atiyah suggested a set of axioms for topological quantum field theory (Atiyah 1988a) which was inspired by Segal's proposed axioms for conformal field theory, and Witten's idea of the geometric meaning of supersymmetry, (Witten 1982). Atiyah's axioms are constructed on gluing the boundary with differentiable (topological or continuous) transformation, while Segal's are with conformal transformation. These axioms have been relatively useful for mathematical treatments of Schwarz-type QFTs, although it isn't clear that they capture the whole structure of Witten-type QFTs. The basic idea is that a TQFT is a functor from a certain category ofcobordisms to the category of vector spaces.

There are in fact two different sets of axioms which could reasonably be called the Atiyah axioms. These axioms differ basically in whether or not they study a TQFT defined on a single fixed n-dimensional Riemannian / Lorentzian spacetime M or a TQFT defined on all n-dimensional spacetimes at once.

Let Λ be a commutative ring with 1 (for almost all real-world purposes we will have Λ = Z, R or C). Atiyah originally proposed the axioms of a topological quantum field theory (TQFT) in dimension d defined over a ground ring Λ as following:

A finitely generated Λ-module $Z(\Sigma)$ associated to each oriented closed smooth d-dimensional manifold Σ (corresponding to the homotopy axiom),

An element $Z(M) \in Z(\partial M)$ associated to each oriented smooth (d+1)-dimensional manifold (with boundary) M (corresponding to an additive axiom).

These data are subject to the following axioms (4 and 5 were added by Atiyah):

Z is functorial with respect to orientation preserving diffeomorphisms of Σ and M,

Z is involutory, i.e. $Z(\Sigma^*) = Z(\Sigma)^*$ where Σ^* is Σ with opposite orientation and $Z(\Sigma)^*$ denotes the dual module,

Z is multiplicative.

$Z(\varphi) = \Lambda$ for the d-dimensional empty manifold and $Z(\varphi) = 1$ for the (d+1)-dimensional empty manifold.

$Z(M^*) = Z(M)$ (the hermitian axiom). Equivalently, $Z(M^*)$ is the disjoint of $Z(M)$

Remark. If for a closed manifold M we view Z(M) as a numerical invariant, then for a manifold with boundary we should think of $Z(M) \in Z(\partial M)$ as a "relative" invariant. Let $f : \Sigma \times I \to \Sigma \times I$ be an orientation preserving diffeomorphism, and identify opposite ends of $\Sigma \times I$ by f. This gives a manifold Σf and our

axioms imply

$$Z(\Sigma_f) = \text{Trace } \Sigma(f)$$

where Σ(f) is the induced automorphism of Z(Σ).

Remark. For a manifold M with boundary Σ we can always form the double $M \cup_\Sigma M^*$ which is a closed manifold. The fifth shows that

$$Z(M \cup_\Sigma M^*) = |Z(M)|^2$$

Here on the right we compute the norm in the hermitian (possibly indefinite) metric.

The relation to physics

Physically (2)+(4) is related to relativistic invariance while (3)+(5) is indicative of the quantum nature of the theory.

Σ is meant to indicate the physical space (usually, d = 3 for standard physics) and the extra dimension in Σ × I is "imaginary" time. The space Z(M) is the Hilbert space of the quantum theory and a physical theory, with a Hamiltonian H, will have an time evolution operator eitH or an "imaginary time" operator e−tH. The main feature of topological QFTs is that H = 0, which implies that there is no real dynamics or propagation, along the cylinder Σ × I. However, there can be non-trivial "propagation" (or tunneling amplitudes) from Σ0 to Σ1 through an intervening manifold M with $\partial M = \Sigma_0^* \cup \Sigma_1$; this reflects the topology of M.

If ∂M = Σ, then the distinguished vector Z(M) in the Hilbert space Z(Σ) is thought of as the vacuum state defined by M. For a closed manifold M the number Z(M) is the vacuum expectation value. In analogy with statistical mechanics it is also called the partition function.

The reason why a theory with zero Hamiltonian can be sensibly formulated in the Feynman path integral approach to QFT. This incorporates relativistic invariance (which caters for general (d+1)-dimensional "spacetimes") and the theory is formally defined by writing down a suitable Lagrangian - a functional of the classical fields of the theory. A Lagrangian which involves only first derivatives in time formally leads to a zero Hamiltonian, but the Lagrangian itself may have non-trivial features which relate it to the topology of M.

G_{24} : Category one of quantum theories; LHS of the above equations

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of topological quantum field theories in (eb) a generalized sense, associating state spaces with (e&eb) boundaries of arbitrary (and possibly finite) regions of space–time; RHS of the above equations

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

quantum theories as topological quantum field theories in (eb) a generalized sense, associating state spaces

with (e&eb) boundaries of arbitrary (and possibly finite) regions of space–time

G_{28} : Category one of quantum theories as topological quantum field theories

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of generalized sense, associating state spaces with (e&eb) boundaries of arbitrary (and possibly finite) regions of space–time

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

quantum theories as topological quantum field theories in a generalized sense, associating state spaces with (e&eb) boundaries of arbitrary (and possibly finite) regions of space–time

G_{32} : Category one of quantum theories as topological quantum field theories in a generalized sense, associating state spaces; boundaries of arbitrary (and possibly finite) regions of space–time

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of boundaries of arbitrary (and possibly finite) regions of space–time; quantum theories as topological quantum field theories in a generalized sense, associating state spaces

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

They further propose to obtain such "general boundary" quantum theories through (e&eb) a generalized path integral quantization

G_{36} : Category one of "general boundary" quantum theories; generalized path integral quantization

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of generalized path integral quantization ; "general boundary" quantum theories

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

They show how both,

non-relativistic quantum mechanics and (e&eb) quantum field theory can be given (eb) a "general boundary" formulation

G_{40} : Category one of non-relativistic quantum mechanics; quantum field theory can be given (eb) a "general boundary" formulation

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of quantum field theory can be given (eb) a "general boundary" formulation; on-relativistic quantum mechanics

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

non-relativistic quantum mechanics and quantum field theory can be given (eb) a "general boundary" formulation

G_{44} : Category one of non-relativistic quantum mechanics and quantum field theory

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of "general boundary" formulation

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,

$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$
$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor
$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}),t)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}),t)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}),t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}),t) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth

detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & \boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} & \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & \boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} & \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & \boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} & \end{bmatrix} G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & \boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & \boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} & \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} & \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a_{40}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a_{44}'')^{(9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b_{20}')^{(3)} & -(b_{20}'')^{(3)}(G_{23},t) & -(b_{16}'')^{(2,2,2)}(G_{19},t) & -(b_{13}'')^{(1,1,1,)}(G,t) \\ -(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b_{36}'')^{(7,7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8,8)}(G_{43},t) & -(b_{44}'')^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b_{21}')^{(3)} & -(b_{21}'')^{(3)}(G_{23},t) & -(b_{17}'')^{(2,2,2)}(G_{19},t) & -(b_{14}'')^{(1,1,1,)}(G,t) \\ -(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b_{37}'')^{(7,7,7,7)}(G_{39},t) & -(b_{41}'')^{(8,8,8,8)}(G_{43},t) & -(b_{45}'')^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b_{22}')^{(3)} & -(b_{22}'')^{(3)}(G_{23},t) & -(b_{18}'')^{(2,2,2)}(G_{19},t) & -(b_{15}'')^{(1,1,1,)}(G,t) \\ -(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b_{38}'')^{(7,7,7,7)}(G_{39},t) & -(b_{42}'')^{(8,8,8,8)}(G_{43},t) & -(b_{46}'')^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b_{20}'')^{(3)}(G_{23},t)$, $-(b_{21}'')^{(3)}(G_{23},t)$, $-(b_{22}'')^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b_{16}'')^{(2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b_{13}'')^{(1,1,1,)}(G,t)$, $-(b_{14}'')^{(1,1,1,)}(G,t)$, $-(b_{15}'')^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b_{36}'')^{(7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b_{40}'')^{(8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)} & \boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{24} \quad 73$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)} & \boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{25} \quad 74$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)} & \boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
are sixth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
are seventh augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
are eighth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)} & \boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad 76$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)} & \boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad (78)$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
are fourth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad (81)$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation

coefficients for category 1,2, and 3
$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3
$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3
$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3
$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3
$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3
$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3
$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3
$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3
$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3
$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3
$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3
$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3
$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \quad 85$$

$$- \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & \boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} & \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} & \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)} & \boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} & \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)} & \boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} & \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)} & \boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} & \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}, \boxed{(a''_{37})^{(7)}(T_{37},t)}, \boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

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$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq \ (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = \ (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \ , \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\,(a_i)^{(1)} + (a_i')^{(1)} + \ (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}\,(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i')^{(1)} + \ (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \leq (p_i)^{(2)} \leq \left(\hat{A}_{16}\right)^{(2)} \qquad 102$$

$$(b_i'')^{(2)}(G_{19},t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\big((G_{19}),t\big) = \ (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - \ T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\big((G_{19}),t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\big[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\big]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}} \ , \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\,(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + \ (\hat{P}_{16})^{(2)}\,(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + \ (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \le (p_i)^{(3)} \le (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \le (r_i)^{(3)} \le (b_i')^{(3)} \le (\hat{B}_{20})^{(3)}$$

$$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with (116)
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \quad (117)$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\widehat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\widehat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\widehat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\,(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and \ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\,(a_i)^{(6)} + (a_i')^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)}\,(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(MMMMMMM) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(NNNNNNN) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37},t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39},t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

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(OOOOOOO) $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

(PPPPPPP)

$\lim_{G\to\infty}(b_i'')^{(7)}((G_{39}),t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}((G_{39}),t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$. (T_{37}',t) and (T_{37},t) are points belonging to the interval $\left[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\right]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(QQQQQQQ) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(RRRRRRR) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$ 138

$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$ 139

$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$ 140

$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t}$ 142

$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t}$ 143

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ 144

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}$, $(r_i)^{(8)}, i = 40,41,42$,
Satisfy the inequalities

$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 145

$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 146

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\hat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)} ||(G_{47}) - (G_{47})'|| e^{-(\hat{M}_{44})^{(9)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and $(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)}$ *and* $(\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}\left[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)} (\hat{k}_{44})^{(9)}\right] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)} t}$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)} t}$, $T_i(0) = T_i^0 > 0$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$, $G_i(0) = G_i^0 > 0$

$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$, $T_i(0) = T_i^0 > 0$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \ , \ T_i(0) = T_i^0 \ , \ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} , T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\overline{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^{0} + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^{0} + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^{0} + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^{0} + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^{0} + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^{0} + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By

$$\bar{G}_{32}(t) = G_{32}^{0} + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^{0} + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^{0} + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^{0} + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^{0} + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^{0} + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad (169)$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that (170)

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that (171)

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that (172)

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that (173)

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that (174)

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \leq G_{32}^{0}+\int_0^t\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)} =$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(cc) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \leq G_{36}^{0}+\int_0^t\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)} =$$
$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \leq G_{40}^{0}+\int_0^t\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)} = \quad 180$$
$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t} \leq \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for G_{41} , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \leq G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \leq \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for G_{45} , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \left\{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\right\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)}\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) - (a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right| \; e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} \} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \qquad (186)$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13{,}14{,}15$ depend only on T_{14} and respectively on G(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: (187)

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. (188)

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$

$sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\hat{M}_{16})^{(2)}t}\}$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $(\widetilde{G_{19}}, \widetilde{T_{19}}) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t (a_{16})^{(2)}|G_{17}^{(1)} - G_{17}^{(2)}|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}ds_{(16)} +$

$\int_0^t\{(a_{16}')^{(2)}|G_{16}^{(1)} - G_{16}^{(2)}|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}\left|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t} \leq$
$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)d\left(\left((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)}\right)\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\}ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0$ for $t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)}G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$: $\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0$ *and* $T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$G_{24}^{(2)} \left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)}\right) d\left(((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on T_{25} and respectively on (G_{27})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (228)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_2 / (a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)}, (T_{35})^{(1)}), ((G_{35})^{(2)}, (T_{35})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}), (\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}), (\widetilde{T_{35}}) \right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right| \; e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} \} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)} (\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)}; \; (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t} \; and \; (\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$ (250)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2 \; and \; \left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33} \; and \; G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$$
$$sup_i\{max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}, max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)} \left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t} \; and \; (\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)} (T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2 \; and \; \left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37} \; and \; G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\,((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1,\left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right),\quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$

From 99 to 44 it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{(-(b_i')^{(9)}t)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)}, , (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)} t} - e^{-(a'_{26})^{(4)} t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)} t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28}e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G^0_{28}e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}}G^0_{28}e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}}G^0_{28}e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)}G^0_{28}}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G^0_{30}e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

$$\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_2)^{(5)}((S_1)^{(5)}-(a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t}-e^{-(a'_{30})^{(5)}t}\right]+\ G_{30}^{0}e^{-(a'_{30})^{(5)}t}\Big)$$

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\le T_{30}(t)\le \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$:

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$-(\sigma_2)^{(6)}\le-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\le-(\sigma_1)^{(6)}$

$-(\tau_2)^{(6)}\le-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\le-(\tau_1)^{(6)}$

Definition of $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$: 350

By $(v_1)^{(6)}>0$, $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$, $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

Definition of $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)}>0$, $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$, $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

Definition of $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$:-

If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)}=(v_0)^{(6)},(m_1)^{(6)}=(v_1)^{(6)},$ ***if*** $(v_0)^{(6)}<(v_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}$ $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}$ $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}$ $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}$ $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}$ $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})} \left[e^{(S_1)^{(6)} t} - e^{-(a'_{34})^{(6)} t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)} t} \right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^{0}e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}}G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}}G_{36}^{0}e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t}-e^{-(S_2)^{(7)}t}\right]+G_{38}^{0}e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left.\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_2)^{(7)}((S_1)^{(7)}-(a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t}-e^{-(a'_{38})^{(7)}t}\right]+\ G_{38}^{0}e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}}T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}}T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right) - (b''_{41})^{(8)}\left((G_{43}), t\right) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(v_1)^{(8)} > 0$, $(v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$, $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{v}_1)^{(8)}$,, $(\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{v}_1)^{(8)} > 0$, $(\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$, $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$:-

If we define $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$, $\boldsymbol{if}$ $(v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$, $\boldsymbol{if}$ $(v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$,
and $\boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}$, $\boldsymbol{if}$ $(\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, $\boldsymbol{if}$ $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, $\boldsymbol{if}$ $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}$ $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t})$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t}\leq T_{42}(t)\leq \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)},(S_2)^{(8)},(R_1)^{(8)},(R_2)^{(8)}$:- 381

Where $(S_1)^{(8)}=(a_{40})^{(8)}(m_2)^{(8)}-(a'_{40})^{(8)}$

$(S_2)^{(8)}=(a_{42})^{(8)}-(p_{42})^{(8)}$

$(R_1)^{(8)}=(b_{40})^{(8)}(\mu_2)^{(8)}-(b'_{40})^{(8)}$

$(R_2)^{(8)}=(b'_{42})^{(8)}-(r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$:

$(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)}\leq -(a'_{44})^{(9)}+(a'_{45})^{(9)}-(a''_{44})^{(9)}(T_{45},t)+(a''_{45})^{(9)}(T_{45},t)\leq -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)}\leq -(b'_{44})^{(9)}+(b'_{45})^{(9)}-(b''_{44})^{(9)}\big((G_{47}),t\big)-(b''_{45})^{(9)}\big((G_{47}),t\big)\leq -(\tau_1)^{(9)}$

Definition of $(v_1)^{(9)},(v_2)^{(9)},(u_1)^{(9)},(u_2)^{(9)},v^{(9)},u^{(9)}$:

By $(v_1)^{(9)}>0$, $(v_2)^{(9)}<0$ and respectively $(u_1)^{(9)}>0$, $(u_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_1)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)}>0$, $(\bar{v}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$, $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

Definition of $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$:-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)},\ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$
and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \le G_{46}(t) \le \right.$$
$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (v_1)^{(1)} \le (\bar{v}_1)^{(1)} \le \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$, we obtain 386

$$(v_1)^{(1)} \le v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $v^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\right) - (a''_{17})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{\mathrm{G}_{16}}{\mathrm{G}_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (\mathrm{C})^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (\mathrm{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)}\right) \le \frac{dv^{(3)}}{dt} \le -\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \ge \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}}$$

it follows $(v_0)^{(3)} \le v^{(3)}(t) \le (v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \le v^{(3)}(t) \le (\bar{v}_1)^{(3)}$

If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)} \le \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}} \le v^{(3)}(t) \le$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (\bar{v}_1)^{(3)}$$

If $0 < (v_1)^{(3)} \le (\bar{v}_1)^{(3)} \le (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$, we obtain 403

$$(v_1)^{(3)} \le v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}$, $\boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

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$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(8)} and\ (b''_i)^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\ ]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\ ]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\ ]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0$ 479

$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0$ 480

$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0$ 481

$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0$ 482

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(cc) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first

equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T^*_{33} for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$$

Definition and uniqueness of T^*_{37} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T^*_{37} for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$$

Definition and uniqueness of T^*_{41} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{25} such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507
$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{29} such that $\varphi((G_{31})^*) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{33} such that $\varphi({G_{35}}^*) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{37} such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{41} such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right]+(b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{45} such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)} G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)} G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \qquad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i \quad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \quad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \quad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}\mathrm{T}^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}\mathrm{T}^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}\left((G_{27})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

ASYMPTOTIC STABILITY ANALYSIS 555

Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^{*}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^{*}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^{*}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^{*}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^{*}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^{*}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^{*}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^{*}\mathbb{G}_j\right) \quad 570$$

ASYMPTOTIC STABILITY ANALYSIS 571

Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS

Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

(586 A)

ASYMPTOTIC STABILITY ANALYSIS

Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\text{ B})$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\Big)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\Big]$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\Big)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\Big)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\Big]$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\Big)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\Big)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\Big]$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\Big)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\Big)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\Big)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\Big)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION FORTY THREE

Time As An, "Indirect Representation," (C2 35-6), The Viewer Experiences The Movement Of Time Itself, Which Images, Scenes, Plots And Characters Presuppose Or Manifest In Order To Gain Any Sort Of Movement Whatsoever

INTRODUCTION—VARIABLES USED

(**Wikipedia: Gilles Deleuze)**

(1) 'Natural' Links Precisely Lost (e) Their Efficacy.
(2) And With The Use Of Unnatural Or False Links, Which Do Not Follow The Sequence Or Narrative Affect Of The Movement-Image, Time Itself, The Time-Image, Is Manifested In (eb) Cinema

(Deleuze Considers Orson Welles To Be The First Auteur To Make Use Of The Time-Image (C2 137)).

(3) Rather Than Finding **Time As An, "Indirect Representation," (C2 35-6), The Viewer Experiences (e) The Movement Of Time Itself, Which Images, Scenes, Plots And Characters Presuppose Or Manifest (eb) In Order To Gain Any Sort Of Movement Whatsoever**.

(4) Along With This 'External' Reason, There Is Also for Deleuze a Motivation within Cinema Itself to Go from the Movement-Image To (e&eb) The Time-Image.

(5) The Movement Image Has (e) The Tendency, Thanks To The Habitual Experience Of Movement As (=) Normal And Centered, To Justify Itself In Relation To (e&eb) Truth:

(6) As Deleuze Argues With Regard To The Dogmatic Image Of Thought (See (3)(D) Above), There Is (=) The Presupposition That **Thought Naturally Moves Towards Truth**

(7) Of Course, Deleuze Suggests, Cinema, When Truly Creative, Never relied upon (e) This Presupposition, And Yet, "The Movement-Image, In Its Very Essence, Is Answerable To (e) The Effect of Truth Which It Invokes (eb) While Movement Preserves Its Centres." (C2 142).

(8) Deleuze Argues, Cinema Moved towards a New, Different, Way of Understanding Movement Itself, In (eb) Questioning Its Own Presuppositions As Subordinate To Time. (Wikipedia: Gilles Deleuze) (have made slight changes in this sentence)

NOTATION

Module One

'Natural' Links Precisely Lost (e) Their Efficacy

G_{13} : Category one of Their Efficacy

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of 'Natural' Links Precisely

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Time Itself, The Time-Image, And With The Use Of Unnatural Or False Links, Which Do Not Follow The Sequence Or Narrative Affect Of The Movement-Image Is Manifested In (eb) Cinema (Deleuze Considers Orson Welles To Be The First Auteur To Make Use Of The Time-Image (C2 137))

G_{16} : Category one of Time Itself, The Time-Image, And With The Use Of Unnatural Or False Links, Which Do Not Follow The Sequence Or Narrative Affect Of The Movement-Image

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of Cinema (Deleuze Considers Orson Welles To Be The First Auteur To Make Use Of The Time-Image (C2 137))

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Time Itself, The Time-Image, And With The Use Of Unnatural Or False Links, Which Do Not (e) Follow The Sequence Or Narrative Affect Of The Movement-Image Is Manifested In Cinema (Deleuze Considers Orson Welles To Be The First Auteur To Make Use Of The Time-Image (C2 137))

G_{20} : Category one of Follow The Sequence Or Narrative Affect Of The Movement-Image Is Manifested In Cinema (Deleuze Considers Orson Welles To Be The First Auteur To Make Use Of The Time-Image (C2 137))

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of Time Itself, The Time-Image, And With The Use Of Unnatural Or False Links

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Rather Than Finding **Time As An, "Indirect Representation," (C2 35-6), The Viewer Experiences (e) The Movement Of Time Itself, Which Images, Scenes, Plots And Characters Presuppose Or Manifest (eb) In Order To Gain Any Sort Of Movement Whatsoever**

G_{24} : Category one of **The Movement Of Time Itself, Which Images, Scenes, Plots And Characters Presuppose Or Manifest (eb) In Order To Gain Any Sort Of Movement Whatsoever**

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of **Time NOT As An, "Indirect Representation," (C2 35-6), The Viewer Experiences**

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Time Rather **As An, "Indirect Representation," (C2 35-6), The Viewer Experiences The Movement Of Time Itself, Which Images, Scenes, Plots And Characters Presuppose Or Manifest (eb) In Order To Gain Any Sort Of Movement Whatsoever**

G_{28} : Category one of **Time** Rather **As An, "Indirect Representation," (C2 35-6), The Viewer Experiences The Movement Of Time Itself, Which Images, Scenes, Plots And Characters Presuppose Or Manifest**

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of **Gain Any Sort Of Movement Whatsoever**

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Along With This 'External' Reason, There Is Also for Deleuze a Motivation within Cinema Itself to Go from the Movement-Image To (e&eb) The Time-Image

G_{32} : Category one of 'External' Reason, There Is Also for Deleuze a Motivation within Cinema Itself to Go from the Movement-Image; Time-Image

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of Time-Image; 'External' Reason, There Is Also for Deleuze a Motivation within Cinema Itself to Go from the Movement-Image

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

The Movement Image Has (e) The Tendency, Thanks To The Habitual Experience Of Movement As (=) Normal And Centered, To Justify Itself In Relation To (e&eb) Truth

G_{36} : Category one of Tendency, Thanks To The Habitual Experience Of Movement As (=) Normal And Centered, To Justify Itself In Relation To (e&eb) Truth

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of Movement Image

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

Movement Image Has The Tendency, Thanks To The Habitual Experience Of Movement As (=) Normal And Centered, To Justify Itself In Relation To (e&eb) Truth

G_{40} : Category one of Movement Image Has The Tendency, Thanks To The Habitual Experience Of Movement

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of Normal And Centered, To Justify Itself In Relation To (e&eb) Truth

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

Movement Image Has The Tendency, Thanks To The Habitual Experience Of Movement As Normal And Centered, To Justify Itself In Relation To (e&eb) Truth

G_{44} : Category one of Movement Image Has The Tendency, Thanks To The Habitual Experience Of Movement As Normal And Centered, To Justify Itself In Relation; Truth

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of Truth; Movement Image Has The Tendency, Thanks To The Habitual Experience Of Movement As Normal And Centered, To Justify Itself In Relation

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,

are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right]G_{13} \quad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14},t)\right]G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14},t)\right]G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G,t)\right]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t)\right]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}),t\big)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}),t\big)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}),t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\big((G_{19}),t\big) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}),t)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}),t)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}),t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}),t) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & \boxed{+(a''_{13})^{(1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & \boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & \boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation

coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\left[-(b''_{13})^{(1)}(G,t)\right] & \left[-(b''_{16})^{(2,2,)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,)}(G_{35},t)\right] \\ \left[-(b''_{36})^{(7,7,)}(G_{39},t)\right] & \left[-(b''_{40})^{(8,8)}(G_{43},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\left[-(b''_{14})^{(1)}(G,t)\right] & \left[-(b''_{17})^{(2,2,)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,)}(G_{35},t)\right] \\ \left[-(b''_{37})^{(7,7,)}(G_{39},t)\right] & \left[-(b''_{41})^{(8,8)}(G_{43},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\left[-(b''_{15})^{(1)}(G,t)\right] & \left[-(b''_{18})^{(2,2,)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,)}(G_{35},t)\right] \\ \left[-(b''_{38})^{(7,7,)}(G_{39},t)\right] & \left[-(b''_{42})^{(8,8)}(G_{43},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t) & \\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t) & \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & +(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) & +(a''_{21})^{(3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t) & \\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t) & \end{bmatrix} G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & +(a''_{18})^{(2)}(T_{17},t) & +(a''_{15})^{(1,1,)}(T_{14},t) & +(a''_{22})^{(3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33},t) & \\ +(a''_{38})^{(7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9)}(T_{45},t) & \end{bmatrix} G_{18} \quad 63$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)} & \boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{24} \quad 73$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)} & \boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{25} \quad 74$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)} & \boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
are sixth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
are seventh augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
are eighth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)} & \boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad 76$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)} & \boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad 78$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
are fourth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad 81$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation

coefficients for category 1,2, and 3
$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3
$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3
$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3
$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3
$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3
$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3
$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3
$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3
$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3
$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3
$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3
$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3
$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \quad 85$$

$$- \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & \boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} & \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} & \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)} & \boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} & \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)} & \boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} & \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)} & \boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} & \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}, \boxed{(a''_{37})^{(7)}(T_{37},t)}, \boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a_{46}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a_{44}'')^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b_{36}')^{(7)}\boxed{-(b_{36}'')^{(7)}(G_{39},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{44}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b_{37}')^{(7)}\boxed{-(b_{37}'')^{(7)}(G_{39},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{45}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b_{38}')^{(7)}\boxed{-(b_{38}'')^{(7)}(G_{39},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b_{46}'')^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\,-(b''_{40})^{(8)}(G_{43},t) & -(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\,-(b''_{41})^{(8)}(G_{43},t) & -(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\,-(b''_{42})^{(8)}(G_{43},t) & -(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)}\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)}\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)}\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq \ (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = \ (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \ , \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\,(a_i)^{(1)} + (a_i')^{(1)} + \ (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}\,(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i')^{(1)} + \ (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \leq (p_i)^{(2)} \leq \left(\hat{A}_{16}\right)^{(2)} \qquad 102$$

$$(b_i'')^{(2)}(G_{19},t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\big((G_{19}),t\big) = \ (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - \ T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\big((G_{19}),t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\big[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\big]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}} \ , \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\,(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + \ (\hat{P}_{16})^{(2)}\,(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + \ (\hat{Q}_{16})^{(2)}\ \ (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \le (p_i)^{(3)} \le (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \le (r_i)^{(3)} \le (b_i')^{(3)} \le (\hat{B}_{20})^{(3)}$$

$$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with (116)
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \quad (117)$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\widehat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\widehat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\widehat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - \ T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\,(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and \ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\,(a_i)^{(6)} + (a_i')^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)}\,(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(SSSSSSS) $\quad (a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(TTTTTTT) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$(a_i'')^{(7)}(T_{37},t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$

$(b_i'')^{(7)}(G_{39},t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$ 132

(UUUUUUU) $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

(VVVVVVV)

$\lim_{G\to\infty}(b_i'')^{(7)}((G_{39}),t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \leq (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$

$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}((G_{39}),t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$. (T_{37}',t) and (T_{37},t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(WWWWWWW) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(XXXXXXX) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$

$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$

Where we suppose

$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$ 138

$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$ 139

$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$ 140

$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t}$ 142

$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t}$ 143

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ 144

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}$, $(r_i)^{(8)}, i = 40,41,42$,
Satisfy the inequalities

$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 145

$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 146

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\hat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)} ||(G_{47}) - (G_{47})'|| e^{-(\hat{M}_{44})^{(9)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}\left[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)} (\hat{k}_{44})^{(9)}\right] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t} \ , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t} \ , \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \ , \ T_i(0) = T_i^0 \ , \ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} , T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\overline{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that 174

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \leq G_{32}^{0}+\int_0^t\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(dd) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \leq G_{36}^{0}+\int_0^t\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \leq G_{40}^{0}+\int_0^t\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}= \quad 180$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t} \leq \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for G_{41} , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \leq G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \leq \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for G_{45} , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \left\{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\right\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+\ (a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\ (t)=0\ and\ T_i\ (t)=0$

From 19 to 24 it results

$$G_i\ (t)\geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\geq 0$$

$$T_i\ (t)\geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13}<(\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt}\leq\left((\widehat{M}_{13})^{(1)}\right)_1-(a'_{14})^{(1)}G_{14}$ and by integrating

$$G_{14}\leq\left((\widehat{M}_{13})^{(1)}\right)_2=G_{14}^0+2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15}\leq\left((\widehat{M}_{13})^{(1)}\right)_3=G_{15}^0+2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a'_{15})^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}} \leq$
$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{\mathrm{A}}_{16})^{(2)} + (\widehat{\mathrm{P}}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)\mathrm{d}\left(\left((G_{19})^{(1)}, (T_{19})^{(1)};\ (G_{19})^{(2)}, (T_{19})^{(2)}\right)\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $\mathrm{G}_i\,(\mathrm{t}) = 0$ and $\mathrm{T}_i\,(\mathrm{t}) = 0$ 200

it results

$\mathrm{G}_i\,(\mathrm{t}) \geq \mathrm{G}_i^0\mathrm{e}^{\left[-\int_0^{\mathrm{t}}\{(a_i')^{(2)} - (a_i'')^{(2)}(\mathrm{T}_{17}(s_{(16)}), s_{(16)})\}\mathrm{d}s_{(16)}\right]} \geq 0$

$\mathrm{T}_i\,(\mathrm{t}) \geq \mathrm{T}_i^0\mathrm{e}^{(-(b_i')^{(2)}\mathrm{t})} > 0 \quad \text{for t} > 0$

Definition of $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1$, $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2$ and $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$\mathrm{G}_{16} < (\widehat{\mathrm{M}}_{16})^{(2)}$ it follows $\frac{\mathrm{dG}_{17}}{\mathrm{dt}} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)}\mathrm{G}_{17}$ and by integrating

$\mathrm{G}_{17} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 = \mathrm{G}_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$

In the same way , one can obtain

$\mathrm{G}_{18} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3 = \mathrm{G}_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{\mathrm{t}\to\infty}((b_i'')^{(2)}\,((G_{19})(\mathrm{t}), \mathrm{t})) = (b_{17}')^{(2)}$ then $\mathrm{T}_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$: $\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$G_{20}^{(2)} \left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)} + (a'_{20})^{(3)} + (\widehat{A}_{20})^{(3)} + (\widehat{P}_{20})^{(3)} (\widehat{k}_{20})^{(3)}\right) d\left(((G_{23})^{(1)}, (T_{23})^{(1)};\ (G_{23})^{(2)}, (T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ *and* $(\widehat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(3)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_1 / (a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)} \left((\widehat{M}_{20})^{(3)}\right)_2 / (a'_{22})^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$G_{24}^{(2)} \left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)}\right) d\left(((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on T_{25} and respectively on (G_{27})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (228)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_2 / (a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\tilde{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\tilde{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\tilde{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\tilde{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)}, (T_{35})^{(1)}), ((G_{35})^{(2)}, (T_{35})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})}, \widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})}, \widetilde{(T_{35})}\right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right)\left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)};\ (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ *and* $(\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ (250)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$$
$$sup_i\{max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}, max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)} \left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)}; (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on (G_{39})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)} (T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(7)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\,((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1,\left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right),\quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$

From 99 to 44 it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(9)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)},, (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)}, , (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad (333)$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad (334)$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)} t} - e^{-(a'_{26})^{(4)} t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)} t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad (335)$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad (336)$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G^0_{28} e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G^0_{28}}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G^0_{30} e^{-(S_2)^{(5)} t} \le G_{30}(t) \le \right. \quad 344$$

$$\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_2)^{(5)}((S_1)^{(5)}-(a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t}-e^{-(a'_{30})^{(5)}t}\right]+\ G_{30}^{0}e^{-(a'_{30})^{(5)}t}\Big)$$

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\le T_{30}(t)\le \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$:

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)}\le-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\le-(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)}\le-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\le-(\tau_1)^{(6)}$$

Definition of $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$: 350

By $(v_1)^{(6)}>0$, $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$, $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

Definition of $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)}>0$, $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$, $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

Definition of $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$:-

If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)}=(v_0)^{(6)},(m_1)^{(6)}=(v_1)^{(6)},$ ***if*** $(v_0)^{(6)}<(v_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}$ $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}$ $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}$ $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}$ $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}$ $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})} \left[e^{(S_1)^{(6)} t} - e^{-(a'_{34})^{(6)} t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)} t} \right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^{0}e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}}G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}}G_{36}^{0}e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t}-e^{-(S_2)^{(7)}t}\right]+G_{38}^{0}e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left.\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_2)^{(7)}((S_1)^{(7)}-(a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t}-e^{-(a'_{38})^{(7)}t}\right]+\ G_{38}^{0}e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}}T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}}T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right) - (b''_{41})^{(8)}\left((G_{43}), t\right) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(v_1)^{(8)} > 0$, $(v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$, $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{v}_1)^{(8)}$,, $(\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{v}_1)^{(8)} > 0$, $(\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$, $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$:-

If we define $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$, $\boldsymbol{if}$ $(v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$, $\boldsymbol{if}$ $(v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$,
and $\boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}$, $\boldsymbol{if}$ $(\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, $\boldsymbol{if}$ $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, $\boldsymbol{if}$ $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}$ $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t})$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t}\leq T_{42}(t)\leq \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)},(S_2)^{(8)},(R_1)^{(8)},(R_2)^{(8)}$:- 381

Where $(S_1)^{(8)}=(a_{40})^{(8)}(m_2)^{(8)}-(a'_{40})^{(8)}$

$(S_2)^{(8)}=(a_{42})^{(8)}-(p_{42})^{(8)}$

$(R_1)^{(8)}=(b_{40})^{(8)}(\mu_2)^{(8)}-(b'_{40})^{(8)}$

$(R_2)^{(8)}=(b'_{42})^{(8)}-(r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$:

$(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)}\leq -(a'_{44})^{(9)}+(a'_{45})^{(9)}-(a''_{44})^{(9)}(T_{45},t)+(a''_{45})^{(9)}(T_{45},t)\leq -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)}\leq -(b'_{44})^{(9)}+(b'_{45})^{(9)}-(b''_{44})^{(9)}\big((G_{47}),t\big)-(b''_{45})^{(9)}\big((G_{47}),t\big)\leq -(\tau_1)^{(9)}$

Definition of $(v_1)^{(9)},(v_2)^{(9)},(u_1)^{(9)},(u_2)^{(9)},v^{(9)},u^{(9)}$:

By $(v_1)^{(9)}>0$, $(v_2)^{(9)}<0$ and respectively $(u_1)^{(9)}>0$, $(u_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_1)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)}>0$, $(\bar{v}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$, $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

Definition of $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$:-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)},\ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$
and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \le G_{46}(t) \le \right.$$
$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (v_1)^{(1)} \le (\bar{v}_1)^{(1)} \le \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$, we obtain 386

$$(v_1)^{(1)} \le v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $v^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\right) - (a''_{17})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{\mathrm{G}_{16}}{\mathrm{G}_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (\mathrm{C})^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (\mathrm{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)}\right) \le \frac{dv^{(3)}}{dt} \le -\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \ge \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}}$$

it follows $(v_0)^{(3)} \le v^{(3)}(t) \le (v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \le v^{(3)}(t) \le (\bar{v}_1)^{(3)}$

If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)} \le \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}} \le v^{(3)}(t) \le$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (\bar{v}_1)^{(3)}$$

If $0 < (v_1)^{(3)} \le (\bar{v}_1)^{(3)} \le (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$, we obtain 403

$$(v_1)^{(3)} \le v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

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$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(8)} and\ (b''_i)^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\ ]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\ ]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\ ]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0$ 479

$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0$ 480

$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0$ 481

$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0$ 482

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(dd) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first

equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T^*_{33} for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$$

Definition and uniqueness of T^*_{37} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T^*_{37} for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$$

Definition and uniqueness of T^*_{41} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{25} such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507
$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{29} such that $\varphi((G_{31})^*) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{33} such that $\varphi({G_{35}}^*) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{37} such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{41} such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right]+(b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{45} such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)} G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)} G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \qquad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad (525)$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad (526)$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad (527)$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \quad (528)$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \quad (529)$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \quad (530)$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i \quad (532)$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \quad (533)$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \quad (534)$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \quad (535)$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \quad (536)$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \quad (537)$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}\mathrm{T}^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}\mathrm{T}^*_{18}\mathbb{G}_j\right) \quad 539$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}\left((G_{27})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^{*}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^{*}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^{*}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^{*}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^{*}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^{*}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^{*}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^{*}\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\ B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\Big)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\Big]$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\Big)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\Big)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\Big]$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\Big)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\Big)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\right)$$

$$\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\Big)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\right)$$

$$\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\Big)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION FORTY FOUR

Doer" Is "Merely A Fiction Added To The Deed -- The Deed Is Everything" (1968b: 45)

INTRODUCTION—VARIABLES USED

POSTMODERN TURN IN PHILOSOPHY: THEORETICAL PROVOCATIONS AND NORMATIVE DEFICITS by Steven Best and Douglas Kelner Http://Www.Gseis.Ucla.Edu/Faculty/Kellner/Kellner.Html

(1) From His Early Writings On, Nietzsche, Like Kierkegaard, Rails Against A Life-Denying Rationalism And Idealist Philosophy Which Champions' Reason Over The Passions. Nietzsche

Interprets The "Subject" As (=) A Mere Construct, An Idealized Sublimation Of Bodily Drives, Experiences, And A Multiplicity Of Thoughts And Impulses.

(2) This "Little Changeling," On Nietzsche's View, This Subject, "Is Believed in (eb) More Firmly than Anything Else on Earth," But Is (=) For Him A Simple Illusion Created Out Of (e)Modern Desperation To Have (e) A Well-Grounded Identity

(3) Belief In The Subject Is Promoted By (e) The Exigencies Of Grammar Which Utilize (e) A Subject/Predicate Form, Giving Rise To (eb) The Fallacy That The "I" Is (=) A Substance, Whereas It Is Really Only A Convention Of Grammar (Nietzsche 1968b: 37-38).

(4) For Nietzsche, "The **Doer" Is (=) "Merely A Fiction Added To The Deed -- The Deed Is Everything**" (1968b: 45).

(5) "The Subject," He Concludes, Is (=) Thus But A Shorthand Expression For (e) A Multiplicity Of Drives, Experiences, And Ideas.

(6) In The Spirit Of Enlightenment, Nietzsche Also Polemicizes Against Metaphysics, Arguing That It Illicitly Generalizes From (e) Ideas In One Historical Epoch To The Entirety Of History. Against This Form Of Philosophical Universalism

(7) Nietzsche Argues "There Are (=) **No Eternal Facts, Just As There Are (=) No Absolute Truths**.

(8) Consequently, What Is Needed From Now On Is Historical Philosophizing, And With It The Virtue Of Modesty" (Nietzsche 1986: 13). Castigating Traditional Philosophy And Values From A Critical Enlightenment Perspective, Nietzsche Anticipates Later Postmodern Critiques Of Metaphysics, Assailing (e) The Concept Of Enduring Knowledge, The Notion Of A Transcendental World, And Presenting Metaphysical Thought As (=) A Thoroughly Obsolete Mode Of Thinking.

NOTATION

Module One

From His Early Writings On, Nietzsche, Like Kierkegaard, Rails Against A Life-Denying Rationalism And Idealist Philosophy Which Champions' Reason Over The Passions

Nietzsche Interprets The "Subject" As (=) A Mere Construct, An Idealized Sublimation Of Bodily Drives, Experiences, And A Multiplicity Of Thoughts And Impulses

G_{13} : Category one of "Subject"

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of Mere Construct, An Idealized Sublimation Of Bodily Drives, Experiences, And A Multiplicity Of Thoughts And Impulses

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

This "Little Changeling," On Nietzsche's View, This Subject, "Is Believed in (eb) More Firmly than Anything Else on Earth," But Is (=) For Him A Simple Illusion Created Out Of (e)Modern Desperation To Have (e) A Well-Grounded Identity

G_{16} : Category one of Subject(s)

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of Simple Illusion Created Out Of (e)Modern Desperation To Have A Well-Grounded Identity

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Belief In The Subject Is Promoted By (e) The Exigencies Of Grammar Which Utilize (e) A Subject/Predicate Form, Giving Rise To (eb) The Fallacy That The "I" Is (=) A Substance, Whereas It Is Really Only A Convention Of Grammar (Nietzsche 1968b: 37-38)

G_{20} : Category one of Exigencies Of Grammar Which Utilize (e) A Subject/Predicate Form, Giving Rise To (eb) The Fallacy That The "I" Is (=) A Substance, Whereas It Is Really Only A Convention Of Grammar (Nietzsche 1968b: 37-38)

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of Belief In The Subject Is Promoted

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Belief In The Subject Is Promoted By The Exigencies Of Grammar Which Utilize (e) A Subject/Predicate Form, Giving Rise To (eb) The Fallacy That The "I" Is (=) A Substance, Whereas It Is Really Only A Convention Of Grammar (Nietzsche 1968b: 37-38)

G_{24} : Category one of Subject/Predicate Form, Giving Rise To (eb) The Fallacy That The "I" Is (=) A Substance, Whereas It Is Really Only A Convention Of Grammar (Nietzsche 1968b: 37-38)

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of Belief In The Subject Is Promoted By The Exigencies Of Grammar

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Belief In The Subject Is Promoted By The Exigencies Of Grammar Which Utilize A Subject/Predicate Form, Giving Rise To (eb) The Fallacy That The "I" Is (=) A Substance, Whereas It Is Really Only A Convention Of Grammar (Nietzsche 1968b: 37-38)

G_{28} : Category one of Belief In The Subject Is Promoted By The Exigencies Of Grammar Which Utilize A

Subject/Predicate Form

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of Fallacy That The "I" Is (=) A Substance, Whereas It Is Really Only A Convention Of Grammar (Nietzsche 1968b: 37-38)

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Belief In The Subject Is Promoted By The Exigencies Of Grammar Which Utilize A Subject/Predicate Form, Giving Rise To The Fallacy That The "I" Is (=) A Substance, Whereas It Is Really Only A Convention Of Grammar (Nietzsche 1968b: 37-38)

G_{32} : Category one of Belief In The Subject Is Promoted By The Exigencies Of Grammar Which Utilize A Subject/Predicate Form, Giving Rise To The Fallacy That The "I"

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of Substance, Whereas It Is Really Only A Convention Of Grammar (Nietzsche 1968b: 37-38) (axioms and postulates)

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

For Nietzsche, "The **Doer" Is (=) "Merely A Fiction Added To The Deed -- The Deed Is Everything**" (1968b: 45)

G_{36} : Category one of **Doer**

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of **"Merely A Fiction Added To The Deed -- The Deed Is Everything**" (1968b: 45)

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

The Deed Is Everything" (1968b: 45)

G_{40} : Category one of **Deed**

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of **Everything**" (1968b: 45)

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

"The Subject," He Concludes, Is (=) Thus But A Shorthand Expression For (e) A Multiplicity Of Drives, Experiences, And Ideas

G_{44} : Category one of Subject (s)

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of Shorthand Expression For A Multiplicity Of Drives, Experiences, And Ideas

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION FORTY FIVE

There Are No Facts, Only Interpretations

INTRODUCTION—VARIABLES USED

POSTMODERN TURN IN PHILOSOPHY: THEORETICAL PROVOCATIONS AND NORMATIVE DEFICITS by Steven Best and Douglas Kelner Http://Www.Gseis.Ucla.Edu/Faculty/Kellner/Kellner.Html

(1) He attributes The "Metaphysical Need," At the Heart of Philosophies Such As Schopenhauer's, To Primitive Yearnings for Religious Consolation For (e) The Sufferings of Life and He Urges "Free Spirits" To Liberate Them and Pursue (eb&eb) Thinking and Living Experimentally (1986: 8).

(2) Nietzsche's Attack on Foundationalism, Universalizing Thought, And Metaphysics Thus Undertakes A "Postmodern" Turn In Philosophy Through (e&eb) A Radical Deconstruction Of Modern Theory.

(3) But While Deconstructionist Philosophies typically terminates In the No, Merely Seeking to Unravel a Positive Modern Value System Into (e&eb) A Heap of Disconnected Fragments, Nietzsche Starts and Finishes with a Big Yes, A Life-Affirming Value, Deconstructing only To (eb) Reconstruct.

(4) Moving Far Away From Schopenhauerian Pessimism, Back Toward A Greek View Of Tragedy,

Toward A Dionysian View Of Existence, Nietzsche Seeks "A Justification Of Life, Even At (eb) Its Most Terrible, Ambiguous, And Mendacious" (1968a: 521), A Justification Found In Art, Creativity, Independence, And The Emergence Of "Higher Types" Of Humanity.

(5) Yet Nietzsche's Perspectivism Denies (e) The Possibility Of Affirming Any Absolute Or Universal Values:

(6) All Ideas, Values, Positions, And So On Are (=) Posits Of Individual Constructs Of A Will To Power, Which Are (=) To Be Judged According To The Extent To (e) Which They Do Or Do Not Serve (e) The Values Of Life, Creativity, And Strong Individuality.

(7) For Nietzsche There Are No Facts, Only Interpretations, And He Argues That All Interpretation Is (=) Constituted by (e) the Individual's Perspectives and Is (=) Thus Inevitably Laden with (e&eb) Presuppositions, Biases, And Limitations.

(8) For Nietzsche, A Perspective Is (=) Thus An Optic, A Way Of Seeing, And The More Perspectives One Has At One's Disposal, (eb) The More One Can See, And The Better One Can Understand And Grasp (e) Specific Phenomena.

(9) To Avoid Limited And Partial Vision One Should Learn "How To Employ (e) A Variety Of Perspectives And Interpretations In The Service Of (e) Knowledge" (Nietzsche 1968a: 119). THE POSTMODERN TURN IN PHILOSOPHY: THEORETICAL PROVOCATIONS AND NORMATIVE DEFICITS By Steven Best And Douglas Kelner Http://Www.Gseis.Ucla.Edu/Faculty/Kellner/Kellner.Html

NOTATION

Module One

There Are (=) **No Eternal Facts, Just As There Are (=) No Absolute Truths**

G_{13} : Category one of existence thereof

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of **Eternal Facts**

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

G_{16} : Category one of existence thereof

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of **Absolute Truths**

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

He attributes The "Metaphysical Need," At the Heart of Philosophies Such As Schopenhauer's, To Primitive Yearnings for Religious Consolation For (e) The Sufferings of Life and He Urges "Free Spirits" To Liberate

Them And Pursue (eb&eb) Thinking And Living Experimentally (1986: 8)

G_{20} : Category one of Sufferings of Life and He Urges "Free Spirits" To Liberate Them And Pursue (eb&eb) Thinking And Living Experimentally (1986: 8)

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of Metaphysical Need," At the Heart of Philosophies Such As Schopenhauer's, To Primitive Yearnings for Religious Consolation

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

"Metaphysical Need," At the Heart of Philosophies Such As Schopenhauer's, To Primitive Yearnings for Religious Consolation For The Sufferings of Life and He Urges "Free Spirits" To Liberate Them And Pursue (eb&eb) Thinking And Living Experimentally (1986: 8)

G_{24} : Category one of "Metaphysical Need," At the Heart of Philosophies Such As Schopenhauer's, To Primitive Yearnings for Religious Consolation For The Sufferings of Life and He Urges "Free Spirits" To Liberate Them; Thinking And Living Experimentally (1986: 8)

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of Thinking And Living Experimentally (1986: 8); "Metaphysical Need," At the Heart of Philosophies Such As Schopenhauer's, To Primitive Yearnings for Religious Consolation For The Sufferings of Life and He Urges "Free Spirits" To Liberate Them

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Nietzsche's Attack on Foundationalism, Universalizing Thought, And Metaphysics Thus Undertakes A "Postmodern" Turn In Philosophy Through (e&eb) A Radical Deconstruction Of Modern Theory.

G_{28} : Category one of Attack on Foundationalism, Universalizing Thought, And Metaphysics Thus Undertakes A "Postmodern" Turn In Philosophy; Radical Deconstruction Of Modern Theory

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of Radical Deconstruction Of Modern Theory ;Attack on Foundationalism, Universalizing Thought, And Metaphysics Thus Undertakes A "Postmodern" Turn In Philosophy

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

But While Deconstructionist Philosophies Typically terminates In the No, Merely Seeking to Unravel a Positive Modern Value System Into (e&eb) A Heap Of Disconnected Fragments, Nietzsche Starts And Finishes With A Big Yes, A Life-Affirming Value, Deconstructing Only To (eb) Reconstruct

G_{32} : Category one of Deconstructionist Philosophies Typically terminates In the No, Merely Seeking to Unravel a Positive Modern Value System; Heap Of Disconnected Fragments, Nietzsche Starts And Finishes With A Big Yes, A Life-Affirming Value, Deconstructing Only To (eb) Reconstruct

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of Heap Of Disconnected Fragments, Nietzsche Starts And Finishes With A Big Yes, A Life-Affirming Value, Deconstructing Only To (eb) Reconstruct ;Deconstructionist Philosophies Typically terminates In the No, Merely Seeking to Unravel a Positive Modern Value System

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

But While Deconstructionist Philosophies Typically terminates In the No, Merely Seeking to Unravel a Positive Modern Value System Into A Heap Of Disconnected Fragments, Nietzsche Starts And Finishes With A Big Yes, A Life-Affirming Value, Deconstructing Only To (eb) Reconstruct

G_{36} : Category one of Deconstructionist Philosophies Typically terminates In the No, Merely Seeking to Unravel a Positive Modern Value System Into A Heap Of Disconnected Fragments, Nietzsche Starts And Finishes With A Big Yes, A Life-Affirming Value, Deconstructing

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of Reconstruct

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

Yet Nietzsche's Perspectivism Denies (e) The Possibility Of Affirming Any Absolute Or Universal Values

G_{40} : Category one of Possibility Of Affirming Any Absolute Or Universal Values

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of Perspectivism

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

For Nietzsche, A Perspective Is (=) Thus An Optic, A Way Of Seeing, And The More Perspectives One Has At One's Disposal, (eb) The More One Can See, And The Better One Can Understand And Grasp (e) Specific Phenomena

Note we hold a very different view ;the more you see more you are confused; reality is the casualty

G_{44} : Category one of Perspective

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of Optic, A Way Of Seeing, And The More Perspectives One Has At One's Disposal, (eb) The More One Can See, And The Better One Can Understand And Grasp (e) Specific Phenomena

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,

$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$
$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor
$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}),t)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}),t)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}),t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}),t) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & \boxed{+(a''_{13})^{(1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & \boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & \boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation

coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\left[-(b''_{13})^{(1)}(G,t)\right] & \left[-(b''_{16})^{(2,2,)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,)}(G_{35},t)\right] \\ \left[-(b''_{36})^{(7,7,)}(G_{39},t)\right] & \left[-(b''_{40})^{(8,8)}(G_{43},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\left[-(b''_{14})^{(1)}(G,t)\right] & \left[-(b''_{17})^{(2,2,)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,)}(G_{35},t)\right] \\ \left[-(b''_{37})^{(7,7,)}(G_{39},t)\right] & \left[-(b''_{41})^{(8,8)}(G_{43},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\left[-(b''_{15})^{(1)}(G,t)\right] & \left[-(b''_{18})^{(2,2,)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,)}(G_{35},t)\right] \\ \left[-(b''_{38})^{(7,7,)}(G_{39},t)\right] & \left[-(b''_{42})^{(8,8)}(G_{43},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} & \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad 70$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{22} \quad 72$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)} & \boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{24} \quad 73$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)} & \boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{25} \quad 74$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)} & \boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
are sixth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
are seventh augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
are eighth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)} & \boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad 76$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)} & \boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* $1, 2$ *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* $1, 2$ *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* $1, 2$ *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
are fourth detrition coefficients for category $1, 2$ *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
are fifth detrition coefficients for category $1, 2$ *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
are sixth detrition coefficients for category $1, 2$ *and* 3

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
are seventh detrition coefficients for category $1, 2$ *and* 3

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category $1, 2$ *and* 3

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{30} \quad 81$$

Where $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* $1, 2$ *and* 3

And $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation coefficient for category* $1, 2$ *and* 3

$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ *are third augmentation coefficient for category* $1, 2$ *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation

coefficients for category 1,2, and 3
$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3
$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3
$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3
$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3
$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3
$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3
$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3
$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3
$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3
$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3
$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3
$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3
$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \quad 85$$

$$- \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & \boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} & \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} & \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)} & \boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} & \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)} & \boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} & \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)} & \boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} & \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\left[-(b''_{40})^{(8)}(G_{43},t)\right] & -(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\left[-(b''_{41})^{(8)}(G_{43},t)\right] & -(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\left[-(b''_{42})^{(8)}(G_{43},t)\right] & -(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t) & -(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$(b_i'')^{(1)}(G,t) \leq \ (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$

$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)}$ 98

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = \ (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$

$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (a_i)^{(1)} + (a_i')^{(1)} + \ (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$

$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i')^{(1)} + \ (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$(a_i'')^{(2)}(T_{17},t) \leq (p_i)^{(2)} \leq \left(\hat{A}_{16}\right)^{(2)}$ 102

$$(b_i'')^{(2)}(G_{19},t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\big((G_{19}),t\big) = \ (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - \ T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\big((G_{19}),t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\big[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\big]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}} \ , \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\,(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + \ (\hat{P}_{16})^{(2)}\,(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + \ (\hat{Q}_{16})^{(2)}\ \ (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \le (p_i)^{(3)} \le (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \le (r_i)^{(3)} \le (b_i')^{(3)} \le (\hat{B}_{20})^{(3)}$$

$$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with (116)
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \quad (117)$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\widehat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\widehat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\widehat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - \ T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and \ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\,(a_i)^{(6)} + (a_i')^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)}\,(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(YYYYYYY) $\quad (a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(ZZZZZZZ) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$(a_i'')^{(7)}(T_{37},t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$

$(b_i'')^{(7)}(G_{39},t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$ 132

(AAAAAAAA) $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

(BBBBBBBB)

$\lim_{G\to\infty}(b_i'')^{(7)}((G_{39}),t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \leq (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$

$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}((G_{39}),t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$. (T_{37}',t) and (T_{37},t) are points belonging to the interval $\left[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\right]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(CCCCCCCC) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(DDDDDDDD) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$

$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$

Where we suppose

$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$ 138

$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$ 139

$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$ 140

$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t}$ 142

$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t}$ 143

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ 144

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}$, $(r_i)^{(8)}, i = 40,41,42$,
Satisfy the inequalities

$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 145

$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 146

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\hat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)} ||(G_{47}) - (G_{47})'|| e^{-(\hat{M}_{44})^{(9)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}\left[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)} (\hat{k}_{44})^{(9)}\right] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t} \ , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t} \ , \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \ , \ T_i(0) = T_i^0 \ , \ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} , T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\overline{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that 174

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \leq G_{32}^{0}+\int_0^t\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(ee) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \leq G_{36}^{0}+\int_0^t\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)}=$$
$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \leq G_{40}^{0}+\int_0^t\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)}= \quad 180$$
$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t} \leq \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for G_{41} , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \leq G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \leq \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for G_{45} , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \left\{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\right\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+\ (a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\ (t)=0\ and\ T_i\ (t)=0$

From 19 to 24 it results

$$G_i\ (t)\geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\geq 0$$

$$T_i\ (t)\geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13}<(\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt}\leq\left((\widehat{M}_{13})^{(1)}\right)_1-(a'_{14})^{(1)}G_{14}$ and by integrating

$$G_{14}\leq\left((\widehat{M}_{13})^{(1)}\right)_2=G_{14}^0+2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15}\leq\left((\widehat{M}_{13})^{(1)}\right)_3=G_{15}^0+2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a'_{15})^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}\left|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right| \; e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\} ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right| e^{-(\widehat{M}_{16})^{(2)}t} \leq$
$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right) d\left(\left((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)}\right)\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$: $\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 216

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$G_{24}^{(2)} \left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)}\right) d\left(((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on T_{25} and respectively on (G_{27})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (228)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_2 / (a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)}, (T_{35})^{(1)}), ((G_{35})^{(2)}, (T_{35})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}), (\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}), (\widetilde{T_{35}}) \right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right| \; e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} \} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)} (\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)}; \; (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t} \; and \; (\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$ (250)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)} (T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2 \; and \; \left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33} \; and \; G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right)$, $t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$ and to choose
$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[(\hat{P}_{36})^{(7)} + \left((\hat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)}\right] \leq (\hat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$$
$$sup_i\{max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}, max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)} \left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)} (T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(7)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1,\left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right),\quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{47}}),(\widetilde{T_{47}}):\left((\widetilde{G_{47}}),(\widetilde{T_{47}})\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$

From 99 to 44 it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(9)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)},, (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)}, , (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad (333)$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad (334)$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)} t} - e^{-(a'_{26})^{(4)} t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)} t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad (335)$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad (336)$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28}e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G^0_{28}e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}}G^0_{28}e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}}G^0_{28}e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)}G^0_{28}}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G^0_{30}e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

$$\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_2)^{(5)}((S_1)^{(5)}-(a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t}-e^{-(a'_{30})^{(5)}t}\right]+\ G_{30}^{0}e^{-(a'_{30})^{(5)}t}\Big)$$

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\le T_{30}(t)\le \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$:

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$-(\sigma_2)^{(6)}\le-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\le-(\sigma_1)^{(6)}$

$-(\tau_2)^{(6)}\le-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\le-(\tau_1)^{(6)}$

Definition of $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$: 350

By $(v_1)^{(6)}>0$, $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$, $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

Definition of $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)}>0$, $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$, $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

Definition of $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$:-

If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)}=(v_0)^{(6)},(m_1)^{(6)}=(v_1)^{(6)},$ ***if*** $(v_0)^{(6)}<(v_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}$ $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}$ $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}$ $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}$ $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}$ $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})} \left[e^{(S_1)^{(6)} t} - e^{-(a'_{34})^{(6)} t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)} t} \right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}$, $\boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$

$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)}$,
and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}$, $\boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)}$,
and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^{0}e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}}G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}}G_{36}^{0}e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t}-e^{-(S_2)^{(7)}t}\right]+G_{38}^{0}e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left.\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_2)^{(7)}((S_1)^{(7)}-(a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t}-e^{-(a'_{38})^{(7)}t}\right]+\ G_{38}^{0}e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}}T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}}T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right) - (b''_{41})^{(8)}\left((G_{43}), t\right) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(v_1)^{(8)} > 0$, $(v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$, $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{v}_1)^{(8)}$,, $(\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{v}_1)^{(8)} > 0$, $(\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$, $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$:-

If we define $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$, $\boldsymbol{if}$ $(v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$, $\boldsymbol{if}$ $(v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$,
and $\boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}$, $\boldsymbol{if}$ $(\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, $\boldsymbol{if}$ $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, $\boldsymbol{if}$ $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}$ $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t})$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t}\leq T_{42}(t)\leq \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)},(S_2)^{(8)},(R_1)^{(8)},(R_2)^{(8)}$:- 381

Where $(S_1)^{(8)}=(a_{40})^{(8)}(m_2)^{(8)}-(a'_{40})^{(8)}$

$(S_2)^{(8)}=(a_{42})^{(8)}-(p_{42})^{(8)}$

$(R_1)^{(8)}=(b_{40})^{(8)}(\mu_2)^{(8)}-(b'_{40})^{(8)}$

$(R_2)^{(8)}=(b'_{42})^{(8)}-(r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$:

$(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)}\leq -(a'_{44})^{(9)}+(a'_{45})^{(9)}-(a''_{44})^{(9)}(T_{45},t)+(a''_{45})^{(9)}(T_{45},t)\leq -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)}\leq -(b'_{44})^{(9)}+(b'_{45})^{(9)}-(b''_{44})^{(9)}\big((G_{47}),t\big)-(b''_{45})^{(9)}\big((G_{47}),t\big)\leq -(\tau_1)^{(9)}$

Definition of $(v_1)^{(9)},(v_2)^{(9)},(u_1)^{(9)},(u_2)^{(9)},v^{(9)},u^{(9)}$:

By $(v_1)^{(9)}>0$, $(v_2)^{(9)}<0$ and respectively $(u_1)^{(9)}>0$, $(u_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_1)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)}>0$, $(\bar{v}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$, $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

Definition of $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$:-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)},\ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$
and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \le G_{46}(t) \le \right.$$
$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (v_1)^{(1)} \le (\bar{v}_1)^{(1)} \le \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$, we obtain 386

$$(v_1)^{(1)} \le v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $v^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\right) - (a''_{17})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{\mathrm{G}_{16}}{\mathrm{G}_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (\mathrm{C})^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (\mathrm{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)}\right) \le \frac{dv^{(3)}}{dt} \le -\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \ge \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}}$$

it follows $(v_0)^{(3)} \le v^{(3)}(t) \le (v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \le v^{(3)}(t) \le (\bar{v}_1)^{(3)}$

If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)} \le \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}} \le v^{(3)}(t) \le$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (\bar{v}_1)^{(3)}$$

If $0 < (v_1)^{(3)} \le (\bar{v}_1)^{(3)} \le (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$, we obtain 403

$$(v_1)^{(3)} \le v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

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$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(8)} and\ (b''_i)^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\,]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\,]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\,]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0$ 479

$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0$ 480

$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0$ 481

$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0$ 482

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(ee) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first

equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T^*_{33} for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$$

Definition and uniqueness of T^*_{37} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T^*_{37} for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$$

Definition and uniqueness of T^*_{41} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{25} such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507
$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{29} such that $\varphi((G_{31})^*) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{33} such that $\varphi({G_{35}}^*) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{37} such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{41} such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right]+(b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{45} such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)} G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)} G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)} T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)} T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)} G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)} G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)} T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)} T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)} G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)} G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)} T_{21}^*}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T_{22}^* = \frac{(b_{22})^{(3)} T_{21}^*}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]} \quad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \qquad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \qquad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \qquad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \qquad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \qquad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \qquad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i \qquad 532$$

$$\frac{\partial (a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial (b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \qquad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \qquad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \qquad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \qquad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \qquad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}\mathrm{T}^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}\mathrm{T}^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}\left((G_{27})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^{*}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^{*}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^{*}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^{*}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^{*}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^{*}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^{*}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^{*}\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\Big)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\Big]$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\Big)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\Big)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\Big]$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\Big)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\Big)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\Big]$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\Big)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\Big)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\Big)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\Big)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION FORTY SIX

There's Something Deeply Wrong With Our Brains. If What We Had Now Was OK, We Wouldn't Try To Change it

INTRODUCTION—VARIABLES USED

A Defence Of Paradise-Engineering Brave New World By Aldous Huxley Brave New World (1932) Is One Of The Most Bewitching And Insidious Works Of Literature Ever Written

(1) And Just As During Much Of The Twentieth Century, Any Plea For(e) Greater Social Justice Could Be(=) Successfully Damned As Communist, Likewise Today, Any Strategy To Eradicate Suffering Is Likely To Be Condemned (e) In Similar Reactionary Terms Either Wire Head

Hedonism Or Revamped Brave New World.

(2) This Response Is Not Just Facile And Simplistic. If It Gains Currency, The Result Is Morally Catastrophic. Of Course, The Abolitionist Issue Rarely Arises. Typically, Universal Bliss Is (=) Still More-Or-Less Unthinkingly Dismissed As Technically Impossible. Insofar As The Prospect Is Even Contemplated - Grudgingly - It Is Usually Assumed That The New Regime Would Be Underwritten Day-By-Day With Drugs Or, More Crudely, Electrodes In The Pleasure-Centres

(3) These Techniques Have Their Uses. Yet In The Medium-To-Long-Term, Stopgaps Won't Be Enough. All Use Of Psychoactive Drugs May Be Conceived As (=) An Attempt To Correct Something Pathological With One's State Of Consciousness.

(4) There's Something Deeply Wrong With Our Brains. If What We Had Now Was OK, We Wouldn't Try To Change (e&eb) it.

(5) But It Isn't, So We Do. Mature Biological Psychiatry Will Recognise Inadequate Innate Bliss As (=) A Pandemic Form Of Mental Ill-Health

(6) Good for Selfish DNA in (eb) the Ancestral Environment Where The Adaptation Arose, But Bad (e) For Its Throwaway Vehicles, Notably Us.

(7) The Whole Gamuts Of Behavioural Conditioning, Socio-Economic Reform, Talk-Therapies - And Even Euphoriant Superdrugs - Are Just Palliatives (e) Not Cures, For A Festering Global Illness.

(8) It's Existence Demands (eb) A Global Eradication Program, Not Idle Philosophical Manifestos And Scientific Belle's Letters.

(9) But One Does One's Best. The Ideological Obstacles to Genetically Pre-Programmed Mental Super-Health Are (=) Actually More Daunting Than the Technical Challenges.

NOTATION

Module One

Perspective Is Thus An Optic, A Way Of Seeing, And The More Perspectives One Has At One's Disposal, (eb) The More One Can See, And The Better One Can Understand And Grasp (e) Specific Phenomena

G_{13} : Category one of Perspective Is Thus An Optic, A Way Of Seeing, And The More Perspectives One Has At One's Disposal The More One Can See

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of The Better One Can Understand And Grasp Specific Phenomena

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

To Avoid Limited And Partial Vision One Should Learn "How To Employ (e) A Variety Of Perspectives And Interpretations In The Service Of (e) Knowledge" (Nietzsche 1968a: 119). THE POSTMODERN TURN IN PHILOSOPHY: THEORETICAL PROVOCATIONS AND NORMATIVE DEFICITS By Steven Best And Douglas Kelner Http://Www.Gseis.Ucla.Edu/Faculty/Kellner/Kellner.Html

G_{16} : Category one of Variety Of Perspectives And Interpretations

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of Knowledge" (Nietzsche 1968a: 119). We have espoused the cause of the fact that what we see is not reality; Bohm's variables are required; more you see more you start seeing, nonexistent realities.

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

And Just As During Much Of The Twentieth Century, Any

Plea For(e) Greater Social Justice Could Be(=) Successfully Damned As Communist, Likewise Today, Any Strategy To Eradicate Suffering Is Likely To Be Condemned (e) In Similar Reactionary Terms Either Wire Head Hedonism Or Revamped Brave New World

G_{20} : Category one of Greater Social Justice Could Be(=) Successfully Damned As Communist, Likewise Today, Any Strategy To Eradicate Suffering Is Likely To Be Condemned (e) In Similar Reactionary Terms Either Wire Head Hedonism Or Revamped Brave New World

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of Plea

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Plea For Greater Social Justice Could Be Successfully Damned As Communist, Likewise Today, Any Strategy To Eradicate Suffering Is Likely To Be Condemned (e) In Similar Reactionary Terms Either Wire Head Hedonism Or Revamped Brave New World

G_{24} : Category one of Strategy To Eradicate Suffering

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of Wire Head Hedonism Or Revamped Brave New World

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

This Response Is Not Just Facile And Simplistic. If It Gains Currency, The Result Is Morally Catastrophic. Of Course, The Abolitionist Issue Rarely Arises. Typically, Universal Bliss Is (=) Still More-Or-Less Unthinkingly Dismissed As Technically Impossible. Insofar As The Prospect Is Even Contemplated - Grudgingly - It Is Usually Assumed That The New Regime Would Be Underwritten Day-By-Day With Drugs Or, More Crudely, Electrodes In The Pleasure-Centres

G_{28} : Category one of Universal Bliss

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of Still More-Or-Less Unthinkingly Dismissed As Technically Impossible.

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

There's Something Deeply Wrong With Our Brains. If What We Had Now Was OK, We Wouldn't Try To Change (e&eb) it

G_{32} : Category one of human beings; brain processes

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of brain processes ;human beings

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Good for Selfish DNA in (eb) the Ancestral Environment Where The Adaptation Arose, But Bad (e) For Its Throwaway Vehicles, Notably Us

G_{36} : Category one of change in the process of brain functioning

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of DNA betterment

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

The Whole Gamuts Of Behavioural Conditioning, Socio-Economic Reform, Talk-Therapies - And Even Euphoriant Superdrugs - Are Just Palliatives (e) Not Cures, For A Festering Global Illness

G_{40} : Category one of Whole Gamuts Of Behavioural Conditioning, Socio-Economic Reform, Talk-Therapies - And Even Euphoriant Superdrugs

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of Palliatives Not Cures, For A Festering Global Illness

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

G_{44} : Category one of change in the brain processes; interface with soft ware

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of interface with soft ware; change in the brain processes

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}), t\right)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}), t\right)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\left((G_{39}), t\right)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\begin{array}{ccc} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) \quad +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{array}\right]G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[\begin{array}{ccc} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) \quad +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{array}\right]G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\begin{array}{ccc} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) \quad +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{array}\right]G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\,-(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad (58)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\,-(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad (59)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\,-(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad (60)$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)}\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{bmatrix} G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}\boxed{-(b''_{13})^{(1,1,)}(G,t)}\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)}\boxed{-(b''_{14})^{(1,1,)}(G,t)}\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)}\boxed{-(b''_{15})^{(1,1,)}(G,t)}\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b''_{40})^{(8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b''_{44})^{(9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{20})^{(3)}\,+(a''_{20})^{(3)}(T_{21},t)\,+(a''_{16})^{(2,2,2)}(T_{17},t)\,+(a''_{13})^{(1,1,1,)}(T_{14},t)\\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)\,+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)\,+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a''_{36})^{(7,7,7,7)}(T_{37},t)\,+(a''_{40})^{(8,8,8,8)}(T_{41},t)\,+(a''_{44})^{(9,9,9)}(T_{45},t)\end{array}\right]G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c}(a'_{21})^{(3)}\,+(a''_{21})^{(3)}(T_{21},t)\,+(a''_{17})^{(2,2,2)}(T_{17},t)\,+(a''_{14})^{(1,1,1,)}(T_{14},t)\\ +(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)\,+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)\,+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a''_{37})^{(7,7,7,7)}(T_{37},t)\,+(a''_{41})^{(8,8,8,8)}(T_{41},t)\,+(a''_{45})^{(9,9,9)}(T_{45},t)\end{array}\right]G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c}(a'_{22})^{(3)}\,+(a''_{22})^{(3)}(T_{21},t)\,+(a''_{18})^{(2,2,2)}(T_{17},t)\,+(a''_{15})^{(1,1,1,)}(T_{14},t)\\ +(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)\,+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)\,+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a''_{38})^{(7,7,7,7)}(T_{37},t)\,+(a''_{42})^{(8,8,8,8)}(T_{41},t)\,+(a''_{46})^{(9,9,9)}(T_{45},t)\end{array}\right]G_{22} \quad (69)$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)}\,-(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)}\,-(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\,-(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$ $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad 75$$

$(a''_{24})^{(4)}(T_{25},t)$, $(a''_{25})^{(4)}(T_{25},t)$, $(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$
are fourth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$
are fifth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$
are sixth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$
are seventh augmentation coefficients for category 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$
are eighth augmentation coefficients for category 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

$Where\ \boxed{-(b_{24}'')^{(4)}(G_{27},t)}, \boxed{-(b_{25}'')^{(4)}(G_{27},t)}, \boxed{-(b_{26}'')^{(4)}(G_{27},t)}\ are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{28}'')^{(5,5,)}(G_{31},t)}, \boxed{-(b_{29}'')^{(5,5,)}(G_{31},t)}, \boxed{-(b_{30}'')^{(5,5,)}(G_{31},t)}\ are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{32}'')^{(6,6,)}(G_{35},t)}, \boxed{-(b_{33}'')^{(6,6,)}(G_{35},t)}, \boxed{-(b_{34}'')^{(6,6,)}(G_{35},t)}\ are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{13}'')^{(1,1,1,1)}(G,t)}, \boxed{-(b_{14}'')^{(1,1,1,1)}(G,t)}, \boxed{-(b_{15}'')^{(1,1,1,1)}(G,t)}$

$are\ fourth\ \ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{16}'')^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b_{17}'')^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b_{18}'')^{(2,2,2,2)}(G_{19},t)}$

$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{20}'')^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b_{21}'')^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b_{22}'')^{(3,3,3,3)}(G_{23},t)}$

$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{36}'')^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b_{37}'')^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b_{38}'')^{(7,7,7,7,7)}(G_{39},t)}$

$are\ seventh\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{40}'')^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b_{41}'')^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b_{42}'')^{(8,8,8,8,8)}(G_{43},t)}$

$are\ eighth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{46}'')^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b_{45}'')^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b_{44}'')^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{c} (a_{28}')^{(5)}\boxed{+(a_{28}'')^{(5)}(T_{29},t)}\ \boxed{+(a_{24}'')^{(4,4,)}(T_{25},t)}\ \boxed{+(a_{32}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{36}'')^{(7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a_{40}'')^{(8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a_{44}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{c} (a_{29}')^{(5)}\boxed{+(a_{29}'')^{(5)}(T_{29},t)}\ \boxed{+(a_{25}'')^{(4,4,)}(T_{25},t)}\ \boxed{+(a_{33}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a_{45}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{c} (a_{30}')^{(5)}\boxed{+(a_{30}'')^{(5)}(T_{29},t)}\ \boxed{+(a_{26}'')^{(4,4,)}(T_{25},t)}\ \boxed{+(a_{34}'')^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a_{46}'')^{(9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{30} \quad (81)$$

$Where\ \boxed{+(a_{28}'')^{(5)}(T_{29},t)}, \boxed{+(a_{29}'')^{(5)}(T_{29},t)}, \boxed{+(a_{30}'')^{(5)}(T_{29},t)}\ are\ first\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$And\ \boxed{+(a_{24}'')^{(4,4,)}(T_{25},t)}, \boxed{+(a_{25}'')^{(4,4,)}(T_{25},t)}, \boxed{+(a_{26}'')^{(4,4,)}(T_{25},t)}\ are\ second\ augmentation\ coefficient\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a_{32}'')^{(6,6,6)}(T_{33},t)}, \boxed{+(a_{33}'')^{(6,6,6)}(T_{33},t)}, \boxed{+(a_{34}'')^{(6,6,6)}(T_{33},t)}\ are\ third\ \ augmentation\ coefficient\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8\ ,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad 82$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad 83$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad 84$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \quad 85$$

$$- \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & \boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} & \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} & \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)} & \boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} & \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)} & \boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} & \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)} & \boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} & \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} & \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq \ (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = \ (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \ , \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\,(a_i)^{(1)} + (a_i')^{(1)} + \ (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}\,(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i')^{(1)} + \ (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \leq (p_i)^{(2)} \leq \left(\hat{A}_{16}\right)^{(2)} \qquad 102$$

$$(b_i'')^{(2)}(G_{19},t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\big((G_{19}),t\big) = \ (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\big((G_{19}),t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\big[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\big]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \le (p_i)^{(3)} \le (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \le (r_i)^{(3)} \le (b_i')^{(3)} \le (\hat{B}_{20})^{(3)}$$

$$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with (116)
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \quad (117)$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\widehat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\widehat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\widehat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - \ T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\,(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and \ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\,(a_i)^{(6)} + (a_i')^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)}\,(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(EEEEEEEE) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(FFFFFFFF) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$(a_i'')^{(7)}(T_{37},t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)}$

$(b_i'')^{(7)}(G_{39},t) \leq (r_i)^{(7)} \leq (b_i')^{(7)} \leq (\hat{B}_{36})^{(7)}$ 132

(GGGGGGGG) $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

(HHHHHHHH)

$\lim_{G\to\infty}(b_i'')^{(7)}((G_{39}),t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \leq (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$

$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}((G_{39}),t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$. (T_{37}',t) and (T_{37},t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(IIIIIIII) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)},$ are positive constants

$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(JJJJJJJJ) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$

$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$

Where we suppose

$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$ 138

$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$ 139

$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$ 140

$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t}$ 142

$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t}$ 143

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ 144

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}$, $(r_i)^{(8)}, i = 40,41,42$,
Satisfy the inequalities

$\frac{1}{(\hat{M}_{40})^{(8)}} [(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 145

$\frac{1}{(\hat{M}_{40})^{(8)}} [(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 146

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\hat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)} ||(G_{47}) - (G_{47})'|| e^{-(\hat{M}_{44})^{(9)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}\left[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)} (\hat{k}_{44})^{(9)}\right] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)} t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)} t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)} t} \ , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t} \ , \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \ , \ T_i(0) = T_i^0 \ , \ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} , T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\overline{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad (169)$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that (170)

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that (171)

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that (172)

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that (173)

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that (174)

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \leq G_{32}^{0}+\int_0^t\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)} =$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(ff) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \leq G_{36}^{0}+\int_0^t\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)} =$$
$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \leq G_{40}^{0}+\int_0^t\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)} = \quad 180$$
$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t} \leq \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for G_{41} , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \leq G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \leq \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for G_{45} , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \left\{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\right\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+\ (a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\ (t)=0\ and\ T_i\ (t)=0$

From 19 to 24 it results

$$G_i\ (t)\geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\geq 0$$

$$T_i\ (t)\geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13}<(\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt}\leq\left((\widehat{M}_{13})^{(1)}\right)_1-(a'_{14})^{(1)}G_{14}$ and by integrating

$$G_{14}\leq\left((\widehat{M}_{13})^{(1)}\right)_2=G_{14}^0+2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15}\leq\left((\widehat{M}_{13})^{(1)}\right)_3=G_{15}^0+2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a'_{15})^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|\mathrm{e}^{-(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}} \leq$
$\frac{1}{(\widehat{\mathrm{M}}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{\mathrm{A}}_{16})^{(2)} + (\widehat{\mathrm{P}}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)\mathrm{d}\left(\left((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)}\right)\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{\mathrm{P}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ and $(\widehat{\mathrm{Q}}_{16})^{(2)}\mathrm{e}^{(\widehat{\mathrm{M}}_{16})^{(2)}\mathrm{t}}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $\mathrm{G}_i\,(\mathrm{t}) = 0$ and $\mathrm{T}_i\,(\mathrm{t}) = 0$ 200

it results

$\mathrm{G}_i\,(\mathrm{t}) \geq \mathrm{G}_i^0\mathrm{e}^{\left[-\int_0^{\mathrm{t}}\{(a_i')^{(2)} - (a_i'')^{(2)}(\mathrm{T}_{17}(s_{(16)}), s_{(16)})\}\mathrm{d}s_{(16)}\right]} \geq 0$

$\mathrm{T}_i\,(\mathrm{t}) \geq \mathrm{T}_i^0\mathrm{e}^{(-(b_i')^{(2)}\mathrm{t})} > 0 \quad \text{for t} > 0$

Definition of $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1$, $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2$ and $\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$\mathrm{G}_{16} < (\widehat{\mathrm{M}}_{16})^{(2)}$ it follows $\frac{\mathrm{dG}_{17}}{\mathrm{dt}} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)}\mathrm{G}_{17}$ and by integrating

$\mathrm{G}_{17} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2 = \mathrm{G}_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$

In the same way , one can obtain

$\mathrm{G}_{18} \leq \left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_3 = \mathrm{G}_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{\mathrm{M}}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{\mathrm{t}\to\infty}((b_i'')^{(2)}\,((G_{19})(\mathrm{t}), \mathrm{t})) = (b_{17}')^{(2)}$ then $\mathrm{T}_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$: $\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_{0}^{t}(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\, ds_{(20)} +$$

$$\int_{0}^{t}\{(a_{20}')^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a_{20}'')^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)}\left|(a_{20}'')^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a_{20}'')^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a_{20}')^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a_{20}'')^{(3)}$ and $(b_{20}'')^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_{+}$. 215

If instead of proving the existence of the solution on $\mathbb{R}_{+}$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0$ *and* $T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a_{21}')^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a_{22}')^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$G_{24}^{(2)} \left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)}\right) d\left(((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on T_{25} and respectively on (G_{27})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (228)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_2 / (a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right| \; e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)}; (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28}, G_{30} and G_{28}, G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30}. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}|T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{35}}), (\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}), (\widetilde{T_{35}}) \right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right| \; e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} \} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)} (\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)}; \; (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t} \; and \; (\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$ (250)

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2 \; and \; \left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33} \; and \; G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$$
$$sup_i\{max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}, max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)} \left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(7)}$ and $(b''_i)^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(7)} - (a''_i)^{(7)} (T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b'_i)^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\,((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_7}$$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)}$$ 267

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)}$$ 268

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1,\left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right),\quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{47})},\widetilde{(T_{47})}:\left(\widetilde{(G_{47})},\widetilde{(T_{47})}\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$

From 99 to 44 it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(9)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$: 281

By $(v_1)^{(1)} > 0$, $(v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{v}_1)^{(1)},, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{v}_1)^{(1)} > 0$, $(\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)}, , (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)} t} - e^{-(a'_{26})^{(4)} t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)} t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G^0_{28} e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G^0_{28}}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G^0_{30} e^{-(S_2)^{(5)} t} \le G_{30}(t) \le \right. \quad 344$$

$$\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_2)^{(5)}((S_1)^{(5)}-(a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t}-e^{-(a'_{30})^{(5)}t}\right]+\ G_{30}^{0}e^{-(a'_{30})^{(5)}t}\Big)$$

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\le T_{30}(t)\le \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$:

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)}\le-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\le-(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)}\le-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\le-(\tau_1)^{(6)}$$

Definition of $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$: 350

By $(v_1)^{(6)}>0$, $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$, $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

Definition of $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)}>0$, $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$, $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

Definition of $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$:-

If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)}=(v_0)^{(6)},(m_1)^{(6)}=(v_1)^{(6)},$ ***if*** $(v_0)^{(6)}<(v_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}$ $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}$ $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}$ $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}$ $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}$ $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})} \left[e^{(S_1)^{(6)} t} - e^{-(a'_{34})^{(6)} t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)} t} \right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^{0}e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}}G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}}G_{36}^{0}e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t}-e^{-(S_2)^{(7)}t}\right]+G_{38}^{0}e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left.\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_2)^{(7)}((S_1)^{(7)}-(a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t}-e^{-(a'_{38})^{(7)}t}\right]+\ G_{38}^{0}e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}}T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}}T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right) - (b''_{41})^{(8)}\left((G_{43}), t\right) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(v_1)^{(8)} > 0$, $(v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$, $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{v}_1)^{(8)}$,, $(\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{v}_1)^{(8)} > 0$, $(\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$, $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$:-

If we define $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$, $\boldsymbol{if}$ $(v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$, $\boldsymbol{if}$ $(v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$,
and $\boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}$, $\boldsymbol{if}$ $(\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, $\boldsymbol{if}$ $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, $\boldsymbol{if}$ $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}$ $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t})$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \qquad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \qquad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t}\leq T_{42}(t)\leq \qquad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)},(S_2)^{(8)},(R_1)^{(8)},(R_2)^{(8)}$:- 381

Where $(S_1)^{(8)}=(a_{40})^{(8)}(m_2)^{(8)}-(a'_{40})^{(8)}$

$(S_2)^{(8)}=(a_{42})^{(8)}-(p_{42})^{(8)}$

$(R_1)^{(8)}=(b_{40})^{(8)}(\mu_2)^{(8)}-(b'_{40})^{(8)}$

$(R_2)^{(8)}=(b'_{42})^{(8)}-(r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$:

$(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)}\leq -(a'_{44})^{(9)}+(a'_{45})^{(9)}-(a''_{44})^{(9)}(T_{45},t)+(a''_{45})^{(9)}(T_{45},t)\leq -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)}\leq -(b'_{44})^{(9)}+(b'_{45})^{(9)}-(b''_{44})^{(9)}\big((G_{47}),t\big)-(b''_{45})^{(9)}\big((G_{47}),t\big)\leq -(\tau_1)^{(9)}$

Definition of $(v_1)^{(9)},(v_2)^{(9)},(u_1)^{(9)},(u_2)^{(9)},v^{(9)},u^{(9)}$:

By $(v_1)^{(9)}>0$, $(v_2)^{(9)}<0$ and respectively $(u_1)^{(9)}>0$, $(u_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_1)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)}>0$, $(\bar{v}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$, $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

Definition of $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$:-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)},\ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$
and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \le G_{46}(t) \le \right.$$
$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (v_1)^{(1)} \le (\bar{v}_1)^{(1)} \le \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$, we obtain 386

$$(v_1)^{(1)} \le v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $v^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\right) - (a''_{17})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{\mathrm{G}_{16}}{\mathrm{G}_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (\mathrm{C})^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (\mathrm{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)}\right) \le \frac{dv^{(3)}}{dt} \le -\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \ge \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}}$$

it follows $(v_0)^{(3)} \le v^{(3)}(t) \le (v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \le v^{(3)}(t) \le (\bar{v}_1)^{(3)}$

If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)} \le \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}} \le v^{(3)}(t) \le$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (\bar{v}_1)^{(3)}$$

If $0 < (v_1)^{(3)} \le (\bar{v}_1)^{(3)} \le (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$, we obtain 403

$$(v_1)^{(3)} \le v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

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$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(8)} and\ (b''_i)^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\ ]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\ ]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\ ]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0$ 479

$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0$ 480

$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0$ 481

$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0$ 482

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(ff) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first

equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T^*_{33} for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$$

Definition and uniqueness of T^*_{37} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T^*_{37} for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$$

Definition and uniqueness of T^*_{41} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{25} such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507
$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{29} such that $\varphi((G_{31})^*) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{33} such that $\varphi({G_{35}}^*) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{37} such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{41} such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right]+(b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{45} such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)} G_{14}^{*}}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)} G_{14}^{*}}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)} T_{14}^{*}}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)} T_{14}^{*}}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)} G_{17}^{*}}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)} G_{17}^{*}}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^{*})]} \qquad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)} T_{17}^{*}}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)} T_{17}^{*}}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^{*})]} \qquad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)} G_{21}^{*}}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)} G_{21}^{*}}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)} T_{21}^{*}}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)} T_{21}^{*}}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^{*} given by $\varphi(G_{27}) = 0$, T_{25}^{*} given by $f(T_{25}^{*}) = 0$ and

$$G_{24}^{*} = \frac{(a_{24})^{(4)} G_{25}^{*}}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^{*})]} \quad , \quad G_{26}^{*} = \frac{(a_{26})^{(4)} G_{25}^{*}}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^{*})]}$$

$$T_{24}^{*} = \frac{(b_{24})^{(4)} T_{25}^{*}}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^{*})]} \quad , \quad T_{26}^{*} = \frac{(b_{26})^{(4)} T_{25}^{*}}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^{*})]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^{*} given by $\varphi((G_{31})^{*}) = 0$, T_{29}^{*} given by $f(T_{29}^{*}) = 0$ and

$$G_{28}^{*} = \frac{(a_{28})^{(5)} G_{29}^{*}}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^{*})]} \quad , \quad G_{30}^{*} = \frac{(a_{30})^{(5)} G_{29}^{*}}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^{*})]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \qquad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \qquad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \qquad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \qquad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \qquad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \qquad 530$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i \qquad 532$$

$$\frac{\partial (a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial (b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \qquad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \qquad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \qquad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \qquad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \qquad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}\mathrm{T}^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}\mathrm{T}^*_{18}\mathbb{G}_j\right) \quad 539$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}\left((G_{27})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\ B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\Big)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\Big]$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\Big)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\Big)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\Big]$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\Big)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\Big)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\Big]$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\Big)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\Big)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\Big)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\Big)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION FORTY SEVEN

Aversive Experience, And The Poisonous Metabolic Pathways

INTRODUCTION—VARIABLES USED

POSTMODERN TURN IN PHILOSOPHY: THEORETICAL PROVOCATIONS AND NORMATIVE DEFICITS by Steven Best and Douglas Kelner Http://Www.Gseis.Ucla.Edu/Faculty/Kellner/Kellner.Html

(1) To Be Cured, Hypo-Hedonia Must Be Recognised As (=) A Primarily Genetic Deficiency-Disorder.
(2) Designer Mood-Brighteners And Anti-Anxiety Agents To Alleviate It Are (=) Sometimes Branded

"Lifestyle-Drugs";

(3) But This Is To Trivialize A Serious Medical Condition Which Must Be Corrected (e&eb) At Source.

(4) Happily, Our Hereditary Neuropsychiatric Disorder Is Likely To Become (=) Extinct Within A Few Generations As The Reproductive Revolution Unfolds.

(5) Aversive Experience, And The Poisonous Metabolic Pathways That Mediate (e&eb) Its Textures, Will Become (=) Physiologically Impossible Once The Genes Coding (e&eb) Its Neural Substrates Have Been Eliminated.

(6) We Won't Miss Its Corrupting Effect When It's Gone. In The Medium-Term, The Functional Equivalent Of Aversive Experience Can Help Animate (eb) Us Instead.

(7) Late In the Third Millennium And Beyond, Its Functional Successors May Be Expressed As (=) Gradients Of Majestic Well-Being.

(8) On This Scenario, Our Descendants Will Enjoy A Civilisation Based On (e) Information-Bearing Pleasure-Gradients: Whether Steep Or Shallow, We Simply Don't Know.

(9) Such A Global Species-Project Does Not (e) Have The Desperate Moral Urgency Of Eliminating (e) The Phenomenon Of Darwinian Pain - Both "Mental" And "Physical", Human And Non-Human Alike

NOTATION

Module One

Hypo-Hedonia Must Be Recognised As (=) A Primarily Genetic Deficiency-Disorder.

G_{13} : Category one of Hypo-Hedonia

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of Primarily Genetic Deficiency-Disorder

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Functional Successors May Be Expressed As (=) Gradients Of Majestic Well-Being

G_{16} : Category one of Functional Successors

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of Gradients Of Majestic Well-Being

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Embodied embedded cognition from Wikipedia, the free encyclopedia

Embodied Embedded Cognition (EEC) is a philosophical theoretical position in cognitive science, closely

related to situated cognition, embodied cognition, embodied cognitive science and dynamical systems theory. The theory states that intelligent behaviour emerges from the interplay between brain, body and world. The world is not just the 'play-ground' on which the brain is acting. Rather, brain, body and world are equally important factors in the explanation of how particular intelligent behaviours come about in practice. There are concerns about whether EEC constitutes a novel and substantive approach to cognition or whether it is merely a manifestation of frustration with the classical cognitivist approach.

Embodiment and Embeddedness

EEC is divided into two aspects: embodiment and embeddedness (or situatedness). Embodiment refers to the idea that the bodies' internal milieu (a.o. homeostatic and hormonal states) heavily influences the higher 'cognitive' processes in the brain, presumably via the emotional system (see e.g. Antonio Damasio's theory of somatic markers). To put it simply: the state of your body is a direct factor of importance on the kinds of cognitive processes that may arise in the higher parts of your brain. Embeddedness refers to the idea that physical interaction between the body and the world strongly constrain the possible behaviours of the organism, which in turn influences (indeed, partly constitutes) the cognitive processes that emerge from the interaction between organism and world. The theory is an explicit reaction to the currently dominant cognitivist paradigm, which states that cognitive systems are essentially computational-representational systems (like computer software), processing input and generating output (behaviour) on the basis of internal information processing. In cognitivism, the causal root of behaviour lies in the 'virtual' processes governed by the software that runs on our brains. The brain is purely the hardware on which the software is implemented. The body (sensors and actors) are purely input-output devices that are in service of the brain. The world is merely the play-ground (the object) in which the cognitive agent acts.

In contrast, EEC holds that the actual physical processes in body and in body-world interaction partly constitute whatever it is that we call 'the cognitive system' as a whole. Body, world and brain form a system. Together these system-parts 'cause' intelligent behaviour to arise as a system property. Dynamical Systems Theory is a way of modeling behaviour that teams up quite naturally with the theoretical concepts of EEC.

Current discussions include:

Is EEC really a (positive) theory of itself, or merely a bunch of complaints about what is wrong about (a too extreme version of) cognitivism?

Is EEC too 'descriptive', instead of really explaining anything about cognition?

How can EEC explain linguistic processes and processes of explicit conscious reasoning?

What would be the most informative empirical hypotheses, starting from an EEC perspective?

Can we use traditional methods (stimulus-response paradigms) of experimental psychology to test EEC hypotheses?

Autonomous Robot Vehicles 1990, pp 167-193 Estimating Uncertain Spatial Relationships in Robotics Randall Smith, Matthew Self, Peter Cheeseman

Randall Smith, Matthew Self, Peter Cheeseman describe a representation for (e) spatial information, called the stochastic map, and associated procedures for building it, reading information from it, and revising it (e&eb) incrementally as new information is obtained

(1) The map contains (e) the estimates of relationships among objects in the map, and their

uncertainties, given all the available information.

(2) The procedures provide (eb) a general solution to the problem of estimating (eb) uncertain relative spatial relationships.

(3) The estimates are probabilistic in nature, an advance over the previous, very conservative, worst-case approaches to the problem. Finally, the procedures are developed in the context of (e&eb) state-estimation and filtering theory, which provides (eb) a solid basis for numerous extensions.

G_{20} : Category one of spatial information, called the stochastic map, and associated procedures for building it, reading information from it, and revising it(e&eb) incrementally as new information is obtained

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of representation

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Randall Smith, Matthew Self, Peter Cheeseman describe a

representation for spatial information, called the stochastic map, and associated procedures for building it, reading information from it, and revising it (e&eb) incrementally as new information is obtained

G_{24} : Category one of representation for spatial information, called the stochastic map, and associated procedures for building it, reading information from it; incrementally as new information is obtained

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of incrementally as new information is obtained; representation for spatial information, called the stochastic map, and associated procedures for building it, reading information from it

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

world is merely the play-ground (the object) in which the cognitive agent acts

G_{28} : Category one of world is merely the play-ground (the object)

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of cognitive agent acts

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Cognitive systems are essentially computational-representational systems (like computer software), processing input and generating output (behaviour) on the basis of internal information processing.

G_{32} : Category one of Cognitive systems

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of computational-representational systems (like computer software), processing input and generating output (behaviour) on the basis of internal information processing

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Cognitive systems are essentially computational-representational systems (like computer software), processing input and generating output (behaviour) on the basis of internal information processing.

G_{36} : Category one of Cognitive systems are essentially computational-representational systems (like computer software), processing input and generating output (behaviour); internal information processing

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of internal information processing ;Cognitive systems are essentially computational-representational systems (like computer software), processing input and generating output (behaviour)

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

map contains (e) the estimates of relationships among objects in the map, and their uncertainties, given all the available information

G_{40} : Category one of estimates of relationships among objects in the map, and their uncertainties, given all the available information

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of maps

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

The procedures provide (eb) a general solution to the problem of estimating (eb) uncertain relative spatial relationships

G_{44} : Category one of procedures

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of general solution to the problem of estimating (eb) uncertain relative spatial relationships

T_{45} : Category two of SAS

T_{46} : Category three of SAS

SECTION FORTY EIGHT

Maximizing Pleasure

INTRODUCTION—VARIABLES USED

POSTMODERN TURN IN PHILOSOPHY: THEORETICAL PROVOCATIONS AND NORMATIVE DEFICITS by Steven Best and Douglas Kelner Http://Www.Gseis.Ucla.Edu/Faculty/Kellner/Kellner.Html

(1) Abolishing Raw Nastiness - Sometimes Vile Beyond Belief - Remains (eb) The Over-Riding Ethical Priority.

(2) One Doesn't Have To Be (=) An Outright Negative Utilitarian To Acknowledge That Getting Rid Of (e) Agony Takes (e) Moral Precedence Over Maximizing Pleasure.

(3) But Both Genetic Fundamentalists And Gung-Ho Advocates Of Better Living Through Chemistry Today Agree On One Crucial Issue. There Is No Sense In Sustaining A Legacy Of (e) Mood-Darkening Metabolic Pathways Out Of (e) Superstitious Deference To Our Savage Past.

(4) When Bernard Marx Tells The Savage He Will Try To Secure Permission For Him And His Mother To Visit The Other Place, John Is Initially Pleased And Excited. Echoing Miranda In The Tempest, He Exclaims: "O Brave New World That Has Such People In It." Heavy Irony. Like Innocent Miranda, He Is Eager To Embrace A Way Of Life He Neither Knows Nor Understands. And Of Course He Comes Unstuck. Yet If We Swallow Such Fancy Literary Conceits, Then Ultimately The Joke Is On Us. It Is Only Funny In The Sense There Are "Jokes" About Auschwitz.

For It Is Huxley Who Neither Knows Nor Understands The Glory Of What Lies Ahead. A Utopian Society In Which We Are Sublimely Happy Will Be Far Better Than We Can Presently Imagine, Not Worse. And It Is We, Trapped In The Emotional Squalor Of Late-Darwinian Antiquity, Which Neither (e) Know Nor Understand The Lives Of The God-Like Super-Beings We Are Destined To Become. A Defence Of Paradise-Engineering Brave New World By Aldous Huxley Brave New World (1932) Is One Of The Most Bewitching And Insidious Works Of Literature Ever Written

NOTATION

Module One

Computational neuroscience from Wikipedia, the free encyclopedia: for references please see Wikipedia

Single-neuron modeling

Main article: Biological neuron models

(1) Even single neurons have complex biophysical characteristics and can perform computations (e.g. [7]). Hodgkin and Huxley's original model only employed two voltage-sensitive currents (Voltage sensitive ion channels are (=) glycoprotein molecules which extend through (e&eb) the lipid bilayer, allowing (eb) ions to traverse under certain conditions through (e&eb) the axolemma), the fast-acting sodium and the inward-rectifying potassium.
(2) The computational functions of complex dendrites are also under intense investigation. There is a large body of literature regarding how different currents interact with (e&eb) geometric properties of neurons.[9]

Development, axonal patterning, and guidance

How do axons and dendrites form during development? How do axons know where to target and how to reach these targets? How do neurons migrate to the proper position in the central and peripheral systems? How do synapses form? We know from molecular biology that distinct parts of the nervous system release distinct chemical cues, from growth factors to hormones that modulate and influence the growth and development of functional connections between neurons.

(3) Theoretical investigations into the formation and patterning of (e) synaptic connection and morphology are still nascent. One hypothesis that has recently garnered some attention is the minimal wiring hypothesis, which postulates that the formation of axons and dendrites effectively minimizes (e) resource allocation while maintaining maximal information storage. [10]

Sensory processing

(4) Early models of sensory processing understood within a theoretical framework are credited to Horace Barlow. Somewhat similar to the minimal wiring hypothesis described in the preceding section, Barlow understood the processing of the early sensory systems to be a (=) form of efficient coding, where the neurons encoded (e) information which minimized (e) the number of spikes. Experimental and computational work has since supported this hypothesis in one form or another.
(5) Current research in sensory processing is divided among a biophysical modelling of different subsystems and a more theoretical modelling of perception. Current models of perception have suggested that the brain performs some form of Bayesian inference and integration of different sensory information in generating (eb) our perception of the physical world

Memory and synaptic plasticity

(6) Earlier models of memory are primarily based on the postulates of Hebbian learning. Biologically relevant models such as Hopfield net have been developed to address the properties of associative, rather than content-addressable, style of memory that occur in (eb) biological systems.
(7) These attempts are primarily focusing on the formation of medium- and long-term memory, localizing in (eb) the hippocampus. Models of working memory, relying on theories of network oscillations and persistent activity, have been built to capture (e) some features of the prefrontal cortex in context-related memory. [11]
(8) One of the major problems in neurophysiological memory is how it is maintained and changed through multiple time scales. Unstable synapses are easy to train but also prone to (e) stochastic disruption.
(9) Stable synapses forget less easily, but they are (=) also harder to consolidate.

Even single neurons have complex biophysical characteristics and can perform computations (e.g. [7]). Hodgkin and Huxley's original model only employed two voltage-sensitive currents (Voltage sensitive ion channels are (=) glycoprotein molecules which extend through (e&eb) the lipid bilayer, allowing (eb) ions to traverse under certain conditions through (e&eb) the axolemma), the fast-acting sodium and the inward-rectifying potassium

G_{13} : Category one of Voltage sensitive ion channels

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of glycoprotein molecules which extend through (e&eb) the lipid bilayer, allowing (eb) ions to traverse under certain conditions through (e&eb) the axolemma

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Voltage sensitive ion channels are glycoprotein molecules which extend through (e&eb) the lipid bilayer, allowing (eb) ions to traverse under certain conditions through (e&eb) the axolemma

G_{16} : Category one of Voltage sensitive ion channels are glycoprotein molecules; lipid bilayer, allowing (eb) ions to traverse under certain conditions through (e&eb) the axolemma

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of lipid bilayer, allowing (eb) ions to traverse under certain conditions through (e&eb) the axolemma ;Voltage sensitive ion channels are glycoprotein molecules

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

Voltage sensitive ion channels are glycoprotein molecules which extend through the lipid bilayer, allowing (eb) ions to traverse under certain conditions through (e&eb) the axolemma

G_{20} : Category one of Voltage sensitive ion channels are glycoprotein molecules which extend through the lipid bilayer

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of ions to traverse under certain conditions through (e&eb) the axolemma

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

The computational functions of complex dendrites are also under intense investigation. There is a large body of literature regarding how

different currents interact with (e&eb) geometric properties of neurons.[9]

Model gives explanation of how they react

G_{24} : Category one of different currents; geometric properties of neurons

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of geometric properties of neurons ;different currents

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Theoretical investigations into the

formation and patterning of (e) synaptic connection and morphology are still nascent.

This is a thorough investigation theoretically of such an interaction

One hypothesis that has recently garnered some attention is the minimal wiring hypothesis, which postulates that the

formation of axons and dendrites effectively minimizes (e) resource allocation while maintaining maximal information storage

G_{28} : Category one of formation and patterning; synaptic connection and morphology

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of synaptic connection and morphology ;formation and patterning

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

formation of axons and dendrites effectively minimizes (e) resource allocation while maintaining maximal information storage

G_{32} : Category one of resource allocation while maintaining maximal information storage

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of formation of axons and dendrites

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Early models of sensory processing understood within a theoretical framework are credited to Horace Barlow. Somewhat similar to the minimal wiring hypothesis described in the preceding section, Barlow understood the

processing of the early sensory systems to be a (=) form of efficient coding, where the neurons encoded (e) information which minimized (e) the number of spikes.

Experimental and computational work has since supported this hypothesis in one form or another

G_{36} : Category one of processing of the early sensory systems

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of form of efficient coding, where the neurons encoded (e) information which minimized (e) the number of spikes

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

processing of the early sensory systems to be a form of efficient coding, where the neurons encoded (e) information which minimized (e) the number of spikes

G_{40} : Category one of information which minimized (e) the number of spikes

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of processing of the early sensory systems to be a form of efficient coding, where the neurons encoded

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

processing of the early sensory systems to be a form of efficient coding, where the neurons encoded information which minimized (e) the number of spikes

G_{44} : Category one of number of spikes

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of processing of the early sensory systems to be a form of efficient coding, where the neurons encoded information

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,

are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right]G_{13} \quad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14},t)\right]G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14},t)\right]G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G,t)\right]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t)\right]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}),t\big)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}),t\big)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}),t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\big((G_{19}),t\big) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}), t)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}), t)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}), t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}), t) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}), t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & \boxed{+(a''_{13})^{(1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & \boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & \boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation

coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\left[-(b''_{13})^{(1)}(G,t)\right] & \left[-(b''_{16})^{(2,2,)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,)}(G_{35},t)\right] \\ \left[-(b''_{36})^{(7,7,)}(G_{39},t)\right] & \left[-(b''_{40})^{(8,8)}(G_{43},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\left[-(b''_{14})^{(1)}(G,t)\right] & \left[-(b''_{17})^{(2,2,)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,)}(G_{35},t)\right] \\ \left[-(b''_{37})^{(7,7,)}(G_{39},t)\right] & \left[-(b''_{41})^{(8,8)}(G_{43},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\left[-(b''_{15})^{(1)}(G,t)\right] & \left[-(b''_{18})^{(2,2,)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,)}(G_{35},t)\right] \\ \left[-(b''_{38})^{(7,7,)}(G_{39},t)\right] & \left[-(b''_{42})^{(8,8)}(G_{43},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t) \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & +(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) & +(a''_{21})^{(3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t) \end{bmatrix} G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & +(a''_{18})^{(2)}(T_{17},t) & +(a''_{15})^{(1,1,)}(T_{14},t) & +(a''_{22})^{(3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{38})^{(7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9)}(T_{45},t) \end{bmatrix} G_{18} \quad 63$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad 70$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad 72$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)} & \boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{24} \quad 73$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)} & \boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{25} \quad 74$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)} & \boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
are sixth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
are seventh augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
are eighth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)} & \boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad 76$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)} & \boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* $1, 2$ *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* $1, 2$ *and* 3

$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* $1, 2$ *and* 3

$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
are fourth detrition coefficients for category $1, 2$ *and* 3

$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
are fifth detrition coefficients for category $1, 2$ *and* 3

$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
are sixth detrition coefficients for category $1, 2$ *and* 3

$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
are seventh detrition coefficients for category $1, 2$ *and* 3

$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category $1, 2$ *and* 3

$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} & +(a''_{28})^{(5)}(T_{29},t) & +(a''_{24})^{(4,4,)}(T_{25},t) & +(a''_{32})^{(6,6,6)}(T_{33},t) \\ & +(a''_{13})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} & +(a''_{29})^{(5)}(T_{29},t) & +(a''_{25})^{(4,4,)}(T_{25},t) & +(a''_{33})^{(6,6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} & +(a''_{30})^{(5)}(T_{29},t) & +(a''_{26})^{(4,4,)}(T_{25},t) & +(a''_{34})^{(6,6,6)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{30} \quad 81$$

Where $+(a''_{28})^{(5)}(T_{29},t)$, $+(a''_{29})^{(5)}(T_{29},t)$, $+(a''_{30})^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* $1, 2$ *and* 3

And $+(a''_{24})^{(4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,)}(T_{25},t)$ *are second augmentation coefficient for category* $1, 2$ *and* 3

$+(a''_{32})^{(6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6)}(T_{33},t)$ *are third augmentation coefficient for category* $1, 2$ *and* 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation

coefficients for category 1,2, and 3
$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3
$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3
$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3
$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3
$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3
$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3
$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3
$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3
$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3
$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3
$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3
$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3
$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \quad 85$$

$$-\left[\begin{matrix}(a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)}\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}\end{matrix}\right]G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{matrix}(a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)}\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}\end{matrix}\right]G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{matrix}(a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)}\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}\end{matrix}\right]G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{matrix}(b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)}\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}\end{matrix}\right]T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[\begin{matrix}(b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}\end{matrix}\right]T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & \boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}, \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)} & \boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)} & \boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)} & \boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}, \boxed{(a''_{37})^{(7)}(T_{37},t)}, \boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\left[-(b''_{40})^{(8)}(G_{43},t)\right] & \left[-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\left[-(b''_{41})^{(8)}(G_{43},t)\right] & \left[-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\left[-(b''_{42})^{(8)}(G_{43},t)\right] & \left[-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$(b_i'')^{(1)}(G,t) \leq \ (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$

$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)}$ 98

$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = \ (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$

$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (a_i)^{(1)} + (a_i')^{(1)} + \ (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$

$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i')^{(1)} + \ (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$(a_i'')^{(2)}(T_{17},t) \leq (p_i)^{(2)} \leq \left(\hat{A}_{16}\right)^{(2)}$ 102

$$(b_i'')^{(2)}(G_{19},t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\big((G_{19}),t\big) = \ (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - \ T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\big((G_{19}),t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\big[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\big]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}} \ , \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\,(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + \ (\hat{P}_{16})^{(2)}\,(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + \ (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \le (p_i)^{(3)} \le (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \le (r_i)^{(3)} \le (b_i')^{(3)} \le (\hat{B}_{20})^{(3)}$$

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$ 113

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 114

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 115

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities 116

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 117

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\widehat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\widehat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\widehat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[\ (a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\,(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and \ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\,(a_i)^{(6)} + (a_i')^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)}\,(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(KKKKKKKK) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(LLLLLLLL) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$(a_i'')^{(7)}(T_{37},t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$

$(b_i'')^{(7)}(G_{39},t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$

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(MMMMMMMM) $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

(NNNNNNNN)

$\lim_{G\to\infty}(b_i'')^{(7)}\left((G_{39}),t\right) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$

$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}\left((G_{39}),t\right)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$. (T_{37}',t) and (T_{37},t) are points belonging to the interval $\left[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\right]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(OOOOOOOO) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(PPPPPPPP) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$

$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \tag{136}$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: (137)

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \tag{138}$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \tag{139}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \tag{140}$$

$$\lim_{G \to \infty} (b_i'')^{(8)}\big((G_{43}), t\big) = (r_i)^{(8)} \tag{141}$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \tag{142}$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}\big((G_{43}), t\big)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \tag{143}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1 \tag{144}$$

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}$, $(r_i)^{(8)}, i = 40,41,42$,
Satisfy the inequalities

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \tag{145}$$

$$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1 \tag{146}$$

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\hat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)} ||(G_{47}) - (G_{47})'|| e^{-(\hat{M}_{44})^{(9)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)} (\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t} \ , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t} \ , \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \ , \ T_i(0) = T_i^0 \ , \ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} , T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\overline{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{16})^{(2)}, T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{20})^{(3)}, T_i^0 \le (\hat{Q}_{20})^{(3)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$$

By

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^{0} + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^{0} + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^{0} + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^{0} + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^{0} + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^{0} + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By

$$\bar{G}_{32}(t) = G_{32}^{0} + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^{0} + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^{0} + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^{0} + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^{0} + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^{0} + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that 174

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \leq G_{32}^{0}+\int_0^t\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)} =$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(gg) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \leq G_{36}^{0}+\int_0^t\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)} =$$

$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \leq G_{40}^{0}+\int_0^t\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)} = \quad 180$$

$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t} \leq \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for G_{41} , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \leq G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \leq \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for G_{45} , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \left\{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\right\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T} : \left(\tilde{G}, \tilde{T}\right) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \qquad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+\ (a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on $G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\ (t)=0\ and\ T_i\ (t)=0$

From 19 to 24 it results

$$G_i\ (t)\geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(1)}-(a_i'')^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]}\geq 0$$

$$T_i\ (t)\geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)}>0\quad \text{for t}>0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13}<(\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt}\leq\left((\widehat{M}_{13})^{(1)}\right)_1-(a'_{14})^{(1)}G_{14}$ and by integrating

$$G_{14}\leq\left((\widehat{M}_{13})^{(1)}\right)_2=G_{14}^0+2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15}\leq\left((\widehat{M}_{13})^{(1)}\right)_3=G_{15}^0+2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a'_{15})^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}\left|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t} \leq$
$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)d\left(\left((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)}\right)\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\}ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0$ for $t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)}G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$: $\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0$ *and* $T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$G_{24}^{(2)} \left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)}\right) d\left(((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on T_{25} and respectively on (G_{27})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (228)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_2 / (a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)}, (T_{35})^{(1)}), ((G_{35})^{(2)}, (T_{35})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})}, \widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})}, \widetilde{(T_{35})}\right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right)\left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)};\ (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ *and* $(\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ (250)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)} \left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t} \; and \; (\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)} (T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2 \; and \; \left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37} \; and \; G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\widehat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{40})^{(8)}$$ 267

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\widehat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{40})^{(8)}\right] \leq (\widehat{Q}_{40})^{(8)}$$ 268

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $(\widetilde{G_{43}}),(\widetilde{T_{43}})$: $\left((\widetilde{G_{43}}),(\widetilde{T_{43}})\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1,\left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right),\quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{47}}),(\widetilde{T_{47}}):\left((\widetilde{G_{47}}),(\widetilde{T_{47}})\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$

From 99 to 44 it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(9)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)}, , (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)}, , (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)} t} - e^{-(a'_{26})^{(4)} t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)} t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28}e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G^0_{28}e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}}G^0_{28}e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}}G^0_{28}e^{(S_1)^{(5)}t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)}G^0_{28}}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G^0_{30}e^{-(S_2)^{(5)}t} \le G_{30}(t) \le \right. \quad 344$$

$$\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_2)^{(5)}((S_1)^{(5)}-(a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t}-e^{-(a'_{30})^{(5)}t}\right]+\ G_{30}^{0}e^{-(a'_{30})^{(5)}t}\Big)$$

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\le T_{30}(t)\le \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$:

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)}\le-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\le-(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)}\le-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\le-(\tau_1)^{(6)}$$

Definition of $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$: 350

By $(v_1)^{(6)}>0$, $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$, $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

Definition of $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)}>0$, $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$, $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

Definition of $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$:-

If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)}=(v_0)^{(6)},(m_1)^{(6)}=(v_1)^{(6)},$ ***if*** $(v_0)^{(6)}<(v_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}$ $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}$ $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}$ $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}$ $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}$ $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})} \left[e^{(S_1)^{(6)} t} - e^{-(a'_{34})^{(6)} t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)} t} \right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)} \text{ where } (u_1)^{(7)}, (\bar{u}_1)^{(7)}$$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^{0}e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}}G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}}G_{36}^{0}e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t}-e^{-(S_2)^{(7)}t}\right]+G_{38}^{0}e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left.\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_2)^{(7)}((S_1)^{(7)}-(a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t}-e^{-(a'_{38})^{(7)}t}\right]+\ G_{38}^{0}e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}}T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}}T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right) - (b''_{41})^{(8)}\left((G_{43}), t\right) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(v_1)^{(8)} > 0$, $(v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$, $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{v}_1)^{(8)}$,, $(\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{v}_1)^{(8)} > 0$, $(\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$, $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$:-

If we define $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$, $\boldsymbol{if}$ $(v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$, $\boldsymbol{if}$ $(v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$,
and $\boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}$, $\boldsymbol{if}$ $(\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, $\boldsymbol{if}$ $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, $\boldsymbol{if}$ $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}$ $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$$\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t})$$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \qquad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq\frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \qquad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t}\leq T_{42}(t)\leq \qquad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)},(S_2)^{(8)},(R_1)^{(8)},(R_2)^{(8)}$:- 381

Where $(S_1)^{(8)}=(a_{40})^{(8)}(m_2)^{(8)}-(a'_{40})^{(8)}$

$(S_2)^{(8)}=(a_{42})^{(8)}-(p_{42})^{(8)}$

$(R_1)^{(8)}=(b_{40})^{(8)}(\mu_2)^{(8)}-(b'_{40})^{(8)}$

$(R_2)^{(8)}=(b'_{42})^{(8)}-(r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$:

$(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)}\leq-(a'_{44})^{(9)}+(a'_{45})^{(9)}-(a''_{44})^{(9)}(T_{45},t)+(a''_{45})^{(9)}(T_{45},t)\leq-(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)}\leq-(b'_{44})^{(9)}+(b'_{45})^{(9)}-(b''_{44})^{(9)}\big((G_{47}),t\big)-(b''_{45})^{(9)}\big((G_{47}),t\big)\leq-(\tau_1)^{(9)}$

Definition of $(v_1)^{(9)},(v_2)^{(9)},(u_1)^{(9)},(u_2)^{(9)},v^{(9)},u^{(9)}$:

By $(v_1)^{(9)}>0$, $(v_2)^{(9)}<0$ and respectively $(u_1)^{(9)}>0$, $(u_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_1)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)}>0$, $(\bar{v}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$, $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

Definition of $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$:-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)},\ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$
and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \le G_{46}(t) \le \right.$$
$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (v_1)^{(1)} \le (\bar{v}_1)^{(1)} \le \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$, we obtain 386

$$(v_1)^{(1)} \le v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $v^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\right) - (a''_{17})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{\mathrm{G}_{16}}{\mathrm{G}_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (\mathrm{C})^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (\mathrm{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)}\right) \le \frac{dv^{(3)}}{dt} \le -\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \ge \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}}$$

it follows $(v_0)^{(3)} \le v^{(3)}(t) \le (v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \le v^{(3)}(t) \le (\bar{v}_1)^{(3)}$

If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)} \le \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}} \le v^{(3)}(t) \le$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (\bar{v}_1)^{(3)}$$

If $0 < (v_1)^{(3)} \le (\bar{v}_1)^{(3)} \le (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$, we obtain 403

$$(v_1)^{(3)} \le v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}$, $\boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

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$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{36}'')^{(7)} = (a_{37}'')^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b_{36}'')^{(7)} = (b_{37}'')^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a_{40}')^{(8)} - (a_{41}')^{(8)} + (a_{40}'')^{(8)}(T_{41}, t)\right) - (a_{41}'')^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(8)} and\ (b''_i)^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\ ]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\ ]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\ ]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0$ 479

$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0$ 480

$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0$ 481

$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0$ 482

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(gg) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first

equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T^*_{33} for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$$

Definition and uniqueness of T^*_{37} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T^*_{37} for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$$

Definition and uniqueness of T^*_{41} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{25} such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507
$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{29} such that $\varphi((G_{31})^*) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{33} such that $\varphi({G_{35}}^*) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{37} such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{41} such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right]+(b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{45} such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)} G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)} G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)} T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)} T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)} G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)} G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)} T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)} T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)} G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)} G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)} T_{21}^*}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T_{22}^* = \frac{(b_{22})^{(3)} T_{21}^*}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]} \quad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \qquad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \qquad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \qquad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \qquad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \qquad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \qquad 530$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i \qquad 532$$

$$\frac{\partial (a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial (b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \qquad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \qquad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \qquad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \qquad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \qquad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}\mathrm{T}^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}\mathrm{T}^*_{18}\mathbb{G}_j\right) \quad 539$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}\left((G_{27})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)}$, $\frac{\partial(b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\ B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\Big)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\Big]$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\Big)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\Big)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\Big]$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\Big)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\right)$$

$$\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left(\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\right]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\right)$$

$$\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\Big)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION FORTY NINE

Computational Neuroscience

INTRODUCTION—VARIABLES USED

Computational neuroscience from Wikipedia, the free encyclopedia: for references please see Wikipedia

(1) One recent computational hypothesis involves cascades of plasticity that allow (eb) synapses to function at multiple time scales.[12]
(2) Stereo chemically detailed models of the acetylcholine receptor-based synapse with (e&eb) the Monte Carlo method, working at the time scale of microseconds, have been built. [13]

(3) It is likely that computational tools will contribute greatly to our understanding of how synapses function and change in relation to (e&eb) external stimulus in the coming decades.

Behaviors of networks

(4) Biological neurons are connected to (e&eb) each other in a complex, recurrent fashion.
(5) These connections are, unlike most artificial neural networks, sparse and usually specific. It is not known how information is transmitted through (e&eb) such sparsely connected networks.
(6) It is also unknown what the computational functions of (e) these specific connectivity patterns are, if any.
(7) The interactions of neurons in a small network can be often reduced to (eb) simple models such as the Ising model.
(8) The statistical mechanics of such simple systems are well-characterized theoretically. There has been some recent evidence that suggests that dynamics of arbitrary neuronal networks can be reduced to (eb) pairwise interactions.[14]
(9) It is not known, however, whether such descriptive dynamics impart (e&eb) any important computational function. With the emergence of two-photon microscopy and calcium imaging, we now have powerful experimental methods with which to test the new theories regarding neuronal networks.

NOTATION

Module One

One recent computational hypothesis involves cascades of plasticity that allow (eb) synapses to function at multiple time scales.[12]

G_{13} : Category one of computational hypothesis involves cascades of plasticity

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of synapses to function at multiple time scales.[12]

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Stereo chemically detailed models of the acetylcholine receptor-based synapse with (e&eb) the Monte Carlo method, working at the time scale of microseconds, have been built. [13]

G_{16} : Category one of acetylcholine receptor-based synapse; Monte Carlo method, working at the time scale of microseconds

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of Monte Carlo method, working at the time scale of microseconds; acetylcholine receptor-based synapse

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

It is likely that computational tools will contribute greatly to our understanding of how

synapses function and change in relation to (e&eb) external stimulus in the coming decades

G_{20} : Category one of synapses function and change; external stimulus in the coming decades

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of external stimulus in the coming decades; synapses function and change

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

Biological neurons are connected to (e&eb) each other in a complex, recurrent fashion

Model gives complexity and recurreceness of the system

G_{24} : Category one of Biological neurons; each other in a complex, recurrent fashion

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of each other in a complex, recurrent fashion; Biological neurons

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

These connections are, unlike most artificial neural networks, sparse and usually specific. It is not known how

information is transmitted through (e&eb) such sparsely connected networks

Model gives how information is transmitted

G_{28} : Category one of information is transmitted; sparsely connected networks

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of sparsely connected networks; information is transmitted

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

It is also unknown what the computational functions of (e) these specific connectivity patterns are, if any

G_{32} : Category one of these specific connectivity patterns

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of computational functions

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

The statistical mechanics of such simple systems are well-characterized theoretically.

There has been some recent evidence that suggests that

dynamics of arbitrary neuronal networks can be reduced to (eb) pairwise interactions.[14

G_{36} : Category one of dynamics of arbitrary neuronal networks

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of pairwise interactions

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

It is not known, however, whether

such descriptive dynamics impart (e&eb) any important computational function.

We assume it does, and give a model.

With the emergence of two-photon microscopy and calcium imaging, we now have powerful experimental methods with which to test the new theories regarding neuronal networks.

G_{40} : Category one of such descriptive dynamics; important computational function

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of important computational function; such descriptive dynamics

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

One of the major problems in neurophysiological memory is how it is maintained and changed through multiple time scales.

Unstable synapses are easy to train but also prone to (e) stochastic disruption

G_{44} : Category one of neurophysiological memory maintenance and change; stochastic disruption

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of stochastic disruption; neurophysiological memory maintenance and change;

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right]G_{13} \quad 1$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14},t)\right]G_{14} \quad 2$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14},t)\right]G_{15} \quad 3$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G,t)\right]T_{13} \quad 4$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t)\right]T_{14} \quad 5$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor

$-(b''_{13})^{(1)}(G,t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\big((G_{19}),t\big)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\big((G_{19}),t\big)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\big((G_{19}),t\big)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor

$-(b''_{16})^{(2)}\big((G_{19}),t\big) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27}),t\big)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27}),t\big)\right]T_{25} \quad 23$$

$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}),t)\right]T_{26}$ 24

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28}$ 25

$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29}$ 26

$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30}$ 27

$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28}$ 28

$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29}$ 29

$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30}$ 30

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}),t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32}$ 31

$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33}$ 32

$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34}$ 33

$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32}$ 34

$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33}$ 35

$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34}$ 36

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36}$ 37

$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37}$ *38*

$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38}$ *39*

$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36}$ *40*

$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37}$ *41*

$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38}$ *42*

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40}$ *43*

$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41}$ *44*

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$
$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}),t\right)\right]T_{40} \quad 46$$
$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}),t\right)\right]T_{41} \quad 47$$
$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}),t\right)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$
$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$
$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$
$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}),t\right)\right]T_{44} \quad 52$$
$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}\left((G_{47}),t\right)\right]T_{45} \quad 53$$
$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}\left((G_{47}),t\right)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}\left((G_{47}),t\right) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & \boxed{+(a''_{13})^{(1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & \boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & \boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7)}(T_{37},t)}$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) & -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) & -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t) \\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t) \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{c} (a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{45})^{(9,9)}(T_{45},t)} \end{array}\right] G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{c} (a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}\ \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}\ \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}\ \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}\ \boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}\ \boxed{+(a''_{46})^{(9,9)}(T_{45},t)} \end{array}\right] G_{18} \quad (63)$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7)}(T_{37},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{40})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8)}(T_{41},t)}$ are eight augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{44})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9,9)}(T_{45},t)}$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[\begin{array}{c} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}\ \boxed{-(b''_{13})^{(1,1,)}(G,t)}\ \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}\ \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}\ \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{array}\right] T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[\begin{array}{c} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)}\ \boxed{-(b''_{14})^{(1,1,)}(G,t)}\ \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}\ \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}\ \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{array}\right] T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[\begin{array}{c} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)}\ \boxed{-(b''_{15})^{(1,1,)}(G,t)}\ \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}\ \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}\ \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}\ \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}\ \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{array}\right] T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category

1,2 and 3

$-(b_{20}'')^{(3,3,3,)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b_{26}'')^{(4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b_{30}'')^{(5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b_{34}'')^{(6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3

$-(b_{36}'')^{(7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2 and 3

$-(b_{40}'')^{(8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1,2 and 3

$-(b_{44}'')^{(9,9,9,9)}(G_{47},t)$, $-(b_{46}'')^{(9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{c}(a_{20}')^{(3)}+(a_{20}'')^{(3)}(T_{21},t)+(a_{16}'')^{(2,2,2)}(T_{17},t)+(a_{13}'')^{(1,1,1,)}(T_{14},t)\\ +(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a_{36}'')^{(7,7,7,7)}(T_{37},t)+(a_{40}'')^{(8,8,8,8)}(T_{41},t)+(a_{44}'')^{(9,9,9)}(T_{45},t)\end{array}\right]G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{c}(a_{21}')^{(3)}+(a_{21}'')^{(3)}(T_{21},t)+(a_{17}'')^{(2,2,2)}(T_{17},t)+(a_{14}'')^{(1,1,1,)}(T_{14},t)\\ +(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a_{37}'')^{(7,7,7,7)}(T_{37},t)+(a_{41}'')^{(8,8,8,8)}(T_{41},t)+(a_{45}'')^{(9,9,9)}(T_{45},t)\end{array}\right]G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{c}(a_{22}')^{(3)}+(a_{22}'')^{(3)}(T_{21},t)+(a_{18}'')^{(2,2,2)}(T_{17},t)+(a_{15}'')^{(1,1,1,)}(T_{14},t)\\ +(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)\\ +(a_{38}'')^{(7,7,7,7)}(T_{37},t)+(a_{42}'')^{(8,8,8,8)}(T_{41},t)+(a_{46}'')^{(9,9,9)}(T_{45},t)\end{array}\right]G_{22} \quad (69)$$

$+(a_{20}'')^{(3)}(T_{21},t)$, $+(a_{21}'')^{(3)}(T_{21},t)$, $+(a_{22}'')^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation

coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) \end{bmatrix} T_{22} \quad (72)$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{46})^{(9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25},t) & +(a''_{28})^{(5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{24} \quad (73)$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25},t) & +(a''_{29})^{(5,5,)}(T_{29},t) & +(a''_{33})^{(6,6)}(T_{33},t) \\ & +(a''_{14})^{(1,1,1,1)}(T_{14},t) & +(a''_{17})^{(2,2,2,2)}(T_{17},t) & +(a''_{21})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{37})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25},t) & +(a''_{30})^{(5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,)}(T_{33},t) \\ & +(a''_{15})^{(1,1,1,1)}(T_{14},t) & +(a''_{18})^{(2,2,2,2)}(T_{17},t) & +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ & +(a''_{38})^{(7,7,7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9)}(T_{45},t) \end{bmatrix} G_{26} \quad 75$$

$(a''_{24})^{(4)}(T_{25},t)$, $(a''_{25})^{(4)}(T_{25},t)$, $(a''_{26})^{(4)}(T_{25},t)$ *are first augmentation coefficients category* 1, 2 3

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{32})^{(6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a''_{13})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14},t)$ *are fourth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$ *are fifth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$ *are sixth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$ *are seventh augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$ *are eighth augmentation coefficients for category* 1, 2 *and* 3

$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad 76$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad 78$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b_{32}'')^{(6,6,)}(G_{35},t)$, $-(b_{33}'')^{(6,6,)}(G_{35},t)$, $-(b_{34}'')^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3

$-(b_{13}'')^{(1,1,1,1)}(G,t)$, $-(b_{14}'')^{(1,1,1,1)}(G,t)$, $-(b_{15}'')^{(1,1,1,1)}(G,t)$

are fourth detrition coefficients for category 1, 2 *and* 3

$-(b_{16}'')^{(2,2,2,2)}(G_{19},t)$, $-(b_{17}'')^{(2,2,2,2)}(G_{19},t)$, $-(b_{18}'')^{(2,2,2,2)}(G_{19},t)$

are fifth detrition coefficients for category 1, 2 *and* 3

$-(b_{20}'')^{(3,3,3,3)}(G_{23},t)$, $-(b_{21}'')^{(3,3,3,3)}(G_{23},t)$, $-(b_{22}'')^{(3,3,3,3)}(G_{23},t)$

are sixth detrition coefficients for category 1, 2 *and* 3

$-(b_{36}'')^{(7,7,7,7,7)}(G_{39},t)$, $-(b_{37}'')^{(7,7,7,7,7)}(G_{39},t)$, $-(b_{38}'')^{(7,7,7,7,7)}(G_{39},t)$

are seventh detrition coefficients for category 1, 2 *and* 3

$-(b_{40}'')^{(8,8,8,8,8)}(G_{43},t)$, $-(b_{41}'')^{(8,8,8,8,8)}(G_{43},t)$, $-(b_{42}'')^{(8,8,8,8,8)}(G_{43},t)$

are eighth detrition coefficients for category 1, 2 *and* 3

$-(b_{46}'')^{(9,9,9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9,9,9)}(G_{47},t)$, $-(b_{44}'')^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a_{28}')^{(5)} & +(a_{28}'')^{(5)}(T_{29},t) & +(a_{24}'')^{(4,4,)}(T_{25},t) & +(a_{32}'')^{(6,6,6)}(T_{33},t) \\ & +(a_{13}'')^{(1,1,1,1,1)}(T_{14},t) & +(a_{16}'')^{(2,2,2,2,2)}(T_{17},t) & +(a_{20}'')^{(3,3,3,3,3)}(T_{21},t) \\ & +(a_{36}'')^{(7,7,7,7,7,7)}(T_{37},t) & +(a_{40}'')^{(8,8,8,8,8,8)}(T_{41},t) & +(a_{44}'')^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a_{29}')^{(5)} & +(a_{29}'')^{(5)}(T_{29},t) & +(a_{25}'')^{(4,4,)}(T_{25},t) & +(a_{33}'')^{(6,6,6)}(T_{33},t) \\ & +(a_{14}'')^{(1,1,1,1,1)}(T_{14},t) & +(a_{17}'')^{(2,2,2,2,2)}(T_{17},t) & +(a_{21}'')^{(3,3,3,3,3)}(T_{21},t) \\ & +(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t) & +(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a_{30}')^{(5)} & +(a_{30}'')^{(5)}(T_{29},t) & +(a_{26}'')^{(4,4,)}(T_{25},t) & +(a_{34}'')^{(6,6,6)}(T_{33},t) \\ & +(a_{15}'')^{(1,1,1,1,1)}(T_{14},t) & +(a_{18}'')^{(2,2,2,2,2)}(T_{17},t) & +(a_{22}'')^{(3,3,3,3,3)}(T_{21},t) \\ & +(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t) & +(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{30} \quad 81$$

Where $+(a_{28}'')^{(5)}(T_{29},t)$, $+(a_{29}'')^{(5)}(T_{29},t)$, $+(a_{30}'')^{(5)}(T_{29},t)$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $+(a_{24}'')^{(4,4,)}(T_{25},t)$, $+(a_{25}'')^{(4,4,)}(T_{25},t)$, $+(a_{26}'')^{(4,4,)}(T_{25},t)$ *are second augmentation coefficient for category* 1, 2 *and* 3

$+(a_{32}'')^{(6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6)}(T_{33},t)$ *are third augmentation coefficient for category* 1, 2 *and* 3

$+(a_{13}'')^{(1,1,1,1,1)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,1,1)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,1,1)}(T_{14},t)$ are fourth augmentation coefficients for category 1,2, and 3

$+(a_{16}'')^{(2,2,2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3

$+(a_{20}'')^{(3,3,3,3,3)}(T_{21},t)$, $+(a_{21}'')^{(3,3,3,3,3)}(T_{21},t)$, $+(a_{22}'')^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3

$+(a_{36}'')^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3

$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3

$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3

$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3

$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3

$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3

$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \begin{bmatrix} (a'_{32})^{(6)} & +(a''_{32})^{(6)}(T_{33},t) & +(a''_{28})^{(5,5,5)}(T_{29},t) & +(a''_{24})^{(4,4,4,)}(T_{25},t) \\ & +(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t) & +(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t) & +(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t) \\ & +(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t) \end{bmatrix} G_{32} \quad (85)$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)}\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)}\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}\boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)}\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)}\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)}\boxed{-(b''_{34})^{(6)}(G_{35},t)}\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}\boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)}\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}, \boxed{-(b''_{33})^{(6)}(G_{35},t)}, \boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b_{28}'')^{(5,5,5)}(G_{31},t)}, \boxed{-(b_{29}'')^{(5,5,5)}(G_{31},t)}, \boxed{-(b_{30}'')^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b_{24}'')^{(4,4,4,)}(G_{27},t)}, \boxed{-(b_{25}'')^{(4,4,4,)}(G_{27},t)}, \boxed{-(b_{26}'')^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b_{13}'')^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b_{14}'')^{(1,1,1,1,1,1)}(G,t)}, \boxed{-(b_{15}'')^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b_{17}'')^{(2,2,2,2,2,2)}(G_{19},t)}, \boxed{-(b_{18}'')^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b_{21}'')^{(3,3,3,3,3,3)}(G_{23},t)}, \boxed{-(b_{22}'')^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b_{36}'')^{(7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b_{37}'')^{(7,7,7,7,7,7)}(G_{39},t)}, \boxed{-(b_{38}'')^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b_{41}'')^{(8,8,8,8,8,8)}(G_{43},t)}, \boxed{-(b_{42}'')^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b_{45}'')^{(9,9,9,9,9,9)}(G_{47},t)}, \boxed{-(b_{44}'')^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a_{36}')^{(7)}\boxed{+(a_{36}'')^{(7)}(T_{37},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a_{37}')^{(7)}\boxed{+(a_{37}'')^{(7)}(T_{37},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a_{38}')^{(7)}\boxed{+(a_{38}'')^{(7)}(T_{37},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a_{36}'')^{(7)}(T_{37},t)}, \boxed{(a_{37}'')^{(7)}(T_{37},t)}, \boxed{(a_{38}'')^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{17}'')^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a_{18}'')^{(2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a_{21}'')^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a_{22}'')^{(3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)}, \boxed{+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\left[-(b''_{36})^{(7)}(G_{39},t)\right] & \left[-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\left[-(b''_{37})^{(7)}(G_{39},t)\right] & \left[-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\left[-(b''_{38})^{(7)}(G_{39},t)\right] & \left[-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)$, $-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \begin{bmatrix} (a'_{42})^{(8)}\boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{38})^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[\begin{array}{ccc} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[\begin{array}{ccc} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq \ (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = \ (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \ , \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (a_i)^{(1)} + (a_i')^{(1)} + \ (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i')^{(1)} + \ (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \leq (p_i)^{(2)} \leq \left(\hat{A}_{16}\right)^{(2)} \qquad 102$$

$$(b_i'')^{(2)}(G_{19},t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\big((G_{19}),t\big) = \ (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - \ T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\big((G_{19}),t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\big[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\big]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}} \ , \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\,(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + \ (\hat{P}_{16})^{(2)}\,(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + \ (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \le (p_i)^{(3)} \le (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \le (r_i)^{(3)} \le (b_i')^{(3)} \le (\hat{B}_{20})^{(3)}$$

$$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with (116)
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \quad (117)$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\widehat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\widehat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\widehat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[\ (a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28,29,30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - \ T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32{,}33{,}34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and \ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32{,}33{,}34,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\,(a_i)^{(6)} + (a_i')^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)}\,(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(QQQQQQQQ) $\quad (a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36{,}37{,}38$ 131

(RRRRRRRR) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37},t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39},t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

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(SSSSSSSS) $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

(TTTTTTTT)

$\lim_{G\to\infty}(b_i'')^{(7)}((G_{39}),t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}((G_{39}),t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$. (T_{37}',t) and (T_{37},t) are points belonging to the interval $\left[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\right]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(UUUUUUUU) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(VVVVVVVV) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$ 138

$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$ 139

$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$ 140

$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t}$ 142

$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t}$ 143

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ 144

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}$, $(r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 145

$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 146

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\hat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)} ||(G_{47}) - (G_{47})'|| e^{-(\hat{M}_{44})^{(9)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)} (\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t} \ , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t} \ , \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \ , \ T_i(0) = T_i^0 \ , \ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} , T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\overline{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^{0} + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^{0} + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^{0} + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^{0} + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^{0} + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^{0} + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By

$$\bar{G}_{32}(t) = G_{32}^{0} + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^{0} + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^{0} + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^{0} + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^{0} + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^{0} + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{40})^{(8)}$, $T_i^0 \leq (\hat{Q}_{40})^{(8)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad (169)$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that (170)

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that (171)

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that (172)

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that (173)

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that (174)

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)} =$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(hh) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \leq G_{36}^{0}+\int_{0}^{t}\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)} =$$
$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \leq G_{40}^{0}+\int_{0}^{t}\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)} = \quad 180$$
$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t} \leq \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for G_{41} , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \leq G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \leq \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for G_{45} , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \left\{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\right\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)}\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) - (a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right| \; e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} \} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \qquad (186)$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on T_{14} and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: (187)

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. (188)

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}\left|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t} \leq$
$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)d\left(\left((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)}\right)\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\}ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0$ for $t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)}G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$: $\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23})\right)$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0$ *and* $T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}} +$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}} e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+\ (a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right) d\left(\left((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on T_{25} and respectively on (G_{27})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (228)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)}-(a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(4)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)}G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1/(a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2/(a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\widehat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{32})^{(6)}\right] \leq (\widehat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})}, \widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})}, \widetilde{(T_{35})}\right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$

$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$

$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right)\left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$

$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\} ds_{(32)}$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq$ 248

$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)};\ (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ *and* $(\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. 249

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 250

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)} \left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for $G_i \; and \; T_i$. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t} \; and \; (\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and \; not \; on \; t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t) = 0 \; and \; T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)} (T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2 \; and \; \left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37} \; and \; G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_7}$$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1,\left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right),\quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{47}}),(\widetilde{T_{47}}):\left((\widetilde{G_{47}}),(\widetilde{T_{47}})\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$

From 99 to 44 it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(9)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$: 281

By $(v_1)^{(1)} > 0$, $(v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{v}_1)^{(1)},, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{v}_1)^{(1)} > 0$, $(\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)},, (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G^0_{16}}{G^0_{17}}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)}, , (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad (333)$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad (334)$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)} t} - e^{-(a'_{26})^{(4)} t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)} t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad (335)$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad (336)$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G^0_{28} e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G^0_{28}}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G^0_{30} e^{-(S_2)^{(5)} t} \le G_{30}(t) \le \right. \quad 344$$

$$\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_2)^{(5)}((S_1)^{(5)}-(a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t}-e^{-(a'_{30})^{(5)}t}\right]+\ G_{30}^{0}e^{-(a'_{30})^{(5)}t}\Big)$$

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\le T_{28}(t)\le\frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\le T_{30}(t)\le \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$:

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)}\le-(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\le-(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)}\le-(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\le-(\tau_1)^{(6)}$$

Definition of $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$: 350

By $(v_1)^{(6)}>0$, $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$, $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

Definition of $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)}>0$, $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$, $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

Definition of $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$:-

If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)}=(v_0)^{(6)},(m_1)^{(6)}=(v_1)^{(6)},$ ***if*** $(v_0)^{(6)}<(v_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}$ $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}$ $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}$ $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}$ $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}$ $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})} \left[e^{(S_1)^{(6)} t} - e^{-(a'_{34})^{(6)} t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)} t} \right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)}, , (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^{0}e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}}G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}}G_{36}^{0}e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t}-e^{-(S_2)^{(7)}t}\right]+G_{38}^{0}e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left.\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_2)^{(7)}((S_1)^{(7)}-(a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t}-e^{-(a'_{38})^{(7)}t}\right]+\ G_{38}^{0}e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}}T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}}T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right) - (b''_{41})^{(8)}\left((G_{43}), t\right) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(v_1)^{(8)} > 0$, $(v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$, $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{v}_1)^{(8)}$,, $(\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{v}_1)^{(8)} > 0$, $(\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$, $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$:-

If we define $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$, $\boldsymbol{if}$ $(v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$, $\boldsymbol{if}$ $(v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$,
and $\boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}$, $\boldsymbol{if}$ $(\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, $\boldsymbol{if}$ $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, $\boldsymbol{if}$ $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}$ $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t})$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t}\leq T_{42}(t)\leq \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)},(S_2)^{(8)},(R_1)^{(8)},(R_2)^{(8)}$:- 381

Where $(S_1)^{(8)}=(a_{40})^{(8)}(m_2)^{(8)}-(a'_{40})^{(8)}$

$(S_2)^{(8)}=(a_{42})^{(8)}-(p_{42})^{(8)}$

$(R_1)^{(8)}=(b_{40})^{(8)}(\mu_2)^{(8)}-(b'_{40})^{(8)}$

$(R_2)^{(8)}=(b'_{42})^{(8)}-(r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$:

$(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)}\leq -(a'_{44})^{(9)}+(a'_{45})^{(9)}-(a''_{44})^{(9)}(T_{45},t)+(a''_{45})^{(9)}(T_{45},t)\leq -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)}\leq -(b'_{44})^{(9)}+(b'_{45})^{(9)}-(b''_{44})^{(9)}\big((G_{47}),t\big)-(b''_{45})^{(9)}\big((G_{47}),t\big)\leq -(\tau_1)^{(9)}$

Definition of $(v_1)^{(9)},(v_2)^{(9)},(u_1)^{(9)},(u_2)^{(9)},v^{(9)},u^{(9)}$:

By $(v_1)^{(9)}>0$, $(v_2)^{(9)}<0$ and respectively $(u_1)^{(9)}>0$, $(u_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_1)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)}>0$, $(\bar{v}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$, $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

Definition of $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$:-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)},\ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$
and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \le G_{46}(t) \le \right.$$
$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (v_1)^{(1)} \le (\bar{v}_1)^{(1)} \le \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$, we obtain 386

$$(v_1)^{(1)} \le v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $v^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\right) - (a''_{17})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{\mathrm{G}_{16}}{\mathrm{G}_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (\mathrm{C})^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (\mathrm{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)}\right) \le \frac{dv^{(3)}}{dt} \le -\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \ge \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}}$$

it follows $(v_0)^{(3)} \le v^{(3)}(t) \le (v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \le v^{(3)}(t) \le (\bar{v}_1)^{(3)}$

If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)} \le \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}} \le v^{(3)}(t) \le$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (\bar{v}_1)^{(3)}$$

If $0 < (v_1)^{(3)} \le (\bar{v}_1)^{(3)} \le (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$, we obtain 403

$$(v_1)^{(3)} \le v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

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If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case,

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

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If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

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$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(8)} and\ (b''_i)^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on t , and the conditions (with the notations 436 A
45,46,27,28)

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\ ]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\ ]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\ ]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\ ]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\ ]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0$ 479

$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0$ 480

$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0$ 481

$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0$ 482

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(hh) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first

equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T^*_{33} for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$$

Definition and uniqueness of T^*_{37} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T^*_{37} for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$$

Definition and uniqueness of T^*_{41} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{25} such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507
$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{29} such that $\varphi((G_{31})^*) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{33} such that $\varphi({G_{35}}^*) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{37} such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{41} such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right]+(b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{45} such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)} G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)} G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)} T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)} T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)} G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)} G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)} T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)} T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)} G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)} G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)} T_{21}^*}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T_{22}^* = \frac{(b_{22})^{(3)} T_{21}^*}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]} \quad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \qquad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \qquad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \qquad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \qquad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \qquad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \qquad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i \qquad 532$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \qquad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \qquad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \qquad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \qquad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \qquad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}\mathrm{T}^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}\mathrm{T}^*_{18}\mathbb{G}_j\right) \quad 539$$

ASYMPTOTIC STABILITY ANALYSIS 540

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

ASYMPTOTIC STABILITY ANALYSIS 547

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}\left((G_{27})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^*\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^*\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^*\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\,B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\Big)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\Big]$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\Big)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\Big)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\Big]$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\Big)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\Big)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\Big]$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\Big)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\Big)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\Big)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\Big)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION FIFTY

Cognition, Discrimination, And Learning

INTRODUCTION—VARIABLES USED

Computational neuroscience from Wikipedia, the free encyclopedia: for references please see Wikipedia

(1) In some cases the complex interactions between inhibitory and (e&eb) excitatory neurons can be simplified using mean field theory, which gives rise to (eb) the population model of neural networks.

(2) While many neurotheorists prefer such models with reduced complexity, others argue that

uncovering structural functional relations depends on (=) including as much neuronal and network structure as possible. Models of this type are typically built in (eb) large simulation platforms like GENESIS or NEURON.

(3) Computational modeling of higher cognitive functions has only recently begun. Experimental data comes primarily from (e) single-unit recording in primates.

(4) The frontal lobe and parietal lobe function as integrators of information from (e) multiple sensory modalities.

(5) There are some tentative ideas regarding how simple mutually inhibitory functional circuits in these areas may carry out (eb) biologically relevant computation.[16]

(6) The brain seems to be able to discriminate and adapt particularly well in certain contexts. For instance, human beings seem to have an enormous capacity for memorizing and recognizing faces. One of the key goals of computational neuroscience is to dissect how biological systems carry out (eb) these complex computations efficiently and potentially replicate these processes in (eb) building intelligent machines.

(7) The brain's large-scale organizational principles are illuminated by (e) many fields, including biology, psychology, and clinical practice. Integrative neuroscience attempts to consolidate these observations through unified descriptive models and databases of behavioral measures and recordings. These are the bases for some quantitative modeling of large-scale brain activity. [17]

(8) The Computational Representational Understanding of Mind (CRUM) is another attempt at modeling human cognition through (e&eb) simulated processes like acquired rule-based systems in decision making and the manipulation of (e&eb) visual representations in decision making.

(9) The Computational Representational Understanding of Mind (CRUM) is another attempt at modeling human cognition through (e&eb) simulated processes like acquired rule-based systems in decision making and the manipulation of (e&eb) visual representations in decision making

NOTATION

Module One

In some cases the complex interactions between inhibitory and (e&eb) excitatory neurons can be simplified using mean field theory, which gives rise to (eb) the population model of neural networks

G_{13} : Category one of inhibitory neurons; excitatory neurons

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of excitatory neurons ;inhibitory neurons;

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

complex interactions between inhibitory and excitatory neurons can be simplified using mean field theory, which gives rise to (eb) the population model of neural networks

G_{16} : Category one of complex interactions between inhibitory and excitatory neurons can be simplified using mean field theory

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of population model of neural networks

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

While many neurotheorists prefer such models with reduced complexity, others argue that uncovering structural functional relations depends on (=) including as much neuronal and network structure as possible. Models of this type are typically built in (eb) large simulation platforms like GENESIS or NEURON.

G_{20} : Category one of uncovering structural functional relations

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of neuronal and network structure X(variable)

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

The frontal lobe and parietal lobe function as integrators of information from (e) multiple sensory modalities.

G_{24} : Category one of multiple sensory modalities

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of frontal lobe and parietal lobe function as integrators of information

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

There are some tentative ideas regarding how

simple mutually inhibitory functional circuits in these areas may carry out (eb) biologically relevant computation.[16]

G_{28} : Category one of simple mutually inhibitory functional circuits in these areas

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of biologically relevant computation

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

The brain seems to be able to discriminate and adapt particularly well in certain contexts. For instance, human beings seem to have an enormous capacity for memorizing and recognizing faces.

One of the key goals of computational neuroscience is to dissect how

biological systems carry out (eb) these complex computations efficiently and potentially replicate these processes in (eb) building intelligent machines

G_{32} : Category one of enormous capacity for memorizing and recognizing faces

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of human beings

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

biological systems carry out (eb) these complex computations efficiently and potentially replicate these processes in (eb) building intelligent machines

G_{36} : Category one of biological systems

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of complex computations efficiently and potentially replicate these processes in (eb) building intelligent machines

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

biological systems carry out these complex computations efficiently and potentially replicate these processes in (eb) building intelligent machines

G_{40} : Category one of biological systems carry out these complex computations efficiently and potentially replicate these processes

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of building intelligent machines

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

The Computational Representational Understanding of Mind (CRUM) is another attempt at modeling

human cognition through (e&eb) simulated processes like acquired rule-based systems in decision making and the manipulation of (e&eb) visual representations in decision making

G_{44} : Category one of human cognition; simulated processes like acquired rule-based systems in decision making and the manipulation of (e&eb) visual representations in decision making

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of simulated processes like acquired rule-based systems in decision making and the manipulation of (e&eb) visual representations in decision making; human cognition

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, \ (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$

$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)},$
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)},$
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)},$
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$
$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$
$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$
$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$
$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$
$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor
$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$
$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$
$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$
$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$
$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$
$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor
$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25},t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}),t)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}),t)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}),t)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}((G_{27}),t) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29},t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29},t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29},t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}),t)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}),t)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}),t)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29},t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}((G_{31}),t) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33},t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33},t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}),t)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}),t)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}),t)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33},t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37},t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}),t)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}),t)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}),t)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41},t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41},t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41},t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}((G_{43}),t)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}((G_{43}),t)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}((G_{43}),t)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45},t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45},t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45},t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}((G_{47}),t)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \begin{bmatrix} (a'_{13})^{(1)} & \boxed{+(a''_{13})^{(1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{bmatrix} (a'_{14})^{(1)} & \boxed{+(a''_{14})^{(1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{bmatrix} (a'_{15})^{(1)} & \boxed{+(a''_{15})^{(1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15} \quad 57$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation

coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{bmatrix} (b'_{13})^{(1)}\left[-(b''_{13})^{(1)}(G,t)\right] & \left[-(b''_{16})^{(2,2,)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,)}(G_{35},t)\right] \\ \left[-(b''_{36})^{(7,7,)}(G_{39},t)\right] & \left[-(b''_{40})^{(8,8)}(G_{43},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{bmatrix} (b'_{14})^{(1)}\left[-(b''_{14})^{(1)}(G,t)\right] & \left[-(b''_{17})^{(2,2,)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,)}(G_{35},t)\right] \\ \left[-(b''_{37})^{(7,7,)}(G_{39},t)\right] & \left[-(b''_{41})^{(8,8)}(G_{43},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)}\left[-(b''_{15})^{(1)}(G,t)\right] & \left[-(b''_{18})^{(2,2,)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,)}(G_{35},t)\right] \\ \left[-(b''_{38})^{(7,7,)}(G_{39},t)\right] & \left[-(b''_{42})^{(8,8)}(G_{43},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15} \quad 60$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t) & \\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t) & \end{bmatrix} G_{16} \quad 61$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & +(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) & +(a''_{21})^{(3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t) & \\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t) & \end{bmatrix} G_{17} \quad 62$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & +(a''_{18})^{(2)}(T_{17},t) & +(a''_{15})^{(1,1,)}(T_{14},t) & +(a''_{22})^{(3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33},t) & \\ +(a''_{38})^{(7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9)}(T_{45},t) & \end{bmatrix} G_{18} \quad 63$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)} & -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{16} \quad 64$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)} & -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9)}(G_{47},t) & \end{bmatrix} T_{17} \quad 65$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad 66$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{44})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a'_{20})^{(3)}\boxed{+(a''_{20})^{(3)}(T_{21},t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{20} \quad 67$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a'_{21})^{(3)}\boxed{+(a''_{21})^{(3)}(T_{21},t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{21} \quad 68$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a'_{22})^{(3)}\boxed{+(a''_{22})^{(3)}(T_{21},t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)} & \boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9)}(T_{45},t)} \end{bmatrix} G_{22} \quad 69$$

$\boxed{+(a''_{20})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,)}(T_{14},t)}$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a''_{36})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a''_{40})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a''_{44})^{(9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b'_{20})^{(3)} & -(b''_{20})^{(3)}(G_{23},t) & -(b''_{16})^{(2,2,2)}(G_{19},t) & -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{36})^{(7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad 70$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b'_{21})^{(3)} & -(b''_{21})^{(3)}(G_{23},t) & -(b''_{17})^{(2,2,2)}(G_{19},t) & -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{37})^{(7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{21} \quad 71$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)} & -(b''_{22})^{(3)}(G_{23},t) & -(b''_{18})^{(2,2,2)}(G_{19},t) & -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b''_{38})^{(7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{22} \quad 72$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detrition coefficients for category 1, 2 and 3*

$-(b''_{36})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt}=(a_{24})^{(4)}G_{25}-\begin{bmatrix}(a'_{24})^{(4)} & \boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{24} \quad 73$$

$$\frac{dG_{25}}{dt}=(a_{25})^{(4)}G_{24}-\begin{bmatrix}(a'_{25})^{(4)} & \boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)}\\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{25} \quad 74$$

$$\frac{dG_{26}}{dt}=(a_{26})^{(4)}G_{25}-\begin{bmatrix}(a'_{26})^{(4)} & \boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}\\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}\\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}\end{bmatrix}G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* 1, 2 3

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$
are fourth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$
are fifth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$
are sixth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)}$
are seventh augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)}$
are eighth augmentation coefficients for category 1, 2 *and* 3

$\boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt}=(b_{24})^{(4)}T_{25}-\begin{bmatrix}(b'_{24})^{(4)} & \boxed{-(b''_{24})^{(4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{24} \quad 76$$

$$\frac{dT_{25}}{dt}=(b_{25})^{(4)}T_{24}-\begin{bmatrix}(b'_{25})^{(4)} & \boxed{-(b''_{25})^{(4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}\\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}\\ \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}\end{bmatrix}T_{25} \quad 77$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{26} \quad 78$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}, \boxed{-(b''_{25})^{(4)}(G_{27},t)}, \boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}, \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}, \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* $1, 2$ *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}, \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
are fourth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}, \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}, \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)}, \boxed{-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)}$
are seventh detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)}, \boxed{-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)}$
are eighth detrition coefficients for category $1, 2$ *and* 3

$\boxed{-(b''_{46})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{45})^{(9,9,9,9)}(G_{47},t)}, \boxed{-(b''_{44})^{(9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad 79$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad 80$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad 81$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation

coefficients for category 1,2, and 3
$+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)$ are fifth augmentation coefficients for category 1,2,and 3
$+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)$ are sixth augmentation coefficients for category 1,2, 3
$+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1,2, 3
$+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)$ are eighth augmentation coefficients for category 1,2, 3
$+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)} & -(b''_{28})^{(5)}(G_{31},t) & -(b''_{24})^{(4,4,)}(G_{27},t) & -(b''_{32})^{(6,6,6)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)} & -(b''_{29})^{(5)}(G_{31},t) & -(b''_{25})^{(4,4,)}(G_{27},t) & -(b''_{33})^{(6,6,6)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31},t) & -(b''_{26})^{(4,4,)}(G_{27},t) & -(b''_{34})^{(6,6,6)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1,1,)}(G,t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{30} \quad (84)$$

where $-(b''_{28})^{(5)}(G_{31},t)$, $-(b''_{29})^{(5)}(G_{31},t)$, $-(b''_{30})^{(5)}(G_{31},t)$ *are first detrition coefficients for category* 1, 2 *and* 3
$-(b''_{24})^{(4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,)}(G_{27},t)$ *are second detrition coefficients for category* 1,2 *and* 3
$-(b''_{32})^{(6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6)}(G_{35},t)$ *are third detrition coefficients for category* 1,2 *and* 3
$-(b''_{13})^{(1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,)}(G,t)$ are fourth detrition coefficients for category 1,2, and 3
$-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1,2, and 3
$-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1,2, and 3
$-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1,2, and 3
$-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1,2, and 3
$-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} \quad 85$$

$$- \begin{bmatrix} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)} & \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{32}$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \begin{bmatrix} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)} & \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{33} \quad 86$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \begin{bmatrix} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)} & \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{34} \quad 87$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}, \boxed{+(a''_{33})^{(6)}(T_{33},t)}, \boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}, \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}, \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}, \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}, \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}, \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)}, \boxed{+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)}$ seventh augmentation coefficients

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{41})^{(8,8,8,8,8,8,8)}(T_{41},t)}, \boxed{+(a''_{42})^{(8,8,8,8,8,8,8)}(T_{41},t)}$ Eighth augmentation coefficients

$\boxed{+(a''_{44})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{45})^{(9,9,9,9,9,9)}(T_{45},t)}, \boxed{+(a''_{46})^{(9,9,9,9,9,9)}(T_{45},t)}$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)}\boxed{-(b''_{32})^{(6)}(G_{35},t)} & \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{32} \quad 88$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)}\boxed{-(b''_{33})^{(6)}(G_{35},t)} & \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{33} \quad 89$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & \boxed{-(b''_{34})^{(6)}(G_{35},t)} & \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{34} \quad 90$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \quad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)} & \boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \quad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)} & \boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \quad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)} & \boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} =$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{40}')^{(8)}\boxed{+(a_{40}'')^{(8)}(T_{41},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{44}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{13}$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[\begin{array}{ccc} (a_{41}')^{(8)}\boxed{+(a_{41}'')^{(8)}(T_{41},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{45}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{14}$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[\begin{array}{ccc} (a_{42}')^{(8)}\boxed{+(a_{42}'')^{(8)}(T_{41},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{46}'')^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array}\right] G_{15}$$

Where $\boxed{+(a_{40}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ $+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)$ are eighth augmentation coefficient for 1,2,3

$+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$

$$(b_{40})^{(8)}T_{41} - \begin{bmatrix} (b'_{40})^{(8)}\left[-(b''_{40})^{(8)}(G_{43},t)\right] & \left[-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{13}$$

$$\frac{dT_{41}}{dt} =$$

$$(b_{41})^{(8)}T_{40} - \begin{bmatrix} (b'_{41})^{(8)}\left[-(b''_{41})^{(8)}(G_{43},t)\right] & \left[-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{14}$$

$$\frac{dT_{42}}{dt} =$$

$$(b_{42})^{(8)}T_{41} - \begin{bmatrix} (b'_{42})^{(8)}\left[-(b''_{42})^{(8)}(G_{43},t)\right] & \left[-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)\right] & \left[-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)\right] \\ \left[-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)\right] & \left[-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)\right] & \left[-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)\right] \\ \left[-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)\right] & \left[-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)\right] & \left[-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)\right] \end{bmatrix} T_{15}$$

Where $-(b''_{36})^{(7)}(G_{39},t)$, $-(b''_{37})^{(7)}(G_{39},t)$, $-(b''_{38})^{(7)}(G_{39},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)$ are eighth detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a'_{44})^{(9)}\boxed{+(a''_{44})^{(9)}(T_{45},t)}\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a'_{45})^{(9)}\boxed{+(a''_{45})^{(9)}(T_{45},t)}\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a'_{46})^{(9)}\boxed{+(a''_{46})^{(9)}(T_{37},t)}\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a''_{44})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9)}(T_{45},t)}$, $\boxed{+(a''_{46})^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} -$$

$$\begin{bmatrix} (b'_{44})^{(9)}\boxed{-(b''_{44})^{(9)}(G_{47},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{45}}{dt} =$$

$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b'_{45})^{(9)}\boxed{-(b''_{45})^{(9)}(G_{47},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{46}}{dt} =$$

$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b'_{46})^{(9)}\boxed{-(b''_{46})^{(9)}(G_{47},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{44})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9)}(G_{47},t)}$, $\boxed{-(b''_{46})^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14},t) \le (p_i)^{(1)} \le (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq \ (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = \ (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \ , \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (a_i)^{(1)} + (a_i')^{(1)} + \ (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[\ (b_i)^{(1)} + (b_i')^{(1)} + \ (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}\ (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17},t) \leq (p_i)^{(2)} \leq \left(\hat{A}_{16}\right)^{(2)} \qquad 102$$

$$(b_i'')^{(2)}(G_{19},t) \leq \ (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G\to\infty}(b_i'')^{(2)}\big((G_{19}),t\big) = \ (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - \ T_{17}'|e^{-(\hat{M}_{16})^{(2)}t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}\big((G_{19}),t\big)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\big[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\big]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}} \ , \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and \ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, \ (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\,(a_i)^{(2)} + (a_i')^{(2)} + \ (\hat{A}_{16})^{(2)} + \ (\hat{P}_{16})^{(2)}\,(\hat{k}_{16})^{(2)}] < 1 \quad 110$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[\ (b_i)^{(2)} + (b_i')^{(2)} + \ (\hat{B}_{16})^{(2)} + \ (\hat{Q}_{16})^{(2)}\ (\hat{k}_{16})^{(2)}] < 1 \quad 111$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22 \quad 112$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \le (p_i)^{(3)} \le (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \le (r_i)^{(3)} \le (b_i')^{(3)} \le (\hat{B}_{20})^{(3)}$$

$$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \le (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with (116)
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \quad (117)$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}$, $(r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25},t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}),t\big) \leq \ (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}(T_{25},t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\big((G_{27}),t\big) = \ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24{,}25{,}26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}',t) - (a_i'')^{(4)}(T_{25},t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\widehat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})',t) - (b_i'')^{(4)}\big((G_{27}),t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\widehat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}',t)$ and$(a_i'')^{(4)}(T_{25},t)$. (T_{25}',t) and (T_{25},t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\widehat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\widehat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24{,}25{,}26$, satisfy the inequalities

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[\,(a_i)^{(4)} + (a_i')^{(4)} + \ (\hat{A}_{24})^{(4)} + \ (\hat{P}_{24})^{(4)}\,(\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\widehat{M}_{24})^{(4)}}[\ (b_i)^{(4)} + (b_i')^{(4)} + \ (\hat{B}_{24})^{(4)} + \ (\hat{Q}_{24})^{(4)}\ (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i,j = 28{,}29{,}30 \qquad 122$$

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29},t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}),t\big) \leq \ (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2\to\infty}(a_i'')^{(5)}(T_{29},t) = (p_i)^{(5)} \qquad 123$$

$$\lim_{G\to\infty}(b_i'')^{(5)}(G_{31},t) = \ (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}',t) - (a_i'')^{(5)}(T_{29},t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})',t) - (b_i'')^{(5)}\big((G_{31}),t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}',t)$ and$(a_i'')^{(5)}(T_{29},t)$. (T_{29}',t) and (T_{29},t) are points belonging to the interval $\left[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\right]$. It is to be noted that $(a_i'')^{(5)}(T_{29},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)},$ are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\,(a_i)^{(5)} + (a_i')^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)}\,(\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i')^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)}\ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i,j = 32,33,34 \qquad 127$$

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}),t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2\to\infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)} \qquad 128$$

$$\lim_{G\to\infty}(b_i'')^{(6)}\big((G_{35}),t\big) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}\big((G_{35}),t\big)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and$(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) and (T_{33},t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)},$ are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and \ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\,(a_i)^{(6)} + (a_i')^{(6)} + \ (\hat{A}_{32})^{(6)} + \ (\hat{P}_{32})^{(6)}\,(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)} + (b_i')^{(6)} + \ (\hat{B}_{32})^{(6)} + \ (\hat{Q}_{32})^{(6)}\ (\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(WWWWWWWW) $\quad (a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i,j = 36,37,38$ 131

(XXXXXXXX) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}, \ (r_i)^{(7)}$:

$(a_i'')^{(7)}(T_{37},t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$

$(b_i'')^{(7)}(G_{39},t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$ 132

(YYYYYYYY) $\lim_{T_2\to\infty} (a_i'')^{(7)}(T_{37},t) = (p_i)^{(7)}$

(ZZZZZZZZ)

$\lim_{G\to\infty}(b_i'')^{(7)}\left((G_{39}),t\right) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$|(a_i'')^{(7)}(T_{37}',t) - (a_i'')^{(7)}(T_{37},t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$

$|(b_i'')^{(7)}((G_{39})',t) - (b_i'')^{(7)}\left((G_{39}),t\right)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}',t)$ and$(a_i'')^{(7)}(T_{37},t)$. (T_{37}',t) and (T_{37},t) are points belonging to the interval $\left[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\right]$. It is to be noted that $(a_i'')^{(7)}(T_{37},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(AAAAAAAAA) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(BBBBBBBBB) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$

$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)}(\hat{k}_{36})^{(7)}] < 1$

Where we suppose

$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42$ 136

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)}$ 138

$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)}$ 139

$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)}$ 140

$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)}$ 141

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t}$ 142

$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t}$ 143

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $\left[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}\right]$. It is to be noted that $(a_i'')^{(8)}(T_{41}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{40})^{(8)} = 1$ then the function $(a_i'')^{(8)}(T_{41}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$:

$(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}$, are positive constants

$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}, \frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ 144

Definition of $(\hat{P}_{40})^{(8)}, (\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\hat{M}_{40})^{(8)}, (\hat{k}_{40})^{(8)}, (\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)}, (a_i')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (p_i)^{(8)}$, $(r_i)^{(8)}, i = 40,41,42,$
Satisfy the inequalities

$\frac{1}{(\hat{M}_{40})^{(8)}}[(a_i)^{(8)} + (a_i')^{(8)} + (\hat{A}_{40})^{(8)} + (\hat{P}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 145

$\frac{1}{(\hat{M}_{40})^{(8)}}[(b_i)^{(8)} + (b_i')^{(8)} + (\hat{B}_{40})^{(8)} + (\hat{Q}_{40})^{(8)} (\hat{k}_{40})^{(8)}] < 1$ 146

Where we suppose

$$(a_i)^{(9)}, (a_i')^{(9)}, (a_i'')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (b_i'')^{(9)} > 0, \quad i,j = 44,45,46 \qquad (146A)$$

The functions $(a_i'')^{(9)}, (b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)}$, $(r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45}, t) \leq (p_i)^{(9)} \leq (\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47}, t) \leq (r_i)^{(9)} \leq (b_i')^{(9)} \leq (\hat{B}_{44})^{(9)}$$

$$\lim_{T_2 \to \infty} (a_i'')^{(9)}(T_{45}, t) = (p_i)^{(9)}$$

$$\lim_{G \to \infty} (b_i'')^{(9)}(G_{47}, t) = (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)}, (\hat{B}_{44})^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}}$ are positive constants and $\boxed{i = 44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}', t) - (a_i'')^{(9)}(T_{45}, t)| \leq (\hat{k}_{44})^{(9)} |T_{45} - T_{45}'| e^{-(\hat{M}_{44})^{(9)} t}$$

$$|(b_i'')^{(9)}((G_{47})', t) - (b_i'')^{(9)}((G_{47}), t)| < (\hat{k}_{44})^{(9)} ||(G_{47}) - (G_{47})'|| e^{-(\hat{M}_{44})^{(9)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}', t)$ and$(a_i'')^{(9)}(T_{45}, t)$. (T_{45}', t) and (T_{45}, t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)}, (\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)} = 1$ then the function $(a_i'')^{(9)}(T_{45}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}$, are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}, \frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}} < 1$$

Definition of $(\hat{P}_{44})^{(9)}, (\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)}, (\hat{k}_{44})^{(9)}, (\hat{A}_{44})^{(9)} and (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)}, (a_i')^{(9)}, (b_i)^{(9)}, (b_i')^{(9)}, (p_i)^{(9)}, (r_i)^{(9)}, i = 44,45,46,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)} + (a_i')^{(9)} + (\hat{A}_{44})^{(9)} + (\hat{P}_{44})^{(9)} (\hat{k}_{44})^{(9)}] < 1$$

$$\frac{1}{(\widehat{M}_{44})^{(9)}}[\ (b_i)^{(9)} + (b_i')^{(9)} + \ (\hat{B}_{44})^{(9)} + (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}] < 1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\widehat{M}_{16})^{(2)}t} \ , \quad T_i(0) = T_i^0 > 0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t} \ , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t} \ , \quad T_i(0) = T_i^0 > 0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t} \ , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t} \ , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{40}\right)^{(8)} e^{(\hat{M}_{40})^{(8)}t} \quad , \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0)$, $T_i(0)$:

$$G_i(t) \le \left(\hat{P}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \le \left(\hat{Q}_{44}\right)^{(9)} e^{(\hat{M}_{44})^{(9)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , T_i: $\mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0 \ , \ T_i(0) = T_i^0 \ , \ G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} , T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\overline{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^{0} + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{16})^{(2)}$, $T_i^0 \le (\hat{Q}_{16})^{(2)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{24})^{(4)}, T_i^0 \le (\hat{Q}_{24})^{(4)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{28})^{(5)}, T_i^0 \le (\hat{Q}_{28})^{(5)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$$

By 163

$$\bar{G}_{28}(t) = G_{28}^{0} + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^{0} + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^{0} + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^{0} + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^{0} + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^{0} + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By

$$\bar{G}_{32}(t) = G_{32}^{0} + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^{0} + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^{0} + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^{0} + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^{0} + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^{0} + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35}(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)})\right) G_{38}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)})\right) T_{38}(s_{(36)})\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\bar{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^{0} + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{40}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^{0} + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{41}(s_{(40)}) \right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^{0} + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)}) \right) T_{42}(s_{(40)}) \right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 166

Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{44})^{(9)},\ T_i^0 \le (\hat{Q}_{44})^{(9)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)} (T_{45}(s_{(44)}), s_{(44)}) \right) G_{46}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{44}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{45}(s_{(44)}) \right] ds_{(44)}$$

$$\bar{T}_{46}(t) = T_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)} (G_{47}(s_{(44)}), s_{(44)}) \right) T_{46}(s_{(44)}) \right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\tilde{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left(e^{(\tilde{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\tilde{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\tilde{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \le \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0\right) e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}\right)} + (\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t) \le G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}}\right)\right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1\right)$$

From which it follows that 174

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \le \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0\right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}\right)} + (\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t) \le G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}}\right)\right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t) \leq G_{32}^{0}+\int_0^t\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)} =$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(ii) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \leq G_{36}^{0}+\int_0^t\left[(a_{36})^{(7)}\left(G_{37}^{0}+(\hat{P}_{36})^{(7)}e^{(\hat{M}_{36})^{(7)}s_{(36)}}\right)\right]ds_{(36)} =$$
$$\left(1+(a_{36})^{(7)}t\right)G_{37}^{0}+\frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left(e^{(\hat{M}_{36})^{(7)}t}-1\right)$$

From which it follows that

$$(G_{36}(t)-G_{36}^{0})e^{-(\hat{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[((\hat{P}_{36})^{(7)}+G_{37}^{0})e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^{0}}{G_{37}^{0}}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \leq G_{40}^{0}+\int_0^t\left[(a_{40})^{(8)}\left(G_{41}^{0}+(\hat{P}_{40})^{(8)}e^{(\hat{M}_{40})^{(8)}s_{(40)}}\right)\right]ds_{(40)} = \quad 180$$
$$\left(1+(a_{40})^{(8)}t\right)G_{41}^{0}+\frac{(a_{40})^{(8)}(\hat{P}_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left(e^{(\hat{M}_{40})^{(8)}t}-1\right)$$

From which it follows that 181

$$(G_{40}(t)-G_{40}^{0})e^{-(\hat{M}_{40})^{(8)}t} \leq \frac{(a_{40})^{(8)}}{(\hat{M}_{40})^{(8)}}\left[((\hat{P}_{40})^{(8)}+G_{41}^{0})e^{\left(-\frac{(\hat{P}_{40})^{(8)}+G_{41}^{0}}{G_{41}^{0}}\right)}+(\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for G_{41} , $G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \leq G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\hat{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$

$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left(e^{(\hat{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\hat{M}_{44})^{(9)} t} \leq \frac{(a_{44})^{(9)}}{(\hat{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for G_{45} , $G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \left((\hat{P}_{13})^{(1)} + G_j^0\right) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{Q}_{13})^{(1)} + T_j^0\right) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)}, T^{(1)}\right), \left(G^{(2)}, T^{(2)}\right)\right) =$$

$$\sup_i \left\{\max_{t \in \mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}, \max_{t \in \mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{13})^{(1)} t}\right\}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$: $(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$\left|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)} \left|G_{14}^{(1)} - G_{14}^{(2)}\right| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)}\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right)\left|G_{13}^{(1)} - G_{13}^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)}, s_{(13)}\right) - (a''_{13})^{(1)}\left(T_{14}^{(2)}, s_{(13)}\right)\right| \; e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} \} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq \qquad (186)$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on T_{14} and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(1)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2$ *and* $\left((\widehat{M}_{13})^{(1)}\right)_3$: (187)

Remark 3: if G_{13} is bounded, the same property have also G_{14} *and* G_{15} . indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} *is* bounded, from below, the same property holds for G_{14} *and* G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. (188)

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + \left((\hat{P}_{16})^{(2)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$$

$$\int_0^t\{(a_{16}')^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$$

$(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$G_{16}^{(2)}\left|(a_{16}'')^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}'')^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right| \; e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 197

From the hypotheses it follows

$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t} \leq$
$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}')^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\right)d\left(\left((G_{19})^{(1)}, (T_{19})^{(1)}; (G_{19})^{(2)}, (T_{19})^{(2)}\right)\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a_{16}'')^{(2)}$ and $(b_{16}'')^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\}ds_{(16)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)}G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1/(a_{17}')^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2/(a_{18}')^{(2)}$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[(\hat{P}_{20})^{(3)} + \left((\hat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)}\right] \leq (\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 210
Equations into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$:$\left(\widetilde{(G_{23})}, \widetilde{(T_{23})}\right) = \mathcal{A}^{(3)}((G_{23}), (T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)}-G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{(a'_{20})^{(3)} \left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} +$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)}t} \leq \qquad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+\ (a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right) d\left(((G_{23})^{(1)},(T_{23})^{(1)};\ (G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0$ *and* $T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)}-(a_i'')^{(3)}(T_{21}(s_{(20)}),s_{(20)})\}ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)}t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a'_{21})^{(3)}G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1/(a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2/(a'_{22})^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t),t\right)=(b_{21}')^{(3)}$ then $T_{21}\to\infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$(b_{21})^{(3)}-(b_i'')^{(3)}\left((G_{23})(t),t\right)<\varepsilon_3, T_{20}(t)>(m)^{(3)}$

Then $\frac{dT_{21}}{dt}\geq(a_{21})^{(3)}(m)^{(3)}-\varepsilon_3 T_{21}$ which leads to 220

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1-e^{-\varepsilon_3 t})+T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t}=\frac{1}{2}$ it results

$T_{21}\geq\left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right),\quad t=log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t),t\right)=(b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}<1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})},\widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})},\widetilde{(T_{27})}\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)}-G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\, ds_{(24)} +$$

$$\int_0^t \{(a'_{24})^{(4)} \left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}} +$$

$$G_{24}^{(2)} \left|(a''_{24})^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a''_{24})^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\} ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} t} \leq \qquad (226)$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)}\right) d\left(((G_{27})^{(1)}, (T_{27})^{(1)};\ (G_{27})^{(2)}, (T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ respectively of $\mathbb{R}_+$. (227)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i = 24{,}25{,}26$ depend only on T_{25} and respectively on (G_{27})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (228)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(4)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$: (229)

Remark 18: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a'_{25})^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_1 / (a'_{25})^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_2 / (a'_{26})^{(4)}$$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}((G_{27})(t),t)) = (b_{25}')^{(4)}$ then $T_{25} \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t),t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t),t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}} < 1$ and to choose 233

$(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[(\hat{P}_{28})^{(5)} + \left((\hat{P}_{28})^{(5)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{28})^{(5)} \quad 234$$

$$\frac{(b_i)^{(5)}}{(\tilde{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)}\right] \leq (\hat{Q}_{28})^{(5)} \quad 235$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 236

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\tilde{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})}, \widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})}, \widetilde{(T_{31})}\right) = \mathcal{A}^{(5)}\left((G_{31}), (T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{28})^{(5)} \left|G_{29}^{(1)} - G_{29}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}}\, ds_{(28)} +$$

$$\int_0^t \{(a'_{28})^{(5)} \left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right)\left|G_{28}^{(1)} - G_{28}^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} +$$

$$G_{28}^{(2)} \left|(a''_{28})^{(5)}\left(T_{29}^{(1)}, s_{(28)}\right) - (a''_{28})^{(5)}\left(T_{29}^{(2)}, s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)} s_{(28)}} e^{(\widehat{M}_{28})^{(5)} s_{(28)}} \} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)} - (G_{31})^{(2)}\right| e^{-(\widehat{M}_{28})^{(5)} t} \leq \qquad (237)$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)}\right) d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)};\ (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ *and* $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)} t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on (G_{31})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ (239)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(5)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$: (240)

Remark 23: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a'_{29})^{(5)}$$

In the same way , one can obtain

$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\hat{P}_{32})^{(6)} + \left((\hat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{32})^{(6)} \quad 245$$

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)}\right] \leq (\hat{Q}_{32})^{(6)} \quad 246$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 247

$d\left(((G_{35})^{(1)}, (T_{35})^{(1)}), ((G_{35})^{(2)}, (T_{35})^{(2)})\right) =$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})}, \widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})}, \widetilde{(T_{35})}\right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a'_{32})^{(6)} \left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right)\left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}} +$$

$$G_{32}^{(2)} \left|(a''_{32})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a''_{32})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)} s_{(32)}} e^{(\widehat{M}_{32})^{(6)} s_{(32)}}\} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)} - (G_{35})^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)} t} \leq \qquad (248)$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right) d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)};\ (G_{35})^{(2)}, (T_{35})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ *and* $(\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)} t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ (250)

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{\left(-(b_i')^{(6)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2$ *and* $\left((\widehat{M}_{32})^{(6)}\right)_3$: (251)

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1 / (a'_{33})^{(6)}$$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} *or* G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for G_{33} *and* G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose
$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)} + \left((\widehat{P}_{36})^{(7)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{36})^{(7)} \quad 256$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{36})^{(7)}\right] \leq (\widehat{Q}_{36})^{(7)} \quad 257$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 258

$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right) =$
$sup_i\{max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}, max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})} : \left(\widetilde{(G_{39})}, \widetilde{(T_{39})} \right) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$
It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{36})^{(7)} \left|G_{37}^{(1)} - G_{37}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}}\, ds_{(36)} +$$
$$\int_0^t \{(a'_{36})^{(7)} \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) \left|G_{36}^{(1)} - G_{36}^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} +$$
$$G_{36}^{(2)} \left|(a''_{36})^{(7)} \left(T_{37}^{(1)}, s_{(36)}\right) - (a''_{36})^{(7)} \left(T_{37}^{(2)}, s_{(36)}\right)\right| \; e^{-(\widehat{M}_{36})^{(7)} s_{(36)}} e^{(\widehat{M}_{36})^{(7)} s_{(36)}} \} ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses on it follows

$$\left|(G_{39})^{(1)} - (G_{39})^{(2)}\right| e^{-(\widehat{M}_{36})^{(7)} t} \leq \qquad 259$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}} \left((a_{36})^{(7)} + (a'_{36})^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}\right) d\left(((G_{39})^{(1)}, (T_{39})^{(1)};\ (G_{39})^{(2)}, (T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ *and* $(\widehat{Q}_{36})^{(7)} e^{(\widehat{M}_{36})^{(7)} t}$ respectively of $\mathbb{R}_+$. 260

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on (G_{39})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 261

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)} (T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(7)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ *and* $\left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also G_{37} *and* G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a'_{37})^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a'_{37})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a'_{38})^{(7)}$$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}((G_{39})(t),t)) = (b'_{37})^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right)$, $t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t),t\right) = (b'_{38})^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}} < 1$ and to choose 266
$(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ large to have

$$\frac{(a_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[(\hat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\hat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)},(T_{43})^{(1)}),((G_{43})^{(2)},(T_{43})^{(2)})\right)= \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})},\widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})},\widetilde{(T_{43})}\right)=\mathcal{A}^{(8)}((G_{43}),(T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{40})^{(8)}\left|G_{41}^{(1)}-G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)}+$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)\left|G_{40}^{(1)}-G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}+$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)},s_{(40)}\right)-(a''_{40})^{(8)}\left(T_{41}^{(2)},s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0,t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)}-(G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t}\le \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)}+(a'_{40})^{(8)}+(\widehat{A}_{40})^{(8)}+(\widehat{P}_{40})^{(8)}(\widehat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)},(T_{43})^{(1)};\ (G_{43})^{(2)},(T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$. 274

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i=40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 275

it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(8)}-(a_i'')^{(8)}(T_{41}(s_{(40)}),s_{(40)})\}ds_{(40)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(8)}t\right)}>0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1,\left((\widehat{M}_{40})^{(8)}\right)_2$ *and* $\left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a'_{41})^{(8)} G_{41}$ and by integrating

$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a'_{41})^{(8)}$

In the same way , one can obtain

$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a'_{42})^{(8)}$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}\ ((G_{43})(t),t)) = (b'_{41})^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t),t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right),\quad t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t),t(t),t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have 279 A

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)},(T_{47})^{(1)}),((G_{47})^{(2)},(T_{47})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{47}}),(\widetilde{T_{47}}):\left((\widetilde{G_{47}}),(\widetilde{T_{47}})\right)=\mathcal{A}^{(9)}\left((G_{47}),(T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)}-\tilde{G}_i^{(2)}\right|\le\int_0^t(a_{44})^{(9)}\left|G_{45}^{(1)}-G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)}+$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)\left|G_{44}^{(1)}-G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}+$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)},s_{(44)}\right)-(a''_{44})^{(9)}\left(T_{45}^{(2)},s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t}\le$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)}+(a'_{44})^{(9)}+(\widehat{A}_{44})^{(9)}+(\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)},(T_{47})^{(1)};\ (G_{47})^{(2)},(T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i=44{,}45{,}46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i(t)=0\ and\ T_i(t)=0$

From 99 to 44 it results

$$G_i(t)\ge G_i^0e^{\left[-\int_0^t\{(a_i')^{(9)}-(a_i'')^{(9)}(T_{45}(s_{(44)}),s_{(44)})\}ds_{(44)}\right]}\ge 0$$

$$T_i(t)\ge T_i^0e^{\left(-(b_i')^{(9)}t\right)}>0\quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1,\left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44}<(\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt}\le\left((\widehat{M}_{44})^{(9)}\right)_1-(a'_{45})^{(9)}G_{45}$ and by integrating

$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1/(a'_{45})^{(9)}$

In the same way , one can obtain

$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2/(a'_{46})^{(9)}$

If G_{45} *or* G_{46} is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for G_{45} *and* G_{46} . The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t),t\right) = (b'_{45})^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t),t\right) < \varepsilon_9, T_{44}(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right)$, $t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b''_{46})^{(9)}\left((G_{47})(t),t\right) = (b'_{46})^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14},t) + (a''_{14})^{(1)}(T_{14},t) \leq -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G,t) - (b''_{14})^{(1)}(G,t) \leq -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)},\ if\ (v_0)^{(1)} < (v_1)^{(1)}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}, if\ (v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)},$

and $\boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)},\ if\ (\bar{v}_1)^{(1)} < (v_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined

Then the solution of global equations satisfies the inequalities 285

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$$

where $(p_i)^{(1)}$ is defined by equation

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right] + G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le \right. \quad 286$$

$$\left. \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}} \quad 287$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t} \quad 288$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le \quad 289$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right] + T_{15}^0 e^{-(R_2)^{(1)} t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 290

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions of equation 291

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 292

$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(\mathrm{T}_{17}, t) + (a''_{17})^{(2)}(\mathrm{T}_{17}, t) \le -(\sigma_1)^{(2)}$ 293

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\big((G_{19}), t\big) - (b''_{17})^{(2)}\big((G_{19}), t\big) \le -(\tau_1)^{(2)}$ 294

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 295

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots 296

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 297

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 298

Definition of $(\bar{\nu}_1)^{(2)},, (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 299

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the 300

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ 301

and $(b_{17})^{(2)}\big(u^{(2)}\big)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)},\ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)} t}$$

$(p_i)^{(2)}$ is defined by equation

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)} t} \quad 311$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)} t}\right] + G_{18}^0 e^{-(S_2)^{(2)} t} \le G_{18}(t) \le \right. \quad 312$$

$$\left. \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)} t} - e^{-(a'_{18})^{(2)} t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)} t}\right)$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}} \quad 313$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t} \quad 314$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)} t} - e^{-(b'_{18})^{(2)} t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)} t} \le T_{18}(t) \le \quad 315$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)} t}\right] + T_{18}^0 e^{-(R_2)^{(2)} t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \quad 318$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(v_1)^{(3)} > 0$, $(v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{v}_1)^{(3)} > 0$, $(\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}$, $(m_2)^{(3)}$, $(\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}$, ***if*** $(v_0)^{(3)} < (v_1)^{(3)}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}$, ***if*** $(v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}$,

and $\boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}$, ***if*** $(\bar{v}_1)^{(3)} < (v_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}$, ***if*** $(\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad (323)$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \le G_{22}(t) \le \right. \quad (324)$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad (325)$$

$$\frac{1}{(\mu_1)^{(3)}}T_{20}^0 e^{(R_1)^{(3)}t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}}T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b'_{22})^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 329

By $(\nu_1)^{(4)} > 0$, $(\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)}, , (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 330

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(\nu^{(4)}\big)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\big(u^{(4)}\big)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}$, ***if*** $(\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}$, ***if*** $(\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$,
and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)},\ \boldsymbol{if}\ (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)},\ \boldsymbol{if}\ (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \boldsymbol{if}\ (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if}\ (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)} t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)} t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)} t}\right] + G_{26}^0 e^{-(S_2)^{(4)} t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)} t} - e^{-(a'_{26})^{(4)} t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)} t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)} t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)} t} - e^{-(b'_{26})^{(4)} t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)} t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})}\left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)} t}\right] + T_{26}^0 e^{-(R_2)^{(4)} t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 337

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

$(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 339

By $(\nu_1)^{(5)} > 0$, $(\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0$, $(u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)} > 0$, $(\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0$, $(\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$
and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)}$, $(m_2)^{(5)}$, $(\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}$, ***if*** $(\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}$, ***if*** $(\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}$, ***if*** $(\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, ***if*** $(u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, ***if*** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,
and $\boxed{(u_0)^{(5)} = \frac{T^0_{28}}{T^0_{29}}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, ***if*** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G^0_{28} e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G^0_{28} e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G^0_{28} e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G^0_{28}}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G^0_{30} e^{-(S_2)^{(5)} t} \le G_{30}(t) \le \right. \quad 344$$

$$\frac{(a_{30})^{(5)}G_{28}^{0}}{(m_2)^{(5)}((S_1)^{(5)}-(a'_{30})^{(5)})}\left[e^{(S_1)^{(5)}t}-e^{-(a'_{30})^{(5)}t}\right]+\ G_{30}^{0}e^{-(a'_{30})^{(5)}t}\Big)$$

$$\boxed{T_{28}^{0}e^{(R_1)^{(5)}t}\leq T_{28}(t)\leq T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}}T_{28}^{0}e^{(R_1)^{(5)}t}\leq T_{28}(t)\leq \frac{1}{(\mu_2)^{(5)}}T_{28}^{0}e^{((R_1)^{(5)}+(r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)}T_{28}^{0}}{(\mu_1)^{(5)}((R_1)^{(5)}-(b'_{30})^{(5)})}\left[e^{(R_1)^{(5)}t}-e^{-(b'_{30})^{(5)}t}\right]+T_{30}^{0}e^{-(b'_{30})^{(5)}t}\leq T_{30}(t)\leq \quad 347$$

$$\frac{(a_{30})^{(5)}T_{28}^{0}}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t}-e^{-(R_2)^{(5)}t}\right]+T_{30}^{0}e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)},(S_2)^{(5)},(R_1)^{(5)},(R_2)^{(5)}$:- 348

Where $(S_1)^{(5)}=(a_{28})^{(5)}(m_2)^{(5)}-(a'_{28})^{(5)}$

$(S_2)^{(5)}=(a_{30})^{(5)}-(p_{30})^{(5)}$

$(R_1)^{(5)}=(b_{28})^{(5)}(\mu_2)^{(5)}-(b'_{28})^{(5)}$

$(R_2)^{(5)}=(b'_{30})^{(5)}-(r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$:

$(\sigma_1)^{(6)},(\sigma_2)^{(6)},(\tau_1)^{(6)},(\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)}\leq -(a'_{32})^{(6)}+(a'_{33})^{(6)}-(a''_{32})^{(6)}(T_{33},t)+(a''_{33})^{(6)}(T_{33},t)\leq -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)}\leq -(b'_{32})^{(6)}+(b'_{33})^{(6)}-(b''_{32})^{(6)}\big((G_{35}),t\big)-(b''_{33})^{(6)}\big((G_{35}),t\big)\leq -(\tau_1)^{(6)}$$

Definition of $(v_1)^{(6)},(v_2)^{(6)},(u_1)^{(6)},(u_2)^{(6)},v^{(6)},u^{(6)}$: 350

By $(v_1)^{(6)}>0$, $(v_2)^{(6)}<0$ and respectively $(u_1)^{(6)}>0$, $(u_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_1)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_1)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$ and

Definition of $(\bar{v}_1)^{(6)},,(\bar{v}_2)^{(6)},(\bar{u}_1)^{(6)},(\bar{u}_2)^{(6)}$: 351

By $(\bar{v}_1)^{(6)}>0$, $(\bar{v}_2)^{(6)}<0$ and respectively $(\bar{u}_1)^{(6)}>0$, $(\bar{u}_2)^{(6)}<0$ the roots of the equations $(a_{33})^{(6)}\big(v^{(6)}\big)^2+(\sigma_2)^{(6)}v^{(6)}-(a_{32})^{(6)}=0$
and $(b_{33})^{(6)}\big(u^{(6)}\big)^2+(\tau_2)^{(6)}u^{(6)}-(b_{32})^{(6)}=0$

Definition of $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)},(v_0)^{(6)}$:-

If we define $(m_1)^{(6)},(m_2)^{(6)},(\mu_1)^{(6)},(\mu_2)^{(6)}$ by

$(m_2)^{(6)}=(v_0)^{(6)},(m_1)^{(6)}=(v_1)^{(6)},$ ***if*** $(v_0)^{(6)}<(v_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}$ $(\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}$ $(\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}$ $(u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}$ $(u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}$ $(\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)} t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)} t} \quad 354$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)} t} \right] + G_{34}^0 e^{-(S_2)^{(6)} t} \leq G_{34}(t) \leq \right. \quad 355$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})} \left[e^{(S_1)^{(6)} t} - e^{-(a'_{34})^{(6)} t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)} t} \right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 356$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)} t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})} \left[e^{(R_1)^{(6)} t} - e^{-(b'_{34})^{(6)} t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)} t} \leq T_{34}(t) \leq \quad 358$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)} t} \right] + T_{34}^0 e^{-(R_2)^{(6)} t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 361

By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)},, (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$$

and $\boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}}$

$$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$$

and analogously 363

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

and $\boxed{(u_0)^{(7)} = \frac{T^0_{36}}{T^0_{37}}}$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^{0}e^{(S_1)^{(7)}t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}}G_{36}^{0}e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}}G_{36}^{0}e^{(S_1)^{(7)}t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t}-e^{-(S_2)^{(7)}t}\right]+G_{38}^{0}e^{-(S_2)^{(7)}t} \le G_{38}(t) \le \right. \quad 366$$

$$\left.\frac{(a_{38})^{(7)}G_{36}^{0}}{(m_2)^{(7)}((S_1)^{(7)}-(a'_{38})^{(7)})}\left[e^{(S_1)^{(7)}t}-e^{-(a'_{38})^{(7)}t}\right]+\ G_{38}^{0}e^{-(a'_{38})^{(7)}t}\right)$$

$$\boxed{T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}}T_{36}^{0}e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}}T_{36}^{0}e^{((R_1)^{(7)}+(r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)}T_{36}^{0}}{(\mu_1)^{(7)}((R_1)^{(7)}-(b'_{38})^{(7)})}\left[e^{(R_1)^{(7)}t}-e^{-(b'_{38})^{(7)}t}\right]+T_{38}^{0}e^{-(b'_{38})^{(7)}t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)}T_{36}^{0}}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t}-e^{-(R_2)^{(7)}t}\right]+T_{38}^{0}e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right) - (b''_{41})^{(8)}\left((G_{43}), t\right) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(v_1)^{(8)} > 0$, $(v_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0$, $(u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_1)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{v}_1)^{(8)}$,, $(\bar{v}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{v}_1)^{(8)} > 0$, $(\bar{v}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0$, $(\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\left(v^{(8)}\right)^2 + (\sigma_2)^{(8)}v^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\left(u^{(8)}\right)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}, (v_0)^{(8)}$:-

If we define $(m_1)^{(8)}$, $(m_2)^{(8)}$, $(\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$(m_2)^{(8)} = (v_0)^{(8)}, (m_1)^{(8)} = (v_1)^{(8)}$, $\boldsymbol{if}$ $(v_0)^{(8)} < (v_1)^{(8)}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (\bar{v}_1)^{(8)}$, $\boldsymbol{if}$ $(v_1)^{(8)} < (v_0)^{(8)} < (\bar{v}_1)^{(8)}$,
and $\boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$

$(m_2)^{(8)} = (v_1)^{(8)}, (m_1)^{(8)} = (v_0)^{(8)}$, $\boldsymbol{if}$ $(\bar{v}_1)^{(8)} < (v_0)^{(8)}$

and analogously 374

$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}$, $\boldsymbol{if}$ $(u_0)^{(8)} < (u_1)^{(8)}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}$, $\boldsymbol{if}$ $(u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)}$,
and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}$ $(\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)}t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)}t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)}t}\right] + G_{42}^0 e^{-(S_2)^{(8)}t} \le G_{42}(t) \le \right. \quad 377$$

$\frac{(a_{42})^{(8)}G_{40}^{0}}{(m_2)^{(8)}((S_1)^{(8)}-(a'_{42})^{(8)})}[e^{(S_1)^{(8)}t}-e^{-(a'_{42})^{(8)}t}]+\ G_{42}^{0}e^{-(a'_{42})^{(8)}t})$

$$\boxed{T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}}T_{40}^{0}e^{(R_1)^{(8)}t}\leq T_{40}(t)\leq \frac{1}{(\mu_2)^{(8)}}T_{40}^{0}e^{((R_1)^{(8)}+(r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)}T_{40}^{0}}{(\mu_1)^{(8)}((R_1)^{(8)}-(b'_{42})^{(8)})}\left[e^{(R_1)^{(8)}t}-e^{-(b'_{42})^{(8)}t}\right]+T_{42}^{0}e^{-(b'_{42})^{(8)}t}\leq T_{42}(t)\leq \quad 380$$

$$\frac{(a_{42})^{(8)}T_{40}^{0}}{(\mu_2)^{(8)}((R_1)^{(8)}+(r_{40})^{(8)}+(R_2)^{(8)})}\left[e^{((R_1)^{(8)}+(r_{40})^{(8)})t}-e^{-(R_2)^{(8)}t}\right]+T_{42}^{0}e^{-(R_2)^{(8)}t}$$

Definition of $(S_1)^{(8)},(S_2)^{(8)},(R_1)^{(8)},(R_2)^{(8)}$:- 381

Where $(S_1)^{(8)}=(a_{40})^{(8)}(m_2)^{(8)}-(a'_{40})^{(8)}$

$(S_2)^{(8)}=(a_{42})^{(8)}-(p_{42})^{(8)}$

$(R_1)^{(8)}=(b_{40})^{(8)}(\mu_2)^{(8)}-(b'_{40})^{(8)}$

$(R_2)^{(8)}=(b'_{42})^{(8)}-(r_{42})^{(8)}$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$:

$(\sigma_1)^{(9)},(\sigma_2)^{(9)},(\tau_1)^{(9)},(\tau_2)^{(9)}$ four constants satisfying

$-(\sigma_2)^{(9)}\leq -(a'_{44})^{(9)}+(a'_{45})^{(9)}-(a''_{44})^{(9)}(T_{45},t)+(a''_{45})^{(9)}(T_{45},t)\leq -(\sigma_1)^{(9)}$

$-(\tau_2)^{(9)}\leq -(b'_{44})^{(9)}+(b'_{45})^{(9)}-(b''_{44})^{(9)}\big((G_{47}),t\big)-(b''_{45})^{(9)}\big((G_{47}),t\big)\leq -(\tau_1)^{(9)}$

Definition of $(v_1)^{(9)},(v_2)^{(9)},(u_1)^{(9)},(u_2)^{(9)},v^{(9)},u^{(9)}$:

By $(v_1)^{(9)}>0$, $(v_2)^{(9)}<0$ and respectively $(u_1)^{(9)}>0$, $(u_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_1)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$ and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_1)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$ and

Definition of $(\bar{v}_1)^{(9)},,(\bar{v}_2)^{(9)},(\bar{u}_1)^{(9)},(\bar{u}_2)^{(9)}$:

By $(\bar{v}_1)^{(9)}>0$, $(\bar{v}_2)^{(9)}<0$ and respectively $(\bar{u}_1)^{(9)}>0$, $(\bar{u}_2)^{(9)}<0$ the roots of the equations $(a_{45})^{(9)}\big(v^{(9)}\big)^2+(\sigma_2)^{(9)}v^{(9)}-(a_{44})^{(9)}=0$ and $(b_{45})^{(9)}\big(u^{(9)}\big)^2+(\tau_2)^{(9)}u^{(9)}-(b_{44})^{(9)}=0$

Definition of $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)},(v_0)^{(9)}$:-

If we define $(m_1)^{(9)},(m_2)^{(9)},(\mu_1)^{(9)},(\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)},\ \boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}, \boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)},$
and $\boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)},\ \boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}, \boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)},$
and $\boxed{(u_0)^{(9)} = \frac{T_{44}^0}{T_{45}^0}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G_{44}^0 e^{(S_1)^{(9)} t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}} G_{44}^0 e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}} G_{44}^0 e^{(S_1)^{(9)} t}$$

$$\left(\frac{(a_{46})^{(9)} G_{44}^0}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})} \left[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)} t} \right] + G_{46}^0 e^{-(S_2)^{(9)} t} \le G_{46}(t) \le \right.$$
$$\left. \frac{(a_{46})^{(9)} G_{44}^0}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})} [e^{(S_1)^{(9)} t} - e^{-(a'_{46})^{(9)} t}] + G_{46}^0 e^{-(a'_{46})^{(9)} t} \right)$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)} t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)} + (r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)} T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)} - (b'_{46})^{(9)})} \left[e^{(R_1)^{(9)} t} - e^{-(b'_{46})^{(9)} t} \right] + T_{46}^0 e^{-(b'_{46})^{(9)} t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)} T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)} + (r_{44})^{(9)} + (R_2)^{(9)})} \left[e^{((R_1)^{(9)} + (r_{44})^{(9)})t} - e^{-(R_2)^{(9)} t} \right] + T_{46}^0 e^{-(R_2)^{(9)} t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right) - (a''_{14})^{(1)}(T_{14},t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}}$$

From which we deduce $(v_0)^{(1)} \le v^{(1)}(t) \le (\bar{v}_1)^{(1)}$

If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}} < (\bar{v}_1)^{(1)}$ we find like in the previous case, 385

$$(v_1)^{(1)} \le \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}}{1 + (C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)} - (v_2)^{(1)}\right)t\right]}} \le v^{(1)}(t) \le$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (\bar{v}_1)^{(1)}$$

If $0 < (v_1)^{(1)} \le (\bar{v}_1)^{(1)} \le \boxed{(v_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$, we obtain 386

$$(v_1)^{(1)} \le v^{(1)}(t) \le \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}\right)t\right]}} \le (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $v^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)} G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)} T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\right) - (a''_{17})^{(2)}(\mathrm{T}_{17}, \mathrm{t})\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{\mathrm{G}_{16}}{\mathrm{G}_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{\mathrm{d}\nu^{(2)}}{\mathrm{dt}} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

For $0 < (\nu_0)^{(2)} = \frac{\mathrm{G}^0_{16}}{\mathrm{G}^0_{17}} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (\mathrm{C})^{(2)}(\nu_2)^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}}{1 + (\mathrm{C})^{(2)} e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)} - (\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\mathrm{C})^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get 391

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$ 392

If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case, 393

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:- 395

$$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a_{16}'')^{(2)} = (a_{17}'')^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)}G_{17}(t)$

Analogously if $(b_{16}'')^{(2)} = (b_{17}'')^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)}\right) \le \frac{dv^{(3)}}{dt} \le -\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \ge \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}}$$

it follows $(v_0)^{(3)} \le v^{(3)}(t) \le (v_1)^{(3)}$

In the same manner , we get 401

$$v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \le v^{(3)}(t) \le (\bar{v}_1)^{(3)}$

If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\bar{v}_1)^{(3)}$ we find like in the previous case, 402

$$(v_1)^{(3)} \le \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)} - (v_2)^{(3)}\right)t\right]}} \le v^{(3)}(t) \le$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (\bar{v}_1)^{(3)}$$

If $0 < (v_1)^{(3)} \le (\bar{v}_1)^{(3)} \le (v_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}$, we obtain 403

$$(v_1)^{(3)} \le v^{(3)}(t) \le \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}\right)t\right]}} \le (v_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G^0_{24}}{G^0_{25}}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}'')^{(4)} = (a_{25}'')^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}'')^{(4)} = (b_{25}'')^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

$$\text{For } 0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 411

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 413

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\bar{\nu}_1)^{(6)}$

If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 414

$$(\nu_1)^{(6)} \le \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \le \nu^{(6)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\bar{\nu}_1)^{(6)}$$

If $0 < (\nu_1)^{(6)} \le (\bar{\nu}_1)^{(6)} \le \boxed{(\nu_0)^{(6)} = \frac{G^0_{32}}{G^0_{33}}}$, we obtain 415

$$(\nu_1)^{(6)} \le \nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \le (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right) - (a''_{37})^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

For $0 < \boxed{(\nu_0)^{(7)} = \frac{G^0_{36}}{G^0_{37}}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get

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$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{36}'')^{(7)} = (a_{37}'')^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b_{36}'')^{(7)} = (b_{37}'')^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a_{40}')^{(8)} - (a_{41}')^{(8)} + (a_{40}'')^{(8)}(T_{41}, t)\right) - (a_{41}'')^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

If $0 < (\nu_1)^{(8)} < (\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{\nu}_1)^{(8)}$ we find like in the previous case, 423

$$(\nu_1)^{(8)} \leq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}}{1 + (C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_2)^{(8)}\right)t\right]}} \leq \nu^{(8)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\bar{\nu}_1)^{(8)}$$

If $0 < (\nu_1)^{(8)} \leq (\bar{\nu}_1)^{(8)} \leq \boxed{(\nu_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain 424

$$(\nu_1)^{(8)} \leq \nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \leq (\nu_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(8)}(t)$:-

$$(m_2)^{(8)} \leq \nu^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{\nu^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}$, $\boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(\nu_1)^{(8)} = (\bar{\nu}_1)^{(8)}$ if in addition $(\nu_0)^{(8)} = (\nu_1)^{(8)}$ then $\nu^{(8)}(t) = (\nu_0)^{(8)}$ and as a consequence $G_{40}(t) = (\nu_0)^{(8)} G_{41}(t)$ **this also defines** $(\nu_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)} T_{41}(t)$ This is an important consequence of the relation between $(\nu_1)^{(8)}$ and $(\bar{\nu}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{d\nu^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)\nu^{(9)} - (a_{45})^{(9)}\nu^{(9)}$$

Definition of $\nu^{(9)}$:- $\boxed{\nu^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right) \leq \frac{d\nu^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(\nu^{(9)}\right)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \le \frac{(\nu_1)^{(9)}+(C)^{(9)}(\nu_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}}{1+(C)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)}-(\nu_2)^{(9)}\right)t\right]}} \le \nu^{(9)}(t) \le$$

$$\frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \le (\bar{\nu}_1)^{(9)} \le \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \le \nu^{(9)}(t) \le \frac{(\bar{\nu}_1)^{(9)}+(\bar{C})^{(9)}(\bar{\nu}_2)^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}}{1+(\bar{C})^{(9)}e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)}-(\bar{\nu}_2)^{(9)}\right)t\right]}} \le (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \le \nu^{(9)}(t) \le (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \le u^{(9)}(t) \le (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)}G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for the special case** .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then $(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(\nu_1)^{(9)}$ and $(\bar{\nu}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions with the notations

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0\,,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions with the notations 426

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(8)} and\ (b''_i)^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a''_i)^{(9)} and\ (b''_i)^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad 439$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\,]T_{13} = 0 \quad 440$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\,]T_{14} = 0 \quad 441$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\,]T_{15} = 0 \quad 442$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0 \quad 443$$

$$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0 \quad 444$$

$$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0 \quad 445$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\,]T_{16} = 0 \quad 446$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\,]T_{17} = 0 \quad 447$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\,]T_{18} = 0 \quad 448$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0 \quad 449$$

$$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0 \quad 450$$

$$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0 \quad 451$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\,]T_{20} = 0 \quad 452$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0$ 479

$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0$ 480

$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0$ 481

$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0$ 482

$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0$ 483

$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0$ 484

$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0$ 484 A

$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$

$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$

$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$

$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$

$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(ii) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$\mathrm{F}(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first

equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]} \qquad 495$$

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T^*_{33} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T^*_{33} for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T^*_{33})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T^*_{33})]}$$

Definition and uniqueness of T^*_{37} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T^*_{37} for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T^*_{37})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T^*_{37})]}$$

Definition and uniqueness of T^*_{41} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^* for which $f(T_{41}^*) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^*)]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^*)]}$$

Definition and uniqueness of T_{45}^* :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^* for which $f(T_{45}^*) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^*)]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^*)]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{25} such that $\varphi((G_{27})^*) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507
$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{29} such that $\varphi((G_{31})^*) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508
$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{33} such that $\varphi({G_{35}}^*) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509
$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$
$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{37} such that $\varphi({G_{39}}^*) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510
$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right]+(b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{41} such that $\varphi({G_{43}}^*) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right]+(b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G^*_{45} such that $\varphi((G_{47})^*) = 0$

Finally we obtain the unique solution 511

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)} G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)} G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)} T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)} T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 512

$$G_{16}^* = \frac{(a_{16})^{(2)} G_{17}^*}{[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)} G_{17}^*}{[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}^*)]} \quad 513$$

$$T_{16}^* = \frac{(b_{16})^{(2)} T_{17}^*}{[(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)} T_{17}^*}{[(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)} G_{21}^*}{[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)} G_{21}^*}{[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)} T_{21}^*}{[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}{}^*)]} \quad , \quad T_{22}^* = \frac{(b_{22})^{(3)} T_{21}^*}{[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]} \quad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$T_{28}^{*} = \frac{(b_{28})^{(5)}T_{29}^{*}}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^{*})]}$, $T_{30}^{*} = \frac{(b_{30})^{(5)}T_{29}^{*}}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^{*})]}$ 519

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]}$, $G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$

$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]}$, $T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$ 521

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^{*} given by $\varphi((G_{39})^{*}) = 0$, T_{37}^{*} given by $f(T_{37}^{*}) = 0$ and

$G_{36}^{*} = \frac{(a_{36})^{(7)}G_{37}^{*}}{[(a'_{36})^{(7)}+(a''_{36})^{(7)}(T_{37}^{*})]}$, $G_{38}^{*} = \frac{(a_{38})^{(7)}G_{37}^{*}}{[(a'_{38})^{(7)}+(a''_{38})^{(7)}(T_{37}^{*})]}$

$T_{36}^{*} = \frac{(b_{36})^{(7)}T_{37}^{*}}{[(b'_{36})^{(7)}-(b''_{36})^{(7)}((G_{39})^{*})]}$, $T_{38}^{*} = \frac{(b_{38})^{(7)}T_{37}^{*}}{[(b'_{38})^{(7)}-(b''_{38})^{(7)}((G_{39})^{*})]}$

Finally we obtain the unique solution 523

G_{41}^{*} given by $\varphi((G_{43})^{*}) = 0$, T_{41}^{*} given by $f(T_{41}^{*}) = 0$ and

$G_{40}^{*} = \frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}$, $G_{42}^{*} = \frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$

$T_{40}^{*} = \frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}$, $T_{42}^{*} = \frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$

Finally we obtain the unique solution of 89 to 99 523 A

G_{45}^{*} given by $\varphi((G_{47})^{*}) = 0$, T_{45}^{*} given by $f(T_{45}^{*}) = 0$ and

$G_{44}^{*} = \frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}$, $G_{46}^{*} = \frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$

$T_{44}^{*} = \frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}$, $T_{46}^{*} = \frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \qquad 525$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \qquad 526$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \qquad 527$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \qquad 528$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \qquad 529$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \qquad 530$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i \qquad 532$$

$$\frac{\partial (a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial (b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \qquad 533$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \qquad 534$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \qquad 535$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \qquad 536$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \qquad 537$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b'_{17})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}\mathrm{T}^*_{17}\mathbb{G}_j\right) \quad 538$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b'_{18})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}\mathrm{T}^*_{18}\mathbb{G}_j\right) \quad 539$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{21})^{(3)}}{\partial T_{21}}(T^*_{21}) = (q_{21})^{(3)} \quad , \frac{\partial(b_i'')^{(3)}}{\partial G_j}\left((G_{23})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a'_{20})^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G^*_{20}\mathbb{T}_{21} \quad 541$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a'_{21})^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G^*_{21}\mathbb{T}_{21} \quad 542$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a'_{22})^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G^*_{22}\mathbb{T}_{21} \quad 543$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b'_{20})^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T^*_{20}\mathbb{G}_j\right) \quad 544$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T^*_{21}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T^*_{22}\mathbb{G}_j\right) \quad 546$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{25})^{(4)}}{\partial T_{25}}(T^*_{25}) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}\left((G_{27})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G^*_{24}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G^*_{25}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G^*_{26}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T^*_{24}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T^*_{25}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T^*_{26}\mathbb{G}_j\right) \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{29})^{(5)}}{\partial T_{29}}(T^*_{29}) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G^*_{28}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

ASYMPTOTIC STABILITY ANALYSIS 563

Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^{*}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^{*}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^{*}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^{*}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^{*}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^{*}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^{*}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^{*}\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**}\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^*) = (q_{41})^{(8)} \quad , \frac{\partial (b_i'')^{(8)}}{\partial G_j}((G_{43})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^*\mathbb{T}_{41} \quad (581)$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^*\mathbb{T}_{41} \quad (582)$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^*\mathbb{T}_{41} \quad (583)$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^*\mathbb{G}_j\right) \quad (584)$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b_{41}')^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T_{41}^*\mathbb{G}_j\right) \quad (585)$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b_{42}')^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T_{42}^*\mathbb{G}_j\right) \quad (586)$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}$ *and* $(b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{45}'')^{(9)}}{\partial T_{45}}(T_{45}^*) = (q_{45})^{(9)} \quad , \frac{\partial (b_i'')^{(9)}}{\partial G_j}((G_{47})^*) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a_{44}')^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G_{44}^*\mathbb{T}_{45} \quad (586\ B)$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad (586C)$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad (586D)$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad (586E)$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad (586F)$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad (586G)$$

The characteristic equation of this system is (587)

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14} + (b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16} + (a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17} + (b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\Big)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\Big)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\Big]$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\Big)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\Big)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\Big)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\Big]$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\Big)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\Big)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\Big)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\Big]$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\Big)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\Big)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\Big)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\Big)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\Big)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\Big)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left(\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\right]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\Big)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\Big)\}=0$$

$$+$$

$$(\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\}\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\Big)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\Big)\}=0$$

$$+$$

$$(\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\}\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\Big)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\Big)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

SECTION FIFTY ONE

Photonic Coupling

INTRODUCTION—VARIABLES USED

Direct Photonic Coupling of a Semiconductor Quantum Dot and a Trapped Ion H. M. Meyer, R. Stockill, M. Steiner, C. Le Gall, C. Matthiesen, E. Clarke, A. Ludwig, J. Reichel, M. Atatüre, and M. Köhl Phys. Rev. Lett. 114, 123001 – Published 23 March 2015 Physics logo See Viewpoint: Single Dot Meets Single Ion

(1) Coupling individual quantum systems lies at (eb) the heart of building scalable quantum networks.
(2) Here authors report the first direct photonic coupling between a semiconductor quantum dot and

(e&eb) a trapped ion and demonstrate that single photons generated by (e) a quantum dot controllably change (e&eb) the internal state of a Yb+ ion.

(3) They ameliorate the effect of the 60-fold mismatch of the radiative linewidths with (e&eb) coherent photon generation and (e&eb) a high-finesse fiber-based optical cavity enhancing (eb) the coupling between the single photon and the ion.

(4) The transfer of information presented here via the classical correlations between the σz projection of the quantum-dot spin and (e&eb) the internal state of the ion provides (eb) a promising step towards quantum-state transfer in a hybrid photonic network. DOI**: http://dx.doi.org/10.1103/PhysRevLett.114.123001**

Viewpoint: Single Dot Meets Single Ion Nika Akopian, Department of Photonics Engineering, Technical University of Denmark, Denmark Published March 23, 2015 | Physics 8, 27 (2015) | DOI: 10.1103/Physics.8.27 Researchers show that a single photon can transfer an excitation from a quantum dot to an ion. APS/Alan Stonebraker

Figure 1 Researchers have shown that a single photon emitted by a quantum dot can excite an ion 25 meters away. (Left) A laser excites an electron in the quantum dot. When the electron relaxes, it emits a photon, which travels through a fiber (not shown) to the ion (right). The ion is prepared in a state that allows it to absorb the photon. The efficiency of this process is only 0.0005%, but various strategies exist to improve it.

Researchers have made many advances in controlling atomic, solid-state, and other types of quantum systems. Hybrid quantum systems bring together two or more such systems and allow them to interact. Hybrid systems combine the advantages of each component and as such, are of interest for a number of proposed technologies, including a hybrid quantum network. One hybrid system of particular interest is a single artificial atom (e.g., a quantum dot) coupled to a natural atom (e.g., a trapped ion). Combining these two components is, however, a challenge: specialized techniques are needed to control single artificial atoms and single natural atoms, while experimentalists have so far mastered only one system or the other. Research groups at the University of Cambridge, UK, led by Mete Atatüre, an expert on quantum dots, and Michael Köhl, an expert on trapped ions, now overcome this challenge to demonstrate a photonic link between a single ion and a single quantum dot—a nanoscale semiconductor that behaves like an atom [1].

Artificial and natural atoms are conceptually the same physical systems. Both have confined electrons that can only occupy well-defined discrete energy levels. And in both systems, an external field (such as the optical field from a laser) can be used to "move" the electrons between the levels to generate single and entangled photons, prepare an electron spin in a quantum superposition, and implement quantum memories.

For particular applications, however, each system has its own unique advantages and limitations. A key difference is that a natural atom is an isolated system while an artificial atom is not. A single trapped atom typically sits in a vacuum, while a single artificial atom is usually embedded in a host material. As a result, natural atoms are "cleaner" systems, and their electrons can be prepared in quantum states that have long coherence times. Artificial atoms are, in contrast, easier to manipulate. For instance, contacts can be fabricated around the host material and used to control the electrons in the artificial atom electrically. But electron quantum states in artificial atoms tend to have shorter coherence times than those associated with ions because of their interaction with the host material.

The first hybrid semiconductor-atomic system [2, 3] was an interface between a single GaAs quantum dot and a vapor of rubidium atoms. In one experiment, the atomic system was used as a slow light medium [2] for single photons generated by an artificial atom. In another experiment, the frequency of single photons emitted by the artificial atoms was locked to the frequency of an atomic optical transition [3]. Ultimately, the nodes in a hybrid quantum network have to be able to exchange quantum information. Research so far

suggests single artificial and natural atoms could, if placed in the nodes of a network, perform this function. The next significant milestone is therefore a "single-to-single" interface that couples a single artificial atom to a single natural atom.

This is exactly what the researchers from the University of Cambridge have demonstrated. The natural atom in their experiment is a single Yb+ ion trapped between two needle-shaped electrodes. The artificial atom is a single InAs quantum dot. The quantum dot is surrounded by a host material (GaAs), which is electrically contacted. An electric field applied through the contacts serves two purposes: it controls the number of electrons present in the dot, and it can be used to tune an optical transition of the quantum dot so it is in resonance with a transition of the Yb+ ion. When the frequencies of the optical transitions are matched, the quantum dot and the ion can exchange single photons.

With a laser, Atatüre, Köhl, and their colleagues excite an electron in the quantum dot, which emits a single photon when the electron relaxes (Fig. 1). A periodic excitation of the quantum dot generates a stream of single photons that are channeled into an optical fiber and sent 25 meters away to the trapped ion. An electron in the ion is prepared in a state such that it can absorb a single photon from the quantum dot. This absorption leads to an electron excitation that the researchers are able to detect.

The experiment demonstrates that an excitation in a single quantum dot can be transferred to an excitation in a single ion by means of a single photon. A great advantage of this type of "single-to-single" interface is that it provides the starting infrastructure for exchanging quantum information between a single quantum dot and a single ion. This, however, is yet to be demonstrated. Atatüre, Köhl, and their colleagues have made a preliminary step forward towards this goal by showing that the spin of an electron in the quantum dot is correlated with the excitation transfer. In this experiment, the quantum dot is first prepared to have one excess electron, and the spin of this electron is controlled to be either up or down. When an excitation laser is turned on, it can only excite the dot if the electron has its spin up. This excitation is then transferred to an ion and detected, exactly as in the first experiment. If the electron's spin is down, however, the laser can't excite the dot, so no photon is transferred to the ion. In other words, if the researchers detect the ion in an excited state, they know the spin state of the electron in the quantum dot.

The photonic link between a quantum dot and an ion, developed in this work, can, in principle, be extended to hybrid quantum networks [4], where the strength of each system can be combined to gain new functionalities. For instance, solid-state systems can be used as scalable and fast processing units, while atomic systems can serve as long-lived quantum memories. But physicists are still in the early stages of developing a hybrid network technology. The first challenge to overcome is the low transfer efficiency between the dot and the ion. In the experiments of Atatüre, Köhl, and their colleagues, only 5 out of 1 million photons from the quantum dot excite the ion. Photon loss occurs at every step in the transfer, but various techniques exist to reduce it. Another challenge is demonstrating that a quantum state can be transferred between the quantum dot and the ion, and that the two objects can be entangled [5]. We should also be able to send information in the reverse direction, from an ion to a quantum dot. This research is published in Physical Review Letters.

NOTATION

Module One

Coupling individual quantum systems lies at (eb) the heart of building scalable quantum networks

G_{13} : Category one of Coupling individual quantum systems

G_{14} : Category two of SAS(same as superior/above)

G_{15} : Category three of SAS

T_{13} : Category one of heart of building scalable quantum networks

T_{14} : Category two of SAS

T_{15} : Category three of SAS

Module Two

Here authors report the first direct photonic coupling between a

semiconductor quantum dot and (e&eb) a trapped ion and demonstrate that single photons generated by (e) a quantum dot controllably change (e&eb) the internal state of a Yb+ ion

G_{16} : Category one of semiconductor quantum dot; trapped ion and demonstrate that single photons generated by (e) a quantum dot controllably change (e&eb) the internal state of a Yb+ ion

G_{17} : Category two of SAS

G_{18} : Category three of SAS

T_{16} : Category one of trapped ion and demonstrate that single photons generated by (e) a quantum dot controllably change (e&eb) the internal state of a Yb+ ion; semiconductor quantum dot

T_{17} : Category two of SAS

T_{18} : Category three of SAS

Module three

semiconductor quantum dot and a trapped ion and demonstrate that single photons generated by (e) a quantum dot controllably change (e&eb) the internal state of a Yb+ ion

G_{20} : Category one of quantum dot controllably change (e&eb) the internal state of a Yb+ ion

G_{21} : Category two of SAS

G_{22} : Category three of SAS

T_{20} : Category one of semiconductor quantum dot and a trapped ion and demonstrate that single photons

T_{21} : Category two of SAS

T_{22} : Category three of SAS

Module four

semiconductor quantum dot and a trapped ion and demonstrate that single photons generated by a quantum dot controllably change (e&eb) the internal state of a Yb+ ion

G_{24} : Category one of semiconductor quantum dot and a trapped ion and demonstrate that single photons generated by a quantum dot; internal state of a Yb+ ion

G_{25} : Category two of SAS

G_{26} : Category three of SAS

T_{24} : Category one of internal state of a Yb+ ion; semiconductor quantum dot and a trapped ion and demonstrate that single photons generated by a quantum dot

T_{25} : Category two of SAS

T_{26} : Category three of SAS

Module five

Amelioration of the effect of the 60-fold mismatch of the radiative linewidths with (e&eb) coherent photon generation and (e&eb) a high-finesse fiber-based optical cavity enhancing (eb) the coupling between the single photon and the ion

G_{28} : Category one of Amelioration of the effect of the 60-fold mismatch of the radiative linewidths; coherent photon generation and (e&eb) a high-finesse fiber-based optical cavity enhancing (eb) the coupling between the single photon and the ion

G_{29} : Category two of SAS

G_{30} : Category three of SAS

T_{28} : Category one of coherent photon generation and (e&eb) a high-finesse fiber-based optical cavity enhancing (eb) the coupling between the single photon and the ion ;Amelioration of the effect of the 60-fold mismatch of the radiative linewidths

T_{29} : Category two of SAS

T_{30} : Category three of SAS

Module six

Amelioration of the effect of the 60-fold mismatch of the radiative linewidths with coherent photon generation and (e&eb) a high-finesse fiber-based optical cavity enhancing (eb) the coupling between the single photon and the ion

G_{32} : Category one of Amelioration of the effect of the 60-fold mismatch of the radiative linewidths with coherent photon generation; high-finesse fiber-based optical cavity enhancing (eb) the coupling between the single photon and the ion

G_{33} : Category two of SAS

G_{34} : Category three of SAS

T_{32} : Category one of high-finesse fiber-based optical cavity enhancing (eb) the coupling between the single photon and the ion; Amelioration of the effect of the 60-fold mismatch of the radiative linewidths with coherent photon generation

T_{33} : Category two of SAS

T_{34} : Category three of SAS

Module seven

Amelioration of the effect of the 60-fold mismatch of the radiative linewidths with coherent photon

generation and a high-finesse fiber-based optical cavity enhancing (eb) the coupling between the single photon and the ion

G_{36} : Category one of Amelioration of the effect of the 60-fold mismatch of the radiative linewidths with coherent photon generation and a high-finesse fiber-based optical cavity; coupling between the single photon and the ion

G_{37} : Category two of SAS

G_{38} : Category three of SAS

T_{36} : Category one of coupling between the single photon and the ion; Amelioration of the effect of the 60-fold mismatch of the radiative linewidths with coherent photon generation and a high-finesse fiber-based optical cavity

T_{37} : Category two of SAS

T_{38} : Category three of SAS

Module eight

transfer of information presented here via the classical correlations between the σz projection of the quantum-dot spin and (e&eb) the internal state of the ion provides (eb) a promising step towards quantum-state transfer in a hybrid photonic network.

DOI**: http://dx.doi.org/10.1103/PhysRevLett.114.123001**

G_{40} : Category one of transfer of information presented here via the classical correlations between the σz projection of the quantum-dot spin; internal state of the ion provides (eb) a promising step towards quantum-state transfer in a hybrid photonic network.

G_{41} : Category two of SAS

G_{42} : Category three of SAS

T_{40} : Category one of internal state of the ion provides (eb) a promising step towards quantum-state transfer in a hybrid photonic network; transfer of information presented here via the classical correlations between the σz projection of the quantum-dot spin

T_{41} : Category two of SAS

T_{42} : Category three of SAS

Module Nine

transfer of information presented here via the classical correlations between the σz projection of the quantum-dot spin and the internal state of the ion provides (eb) a promising step towards quantum-state transfer in a hybrid photonic network.

G_{44} : Category one of transfer of information presented here via the classical correlations between the σz projection of the quantum-dot spin and the internal state of the ion

G_{45} : Category two of SAS

G_{46} : Category three of SAS

T_{44} : Category one of promising step towards quantum-state transfer in a hybrid photonic network.

T_{45} : Category two of SAS

T_{46} : Category three of SAS

The Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}\ (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}\ (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}, (b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$
$(a_{40})^{(8)}, (a_{41})^{(8)}, (a_{42})^{(8)}, (b_{40})^{(8)}, (b_{41})^{(8)}, (b_{42})^{(8)}$
$(a_{44})^{(9)}, (a_{45})^{(9)}, (a_{46})^{(9)}, (b_{44})^{(9)}, (b_{45})^{(9)}, (b_{46})^{(9)}$

are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
$, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}\ (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}, (b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$,
$(a'_{40})^{(8)}, (a'_{41})^{(8)}, (a'_{42})^{(8)}, (b'_{40})^{(8)}, (b'_{41})^{(8)}, (b'_{42})^{(8)}$,
$(a'_{44})^{(9)}, (a'_{45})^{(9)}, (a'_{46})^{(9)}, (b'_{44})^{(9)}, (b'_{45})^{(9)}, (b'_{46})^{(9)}$,
are Dissipation coefficients

Module Numbered One

The differential system of this model is now (Module Numbered one)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right]G_{13} \quad 1$$
$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)\right]G_{14} \quad 2$$
$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)\right]G_{15} \quad 3$$
$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)\right]T_{13} \quad 4$$
$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)\right]T_{14} \quad 5$$
$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)\right]T_{15} \quad 6$$

$+(a''_{13})^{(1)}(T_{14}, t) =$ First augmentation factor
$-(b''_{13})^{(1)}(G, t) =$ First detritions factor

Module Numbered Two

The differential system of this model is now (Module numbered two)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right]G_{16} \quad 7$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)\right]G_{17} \quad 8$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)\right]G_{18} \quad 9$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right)\right]T_{16} \quad 10$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}), t\right)\right]T_{17} \quad 11$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}), t\right)\right]T_{18} \quad 12$$

$+(a''_{16})^{(2)}(T_{17}, t) =$ First augmentation factor

$-(b''_{16})^{(2)}\left((G_{19}), t\right) =$ First detritions factor

Module Numbered Three

The differential system of this model is now (Module numbered three)

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right]G_{20} \quad 13$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)\right]G_{21} \quad 14$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)\right]G_{22} \quad 15$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)\right]T_{20} \quad 16$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)\right]T_{21} \quad 17$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)\right]T_{22} \quad 18$$

$+(a''_{20})^{(3)}(T_{21}, t) =$ First augmentation factor

$-(b''_{20})^{(3)}(G_{23}, t) =$ First detritions factor

Module Numbered Four

The differential system of this model is now (Module numbered Four)

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right]G_{24} \quad 19$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)\right]G_{25} \quad 20$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)\right]G_{26} \quad 21$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}), t\right)\right]T_{24} \quad 22$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27}), t\right)\right]T_{25} \quad 23$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}), t\right)\right]T_{26} \quad 24$$

$+(a''_{24})^{(4)}(T_{25}, t) =$ **First augmentation factor**

$-(b''_{24})^{(4)}\left((G_{27}), t\right) =$ **First detritions factor**

Module Numbered Five:

The differential system of this model is now (Module number five)

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right]G_{28} \quad 25$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)\right]G_{29} \quad 26$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)\right]G_{30} \quad 27$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right)\right]T_{28} \quad 28$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right)\right]T_{29} \quad 29$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right)\right]T_{30} \quad 30$$

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor**

$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor**

Module Numbered Six

The differential system of this model is now (Module numbered Six)

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right]G_{32} \quad 31$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)\right]G_{33} \quad 32$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)\right]G_{34} \quad 33$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right)\right]T_{32} \quad 34$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right)\right]T_{33} \quad 35$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right)\right]T_{34} \quad 36$$

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**

Module Numbered Seven:

The differential system of this model is now (Seventh Module)

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)\right]G_{36} \quad 37$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)\right]G_{37} \quad 38$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)\right]G_{38} \quad 39$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\left((G_{39}), t\right)\right]T_{36} \quad 40$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\left((G_{39}), t\right)\right]T_{37} \quad 41$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\left((G_{39}), t\right)\right]T_{38} \quad 42$$

$+(a''_{36})^{(7)}(T_{37}, t) =$ **First augmentation factor**

Module Numbered Eight

The differential system of this model is now

$$\frac{dG_{40}}{dt} = (a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right]G_{40} \quad 43$$

$$\frac{dG_{41}}{dt} = (a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41}, t)\right]G_{41} \quad 44$$

$$\frac{dG_{42}}{dt} = (a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}, t)\right]G_{42} \quad 45$$

$$\frac{dT_{40}}{dt} = (b_{40})^{(8)}T_{41} - \left[(b'_{40})^{(8)} - (b''_{40})^{(8)}\left((G_{43}), t\right)\right]T_{40} \quad 46$$

$$\frac{dT_{41}}{dt} = (b_{41})^{(8)}T_{40} - \left[(b'_{41})^{(8)} - (b''_{41})^{(8)}\left((G_{43}), t\right)\right]T_{41} \quad 47$$

$$\frac{dT_{42}}{dt} = (b_{42})^{(8)}T_{41} - \left[(b'_{42})^{(8)} - (b''_{42})^{(8)}\left((G_{43}), t\right)\right]T_{42} \quad 48$$

Module Numbered Nine

The differential system of this model is now

$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right]G_{44} \quad 49$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45}, t)\right]G_{45} \quad 50$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}, t)\right]G_{46} \quad 51$$

$$\frac{dT_{44}}{dt} = (b_{44})^{(9)}T_{45} - \left[(b'_{44})^{(9)} - (b''_{44})^{(9)}\left((G_{47}), t\right)\right]T_{44} \quad 52$$

$$\frac{dT_{45}}{dt} = (b_{45})^{(9)}T_{44} - \left[(b'_{45})^{(9)} - (b''_{45})^{(9)}((G_{47}),t)\right]T_{45} \quad 53$$

$$\frac{dT_{46}}{dt} = (b_{46})^{(9)}T_{45} - \left[(b'_{46})^{(9)} - (b''_{46})^{(9)}((G_{47}),t)\right]T_{46} \quad 54$$

$+(a''_{44})^{(9)}(T_{45},t) =$ **First augmentation factor**

$-(b''_{44})^{(9)}((G_{47}),t) =$ **First detrition factor**

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\begin{array}{ccc} (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14},t) & +(a''_{16})^{(2,2,)}(T_{17},t) \quad +(a''_{20})^{(3,3,)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{36})^{(7,7)}(T_{37},t) & +(a''_{40})^{(8,8)}(T_{41},t) & +(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{array}\right]G_{13} \quad 55$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[\begin{array}{ccc} (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14},t) & +(a''_{17})^{(2,2,)}(T_{17},t) \quad +(a''_{21})^{(3,3,)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{37})^{(7,7)}(T_{37},t) & +(a''_{41})^{(8,8)}(T_{41},t) & +(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{array}\right]G_{14} \quad 56$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\begin{array}{ccc} (a'_{15})^{(1)} & +(a''_{15})^{(1)}(T_{14},t) & +(a''_{18})^{(2,2,)}(T_{17},t) \quad +(a''_{22})^{(3,3,)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,)}(T_{33},t) \\ +(a''_{38})^{(7,7)}(T_{37},t) & +(a''_{42})^{(8,8)}(T_{41},t) & +(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t) \end{array}\right]G_{15} \quad 57$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,)}(T_{17},t)$, $+(a''_{17})^{(2,2,)}(T_{17},t)$, $+(a''_{18})^{(2,2,)}(T_{17},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,)}(T_{21},t)$, $+(a''_{21})^{(3,3,)}(T_{21},t)$, $+(a''_{22})^{(3,3,)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{38})^{(7,7)}(T_{37},t)$ $+(a''_{37})^{(7,7)}(T_{37},t)$ $+(a''_{36})^{(7,7)}(T_{37},t)$ are seventh augmentation coefficient for 1,2,3

$+(a''_{40})^{(8,8)}(T_{41},t)$ $+(a''_{41})^{(8,8)}(T_{41},t)$ $+(a''_{42})^{(8,8)}(T_{41},t)$ are eight augmentation coefficient for 1,2,3

$+(a''_{44})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9,9,9)}(T_{45},t)$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[\begin{array}{ccc} (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G,t) & -(b''_{16})^{(2,2,)}(G_{19},t) \quad -(b''_{20})^{(3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{36})^{(7,7,)}(G_{39},t) & -(b''_{40})^{(8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{array}\right]T_{13} \quad 58$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[\begin{array}{ccc} (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G,t) & -(b''_{17})^{(2,2,)}(G_{19},t) \quad -(b''_{21})^{(3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,)}(G_{35},t) \\ -(b''_{37})^{(7,7,)}(G_{39},t) & -(b''_{41})^{(8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) \end{array}\right]T_{14} \quad 59$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} & -(b''_{15})^{(1)}(G,t) & -(b''_{18})^{(2,2,)}(G_{19},t) & -(b''_{22})^{(3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,)}(G_{35},t) & \\ -(b''_{38})^{(7,7,)}(G_{39},t) & -(b''_{42})^{(8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t) & \end{bmatrix} T_{15} \quad (60)$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,)}(G_{19},t)$, $-(b''_{17})^{(2,2,)}(G_{19},t)$, $-(b''_{18})^{(2,2,)}(G_{19},t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,)}(G_{23},t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,)}(G_{27},t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,)}(G_{31},t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,)}(G_{35},t)$ are sixth detrition coefficients for category 1, 2 and 3

$-(b''_{37})^{(7,7,)}(G_{39},t)$, $-(b''_{36})^{(7,7,)}(G_{39},t)$, $-(b''_{38})^{(7,7,)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2 and 3

$-(b''_{40})^{(8,8)}(G_{43},t)$ $-(b''_{41})^{(8,8)}(G_{43},t)$ $-(b''_{42})^{(8,8)}(G_{43},t)$ are eight detrition coefficients for category 1, 2 and 3

$-(b''_{44})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{46})^{(9,9,9,9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17},t) & +(a''_{13})^{(1,1,)}(T_{14},t) & +(a''_{20})^{(3,3,3)}(T_{21},t) \\ +(a''_{24})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33},t) & \\ +(a''_{36})^{(7,7,7)}(T_{37},t) & +(a''_{40})^{(8,8,8)}(T_{41},t) & +(a''_{44})^{(9,9)}(T_{45},t) & \end{bmatrix} G_{16} \quad (61)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} & +(a''_{17})^{(2)}(T_{17},t) & +(a''_{14})^{(1,1,)}(T_{14},t) & +(a''_{21})^{(3,3,3)}(T_{21},t) \\ +(a''_{25})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6)}(T_{33},t) & \\ +(a''_{37})^{(7,7,7)}(T_{37},t) & +(a''_{41})^{(8,8,8)}(T_{41},t) & +(a''_{45})^{(9,9)}(T_{45},t) & \end{bmatrix} G_{17} \quad (62)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} & +(a''_{18})^{(2)}(T_{17},t) & +(a''_{15})^{(1,1,)}(T_{14},t) & +(a''_{22})^{(3,3,3)}(T_{21},t) \\ +(a''_{26})^{(4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6)}(T_{33},t) & \\ +(a''_{38})^{(7,7,7)}(T_{37},t) & +(a''_{42})^{(8,8,8)}(T_{41},t) & +(a''_{46})^{(9,9)}(T_{45},t) & \end{bmatrix} G_{18} \quad (63)$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3)}(T_{21},t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficient for category 1, 2 and 3

$+(a''_{36})^{(7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7)}(T_{37},t)$ are seventh augmentation coefficient for category 1, 2 and 3

$+(a''_{40})^{(8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8)}(T_{41},t)$ are eight augmentation coefficient for category 1, 2 and 3

$+(a''_{44})^{(9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9)}(T_{45},t)$ are ninth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \begin{bmatrix} (b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)} & \boxed{-(b''_{13})^{(1,1,)}(G,t)} & \boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{16} \quad (64)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \begin{bmatrix} (b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)} & \boxed{-(b''_{14})^{(1,1,)}(G,t)} & \boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{17} \quad (65)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{bmatrix} (b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,)}(G,t)} & \boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9)}(G_{47},t)} \end{bmatrix} T_{18} \quad (66)$$

where $\boxed{-(b''_{16})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2)}(G_{19},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{13})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,)}(G,t)}$ are second detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{20})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{36})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2 and 3

$\boxed{-(b''_{40})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1,2 and 3

$-(b_{44}'')^{(9,9)}(G_{47},t)$, $-(b_{46}'')^{(9,9)}(G_{47},t)$, $-(b_{45}'')^{(9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{bmatrix} (a_{20}')^{(3)} & +(a_{20}'')^{(3)}(T_{21},t) & +(a_{16}'')^{(2,2,2)}(T_{17},t) & +(a_{13}'')^{(1,1,1,)}(T_{14},t) \\ +(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t) & +(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t) & +(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a_{36}'')^{(7,7,7,7)}(T_{37},t) & +(a_{40}'')^{(8,8,8,8)}(T_{41},t) & +(a_{44}'')^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{20} \quad (67)$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \begin{bmatrix} (a_{21}')^{(3)} & +(a_{21}'')^{(3)}(T_{21},t) & +(a_{17}'')^{(2,2,2)}(T_{17},t) & +(a_{14}'')^{(1,1,1,)}(T_{14},t) \\ +(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t) & +(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t) & +(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a_{37}'')^{(7,7,7,7)}(T_{37},t) & +(a_{41}'')^{(8,8,8,8)}(T_{41},t) & +(a_{45}'')^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{21} \quad (68)$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{bmatrix} (a_{22}')^{(3)} & +(a_{22}'')^{(3)}(T_{21},t) & +(a_{18}'')^{(2,2,2)}(T_{17},t) & +(a_{15}'')^{(1,1,1,)}(T_{14},t) \\ +(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t) & +(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t) & +(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t) & \\ +(a_{38}'')^{(7,7,7,7)}(T_{37},t) & +(a_{42}'')^{(8,8,8,8)}(T_{41},t) & +(a_{46}'')^{(9,9,9)}(T_{45},t) & \end{bmatrix} G_{22} \quad (69)$$

$+(a_{20}'')^{(3)}(T_{21},t)$, $+(a_{21}'')^{(3)}(T_{21},t)$, $+(a_{22}'')^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a_{16}'')^{(2,2,2)}(T_{17},t)$, $+(a_{17}'')^{(2,2,2)}(T_{17},t)$, $+(a_{18}'')^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a_{13}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{14}'')^{(1,1,1,)}(T_{14},t)$, $+(a_{15}'')^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a_{24}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{25}'')^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a_{26}'')^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a_{28}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{29}'')^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a_{30}'')^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a_{32}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{33}'')^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a_{34}'')^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3

$+(a_{36}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{37}'')^{(7,7,7,7)}(T_{37},t)$, $+(a_{38}'')^{(7,7,7,7)}(T_{37},t)$ are seventh augmentation coefficients for category 1, 2 and 3

$+(a_{40}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{41}'')^{(8,8,8,8)}(T_{41},t)$, $+(a_{42}'')^{(8,8,8,8)}(T_{41},t)$ are eight augmentation coefficients for category 1, 2 and 3

$+(a_{44}'')^{(9,9,9)}(T_{45},t)$, $+(a_{45}'')^{(9,9,9)}(T_{45},t)$, $+(a_{46}'')^{(9,9,9)}(T_{45},t)$ are ninth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{bmatrix} (b_{20}')^{(3)} & -(b_{20}'')^{(3)}(G_{23},t) & -(b_{16}'')^{(2,2,2)}(G_{19},t) & -(b_{13}'')^{(1,1,1,)}(G,t) \\ -(b_{24}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{28}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{32}'')^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b_{36}'')^{(7,7,7,7)}(G_{39},t) & -(b_{40}'')^{(8,8,8,8)}(G_{43},t) & -(b_{44}'')^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{20} \quad (70)$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{bmatrix} (b_{21}')^{(3)} & -(b_{21}'')^{(3)}(G_{23},t) & -(b_{17}'')^{(2,2,2)}(G_{19},t) & -(b_{14}'')^{(1,1,1,)}(G,t) \\ -(b_{25}'')^{(4,4,4,4,4,4)}(G_{27},t) & -(b_{29}'')^{(5,5,5,5,5,5)}(G_{31},t) & -(b_{33}'')^{(6,6,6,6,6,6)}(G_{35},t) & \\ -(b_{37}'')^{(7,7,7,7)}(G_{39},t) & -(b_{41}'')^{(8,8,8,8)}(G_{43},t) & -(b_{45}'')^{(9,9,9)}(G_{47},t) & \end{bmatrix} T_{21} \quad (71)$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix} (b'_{22})^{(3)}\boxed{-(b''_{22})^{(3)}(G_{23},t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)} & \boxed{-(b''_{15})^{(1,1,1,)}(G,t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)} \end{bmatrix} T_{22} \quad 72$$

$\boxed{-(b''_{20})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23},t)}$ *are first detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}$ *are second detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{13})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,)}(G,t)}$ *are third detrition coefficients for category 1,2 and 3*

$\boxed{-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)}$ *are fourth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)}$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{36})^{(7,7,7,7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{38})^{(7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8)}(G_{43},t)}$ are eight detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)}\boxed{+(a''_{24})^{(4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{24} \quad 73$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)}\boxed{+(a''_{25})^{(4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{25} \quad 74$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)}\boxed{+(a''_{26})^{(4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{26} \quad 75$$

$\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ *are first augmentation coefficients category* $1, 2\ 3$

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ *are second augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* $1, 2$ *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$

are fourth augmentation coefficients for category 1, 2 *and* 3
$+(a''_{16})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17},t)$
are fifth augmentation coefficients for category 1, 2 *and* 3
$+(a''_{20})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21},t)$
are sixth augmentation coefficients for category 1, 2 *and* 3
$+(a''_{36})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7)}(T_{37},t)$
are seventh augmentation coefficients for category 1, 2 *and* 3
$+(a''_{40})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8)}(T_{41},t)$
are eighth augmentation coefficients for category 1, 2 *and* 3
$+(a''_{46})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9)}(T_{45},t)$, $+(a''_{44})^{(9,9,9,9)}(T_{45},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27},t) & -(b''_{28})^{(5,5,)}(G_{31},t) & -(b''_{32})^{(6,6,)}(G_{35},t) \\ & -(b''_{13})^{(1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{24} \quad (76)$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27},t) & -(b''_{29})^{(5,5,)}(G_{31},t) & -(b''_{33})^{(6,6,)}(G_{35},t) \\ & -(b''_{14})^{(1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{25} \quad (77)$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27},t) & -(b''_{30})^{(5,5,)}(G_{31},t) & -(b''_{34})^{(6,6,)}(G_{35},t) \\ & -(b''_{15})^{(1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9)}(G_{47},t) \end{bmatrix} T_{26} \quad (78)$$

Where $-(b''_{24})^{(4)}(G_{27},t)$, $-(b''_{25})^{(4)}(G_{27},t)$, $-(b''_{26})^{(4)}(G_{27},t)$ *are first detrition coefficients for category* 1, 2 *and* 3
$-(b''_{28})^{(5,5,)}(G_{31},t)$, $-(b''_{29})^{(5,5,)}(G_{31},t)$, $-(b''_{30})^{(5,5,)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3
$-(b''_{32})^{(6,6,)}(G_{35},t)$, $-(b''_{33})^{(6,6,)}(G_{35},t)$, $-(b''_{34})^{(6,6,)}(G_{35},t)$ *are third detrition coefficients for category* 1, 2 *and* 3
$-(b''_{13})^{(1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1)}(G,t)$
are fourth detrition coefficients for category 1, 2 *and* 3
$-(b''_{16})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19},t)$
are fifth detrition coefficients for category 1, 2 *and* 3
$-(b''_{20})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23},t)$
are sixth detrition coefficients for category 1, 2 *and* 3
$-(b''_{36})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7)}(G_{39},t)$
are seventh detrition coefficients for category 1, 2 *and* 3
$-(b''_{40})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8)}(G_{43},t)$
are eighth detrition coefficients for category 1, 2 *and* 3
$-(b''_{46})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1 2 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} & \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{28} \quad (79)$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} & \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{29} \quad (80)$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} & \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} & \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{30} \quad (81)$$

Where $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7)}(T_{37},t)}$ are seventh augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{40})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8,8,8,8,8,8)}(T_{41},t)}$ are eighth augmentation coefficients for category 1,2, 3

$\boxed{+(a''_{46})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficients for category 1,2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \begin{bmatrix} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)} & \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{28} \quad (82)$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \begin{bmatrix} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)} & \boxed{-(b''_{25})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)} & \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{29} \quad (83)$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \begin{bmatrix} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)} & \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)} & \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)} & \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{30} \quad (84)$$

$where\ \boxed{-(b_{28}'')^{(5)}(G_{31},t)}\ ,\boxed{-(b_{29}'')^{(5)}(G_{31},t)}\ ,\boxed{-(b_{30}'')^{(5)}(G_{31},t)}\ \ are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{24}'')^{(4,4,)}(G_{27},t)}\ ,\boxed{-(b_{25}'')^{(4,4,)}(G_{27},t)}\ ,\boxed{-(b_{26}'')^{(4,4,)}(G_{27},t)}\ are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{32}'')^{(6,6,6)}(G_{35},t)}\ ,\boxed{-(b_{33}'')^{(6,6,6)}(G_{35},t)}\ ,\boxed{-(b_{34}'')^{(6,6,6)}(G_{35},t)}\ are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b_{13}'')^{(1,1,1,1,1)}(G,t)}\ ,\boxed{-(b_{14}'')^{(1,1,1,1,1)}(G,t)}\ ,\boxed{-(b_{15}'')^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2)}(G_{19},t)}\ ,\boxed{-(b_{17}'')^{(2,2,2,2,2)}(G_{19},t)}\ ,\boxed{-(b_{18}'')^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3)}(G_{23},t)}\ ,\boxed{-(b_{21}'')^{(3,3,3,3,3)}(G_{23},t)}\ ,\boxed{-(b_{22}'')^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3

$\boxed{-(b_{36}'')^{(7,7,7,7,7,7)}(G_{39},t)}\ ,\ \boxed{-(b_{37}'')^{(7,7,7,7,7,7)}(G_{39},t)}\ ,\boxed{-(b_{38}'')^{(7,7,7,7,7,7)}(G_{39},t)}$ are seventh detrition coefficients for category 1,2, and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8)}(G_{43},t)}\ ,\boxed{-(b_{41}'')^{(8,8,8,8,8,8)}(G_{43},t)}\ ,\boxed{-(b_{40}'')^{(8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1,2, and 3

$\boxed{-(b_{46}'')^{(9,9,9,9,9)}(G_{47},t)}\ ,\boxed{-(b_{45}'')^{(9,9,9,9,9)}(G_{47},t)}\ ,\boxed{-(b_{44}'')^{(9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt}=(a_{32})^{(6)}G_{33} \qquad 85$$

$$-\left[\begin{matrix}(a_{32}')^{(6)}\boxed{+(a_{32}'')^{(6)}(T_{33},t)}\ \boxed{+(a_{28}'')^{(5,5,5)}(T_{29},t)}\ \boxed{+(a_{24}'')^{(4,4,4,)}(T_{25},t)}\\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{16}'')^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{20}'')^{(3,3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a_{40}'')^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a_{44}'')^{(9,9,9,9,9,9)}(T_{45},t)}\end{matrix}\right]G_{32}$$

$$\frac{dG_{33}}{dt}=(a_{33})^{(6)}G_{32}-\left[\begin{matrix}(a_{33}')^{(6)}\boxed{+(a_{33}'')^{(6)}(T_{33},t)}\ \boxed{+(a_{29}'')^{(5,5,5)}(T_{29},t)}\ \boxed{+(a_{25}'')^{(4,4,4,)}(T_{25},t)}\\ \boxed{+(a_{14}'')^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{17}'')^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{21}'')^{(3,3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a_{41}'')^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a_{45}'')^{(9,9,9,9,9,9)}(T_{45},t)}\end{matrix}\right]G_{33} \qquad 86$$

$$\frac{dG_{34}}{dt}=(a_{34})^{(6)}G_{33}-\left[\begin{matrix}(a_{34}')^{(6)}\boxed{+(a_{34}'')^{(6)}(T_{33},t)}\ \boxed{+(a_{30}'')^{(5,5,5)}(T_{29},t)}\ \boxed{+(a_{26}'')^{(4,4,4,)}(T_{25},t)}\\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a_{18}'')^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a_{22}'')^{(3,3,3,3,3,3)}(T_{21},t)}\\ \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7)}(T_{37},t)}\ \boxed{+(a_{42}'')^{(8,8,8,8,8,8,8)}(T_{41},t)}\ \boxed{+(a_{46}'')^{(9,9,9,9,9,9)}(T_{45},t)}\end{matrix}\right]G_{34} \qquad 87$$

$\boxed{+(a_{32}'')^{(6)}(T_{33},t)}\ ,\boxed{+(a_{33}'')^{(6)}(T_{33},t)}\ ,\boxed{+(a_{34}'')^{(6)}(T_{33},t)}\ \ are\ first\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a_{28}'')^{(5,5,5)}(T_{29},t)}\ ,\boxed{+(a_{29}'')^{(5,5,5)}(T_{29},t)}\ ,\boxed{+(a_{30}'')^{(5,5,5)}(T_{29},t)}\ are\ second\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a_{24}'')^{(4,4,4,)}(T_{25},t)}\ ,\boxed{+(a_{25}'')^{(4,4,4,)}(T_{25},t)}\ ,\boxed{+(a_{26}'')^{(4,4,4,)}(T_{25},t)}\ \ are\ third\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a_{13}'')^{(1,1,1,1,1,1)}(T_{14},t)}\ ,\boxed{+(a_{14}'')^{(1,1,1,1,1,1)}(T_{14},t)}\ ,\boxed{+(a_{15}'')^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2)}(T_{17},t)}\ ,\boxed{+(a_{17}'')^{(2,2,2,2,2,2)}(T_{17},t)}\ ,\boxed{+(a_{18}'')^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation

coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)$ sixth augmentation coefficients

$+(a''_{36})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{37})^{(7,7,7,7,7,7,7)}(T_{37},t)$, $+(a''_{38})^{(7,7,7,7,7,7,7)}(T_{37},t)$ seventh augmentation coefficients

$+(a''_{40})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$, $+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)$ Eighth augmentation coefficients

$+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)$, $+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)$ ninth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \begin{bmatrix} (b'_{32})^{(6)} & -(b''_{32})^{(6)}(G_{35},t) & -(b''_{28})^{(5,5,5)}(G_{31},t) & -(b''_{24})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{13})^{(1,1,1,1,1,1)}(G,t) & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{40})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{32} \quad (88)$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \begin{bmatrix} (b'_{33})^{(6)} & -(b''_{33})^{(6)}(G_{35},t) & -(b''_{29})^{(5,5,5)}(G_{31},t) & -(b''_{25})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{14})^{(1,1,1,1,1,1)}(G,t) & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{33} \quad (89)$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \begin{bmatrix} (b'_{34})^{(6)} & -(b''_{34})^{(6)}(G_{35},t) & -(b''_{30})^{(5,5,5)}(G_{31},t) & -(b''_{26})^{(4,4,4,)}(G_{27},t) \\ & -(b''_{15})^{(1,1,1,1,1,1)}(G,t) & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t) \\ & -(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t) & -(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t) & -(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t) \end{bmatrix} T_{34} \quad (90)$$

$-(b''_{32})^{(6)}(G_{35},t)$, $-(b''_{33})^{(6)}(G_{35},t)$, $-(b''_{34})^{(6)}(G_{35},t)$ *are first detrition coefficients for category* 1, 2 *and* 3

$-(b''_{28})^{(5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5)}(G_{31},t)$ *are second detrition coefficients for category* 1, 2 *and* 3

$-(b''_{24})^{(4,4,4,)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,)}(G_{27},t)$ *are third detrition coefficients for category* 1,2 *and* 3

$-(b''_{13})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1,1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1,1,1,1)}(G,t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)$ are sixth detrition coefficients for category 1, 2, and 3

$-(b''_{36})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{37})^{(7,7,7,7,7,7,7)}(G_{39},t)$, $-(b''_{38})^{(7,7,7,7,7,7,7)}(G_{39},t)$ are seventh detrition coefficients for category 1, 2, and 3

$-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$, $-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)$ are eighth detrition coefficients for category 1, 2, and 3

$-(b''_{46})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{45})^{(9,9,9,9,9,9)}(G_{47},t)$, $-(b''_{44})^{(9,9,9,9,9,9)}(G_{47},t)$ are ninth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{36}}{dt} \qquad 91$$

$$= (a_{36})^{(7)}G_{37} - \begin{bmatrix} (a'_{36})^{(7)}\boxed{+(a''_{36})^{(7)}(T_{37},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{37}}{dt} \qquad 92$$

$$= (a_{37})^{(7)}G_{36} - \begin{bmatrix} (a'_{37})^{(7)}\boxed{+(a''_{37})^{(7)}(T_{37},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{38}}{dt} \qquad 93$$

$$= (a_{38})^{(7)}G_{37} - \begin{bmatrix} (a'_{38})^{(7)}\boxed{+(a''_{38})^{(7)}(T_{37},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{15}$$

Where $\boxed{(a''_{36})^{(7)}(T_{37},t)}$, $\boxed{(a''_{37})^{(7)}(T_{37},t)}$, $\boxed{(a''_{38})^{(7)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{13})^{(1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{42})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a''_{40})^{(8,8,8,8,8,8,8,8,)}(T_{41},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{36}}{dt} = \qquad 94$$

$$(b_{36})^{(7)}T_{37} - \begin{bmatrix} (b'_{36})^{(7)}\boxed{-(b''_{36})^{(7)}(G_{39},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{13}$$

$$\frac{dT_{37}}{dt} =$$

$$(b_{37})^{(7)}T_{36} - \begin{bmatrix} (b'_{37})^{(7)}\boxed{-(b''_{37})^{(7)}(G_{39},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{41})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{14}$$

$$\frac{dT_{38}}{dt} =$$

$$(b_{38})^{(7)}T_{37} - \begin{bmatrix} (b'_{38})^{(7)}\boxed{-(b''_{38})^{(7)}(G_{39},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{42})^{(8,8,8,8,8,8,8)}(G_{43},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9)}(G_{47},t)} \end{bmatrix} T_{15}$$

Where $\boxed{-(b''_{36})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{37})^{(7)}(G_{39},t)}$, $\boxed{-(b''_{38})^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{15})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{13})^{(1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{40})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{41})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b''_{42})^{(8,8,8,8,8,8,8,8)}(G_{43},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{40}}{dt}$$ *95*

$$= (a_{40})^{(8)}G_{41} - \begin{bmatrix} (a'_{40})^{(8)}\boxed{+(a''_{40})^{(8)}(T_{41},t)} & \boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{41}}{dt}$$

$$= (a_{41})^{(8)}G_{40} - \begin{bmatrix} (a'_{41})^{(8)}\boxed{+(a''_{41})^{(8)}(T_{41},t)} & \boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{42}}{dt}$$
$$= (a_{42})^{(8)} G_{41}$$
$$- \left[\begin{array}{ccc} (a'_{42})^{(8)} \boxed{+(a''_{42})^{(8)}(T_{41},t)} & \boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)} \end{array} \right] G_{15}$$

Where $\boxed{+(a''_{40})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{41})^{(8)}(T_{41},t)}$, $\boxed{+(a''_{42})^{(8)}(T_{41},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{14})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ $\boxed{+(a''_{15})^{(1,1,1,1,1,1,1,1)}(T_{14},t)}$ are seventh augmentation coefficient for 1,2,3

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{37})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$, $\boxed{+(a''_{38})^{(7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth augmentation coefficient for 1,2,3

$\boxed{+(a''_{46})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{45})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$, $\boxed{+(a''_{44})^{(9,9,9,9,9,9,9,9)}(T_{45},t)}$ are ninth augmentation coefficient for 1,2,3

$$\frac{dT_{40}}{dt} =$$
$$(b_{40})^{(8)} T_{41} - \left[\begin{array}{ccc} (b'_{40})^{(8)} \boxed{-(b''_{40})^{(8)}(G_{43},t)} & \boxed{-(b''_{16})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{20})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{28})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{32})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{36})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{44})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{13}$$

$$\frac{dT_{41}}{dt} =$$
$$(b_{41})^{(8)} T_{40} - \left[\begin{array}{ccc} (b'_{41})^{(8)} \boxed{-(b''_{41})^{(8)}(G_{43},t)} & \boxed{-(b''_{17})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{21})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{29})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{33})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{37})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{45})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{14}$$

$$\frac{dT_{42}}{dt} =$$
$$(b_{42})^{(8)} T_{41} - \left[\begin{array}{ccc} (b'_{42})^{(8)} \boxed{-(b''_{42})^{(8)}(G_{43},t)} & \boxed{-(b''_{18})^{(2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b''_{22})^{(3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b''_{30})^{(5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b''_{34})^{(6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b''_{38})^{(7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b''_{46})^{(9,9,9,9,9,9,9,9)}(G_{47},t)} \end{array} \right] T_{15}$$

Where $\boxed{-(b_{36}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7)}(G_{39},t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{38}'')^{(7,7,)}(G_{39},t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{44}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9,9,9,9,9,9,9,9)}(G_{47},t)}$ are ninth detrition coefficients for category 1, 2 and 3

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$$\frac{dG_{44}}{dt} = (a_{44})^{(9)}G_{45} - \begin{bmatrix} (a_{44}')^{(9)}\boxed{+(a_{44}'')^{(9)}(T_{45},t)} & \boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{13}$$

$$\frac{dG_{45}}{dt} = (a_{45})^{(9)}G_{44} - \begin{bmatrix} (a_{45}')^{(9)}\boxed{+(a_{45}'')^{(9)}(T_{45},t)} & \boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{14}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{14}$$

$$\frac{dG_{46}}{dt} = (a_{46})^{(9)}G_{45} - \begin{bmatrix} (a_{46}')^{(9)}\boxed{+(a_{46}'')^{(9)}(T_{37},t)} & \boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)} & \boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4)}(T_{25},t)} & \boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5)}(T_{29},t)} & \boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1)}(T_{14},t)} & \boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7)}(T_{37},t)} & \boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8)}(T_{41},t)} \end{bmatrix} G_{15}$$

Where $\boxed{+(a_{44}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{45}'')^{(9)}(T_{45},t)}$, $\boxed{+(a_{46}'')^{(9)}(T_{37},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2,2,2,2,2,2,2,2)}(T_{17},t)}$ are second

augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{20}'')^{(3,3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3,3,3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3,3,3,3,3,3,3,3)}(T_{21},t)}$ are third
augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{24}'')^{(4,4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4,4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4,4,4,4,4,4)}(T_{25},t)}$ are fourth
augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{28}'')^{(5,5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5,5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5,5,5,5,5,5)}(T_{29},t)}$ are fifth
augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{32}'')^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6,6,6,6,6,6)}(T_{33},t)}$ are sixth
augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{13}'')^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{14}'')^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,1,1,1,1,1,1,1,1)}(T_{14},t)}$ are Seventh
augmentation coefficient for category 1, 2 and 3
$\boxed{+(a_{38}'')^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a_{37}'')^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ $\boxed{+(a_{36}'')^{(7,7,7,7,7,7,7,7,7,7)}(T_{37},t)}$ are eighth
augmentation coefficient for 1,2,3
$\boxed{+(a_{40}'')^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{42}'')^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)}$, $\boxed{+(a_{41}'')^{(8,8,8,8,8,8,8,8,8,8)}(T_{41},t)}$ are ninth
augmentation coefficient for 1,2,3

$$\frac{dT_{44}}{dt} =$$
$$(b_{44})^{(9)}T_{45} -$$
$$\begin{bmatrix} (b_{44}')^{(9)}\boxed{-(b_{44}'')^{(9)}(G_{47},t)} & \boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{13}$$
$$\frac{dT_{45}}{dt} =$$
$$(b_{45})^{(9)}T_{44} - \begin{bmatrix} (b_{45}')^{(9)}\boxed{-(b_{45}'')^{(9)}(G_{47},t)} & \boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{14}$$
$$\frac{dT_{46}}{dt} =$$
$$(b_{46})^{(9)}T_{45} - \begin{bmatrix} (b_{46}')^{(9)}\boxed{-(b_{46}'')^{(9)}(G_{47},t)} & \boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)} & \boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)} & \boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5,5)}(G_{31},t)} & \boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6,6)}(G_{35},t)} \\ \boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1,1)}(G,t)} & \boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7,7)}(G_{39},t)} & \boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8,8)}(G_{43},t)} \end{bmatrix} T_{15}$$
Where $\boxed{-(b_{44}'')^{(9)}(G_{47},t)}$, $\boxed{-(b_{45}'')^{(9)}(G_{47},t)}$, $\boxed{-(b_{46}'')^{(9)}(G_{47},t)}$ are first detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{16}'')^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,2,2,2,2,2,2,2,2)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{20}'')^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,3,3,3,3,3,3,3,3)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3
$\boxed{-(b_{24}'')^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,4,4,4,4,4,4)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,5,5,5,5,5)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,6,6,6,6,6)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{15}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b_{13}'')^{(1,1,1,1,1,1,1,1,1)}(G,t)}$ are seventh detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{37}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{36}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$, $\boxed{-(b_{38}'')^{(7,7,7,7,7,7,7,7,7)}(G_{39},t)}$ are eighth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{42}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{41}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$, $\boxed{-(b_{40}'')^{(8,8,8,8,8,8,8,8,8)}(G_{43},t)}$ are ninth detrition coefficients for category 1, 2 and 3

Where we suppose

$$(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0, \quad i,j = 13,14,15 \qquad 97$$

The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

$$lim_{T_2\to\infty}(a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \qquad 98$$

$$\lim_{G\to\infty}(b_i'')^{(1)}(G,t) = (r_i)^{(1)}$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition: 99

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,t)| < (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},t)$. (T_{14}',t) and (T_{14},t) are points belonging to the interval $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 100

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 101

There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$which together With $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

$(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i, j = 16,17,18$

The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}$, $(r_i)^{(2)}$:

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 102$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 103$$

$$\lim_{T_2 \to \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)} \quad 104$$

$$\lim_{G \to \infty} (b_i'')^{(2)}((G_{19}), t) = (r_i)^{(2)} \quad 105$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 106

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T_{17}', t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T_{17} - T_{17}'| e^{-(\hat{M}_{16})^{(2)} t} \quad 107$$

$$|(b_i'')^{(2)}((G_{19})', t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} ||(G_{19}) - (G_{19})'|| e^{-(\hat{M}_{16})^{(2)} t} \quad 108$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}', t)$ and$(a_i'')^{(2)}(T_{17}, t)$. (T_{17}', t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

$(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 109

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (110)$$

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad (111)$$

Where we suppose

$$(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20,21,22 \quad (112)$$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad (113)$$

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: (114)

$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21} - T_{21}'| e^{-(\hat{M}_{20})^{(3)} t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} ||G_{23} - G_{23}'|| e^{-(\hat{M}_{20})^{(3)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and $(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) And (T_{21}, t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the first augmentation coefficient attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: (115)

$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}\ ,\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with 116
$(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants
$(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)},\ (r_i)^{(3)}, i = 20,21,22,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\,(a_i)^{(3)} + (a_i')^{(3)} +\ (\hat{A}_{20})^{(3)} +\ (\hat{P}_{20})^{(3)}\,(\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}}[\ (b_i)^{(3)} + (b_i')^{(3)} +\ (\hat{B}_{20})^{(3)} +\ (\hat{Q}_{20})^{(3)}\ (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

$$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26 \qquad 117$$

The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)},\ (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \le (p_i)^{(4)} \le (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}\big((G_{27}), t\big) \le\ (r_i)^{(4)} \le (b_i')^{(4)} \le (\hat{B}_{24})^{(4)}$$

$$lim_{T_2\to\infty}(a_i'')^{(4)}\,(T_{25}, t) = (p_i)^{(4)} \qquad 118$$

$$\lim_{G\to\infty}(b_i'')^{(4)}\,\big((G_{27}), t\big) =\ (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)},\ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 119

$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \le (\hat{k}_{24})^{(4)}|T_{25} - T_{25}'|e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})'||e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $\big[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\big]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 120

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)},$ are positive constants

$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 121

There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and\ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

$(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, \quad i, j = 28,29,30$ 122

The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}$, $(r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}\big((G_{31}), t\big) \leq (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)}$$ 123

$$\lim_{G \to \infty}(b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 124

$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}'|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i'')^{(5)}((G_{31})', t) - (b_i'')^{(5)}\big((G_{31}), t\big)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})'||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ and$(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $\big[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}\big]$. It is to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 1$ then the function $(a_i'')^{(5)}(T_{29}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 125

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 126

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and\ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)} + (a_i')^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i')^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a_i')^{(6)}, (a_i'')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (b_i'')^{(6)} > 0, \quad i, j = 32,33,34$$ 127

The functions $(a_i'')^{(6)}, (b_i'')^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq \ (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$ 128

$$\lim_{G \to \infty}(b_i'')^{(6)}\left((G_{35}), t\right) = \ (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, \ (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(6)}(T_{33}', t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}\left((G_{35}), t\right)| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}', t)$ and$(a_i'')^{(6)}(T_{33}, t)$. (T_{33}', t) and (T_{33}, t) are points belonging to the interval $\left[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}\right]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 129

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$: 130

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$

Where we suppose

(CCCCCCCCC) $(a_i)^{(7)}, (a_i')^{(7)}, (a_i'')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (b_i'')^{(7)} > 0, \quad i, j = 36,37,38$ 131

(DDDDDDDDD) The functions $(a_i'')^{(7)}, (b_i'')^{(7)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(7)}$, $(r_i)^{(7)}$:

$$(a_i'')^{(7)}(T_{37}, t) \le (p_i)^{(7)} \le (\hat{A}_{36})^{(7)}$$

$$(b_i'')^{(7)}(G_{39}, t) \le (r_i)^{(7)} \le (b_i')^{(7)} \le (\hat{B}_{36})^{(7)}$$

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(EEEEEEEEE) $\lim_{T_2 \to \infty} (a_i'')^{(7)}(T_{37}, t) = (p_i)^{(7)}$

(FFFFFFFFF)

$\lim_{G \to \infty} (b_i'')^{(7)}((G_{39}), t) = (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $\boxed{(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}}$ are positive constants and $\boxed{i = 36,37,38}$

They satisfy Lipschitz condition: 133

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \le (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}((G_{39}), t)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 1$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 134

(GGGGGGGGG) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 135

(HHHHHHHHH) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and\ (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Where we suppose

$$(a_i)^{(8)}, (a_i')^{(8)}, (a_i'')^{(8)}, (b_i)^{(8)}, (b_i')^{(8)}, (b_i'')^{(8)} > 0, \quad i,j = 40,41,42 \qquad 136$$

The functions $(a_i'')^{(8)}, (b_i'')^{(8)}$ are positive continuous increasing and bounded

Definition of $(p_i)^{(8)}$, $(r_i)^{(8)}$: 137

$$(a_i'')^{(8)}(T_{41}, t) \leq (p_i)^{(8)} \leq (\hat{A}_{40})^{(8)} \qquad 138$$

$$(b_i'')^{(8)}((G_{43}), t) \leq (r_i)^{(8)} \leq (b_i')^{(8)} \leq (\hat{B}_{40})^{(8)} \qquad 139$$

$$\lim_{T_2 \to \infty} (a_i'')^{(8)}(T_{41}, t) = (p_i)^{(8)} \qquad 140$$

$$\lim_{G \to \infty} (b_i'')^{(8)}((G_{43}), t) = (r_i)^{(8)} \qquad 141$$

Definition of $(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}$:

Where $\boxed{(\hat{A}_{40})^{(8)}, (\hat{B}_{40})^{(8)}, (p_i)^{(8)}, (r_i)^{(8)}}$ are positive constants and $\boxed{i = 40,41,42}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(8)}(T_{41}', t) - (a_i'')^{(8)}(T_{41}, t)| \leq (\hat{k}_{40})^{(8)} |T_{41} - T_{41}'| e^{-(\hat{M}_{40})^{(8)} t} \qquad 142$$

$$|(b_i'')^{(8)}((G_{43})', t) - (b_i'')^{(8)}((G_{43}), t)| < (\hat{k}_{40})^{(8)} ||(G_{43}) - (G_{43})'|| e^{-(\hat{M}_{40})^{(8)} t} \qquad 143$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(8)}(T_{41}', t)$ and $(a_i'')^{(8)}(T_{41}, t)$. (T_{41}', t) and (T_{41}, t) are points belonging to the interval $[(\hat{k}_{40})^{(8)}, (\hat{M}_{40})^{(8)}]$. It is to be

noted that $(a_i'')^{(8)}(T_{41},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\widehat{M}_{40})^{(8)}=1$ then the function $(a_i'')^{(8)}(T_{41},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$:

$(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)}$, are positive constants

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\ ,\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}<1 \qquad 144$$

Definition of $(\hat{P}_{40})^{(8)},(\hat{Q}_{40})^{(8)}$:

There exists two constants $(\hat{P}_{40})^{(8)}$ and $(\hat{Q}_{40})^{(8)}$ which together with $(\widehat{M}_{40})^{(8)},(\hat{k}_{40})^{(8)},(\hat{A}_{40})^{(8)}$ $(\hat{B}_{40})^{(8)}$ and the constants $(a_i)^{(8)},(a_i')^{(8)},(b_i)^{(8)},(b_i')^{(8)},(p_i)^{(8)},\ (r_i)^{(8)},i=40,41,42,$
Satisfy the inequalities

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[(a_i)^{(8)}+(a_i')^{(8)}+\ (\hat{A}_{40})^{(8)}+(\hat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}]<1 \qquad 145$$

$$\frac{1}{(\widehat{M}_{40})^{(8)}}[\ (b_i)^{(8)}+(b_i')^{(8)}+\ (\hat{B}_{40})^{(8)}+(\hat{Q}_{40})^{(8)}\ (\hat{k}_{40})^{(8)}]<1 \qquad 146$$

Where we suppose

$$(a_i)^{(9)},(a_i')^{(9)},(a_i'')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(b_i'')^{(9)}>0,\quad i,j=44,45,46 \qquad 146\text{A}$$

The functions $(a_i'')^{(9)},(b_i'')^{(9)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(9)},\ (r_i)^{(9)}$:

$$(a_i'')^{(9)}(T_{45},t)\le(p_i)^{(9)}\le(\hat{A}_{44})^{(9)}$$

$$(b_i'')^{(9)}(G_{47},t)\le\ (r_i)^{(9)}\le(b_i')^{(9)}\le(\hat{B}_{44})^{(9)}$$

$$\lim_{T_2\to\infty}(a_i'')^{(9)}(T_{45},t)=(p_i)^{(9)}$$

$$\lim_{G\to\infty}(b_i'')^{(9)}(G_{47},t)=\ (r_i)^{(9)}$$

Definition of $(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)}$:

Where $\boxed{(\hat{A}_{44})^{(9)},(\hat{B}_{44})^{(9)},(p_i)^{(9)},\ (r_i)^{(9)}}$ are positive constants and $\boxed{i=44,45,46}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(9)}(T_{45}',t)-(a_i'')^{(9)}(T_{45},t)|\le(\hat{k}_{44})^{(9)}|T_{45}-\ T_{45}'|e^{-(\widehat{M}_{44})^{(9)}t}$$

$$|(b_i'')^{(9)}((G_{47})',t)-(b_i'')^{(9)}\big((G_{47}),t\big)|<(\hat{k}_{44})^{(9)}||(G_{47})-(G_{47})'||e^{-(\widehat{M}_{44})^{(9)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(9)}(T_{45}',t)$

and $(a_i'')^{(9)}(T_{45},t)$. (T_{45}',t) and (T_{45},t) are points belonging to the interval $\left[(\hat{k}_{44})^{(9)},(\hat{M}_{44})^{(9)}\right]$. It is to be noted that $(a_i'')^{(9)}(T_{45},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{44})^{(9)}=1$ then the function $(a_i'')^{(9)}(T_{45},t)$, the **first augmentation coefficient** attributable to the system, would be absolutely continuous.

Definition of $(\hat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)}$:

$(\hat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},$ are positive constants

$$\frac{(a_i)^{(9)}}{(\hat{M}_{44})^{(9)}}\ ,\frac{(b_i)^{(9)}}{(\hat{M}_{44})^{(9)}}<1$$

Definition of $(\hat{P}_{44})^{(9)},(\hat{Q}_{44})^{(9)}$:

There exists two constants $(\hat{P}_{44})^{(9)}$ and $(\hat{Q}_{44})^{(9)}$ which together with $(\hat{M}_{44})^{(9)},(\hat{k}_{44})^{(9)},(\hat{A}_{44})^{(9)} and\ (\hat{B}_{44})^{(9)}$ and the constants $(a_i)^{(9)},(a_i')^{(9)},(b_i)^{(9)},(b_i')^{(9)},(p_i)^{(9)},\ (r_i)^{(9)},i=44,45,46,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{44})^{(9)}}[(a_i)^{(9)}+(a_i')^{(9)}+\ (\hat{A}_{44})^{(9)}+\ (\hat{P}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

$$\frac{1}{(\hat{M}_{44})^{(9)}}[\ (b_i)^{(9)}+(b_i')^{(9)}+\ (\hat{B}_{44})^{(9)}+\ (\hat{Q}_{44})^{(9)}\ (\hat{k}_{44})^{(9)}]<1$$

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions 147

Definition of $G_i(0)\ ,T_i(0)$:

$$G_i(t)\le\ (\hat{P}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}\ ,\ \boxed{G_i(0)=G_i^0>0}$$

$$T_i(t)\le\ (\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}\ ,\ \boxed{T_i(0)=T_i^0>0}$$

Theorem 2 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 148

Definition of $G_i(0)\ ,T_i(0)$

$$G_i(t)\le\ (\hat{P}_{16})^{(2)}e^{(\hat{M}_{16})^{(2)}t}\ ,\quad G_i(0)=G_i^0>0$$

$$T_i(t)\le\ (\hat{Q}_{16})^{(2)}e^{(\hat{M}_{16})^{(2)}t}\ ,\quad T_i(0)=T_i^0>0$$

Theorem 3 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 149

$$G_i(t)\le\ (\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}t}\ ,\quad G_i(0)=G_i^0>0$$

$$T_i(t)\le\ (\hat{Q}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}t}\ ,\quad T_i(0)=T_i^0>0$$

Theorem 4 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0)\ ,T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{24}\right)^{(4)} e^{\left(\hat{M}_{24}\right)^{(4)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{24}\right)^{(4)} e^{\left(\hat{M}_{24}\right)^{(4)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 5 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 151

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{28}\right)^{(5)} e^{\left(\hat{M}_{28}\right)^{(5)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{28}\right)^{(5)} e^{\left(\hat{M}_{28}\right)^{(5)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 6 : if the conditions above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{32}\right)^{(6)} e^{\left(\hat{M}_{32}\right)^{(6)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{32}\right)^{(6)} e^{\left(\hat{M}_{32}\right)^{(6)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 7: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{36}\right)^{(7)} e^{\left(\hat{M}_{36}\right)^{(7)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{36}\right)^{(7)} e^{\left(\hat{M}_{36}\right)^{(7)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 8: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 A

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{40}\right)^{(8)} e^{\left(\hat{M}_{40}\right)^{(8)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{40}\right)^{(8)} e^{\left(\hat{M}_{40}\right)^{(8)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Theorem 9: if the conditions above are fulfilled, there exists a solution satisfying the conditions 153 B

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{44}\right)^{(9)} e^{\left(\hat{M}_{44}\right)^{(9)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq \left(\hat{Q}_{44}\right)^{(9)} e^{\left(\hat{M}_{44}\right)^{(9)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 154

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{13})^{(1)}, T_i^0 \le (\hat{Q}_{13})^{(1)}, \quad 155$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 156$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \quad 157$$

By 158

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: 159

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0,\ T_i(0) = T_i^0,\ G_i^0 \le (\hat{P}_{16})^{(2)}, T_i^0 \le (\hat{Q}_{16})^{(2)},$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$$

By 160

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{20})^{(3)}$, $T_i^0 \le (\hat{Q}_{20})^{(3)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$

By 161

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{24})^{(4)}$, $T_i^0 \le (\hat{Q}_{24})^{(4)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$

By 162

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{24}(s_{(24)})\right] ds_{(24)}$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{25}(s_{(24)})\right] ds_{(24)}$$

$$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}(s_{(24)}), s_{(24)})\right) T_{26}(s_{(24)})\right] ds_{(24)}$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{28})^{(5)}$, $T_i^0 \le (\hat{Q}_{28})^{(5)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$

By

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$

$$\bar{\mathrm{T}}_{30}(\mathrm{t}) = \mathrm{T}_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{32})^{(6)}$, $T_i^0 \le (\hat{Q}_{32})^{(6)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)} t}$

By 164

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{\mathrm{T}}_{34}(\mathrm{t}) = \mathrm{T}_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G_{35}(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{36})^{(7)}$, $T_i^0 \le (\hat{Q}_{36})^{(7)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{36})^{(7)} e^{(\bar{M}_{36})^{(7)} t}$

By 165

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}(s_{(36)}) - \left((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}) \right) G_{38}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}(s_{(36)}) - \left((b'_{36})^{(7)} - (b''_{36})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{36}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}(s_{(36)}) - \left((b'_{37})^{(7)} - (b''_{37})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{37}(s_{(36)}) \right] ds_{(36)}$$

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}(s_{(36)}) - \left((b'_{38})^{(7)} - (b''_{38})^{(7)} (G_{39}(s_{(36)}), s_{(36)}) \right) T_{38}(s_{(36)}) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0,t)$

Proof:

Consider operator $\mathcal{A}^{(8)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{40})^{(8)}$, $T_i^0 \le (\hat{Q}_{40})^{(8)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} t}$

By 166

$$\bar{G}_{40}(t) = G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} G_{41}(s_{(40)}) - \left((a'_{40})^{(8)} + a''_{40})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{41}(t) = G_{41}^0 + \int_0^t \left[(a_{41})^{(8)} G_{40}(s_{(40)}) - \left((a'_{41})^{(8)} + (a''_{41})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{G}_{42}(t) = G_{42}^0 + \int_0^t \left[(a_{42})^{(8)} G_{41}(s_{(40)}) - \left((a'_{42})^{(8)} + (a''_{42})^{(8)} (T_{41}(s_{(40)}), s_{(40)})\right) G_{42}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{40}(t) = T_{40}^0 + \int_0^t \left[(b_{40})^{(8)} T_{41}(s_{(40)}) - \left((b'_{40})^{(8)} - (b''_{40})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{40}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{41}(t) = T_{41}^0 + \int_0^t \left[(b_{41})^{(8)} T_{40}(s_{(40)}) - \left((b'_{41})^{(8)} - (b''_{41})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{41}(s_{(40)})\right] ds_{(40)}$$

$$\bar{T}_{42}(t) = T_{42}^0 + \int_0^t \left[(b_{42})^{(8)} T_{41}(s_{(40)}) - \left((b'_{42})^{(8)} - (b''_{42})^{(8)} (G_{43}(s_{(40)}), s_{(40)})\right) T_{42}(s_{(40)})\right] ds_{(40)}$$

Where $s_{(40)}$ is the integrand that is integrated over an interval $(0,t)$

Proof: 166
Consider operator $\mathcal{A}^{(9)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy A

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \le (\hat{P}_{44})^{(9)}$, $T_i^0 \le (\hat{Q}_{44})^{(9)}$,

$0 \le G_i(t) - G_i^0 \le (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} t}$

By

$$\bar{G}_{44}(t) = G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} G_{45}(s_{(44)}) - \left((a'_{44})^{(9)} + a''_{44})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{45}(t) = G_{45}^0 + \int_0^t \left[(a_{45})^{(9)} G_{44}(s_{(44)}) - \left((a'_{45})^{(9)} + (a''_{45})^{(9)} (T_{45}(s_{(44)}), s_{(44)})\right) G_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{G}_{46}(t) = G_{46}^0 + \int_0^t \left[(a_{46})^{(9)} G_{45}(s_{(44)}) - \left((a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}(s_{(44)}), s_{(44)})\right) G_{46}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{44}(t) = T_{44}^0 + \int_0^t \left[(b_{44})^{(9)} T_{45}(s_{(44)}) - \left((b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{44}(s_{(44)})\right] ds_{(44)}$$

$$\bar{T}_{45}(t) = T_{45}^0 + \int_0^t \left[(b_{45})^{(9)} T_{44}(s_{(44)}) - \left((b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{45}(s_{(44)})\right] ds_{(44)}$$

$$\bar{\mathrm{T}}_{46}(\mathrm{t}) = \mathrm{T}_{46}^0 + \int_0^t \left[(b_{46})^{(9)} T_{45}(s_{(44)}) - \left((b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47}(s_{(44)}), s_{(44)})\right) T_{46}(s_{(44)})\right] ds_{(44)}$$

Where $s_{(44)}$ is the integrand that is integrated over an interval $(0, t)$

The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 167

$$G_{13}(t) \le G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\right)\right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1\right)$$

From which it follows that 168

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0\right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{16}(t) \le G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\right)\right] ds_{(16)} = \quad 169$$

$$\left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1\right)$$

From which it follows that 170

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \le \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0\right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}\right)} + (\hat{P}_{16})^{(2)}\right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 171

$$G_{20}(t) \le G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}}\right)\right] ds_{(20)} =$$

$$\left(1 + (a_{20})^{(3)} t\right) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1\right)$$

From which it follows that 172

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[((\hat{P}_{20})^{(3)}+G_{21}^{0})e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 173

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$

From which it follows that 174

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[((\hat{P}_{24})^{(4)}+G_{25}^{0})e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem 4

The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$

From which it follows that 175

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[((\hat{P}_{28})^{(5)}+G_{29}^{0})e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem 5

The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 176

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$

From which it follows that 177

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{P}_{32})^{(6)}+G_{33}^{0})e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 6

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

(jj) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that 178

$$G_{36}(t) \le G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\tilde{M}_{36})^{(7)} s_{(36)}}\right)\right] ds_{(36)} =$$
$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left(e^{(\tilde{M}_{36})^{(7)} t} - 1\right)$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\tilde{M}_{36})^{(7)} t} \le \frac{(a_{36})^{(7)}}{(\tilde{M}_{36})^{(7)}} \left[\left((\hat{P}_{36})^{(7)} + G_{37}^0\right) e^{\left(-\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

The operator $\mathcal{A}^{(8)}$ maps the space of functions satisfying Equations into itself .Indeed it is obvious that

$$G_{40}(t) \le G_{40}^0 + \int_0^t \left[(a_{40})^{(8)} \left(G_{41}^0 + (\hat{P}_{40})^{(8)} e^{(\tilde{M}_{40})^{(8)} s_{(40)}}\right)\right] ds_{(40)} = \quad 180$$
$$\left(1 + (a_{40})^{(8)} t\right) G_{41}^0 + \frac{(a_{40})^{(8)} (\hat{P}_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left(e^{(\tilde{M}_{40})^{(8)} t} - 1\right)$$

From which it follows that 181

$$(G_{40}(t) - G_{40}^0) e^{-(\tilde{M}_{40})^{(8)} t} \le \frac{(a_{40})^{(8)}}{(\tilde{M}_{40})^{(8)}} \left[\left((\hat{P}_{40})^{(8)} + G_{41}^0\right) e^{\left(-\frac{(\hat{P}_{40})^{(8)} + G_{41}^0}{G_{41}^0}\right)} + (\hat{P}_{40})^{(8)}\right]$$

(G_i^0) is as defined in the statement of theorem 8

Analogous inequalities hold also for $G_{41}, G_{42}, T_{40}, T_{41}, T_{42}$

The operator $\mathcal{A}^{(9)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{44}(t) \le G_{44}^0 + \int_0^t \left[(a_{44})^{(9)} \left(G_{45}^0 + (\hat{P}_{44})^{(9)} e^{(\tilde{M}_{44})^{(9)} s_{(44)}}\right)\right] ds_{(44)} =$$
$$\left(1 + (a_{44})^{(9)} t\right) G_{45}^0 + \frac{(a_{44})^{(9)} (\hat{P}_{44})^{(9)}}{(\tilde{M}_{44})^{(9)}} \left(e^{(\tilde{M}_{44})^{(9)} t} - 1\right)$$

From which it follows that

$$(G_{44}(t) - G_{44}^0) e^{-(\tilde{M}_{44})^{(9)} t} \le \frac{(a_{44})^{(9)}}{(\tilde{M}_{44})^{(9)}} \left[\left((\hat{P}_{44})^{(9)} + G_{45}^0\right) e^{\left(-\frac{(\hat{P}_{44})^{(9)} + G_{45}^0}{G_{45}^0}\right)} + (\hat{P}_{44})^{(9)}\right]$$

(G_i^0) is as defined in the statement of theorem 9

Analogous inequalities hold also for $G_{45}, G_{46}, T_{44}, T_{45}, T_{46}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\tilde{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\tilde{M}_{13})^{(1)}} < 1$ and to choose 182

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[(\widehat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)} \quad 183$$

$$\frac{(b_i)^{(1)}}{(\widehat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)} \quad 184$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 185

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{13})^{(1)}t}\}$$

Indeed if we denote

Definition of $\tilde{G},\tilde{T}:\left(\tilde{G},\tilde{T}\right)=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)}+$$

$$\int_0^t\{(a'_{13})^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}+$$

$$G_{13}^{(2)}\left|(a''_{13})^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a''_{13})^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t}\leq \quad 186$$
$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a'_{13})^{(1)}+(\widehat{A}_{13})^{(1)}+(\widehat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}\ and\ (\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i=13{,}14{,}15$ depend only on T_{14} and respectively on

$G(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 19 to 24 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(1)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$: 187

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)} G_{14}$ and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a_{15}')^{(1)}$$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If $G_{13}\ \ is$ bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 188

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}\,(G(t), t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 189

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}\,(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.
The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}\,(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 190

$(\hat{P}_{16})^{(2)}\ and\ (\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)}+\left((\hat{P}_{16})^{(2)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{16})^{(2)} \quad 191$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[\left((\hat{Q}_{16})^{(2)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{16})^{(2)}\right]\leq(\hat{Q}_{16})^{(2)} \quad 192$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 193

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 194

$$d\left(((G_{19})^{(1)},(T_{19})^{(1)}),((G_{19})^{(2)},(T_{19})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{16})^{(2)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{16})^{(2)}t}\}$$

Indeed if we denote 195

Definition of $\widetilde{G_{19}},\widetilde{T_{19}}$: $\left(\widetilde{G_{19}},\widetilde{T_{19}}\right)=\mathcal{A}^{(2)}(G_{19},T_{19})$

It results 196

$$\left|\tilde{G}_{16}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{16})^{(2)}\left|G_{17}^{(1)}-G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)}+$$

$$\int_0^t\{(a'_{16})^{(2)}\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)\left|G_{16}^{(1)}-G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$$

$$G_{16}^{(2)}\left|(a''_{16})^{(2)}\left(T_{17}^{(1)},s_{(16)}\right)-(a''_{16})^{(2)}\left(T_{17}^{(2)},s_{(16)}\right)\right|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$ 197

From the hypotheses it follows

$$\left|(G_{19})^{(1)}-(G_{19})^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}t}\leq$$
$$\frac{1}{(\widehat{M}_{16})^{(2)}}\left((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right)d\left(((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)})\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 198

Remark 6: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$. 199

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on

(G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 7: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 200

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(2)} - (a_i'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(2)}t)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1$, $\left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 201

Remark 8: if G_{16} is bounded, the same property have also G_{17} and G_{18}. indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a_{17}')^{(2)} G_{17}$ and by integrating

$$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a_{17}')^{(2)}$$

In the same way , one can obtain

$$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a_{18}')^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16}, G_{18} and G_{16}, G_{17} respectively.

Remark 9: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18}. The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 202

Remark 10: If T_{16} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \to \infty$. 203

Definition of $(m)^{(2)}$ and ε_2:

Indeed let t_2 be so that for $t > t_2$

$$(b_{17})^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 204

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 205
The same property holds for T_{18} if $\lim_{t\to\infty}(b_{18}'')^{(2)}\left((G_{19})(t), t\right) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$ and to choose 207

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\left((\hat{P}_{20})^{(3)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{20})^{(3)} \quad 208$$

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\hat{Q}_{20})^{(3)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{20})^{(3)}\right]\leq(\hat{Q}_{20})^{(3)} \quad 209$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying Equations into itself 210

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 211

$$d\left(((G_{23})^{(1)},(T_{23})^{(1)}),((G_{23})^{(2)},(T_{23})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{20})^{(3)}t}\}$$

Indeed if we denote 212

Definition of $\widetilde{G_{23}},\widetilde{T_{23}}:(\widetilde{(G_{23})},\widetilde{(T_{23})})=\mathcal{A}^{(3)}((G_{23}),(T_{23}))$

It results 213

$$\left|\tilde{G}_{20}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{20})^{(3)}\left|G_{21}^{(1)}-G_{21}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\,ds_{(20)}+$$

$$\int_0^t\{(a'_{20})^{(3)}\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)\left|G_{20}^{(1)}-G_{20}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}+$$

$$G_{20}^{(2)}\left|(a''_{20})^{(3)}\left(T_{21}^{(1)},s_{(20)}\right)-(a''_{20})^{(3)}\left(T_{21}^{(2)},s_{(20)}\right)\right|\;e^{-(\widehat{M}_{20})^{(3)}s_{(20)}}e^{(\widehat{M}_{20})^{(3)}s_{(20)}}\}ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G_{23}{}^{(1)}-G_{23}{}^{(2)}\right|e^{-(\widehat{M}_{20})^{(3)}t}\leq \quad 214$$
$$\frac{1}{(\widehat{M}_{20})^{(3)}}\left((a_{20})^{(3)}+(a'_{20})^{(3)}+(\widehat{A}_{20})^{(3)}+(\widehat{P}_{20})^{(3)}(\widehat{k}_{20})^{(3)}\right)d\left(((G_{23})^{(1)},(T_{23})^{(1)};\,(G_{23})^{(2)},(T_{23})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 11: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ *and* $(\widehat{Q}_{20})^{(3)}e^{(\widehat{M}_{20})^{(3)}t}$ respectively of $\mathbb{R}_+$. 215

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(3)}$ and $(b_i'')^{(3)}, i=20,21,22$ depend only on T_{21} and respectively on

$(G_{23})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 12: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 216

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(3)} - (a_i'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(3)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2\ and\ \left((\widehat{M}_{20})^{(3)}\right)_3$: 217

Remark 13: if G_{20} is bounded, the same property have also $G_{21}\ and\ G_{22}$. indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}')^{(3)} G_{21}$ and by integrating

$$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}')^{(3)}$$

In the same way , one can obtain

$$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}')^{(3)}$$

If $G_{21}\ or\ G_{22}$ is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 14: If G_{20} *is* bounded, from below, the same property holds for $G_{21}\ and\ G_{22}$. The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 218

Remark 15: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(3)}\left((G_{23})(t), t\right) = (b_{21}')^{(3)}$ then $T_{21} \to \infty$. 219

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_i'')^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 220

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right)$, $t = log\,\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded.
The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 221

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)}+\left((\hat{P}_{24})^{(4)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{24})^{(4)} \quad 222$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{24})^{(4)}\right]\leq(\hat{Q}_{24})^{(4)} \quad 223$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself 224

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 225

$$d\left(((G_{27})^{(1)},(T_{27})^{(1)}),((G_{27})^{(2)},(T_{27})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\tilde{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{27}}),(\widetilde{T_{27}})$: $\left((\widetilde{G_{27}}),(\widetilde{T_{27}})\right)=\mathcal{A}^{(4)}((G_{27}),(T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{24})^{(4)}\left|G_{25}^{(1)}-G_{25}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)}+$$

$$\int_0^t\{(a'_{24})^{(4)}\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)\left|G_{24}^{(1)}-G_{24}^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}+$$

$$G_{24}^{(2)}\left|(a''_{24})^{(4)}\left(T_{25}^{(1)},s_{(24)}\right)-(a''_{24})^{(4)}\left(T_{25}^{(2)},s_{(24)}\right)\right|\ e^{-(\widehat{M}_{24})^{(4)}s_{(24)}}e^{(\widehat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses on Equations it follows

$$\left|(G_{27})^{(1)}-(G_{27})^{(2)}\right|e^{-(\widehat{M}_{24})^{(4)}t}\leq \quad 226$$
$$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)}+(a'_{24})^{(4)}+(\widehat{A}_{24})^{(4)}+(\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right)d\left(((G_{27})^{(1)},(T_{27})^{(1)};\ (G_{27})^{(2)},(T_{27})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 16: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ *and* $(\widehat{Q}_{24})^{(4)}e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 227

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(4)}$ and $(b_i'')^{(4)}, i=24,25,26$ depend only on T_{25} and respectively on

$(G_{27})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 17: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 228

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(4)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2\ and\ \left((\widehat{M}_{24})^{(4)}\right)_3$: 229

Remark 18: if G_{24} is bounded, the same property have also $G_{25}\ and\ G_{26}$. indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}')^{(4)} G_{25}$ and by integrating

$$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}')^{(4)}$$

In the same way , one can obtain

$$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}')^{(4)}$$

If $G_{25}\ or\ G_{26}$ is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 19: If G_{24} *is* bounded, from below, the same property holds for $G_{25}\ and\ G_{26}$. The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 230

Remark 20: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(4)}\ ((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25}\ \to \infty$. 231

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t\ > t_4$

$$(b_{25})^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 232

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded.
The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}'')^{(4)}\left((G_{27})(t), t\right) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Analogous inequalities hold also for G_{29} , $G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 233

$(\widehat{P}_{28})^{(5)}$ and $(\widehat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\widehat{P}_{28})^{(5)}+\left((\hat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{28})^{(5)} \tag{234}$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[\left((\hat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{28})^{(5)}\right]\leq(\hat{Q}_{28})^{(5)} \tag{235}$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric (236)

$$d\left(((G_{31})^{(1)},(T_{31})^{(1)}),((G_{31})^{(2)},(T_{31})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{31})},\widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})},\widetilde{(T_{31})}\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}\left|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\leq \tag{237}$$
$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(((G_{31})^{(1)},(T_{31})^{(1)};\ (G_{31})^{(2)},(T_{31})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 21: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. (238)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 22: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 239

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}(T_{29}(s_{(28)}), s_{(28)})\} ds_{(28)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(5)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1, \left((\widehat{M}_{28})^{(5)}\right)_2\ and\ \left((\widehat{M}_{28})^{(5)}\right)_3$: 240

Remark 23: if G_{28} is bounded, the same property have also $G_{29}\ and\ G_{30}$. indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\widehat{M}_{28})^{(5)}\right)_1 - (a_{29}')^{(5)} G_{29}$ and by integrating

$$G_{29} \leq \left((\widehat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\widehat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2 / (a_{30}')^{(5)}$$

If $G_{29}\ or\ G_{30}$ is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 24: If G_{28} is bounded, from below, the same property holds for $G_{29}\ and\ G_{30}$. The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 241

Remark 25: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \to \infty$. 242

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}\,(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 243

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded.
The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t), t\right) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose 244

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)}+\left((\widehat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\widehat{P}_{32})^{(6)} \tag{245}$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\widehat{Q}_{32})^{(6)}\right]\leq(\widehat{Q}_{32})^{(6)} \tag{246}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric (247)

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$

$$\int_0^t\{(a'_{32})^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$

$$G_{32}^{(2)}\left|(a''_{32})^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a''_{32})^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses it follows

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\leq \tag{248}$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 26: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ *and* $(\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. (249)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i = 32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 27: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 250

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}(T_{33}(s_{(32)}), s_{(32)})\} ds_{(32)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(6)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$: 251

Remark 28: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a_{33}')^{(6)} G_{33}$ and by integrating

$$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 + 2(a_{33})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_1 / (a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 + 2(a_{34})^{(6)} \left((\widehat{M}_{32})^{(6)}\right)_2 / (a_{34}')^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 29: If G_{32} *is* bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 252

Remark 30: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}\ ((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \to \infty$. 253

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t), t\right) < \varepsilon_6, T_{32}\,(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to 254

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded.
The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t), t(t), t\right) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations

Analogous inequalities hold also for G_{37} , $G_{38}, T_{36}, T_{37}, T_{38}$ 255

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}$, $\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}} < 1$ and to choose

$(\widehat{P}_{36})^{(7)}$ and $(\widehat{Q}_{36})^{(7)}$ large to have

$$\frac{(a_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[(\widehat{P}_{36})^{(7)}+\left((\widehat{P}_{36})^{(7)}+G_j^0\right)e^{-\left(\frac{(\widehat{P}_{36})^{(7)}+G_j^0}{G_j^0}\right)}\right]\leq(\widehat{P}_{36})^{(7)} \tag{256}$$

$$\frac{(b_i)^{(7)}}{(\widehat{M}_{36})^{(7)}}\left[\left((\widehat{Q}_{36})^{(7)}+T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{36})^{(7)}+T_j^0}{T_j^0}\right)}+(\widehat{Q}_{36})^{(7)}\right]\leq(\widehat{Q}_{36})^{(7)} \tag{257}$$

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself

The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric (258)

$$d\left(((G_{39})^{(1)},(T_{39})^{(1)}),((G_{39})^{(2)},(T_{39})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})},\widetilde{(T_{39})}:\left(\widetilde{(G_{39})},\widetilde{(T_{39})}\right)=\mathcal{A}^{(7)}((G_{39}),(T_{39}))$

It results

$$|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{36})^{(7)}|G_{37}^{(1)}-G_{37}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$
$$\int_0^t\{(a'_{36})^{(7)}|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})|G_{36}^{(1)}-G_{36}^{(2)}|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$
$$G_{36}^{(2)}|(a''_{36})^{(7)}(T_{37}^{(1)},s_{(36)})-(a''_{36})^{(7)}(T_{37}^{(2)},s_{(36)})|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on it follows

$$|(G_{39})^{(1)}-(G_{39})^{(2)}|e^{-(\widehat{M}_{36})^{(7)}t}\leq \tag{259}$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a'_{36})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 31: The fact that we supposed $(a''_{36})^{(7)}$ and $(b''_{36})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ *and* $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. (260)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i = 36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 32: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$ 261

it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{\left(-(b_i')^{(7)} t\right)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2\ and\ \left((\widehat{M}_{36})^{(7)}\right)_3$: 262

Remark 33: if G_{36} is bounded, the same property have also $G_{37}\ and\ G_{38}$. indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}')^{(7)} G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1 / (a_{37}')^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2 / (a_{38}')^{(7)}$$

If $G_{37}\ or\ G_{38}$ is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 34: If G_{36} *is* bounded, from below, the same property holds for $G_{37}\ and\ G_{38}$. The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 263

Remark 35: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(7)}\ ((G_{39})(t), t)) = (b_{37}')^{(7)}$ then $T_{37} \to \infty$. 264

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i'')^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 265

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\,\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded.
The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}'')^{(7)}\left((G_{39})(t), t\right) = (b_{38}')^{(7)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations

It is now sufficient to take $\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}$, $\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}} < 1$ and to choose $(\widehat{P}_{40})^{(8)}$ and $(\widehat{Q}_{40})^{(8)}$ large to have 266

$$\frac{(a_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[(\widehat{P}_{40})^{(8)} + \left((\hat{P}_{40})^{(8)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{40})^{(8)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{40})^{(8)} \quad 267$$

$$\frac{(b_i)^{(8)}}{(\widehat{M}_{40})^{(8)}}\left[\left((\hat{Q}_{40})^{(8)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{40})^{(8)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{40})^{(8)}\right] \leq (\hat{Q}_{40})^{(8)} \quad 268$$

In order that the operator $\mathcal{A}^{(8)}$ transforms the space of sextuples of functions G_i, T_i satisfying Equations into itself
The operator $\mathcal{A}^{(8)}$ is a contraction with respect to the metric

$$d\left(((G_{43})^{(1)}, (T_{43})^{(1)}), ((G_{43})^{(2)}, (T_{43})^{(2)})\right) = \quad 269$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{40})^{(8)}t}\}$$

Indeed if we denote 270

Definition of $\widetilde{(G_{43})}, \widetilde{(T_{43})}$: $\left(\widetilde{(G_{43})}, \widetilde{(T_{43})}\right) = \mathcal{A}^{(8)}((G_{43}), (T_{43}))$

It results 271

$$\left|\tilde{G}_{40}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{40})^{(8)}\left|G_{41}^{(1)} - G_{41}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\,ds_{(40)} +$$
$$\int_0^t\{(a'_{40})^{(8)}\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{-(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right)\left|G_{40}^{(1)} - G_{40}^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}} +$$
$$G_{40}^{(2)}\left|(a''_{40})^{(8)}\left(T_{41}^{(1)}, s_{(40)}\right) - (a''_{40})^{(8)}\left(T_{41}^{(2)}, s_{(40)}\right)\right|\ e^{-(\widehat{M}_{40})^{(8)}s_{(40)}}e^{(\widehat{M}_{40})^{(8)}s_{(40)}}\}ds_{(40)}$$

Where $s_{(40)}$ represents integrand that is integrated over the interval $[0, t]$ 272

From the hypotheses it follows

$$\left|(G_{43})^{(1)} - (G_{43})^{(2)}\right|e^{-(\widehat{M}_{40})^{(8)}t} \leq \quad 273$$
$$\frac{1}{(\widehat{M}_{40})^{(8)}}\left((a_{40})^{(8)} + (a'_{40})^{(8)} + (\widehat{A}_{40})^{(8)} + (\widehat{P}_{40})^{(8)}(\hat{k}_{40})^{(8)}\right)d\left(((G_{43})^{(1)}, (T_{43})^{(1)};\ (G_{43})^{(2)}, (T_{43})^{(2)})\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 36: The fact that we supposed $(a''_{40})^{(8)}$ and $(b''_{40})^{(8)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by 274

$(\widehat{P}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ *and* $(\widehat{Q}_{40})^{(8)}e^{(\widehat{M}_{40})^{(8)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(8)}$ and $(b_i'')^{(8)}, i = 40,41,42$ depend only on T_{41} and respectively on $(G_{43})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 37 There does not exist any t where $G_i(t) = 0\ and\ T_i(t) = 0$ 275

it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(8)} - (a_i'')^{(8)}(T_{41}(s_{(40)}), s_{(40)})\} ds_{(40)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(8)}t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{40})^{(8)}\right)_1, \left((\widehat{M}_{40})^{(8)}\right)_2\ and\ \left((\widehat{M}_{40})^{(8)}\right)_3$: 276

Remark 38: if G_{40} is bounded, the same property have also $G_{41}\ and\ G_{42}$. indeed if

$G_{40} < (\widehat{M}_{40})^{(8)}$ it follows $\frac{dG_{41}}{dt} \leq \left((\widehat{M}_{40})^{(8)}\right)_1 - (a_{41}')^{(8)}G_{41}$ and by integrating

$$G_{41} \leq \left((\widehat{M}_{40})^{(8)}\right)_2 = G_{41}^0 + 2(a_{41})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_1/(a_{41}')^{(8)}$$

In the same way , one can obtain

$$G_{42} \leq \left((\widehat{M}_{40})^{(8)}\right)_3 = G_{42}^0 + 2(a_{42})^{(8)}\left((\widehat{M}_{40})^{(8)}\right)_2/(a_{42}')^{(8)}$$

If $G_{41}\ or\ G_{42}$ is bounded, the same property follows for G_{40} , G_{42} and G_{40} , G_{41} respectively.

Remark 39: If G_{40} *is* bounded, from below, the same property holds for $G_{41}\ and\ G_{42}$. The proof is analogous with the preceding one. An analogous property is true if G_{41} is bounded from below. 277

Remark 40: If T_{40} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(8)}((G_{43})(t), t)) = (b_{41}')^{(8)}$ then $T_{41} \to \infty$. 278

Definition of $(m)^{(8)}$ and ε_8 :

Indeed let t_8 be so that for $t > t_8$

$$(b_{41})^{(8)} - (b_i'')^{(8)}\left((G_{43})(t), t\right) < \varepsilon_8, T_{40}(t) > (m)^{(8)}$$

Then $\frac{dT_{41}}{dt} \geq (a_{41})^{(8)}(m)^{(8)} - \varepsilon_8 T_{41}$ which leads to 279

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{\varepsilon_8}\right)(1 - e^{-\varepsilon_8 t}) + T_{41}^0 e^{-\varepsilon_8 t}$ If we take t such that $e^{-\varepsilon_8 t} = \frac{1}{2}$ it results

$T_{41} \geq \left(\frac{(a_{41})^{(8)}(m)^{(8)}}{2}\right)$, $t = log\frac{2}{\varepsilon_8}$ By taking now ε_8 sufficiently small one sees that T_{41} is unbounded.
The same property holds for T_{42} if $\lim_{t\to\infty}(b''_{42})^{(8)}\left((G_{43})(t), t(t), t\right) = (b'_{42})^{(8)}$

It is now sufficient to take $\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}$, $\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}} < 1$ and to choose $(\widehat{P}_{44})^{(9)}$ and $(\widehat{Q}_{44})^{(9)}$ large to have

$$\frac{(a_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[(\widehat{P}_{44})^{(9)} + \left((\widehat{P}_{44})^{(9)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{44})^{(9)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{44})^{(9)}$$

$$\frac{(b_i)^{(9)}}{(\widehat{M}_{44})^{(9)}}\left[\left((\widehat{Q}_{44})^{(9)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{44})^{(9)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{44})^{(9)}\right] \leq (\widehat{Q}_{44})^{(9)}$$

In order that the operator $\mathcal{A}^{(9)}$ transforms the space of sextuples of functions G_i , T_i satisfying 39,35,36 into itself
The operator $\mathcal{A}^{(9)}$ is a contraction with respect to the metric

$$d\left(((G_{47})^{(1)}, (T_{47})^{(1)}), ((G_{47})^{(2)}, (T_{47})^{(2)})\right) =$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{44})^{(9)}t}\}$$

Indeed if we denote

Definition of $(\widetilde{G_{47}}), (\widetilde{T_{47}}) : \left((\widetilde{G_{47}}), (\widetilde{T_{47}})\right) = \mathcal{A}^{(9)}\left((G_{47}), (T_{47})\right)$
It results

$$\left|\tilde{G}_{44}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{44})^{(9)}\left|G_{45}^{(1)} - G_{45}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\,ds_{(44)} +$$
$$\int_0^t\{(a'_{44})^{(9)}\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{-(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right)\left|G_{44}^{(1)} - G_{44}^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}} +$$
$$G_{44}^{(2)}\left|(a''_{44})^{(9)}\left(T_{45}^{(1)}, s_{(44)}\right) - (a''_{44})^{(9)}\left(T_{45}^{(2)}, s_{(44)}\right)\right|\ e^{-(\widehat{M}_{44})^{(9)}s_{(44)}}e^{(\widehat{M}_{44})^{(9)}s_{(44)}}\}ds_{(44)}$$

Where $s_{(44)}$ represents integrand that is integrated over the interval [0, t]

From the hypotheses on 45,46,47,28 and 29 it follows

$$\left|(G_{47})^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{44})^{(9)}t} \leq$$
$$\frac{1}{(\widehat{M}_{44})^{(9)}}\left((a_{44})^{(9)} + (a'_{44})^{(9)} + (\widehat{A}_{44})^{(9)} + (\widehat{P}_{44})^{(9)}(\widehat{k}_{44})^{(9)}\right)d\left(((G_{47})^{(1)}, (T_{47})^{(1)};\ (G_{47})^{(2)}, (T_{47})^{(2)})\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis (39,35,36) the result follows

Remark 41: The fact that we supposed $(a''_{44})^{(9)}$ and $(b''_{44})^{(9)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}\ and\ (\widehat{Q}_{44})^{(9)}e^{(\widehat{M}_{44})^{(9)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(9)}$ and $(b_i'')^{(9)}, i = 44,45,46$ depend only on T_{45} and respectively on $(G_{47})(and\ not\ on\ \ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 42: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 99 to 44 it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(9)} - (a_i'')^{(9)}(T_{45}(s_{(44)}), s_{(44)})\} ds_{(44)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i')^{(9)} t)} > 0 \quad \text{for t} > 0$$

Definition of $\left((\widehat{M}_{44})^{(9)}\right)_1, \left((\widehat{M}_{44})^{(9)}\right)_2\ and\ \left((\widehat{M}_{44})^{(9)}\right)_3$:

Remark 43: if G_{44} is bounded, the same property have also $G_{45}\ and\ G_{46}$. indeed if

$G_{44} < (\widehat{M}_{44})^{(9)}$ it follows $\frac{dG_{45}}{dt} \leq \left((\widehat{M}_{44})^{(9)}\right)_1 - (a_{45}')^{(9)} G_{45}$ and by integrating

$$G_{45} \leq \left((\widehat{M}_{44})^{(9)}\right)_2 = G_{45}^0 + 2(a_{45})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_1 / (a_{45}')^{(9)}$$

In the same way , one can obtain

$$G_{46} \leq \left((\widehat{M}_{44})^{(9)}\right)_3 = G_{46}^0 + 2(a_{46})^{(9)}\left((\widehat{M}_{44})^{(9)}\right)_2 / (a_{46}')^{(9)}$$

If $G_{45}\ or\ G_{46}$ is bounded, the same property follows for G_{44} , G_{46} and G_{44} , G_{45} respectively.

Remark 44: If G_{44} *is* bounded, from below, the same property holds for $G_{45}\ and\ G_{46}$. The proof is analogous with the preceding one. An analogous property is true if G_{45} is bounded from below.

Remark 45: If T_{44} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(9)}\left((G_{47})(t), t\right)) = (b_{45}')^{(9)}$ then $T_{45} \to \infty$.

Definition of $(m)^{(9)}$ and ε_9 :

Indeed let t_9 be so that for $t > t_9$

$$(b_{45})^{(9)} - (b_i'')^{(9)}\left((G_{47})(t), t\right) < \varepsilon_9, T_{44}\,(t) > (m)^{(9)}$$

Then $\frac{dT_{45}}{dt} \geq (a_{45})^{(9)}(m)^{(9)} - \varepsilon_9 T_{45}$ which leads to

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{\varepsilon_9}\right)(1 - e^{-\varepsilon_9 t}) + T_{45}^0 e^{-\varepsilon_9 t}$ If we take t such that $e^{-\varepsilon_9 t} = \frac{1}{2}$ it results

$T_{45} \geq \left(\frac{(a_{45})^{(9)}(m)^{(9)}}{2}\right), \quad t = log\frac{2}{\varepsilon_9}$ By taking now ε_9 sufficiently small one sees that T_{45} is unbounded.
The same property holds for T_{46} if $\lim_{t\to\infty}(b_{46}'')^{(9)}\left((G_{47})(t), t\right) = (b_{46}')^{(9)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 92

Behavior of the solutions of equation 280

Theorem If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying
$-(\sigma_2)^{(1)} \le -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \le -(\sigma_1)^{(1)}$

$-(\tau_2)^{(1)} \le -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \le -(\tau_1)^{(1)}$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$: 281

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{\nu}_1)^{(1)}, , (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 282

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations
$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:- 283

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by
$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}, \ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)},$

and $\boxed{(\nu_0)^{(1)} = \frac{G^0_{13}}{G^0_{14}}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)}, \ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$

and analogously 284

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$

and $\boxed{(u_0)^{(1)} = \frac{T^0_{13}}{T^0_{14}}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$

are defined

Then the solution of global equations satisfies the inequalities 285

$G^0_{13} e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G^0_{13} e^{(S_1)^{(1)} t}$

where $(p_i)^{(1)}$ is defined by equation

$\frac{1}{(m_1)^{(1)}} G^0_{13} e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G^0_{13} e^{(S_1)^{(1)} t}$

$$\left(\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_1)^{(1)}((S_1)^{(1)}-(p_{13})^{(1)}-(S_2)^{(1)})}\left[e^{((S_1)^{(1)}-(p_{13})^{(1)})t}-e^{-(S_2)^{(1)}t}\right]+G_{15}^{0}e^{-(S_2)^{(1)}t}\le G_{15}(t)\le\right. \quad (286)$$

$$\left.\frac{(a_{15})^{(1)}G_{13}^{0}}{(m_2)^{(1)}((S_1)^{(1)}-(a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t}-e^{-(a'_{15})^{(1)}t}\right]+G_{15}^{0}e^{-(a'_{15})^{(1)}t}\right)$$

$$\boxed{T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t}} \quad (287)$$

$$\frac{1}{(\mu_1)^{(1)}}T_{13}^{0}e^{(R_1)^{(1)}t}\le T_{13}(t)\le\frac{1}{(\mu_2)^{(1)}}T_{13}^{0}e^{((R_1)^{(1)}+(r_{13})^{(1)})t} \quad (288)$$

$$\frac{(b_{15})^{(1)}T_{13}^{0}}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t}-e^{-(b'_{15})^{(1)}t}\right]+T_{15}^{0}e^{-(b'_{15})^{(1)}t}\le T_{15}(t)\le \quad (289)$$

$$\frac{(a_{15})^{(1)}T_{13}^{0}}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})}\left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t}-e^{-(R_2)^{(1)}t}\right]+T_{15}^{0}e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)},(S_2)^{(1)},(R_1)^{(1)},(R_2)^{(1)}$:- (290)

Where $(S_1)^{(1)}=(a_{13})^{(1)}(m_2)^{(1)}-(a'_{13})^{(1)}$

$(S_2)^{(1)}=(a_{15})^{(1)}-(p_{15})^{(1)}$

$(R_1)^{(1)}=(b_{13})^{(1)}(\mu_2)^{(1)}-(b'_{13})^{(1)}$

$(R_2)^{(1)}=(b'_{15})^{(1)}-(r_{15})^{(1)}$

Behavior of the solutions of equation (291)

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$: (292)

$(\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)}\le-(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(T_{17},t)+(a''_{17})^{(2)}(T_{17},t)\le-(\sigma_1)^{(2)} \quad (293)$$

$$-(\tau_2)^{(2)}\le-(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\big((G_{19}),t\big)-(b''_{17})^{(2)}\big((G_{19}),t\big)\le-(\tau_1)^{(2)} \quad (294)$$

Definition of $(\nu_1)^{(2)},(\nu_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: (295)

By $(\nu_1)^{(2)}>0$, $(\nu_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0$, $(u_2)^{(2)}<0$ the roots (296)

of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_1)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ (297)

and $(b_{14})^{(2)}\big(u^{(2)}\big)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and (298)

Definition of $(\bar{\nu}_1)^{(2)},,(\bar{\nu}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: (299)

By $(\bar{\nu}_1)^{(2)}>0$, $(\bar{\nu}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0$, $(\bar{u}_2)^{(2)}<0$ the (300)

roots of the equations $(a_{17})^{(2)}\big(\nu^{(2)}\big)^2+(\sigma_2)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ (301)

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 302

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 303

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 304

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)}$, ***if*** $(\nu_0)^{(2)} < (\nu_1)^{(2)}$ 305

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}$, ***if*** $(\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, 306

and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}$, ***if*** $(\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ 307

and analogously 308

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}$, ***if*** $(u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}$, ***if*** $(u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}$,

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}$, ***if*** $(\bar{u}_1)^{(2)} < (u_0)^{(2)}$ 309

Then the solution of global equations satisfies the inequalities 310

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined by equation

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ 311

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 312

$\left.\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t}\right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}\right)$

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ 313

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \le T_{16}(t) \le \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ 314

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t}\right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \le T_{18}(t) \le$ 315

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t}\right] + T_{18}^0 e^{-(R_2)^{(2)}t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 316

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 317

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$ 318

$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$

Behavior of the solutions 319

Theorem 3: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \le -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \le -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \le -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G_{23}, t) - (b''_{21})^{(3)}\big((G_{23}), t\big) \le -(\tau_1)^{(3)}$$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 320

By $(\nu_1)^{(3)} > 0$, $(\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0$, $(u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0$, $(\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0$, $(\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}\big(\nu^{(3)}\big)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\big(u^{(3)}\big)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 321

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}$, ***if*** $(\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}$, ***if*** $(\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}$, ***if*** $(\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 322

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}$, ***if*** $(u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}$, ***if*** $(u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}$, and $\boxed{(u_0)^{(3)} = \frac{T^0_{20}}{T^0_{21}}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of global equations satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{20}(t) \le G_{20}^0 e^{(S_1)^{(3)} t}$$

$(p_i)^{(3)}$ is defined by equation

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \le G_{21}(t) \le \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)} t} \quad 323$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})}\left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)} t}\right] + G_{22}^0 e^{-(S_2)^{(3)} t} \le G_{22}(t) \le \right. \quad 324$$

$$\left. \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})}\left[e^{(S_1)^{(3)} t} - e^{-(a'_{22})^{(3)} t}\right] + G_{22}^0 e^{-(a'_{22})^{(3)} t}\right)$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 325$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)} t} \le T_{20}(t) \le \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 326$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})}\left[e^{(R_1)^{(3)} t} - e^{-(b'_{22})^{(3)} t}\right] + T_{22}^0 e^{-(b'_{22})^{(3)} t} \le T_{22}(t) \le \quad 327$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})}\left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)} t}\right] + T_{22}^0 e^{-(R_2)^{(3)} t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 328

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$$

$$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$$

Behavior of the solutions of equation

Theorem: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \le -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \le -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \le -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}\big((G_{27}), t\big) - (b''_{25})^{(4)}\big((G_{27}), t\big) \le -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$: 329

By $(v_1)^{(4)} > 0$, $(v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0$, $(u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\big(v^{(4)}\big)^2 + (\sigma_1)^{(4)} v^{(4)} - (a_{24})^{(4)} = 0$

and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)},,(\bar{v}_2)^{(4)},(\bar{u}_1)^{(4)},(\bar{u}_2)^{(4)}$: 330

By $(\bar{v}_1)^{(4)} > 0$, $(\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$ and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}$, $(\mu_2)^{(4)}$, $(v_0)^{(4)}$:-

If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}$, $(\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, ***if*** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, ***if*** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,
and $\boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, ***if*** $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

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$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, ***if*** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, ***if*** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,
and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, ***if*** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

Then the solution of global equations satisfies the inequalities

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$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{24}(t) \le G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \le G_{25}(t) \le \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t} \quad 333$$

$$\left(\frac{(a_{26})^{(4)}G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})}\left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \le G_{26}(t) \le \right. \quad 334$$

$$\left. \frac{(a_{26})^{(4)}G_{24}^0}{(m_2)^{(4)}((S_1)^{(4)} - (a'_{26})^{(4)})}\left[e^{(S_1)^{(4)}t} - e^{-(a'_{26})^{(4)}t}\right] + G_{26}^0 e^{-(a'_{26})^{(4)}t}\right)$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)}t} \le T_{24}(t) \le \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 335$$

$$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b'_{26})^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t}\right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \le T_{26}(t) \le \quad 336$$

$$\frac{(a_{26})^{(4)}T_{24}^{0}}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t}-e^{-(R_2)^{(4)}t}\right]+T_{26}^{0}e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)},(S_2)^{(4)},(R_1)^{(4)},(R_2)^{(4)}$:- 337

Where $(S_1)^{(4)}=(a_{24})^{(4)}(m_2)^{(4)}-(a'_{24})^{(4)}$

$(S_2)^{(4)}=(a_{26})^{(4)}-(p_{26})^{(4)}$

$(R_1)^{(4)}=(b_{24})^{(4)}(\mu_2)^{(4)}-(b'_{24})^{(4)}$

$(R_2)^{(4)}=(b'_{26})^{(4)}-(r_{26})^{(4)}$

Behavior of the solutions of equation 338

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)},(\sigma_2)^{(5)},(\tau_1)^{(5)},(\tau_2)^{(5)}$:

$(\sigma_1)^{(5)},(\sigma_2)^{(5)},(\tau_1)^{(5)},(\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)}\le-(a'_{28})^{(5)}+(a'_{29})^{(5)}-(a''_{28})^{(5)}(T_{29},t)+(a''_{29})^{(5)}(T_{29},t)\le-(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)}\le-(b'_{28})^{(5)}+(b'_{29})^{(5)}-(b''_{28})^{(5)}\big((G_{31}),t\big)-(b''_{29})^{(5)}\big((G_{31}),t\big)\le-(\tau_1)^{(5)}$$

Definition of $(\nu_1)^{(5)},(\nu_2)^{(5)},(u_1)^{(5)},(u_2)^{(5)},\nu^{(5)},u^{(5)}$: 339

By $(\nu_1)^{(5)}>0$, $(\nu_2)^{(5)}<0$ and respectively $(u_1)^{(5)}>0$, $(u_2)^{(5)}<0$ the roots of the equations $(a_{29})^{(5)}\left(\nu^{(5)}\right)^2+(\sigma_1)^{(5)}\nu^{(5)}-(a_{28})^{(5)}=0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2+(\tau_1)^{(5)}u^{(5)}-(b_{28})^{(5)}=0$ and

Definition of $(\bar{\nu}_1)^{(5)},,(\bar{\nu}_2)^{(5)},(\bar{u}_1)^{(5)},(\bar{u}_2)^{(5)}$: 340

By $(\bar{\nu}_1)^{(5)}>0$, $(\bar{\nu}_2)^{(5)}<0$ and respectively $(\bar{u}_1)^{(5)}>0$, $(\bar{u}_2)^{(5)}<0$ the roots of the equations $(a_{29})^{(5)}\left(\nu^{(5)}\right)^2+(\sigma_2)^{(5)}\nu^{(5)}-(a_{28})^{(5)}=0$
and $(b_{29})^{(5)}\left(u^{(5)}\right)^2+(\tau_2)^{(5)}u^{(5)}-(b_{28})^{(5)}=0$

Definition of $(m_1)^{(5)},(m_2)^{(5)},(\mu_1)^{(5)},(\mu_2)^{(5)},(\nu_0)^{(5)}$:-

If we define $(m_1)^{(5)},(m_2)^{(5)},(\mu_1)^{(5)},(\mu_2)^{(5)}$ by

$(m_2)^{(5)}=(\nu_0)^{(5)},(m_1)^{(5)}=(\nu_1)^{(5)}$, $\boldsymbol{if}\ (\nu_0)^{(5)}<(\nu_1)^{(5)}$

$(m_2)^{(5)}=(\nu_1)^{(5)},(m_1)^{(5)}=(\bar{\nu}_1)^{(5)}$, $\boldsymbol{if}\ (\nu_1)^{(5)}<(\nu_0)^{(5)}<(\bar{\nu}_1)^{(5)}$,
and $\boxed{(\nu_0)^{(5)}=\frac{G_{28}^{0}}{G_{29}^{0}}}$

$(m_2)^{(5)}=(\nu_1)^{(5)},(m_1)^{(5)}=(\nu_0)^{(5)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(5)}<(\nu_0)^{(5)}$

and analogously 341

$(\mu_2)^{(5)}=(u_0)^{(5)},(\mu_1)^{(5)}=(u_1)^{(5)}$, $\boldsymbol{if}\ (u_0)^{(5)}<(u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, $\boldsymbol{if}$ $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$,

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if}$ $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$

Then the solution of global equations satisfies the inequalities 342

$$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{28}(t) \leq G_{28}^0 e^{(S_1)^{(5)} t}$$

where $(p_i)^{(5)}$ is defined by equation

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{29}(t) \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t} \quad 343$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)} t}\right] + G_{30}^0 e^{-(S_2)^{(5)} t} \leq G_{30}(t) \leq \right. \quad 344$$

$$\left. \frac{(a_{30})^{(5)} G_{28}^0}{(m_2)^{(5)}((S_1)^{(5)} - (a'_{30})^{(5)})}\left[e^{(S_1)^{(5)} t} - e^{-(a'_{30})^{(5)} t}\right] + G_{30}^0 e^{-(a'_{30})^{(5)} t}\right)$$

$$\boxed{T_{28}^0 e^{(R_1)^{(5)} t} \leq T_{28}(t) \leq T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}} \quad 345$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)} t} \leq T_{28}(t) \leq \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t} \quad 346$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b'_{30})^{(5)})}\left[e^{(R_1)^{(5)} t} - e^{-(b'_{30})^{(5)} t}\right] + T_{30}^0 e^{-(b'_{30})^{(5)} t} \leq T_{30}(t) \leq \quad 347$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)} t}\right] + T_{30}^0 e^{-(R_2)^{(5)} t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 348

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$

$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions of equation 349

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

$(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \leq -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \leq -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \leq -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\big((G_{35}), t\big) - (b''_{33})^{(6)}\big((G_{35}), t\big) \leq -(\tau_1)^{(6)}$$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 350

By $(\nu_1)^{(6)} > 0$, $(\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0$, $(u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 351

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$
and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}$, $(m_2)^{(6)}$, $(\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

If we define $(m_1)^{(6)}$, $(m_2)^{(6)}$, $(\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)}$, $\boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}$, $\boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)}$,
and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 352

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}$, $\boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}$, $\boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}$,
and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

Then the solution of global equations satisfies the inequalities 353

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad (354)$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le \right. \quad (355)$$

$$\left. \frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)} - (a'_{34})^{(6)})}\left[e^{(S_1)^{(6)}t} - e^{-(a'_{34})^{(6)}t}\right] + G_{34}^0 e^{-(a'_{34})^{(6)}t}\right)$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad (356)$$

$$\frac{1}{(\mu_1)^{(6)}}T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}}T_{32}^0 e^{((R_1)^{(6)}+(r_{32})^{(6)})t} \quad 357$$

$$\frac{(b_{34})^{(6)}T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le \quad 358$$

$$\frac{(a_{34})^{(6)}T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 359

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

Behavior of the solutions of equation

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

$(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \le -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \le -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \le -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \le -(\tau_1)^{(7)}$$

Definition of $(v_1)^{(7)}, (v_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, v^{(7)}, u^{(7)}$: 361

By $(v_1)^{(7)} > 0, (v_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{v}_1)^{(7)}, , (\bar{v}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 362

By $(\bar{v}_1)^{(7)} > 0, (\bar{v}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the roots of the equations $(a_{37})^{(7)}\big(v^{(7)}\big)^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$
and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$:-

If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}$, ***if*** $(v_0)^{(7)} < (v_1)^{(7)}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}$, ***if*** $(v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)}$,

and $\boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)},\ \boldsymbol{if}\ (\bar{v}_1)^{(7)} < (v_0)^{(7)}$

and analogously 363

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)},\ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

Then the solution of global equations satisfies the inequalities 364

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)} t}$$

where $(p_i)^{(7)}$ is defined by equation

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)} t} \quad 365$$

$$\left(\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})}\left[e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)} t}\right] + G_{38}^0 e^{-(S_2)^{(7)} t} \le G_{38}(t) \le \right. \quad 366$$

$$\left. \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})}\left[e^{(S_1)^{(7)} t} - e^{-(a'_{38})^{(7)} t}\right] + G_{38}^0 e^{-(a'_{38})^{(7)} t}\right)$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}} \quad 367$$

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)} t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t} \quad 368$$

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})}\left[e^{(R_1)^{(7)} t} - e^{-(b'_{38})^{(7)} t}\right] + T_{38}^0 e^{-(b'_{38})^{(7)} t} \le T_{38}(t) \le \quad 369$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})}\left[e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)} t}\right] + T_{38}^0 e^{-(R_2)^{(7)} t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 370

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)}$$

Behavior of the solutions of equation 371

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$:

$(\sigma_1)^{(8)}, (\sigma_2)^{(8)}, (\tau_1)^{(8)}, (\tau_2)^{(8)}$ four constants satisfying

$$-(\sigma_2)^{(8)} \le -(a'_{40})^{(8)} + (a'_{41})^{(8)} - (a''_{40})^{(8)}(T_{41}, t) + (a''_{41})^{(8)}(T_{41}, t) \le -(\sigma_1)^{(8)}$$

$$-(\tau_2)^{(8)} \le -(b'_{40})^{(8)} + (b'_{41})^{(8)} - (b''_{40})^{(8)}\big((G_{43}), t\big) - (b''_{41})^{(8)}\big((G_{43}), t\big) \le -(\tau_1)^{(8)}$$

Definition of $(\nu_1)^{(8)}, (\nu_2)^{(8)}, (u_1)^{(8)}, (u_2)^{(8)}, \nu^{(8)}, u^{(8)}$: 372

By $(\nu_1)^{(8)} > 0, (\nu_2)^{(8)} < 0$ and respectively $(u_1)^{(8)} > 0, (u_2)^{(8)} < 0$ the roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$
and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_1)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$ and

Definition of $(\bar{\nu}_1)^{(8)}, , (\bar{\nu}_2)^{(8)}, (\bar{u}_1)^{(8)}, (\bar{u}_2)^{(8)}$:

By $(\bar{\nu}_1)^{(8)} > 0, (\bar{\nu}_2)^{(8)} < 0$ and respectively $(\bar{u}_1)^{(8)} > 0, (\bar{u}_2)^{(8)} < 0$ the

roots of the equations $(a_{41})^{(8)}\big(\nu^{(8)}\big)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)} = 0$

and $(b_{41})^{(8)}\big(u^{(8)}\big)^2 + (\tau_2)^{(8)}u^{(8)} - (b_{40})^{(8)} = 0$

Definition of $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}, (\nu_0)^{(8)}$:-

If we define $(m_1)^{(8)}, (m_2)^{(8)}, (\mu_1)^{(8)}, (\mu_2)^{(8)}$ by

$$(m_2)^{(8)} = (\nu_0)^{(8)}, (m_1)^{(8)} = (\nu_1)^{(8)}, \ \boldsymbol{if}\ (\nu_0)^{(8)} < (\nu_1)^{(8)}$$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\bar{\nu}_1)^{(8)}, \boldsymbol{if}\ (\nu_1)^{(8)} < (\nu_0)^{(8)} < (\bar{\nu}_1)^{(8)},$$

and $\boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}}$

$$(m_2)^{(8)} = (\nu_1)^{(8)}, (m_1)^{(8)} = (\nu_0)^{(8)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(8)} < (\nu_0)^{(8)}$$

and analogously 374

$$(\mu_2)^{(8)} = (u_0)^{(8)}, (\mu_1)^{(8)} = (u_1)^{(8)}, \ \boldsymbol{if}\ (u_0)^{(8)} < (u_1)^{(8)}$$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (\bar{u}_1)^{(8)}, \boldsymbol{if}\ (u_1)^{(8)} < (u_0)^{(8)} < (\bar{u}_1)^{(8)},$

and $\boxed{(u_0)^{(8)} = \frac{T_{40}^0}{T_{41}^0}}$

$(\mu_2)^{(8)} = (u_1)^{(8)}, (\mu_1)^{(8)} = (u_0)^{(8)}, \boldsymbol{if}\ (\bar{u}_1)^{(8)} < (u_0)^{(8)}$ where $(u_1)^{(8)}, (\bar{u}_1)^{(8)}$

Then the solution of global equations satisfies the inequalities 375

$$G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{40}(t) \le G_{40}^0 e^{(S_1)^{(8)} t}$$

where $(p_i)^{(8)}$ is defined by equation

$$\frac{1}{(m_1)^{(8)}} G_{40}^0 e^{((S_1)^{(8)} - (p_{40})^{(8)})t} \le G_{41}(t) \le \frac{1}{(m_2)^{(8)}} G_{40}^0 e^{(S_1)^{(8)} t} \quad 376$$

$$\left(\frac{(a_{42})^{(8)} G_{40}^0}{(m_1)^{(8)}((S_1)^{(8)} - (p_{40})^{(8)} - (S_2)^{(8)})}\left[e^{((S_1)^{(8)} - (p_{40})^{(8)})t} - e^{-(S_2)^{(8)} t}\right] + G_{42}^0 e^{-(S_2)^{(8)} t} \le G_{42}(t) \le \right. \quad 377$$

$$\left. \frac{(a_{42})^{(8)} G_{40}^0}{(m_2)^{(8)}((S_1)^{(8)} - (a'_{42})^{(8)})}\left[e^{(S_1)^{(8)} t} - e^{-(a'_{42})^{(8)} t}\right] + G_{42}^0 e^{-(a'_{42})^{(8)} t}\right)$$

$$\boxed{T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t}} \quad 378$$

$$\frac{1}{(\mu_1)^{(8)}} T_{40}^0 e^{(R_1)^{(8)} t} \le T_{40}(t) \le \frac{1}{(\mu_2)^{(8)}} T_{40}^0 e^{((R_1)^{(8)} + (r_{40})^{(8)})t} \quad 379$$

$$\frac{(b_{42})^{(8)} T_{40}^0}{(\mu_1)^{(8)}((R_1)^{(8)} - (b'_{42})^{(8)})}\left[e^{(R_1)^{(8)} t} - e^{-(b'_{42})^{(8)} t}\right] + T_{42}^0 e^{-(b'_{42})^{(8)} t} \le T_{42}(t) \le \quad 380$$

$$\frac{(a_{42})^{(8)} T_{40}^0}{(\mu_2)^{(8)}((R_1)^{(8)} + (r_{40})^{(8)} + (R_2)^{(8)})}\left[e^{((R_1)^{(8)} + (r_{40})^{(8)})t} - e^{-(R_2)^{(8)} t}\right] + T_{42}^0 e^{-(R_2)^{(8)} t}$$

Definition of $(S_1)^{(8)}, (S_2)^{(8)}, (R_1)^{(8)}, (R_2)^{(8)}$:- 381

Where $(S_1)^{(8)} = (a_{40})^{(8)}(m_2)^{(8)} - (a'_{40})^{(8)}$

$$(S_2)^{(8)} = (a_{42})^{(8)} - (p_{42})^{(8)}$$

$$(R_1)^{(8)} = (b_{40})^{(8)}(\mu_2)^{(8)} - (b'_{40})^{(8)}$$

$$(R_2)^{(8)} = (b'_{42})^{(8)} - (r_{42})^{(8)}$$

Behavior of the solutions of equation 37 to 92 382

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$:

$(\sigma_1)^{(9)}, (\sigma_2)^{(9)}, (\tau_1)^{(9)}, (\tau_2)^{(9)}$ four constants satisfying

$$-(\sigma_2)^{(9)} \le -(a'_{44})^{(9)} + (a'_{45})^{(9)} - (a''_{44})^{(9)}(T_{45}, t) + (a''_{45})^{(9)}(T_{45}, t) \le -(\sigma_1)^{(9)}$$

$$-(\tau_2)^{(9)} \le -(b'_{44})^{(9)} + (b'_{45})^{(9)} - (b''_{44})^{(9)}\big((G_{47}),t\big) - (b''_{45})^{(9)}\big((G_{47}),t\big) \le -(\tau_1)^{(9)}$$

Definition of $(\nu_1)^{(9)}, (\nu_2)^{(9)}, (u_1)^{(9)}, (u_2)^{(9)}, \nu^{(9)}, u^{(9)}$:

By $(\nu_1)^{(9)} > 0$, $(\nu_2)^{(9)} < 0$ and respectively $(u_1)^{(9)} > 0$, $(u_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_1)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_1)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$ and

Definition of $(\bar{\nu}_1)^{(9)},, (\bar{\nu}_2)^{(9)}, (\bar{u}_1)^{(9)}, (\bar{u}_2)^{(9)}$:

By $(\bar{\nu}_1)^{(9)} > 0$, $(\bar{\nu}_2)^{(9)} < 0$ and respectively $(\bar{u}_1)^{(9)} > 0$, $(\bar{u}_2)^{(9)} < 0$ the roots of the equations $(a_{45})^{(9)}\big(\nu^{(9)}\big)^2 + (\sigma_2)^{(9)}\nu^{(9)} - (a_{44})^{(9)} = 0$
and $(b_{45})^{(9)}\big(u^{(9)}\big)^2 + (\tau_2)^{(9)}u^{(9)} - (b_{44})^{(9)} = 0$

Definition of $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}, (\nu_0)^{(9)}$:-

If we define $(m_1)^{(9)}$, $(m_2)^{(9)}$, $(\mu_1)^{(9)}, (\mu_2)^{(9)}$ by

$(m_2)^{(9)} = (\nu_0)^{(9)}, (m_1)^{(9)} = (\nu_1)^{(9)}$, $\boldsymbol{if}\ (\nu_0)^{(9)} < (\nu_1)^{(9)}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\bar{\nu}_1)^{(9)}$, $\boldsymbol{if}\ (\nu_1)^{(9)} < (\nu_0)^{(9)} < (\bar{\nu}_1)^{(9)}$,
and $\boxed{(\nu_0)^{(9)} = \frac{G^0_{44}}{G^0_{45}}}$

$(m_2)^{(9)} = (\nu_1)^{(9)}, (m_1)^{(9)} = (\nu_0)^{(9)}$, $\boldsymbol{if}\ (\bar{\nu}_1)^{(9)} < (\nu_0)^{(9)}$

and analogously

$(\mu_2)^{(9)} = (u_0)^{(9)}, (\mu_1)^{(9)} = (u_1)^{(9)}$, $\boldsymbol{if}\ (u_0)^{(9)} < (u_1)^{(9)}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (\bar{u}_1)^{(9)}$, $\boldsymbol{if}\ (u_1)^{(9)} < (u_0)^{(9)} < (\bar{u}_1)^{(9)}$,
and $\boxed{(u_0)^{(9)} = \frac{T^0_{44}}{T^0_{45}}}$

$(\mu_2)^{(9)} = (u_1)^{(9)}, (\mu_1)^{(9)} = (u_0)^{(9)}, \boldsymbol{if}\ (\bar{u}_1)^{(9)} < (u_0)^{(9)}$ where $(u_1)^{(9)}, (\bar{u}_1)^{(9)}$ are defined by 59 and 69 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G^0_{44}e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{44}(t) \le G^0_{44}e^{(S_1)^{(9)}t}$$

where $(p_i)^{(9)}$ is defined by equation 45

$$\frac{1}{(m_9)^{(9)}}G^0_{44}e^{((S_1)^{(9)} - (p_{44})^{(9)})t} \le G_{45}(t) \le \frac{1}{(m_2)^{(9)}}G^0_{44}e^{(S_1)^{(9)}t}$$

(

$$\frac{(a_{46})^{(9)}G^0_{44}}{(m_1)^{(9)}((S_1)^{(9)} - (p_{44})^{(9)} - (S_2)^{(9)})}\Big[e^{((S_1)^{(9)} - (p_{44})^{(9)})t} - e^{-(S_2)^{(9)}t}\Big] + G^0_{46}e^{-(S_2)^{(9)}t} \le G_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}G^0_{44}}{(m_2)^{(9)}((S_1)^{(9)} - (a'_{46})^{(9)})}[e^{(S_1)^{(9)}t} - e^{-(a'_{46})^{(9)}t}] + G^0_{46}e^{-(a'_{46})^{(9)}t})$$

$$\boxed{T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}}$$

$$\frac{1}{(\mu_1)^{(9)}} T_{44}^0 e^{(R_1)^{(9)}t} \le T_{44}(t) \le \frac{1}{(\mu_2)^{(9)}} T_{44}^0 e^{((R_1)^{(9)}+(r_{44})^{(9)})t}$$

$$\frac{(b_{46})^{(9)}T_{44}^0}{(\mu_1)^{(9)}((R_1)^{(9)}-(b'_{46})^{(9)})}\left[e^{(R_1)^{(9)}t} - e^{-(b'_{46})^{(9)}t}\right] + T_{46}^0 e^{-(b'_{46})^{(9)}t} \le T_{46}(t) \le$$

$$\frac{(a_{46})^{(9)}T_{44}^0}{(\mu_2)^{(9)}((R_1)^{(9)}+(r_{44})^{(9)}+(R_2)^{(9)})}\left[e^{((R_1)^{(9)}+(r_{44})^{(9)})t} - e^{-(R_2)^{(9)}t}\right] + T_{46}^0 e^{-(R_2)^{(9)}t}$$

Definition of $(S_1)^{(9)}, (S_2)^{(9)}, (R_1)^{(9)}, (R_2)^{(9)}$:-

Where $(S_1)^{(9)} = (a_{44})^{(9)}(m_2)^{(9)} - (a'_{44})^{(9)}$

$(S_2)^{(9)} = (a_{46})^{(9)} - (p_{46})^{(9)}$

$(R_1)^{(9)} = (b_{44})^{(9)}(\mu_2)^{(9)} - (b'_{44})^{(9)}$

$(R_2)^{(9)} = (b'_{46})^{(9)} - (r_{46})^{(9)}$

Proof : From global equations we obtain 383

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$-\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)}\right) \le \frac{dv^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(v^{(1)}\right)^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \ge \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((v_1)^{(1)}-(v_0)^{(1)}\right)t\right]}} \quad , \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)}-(v_0)^{(1)}}{(v_0)^{(1)}-(v_2)^{(1)}}}$$

it follows $(v_0)^{(1)} \le v^{(1)}(t) \le (v_1)^{(1)}$

In the same manner , we get 384

$$\nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)}-(\nu_0)^{(1)}}{(\nu_0)^{(1)}-(\bar{\nu}_2)^{(1)}}}$$

From which we deduce $(\nu_0)^{(1)} \leq \nu^{(1)}(t) \leq (\bar{\nu}_1)^{(1)}$

If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case, 385

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain 386

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From global equations we obtain 387

$$\frac{dv^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)\right) - (a''_{17})^{(2)}(T_{17}, t)v^{(2)} - (a_{17})^{(2)}v^{(2)}$$

Definition of $v^{(2)}$:- $\boxed{v^{(2)} = \frac{G_{16}}{G_{17}}}$ 388

It follows 389

$$-\left((a_{17})^{(2)}\left(v^{(2)}\right)^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)}\right) \leq \frac{dv^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(v^{(2)}\right)^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains 390

Definition of $(\bar{v}_1)^{(2)}, (v_0)^{(2)}$:-

For $0 < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (v_1)^{(2)} < (\bar{v}_1)^{(2)}$

$$v^{(2)}(t) \geq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_0)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(v_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (v_2)^{(2)}}}$$

it follows $(v_0)^{(2)} \leq v^{(2)}(t) \leq (v_1)^{(2)}$

In the same manner , we get 391

$$v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{v}_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (\bar{v}_2)^{(2)}}}$$

From which we deduce $(v_0)^{(2)} \leq v^{(2)}(t) \leq (\bar{v}_1)^{(2)}$ 392

If $0 < (v_1)^{(2)} < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{v}_1)^{(2)}$ we find like in the previous case, 393

$$(v_1)^{(2)} \leq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_2)^{(2)}\right)t\right]}}{1 + (C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((v_1)^{(2)} - (v_2)^{(2)}\right)t\right]}} \leq v^{(2)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \leq (\bar{v}_1)^{(2)}$$

If $0 < (v_1)^{(2)} \leq (\bar{v}_1)^{(2)} \leq (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 394

$$(v_1)^{(2)} \leq v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}}{1 + (\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}\right)t\right]}} \leq (v_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(2)}(t)$:- 395

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain 396

Definition of $u^{(2)}(t)$:-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case : 397

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

Proof : From global equations we obtain 398

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$ 399

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

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From which one obtains

For $0 < (\nu_0)^{(3)} = \frac{G^0_{20}}{G^0_{21}} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get 401

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case, 402

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)}+(C)^{(3)}(\nu_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)}-(\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 403

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)}+(\bar{C})^{(3)}(\bar{\nu}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)}-(\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{20}'')^{(3)} = (a_{21}'')^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)}G_{21}(t)$

Analogously if $(b_{20}'')^{(3)} = (b_{21}'')^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From global equations we obtain 404

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25}')^{(4)} + (a_{24}'')^{(4)}(T_{25},t)\right) - (a_{25}'')^{(4)}(T_{25},t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 405

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 406

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 407

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then
$(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From global equations we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

For $0 < \boxed{(\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 409

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G^0_{28}}{G^0_{29}} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 410

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

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If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From global equations we obtain 412

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33}, t)\right) - (a_{33}'')^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\quad \boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 413

$$v^{(6)}(t) \le \frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{v}_1)^{(6)}-(v_0)^{(6)}}{(v_0)^{(6)}-(\bar{v}_2)^{(6)}}}$$

From which we deduce $(v_0)^{(6)} \le v^{(6)}(t) \le (\bar{v}_1)^{(6)}$

If $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{v}_1)^{(6)}$ we find like in the previous case, 414

$$(v_1)^{(6)} \le \frac{(v_1)^{(6)}+(C)^{(6)}(v_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)}-(v_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((v_1)^{(6)}-(v_2)^{(6)}\right)t\right]}} \le v^{(6)}(t) \le$$

$$\frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \le (\bar{v}_1)^{(6)}$$

If $0 < (v_1)^{(6)} \le (\bar{v}_1)^{(6)} \le \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain 415

$$(v_1)^{(6)} \le v^{(6)}(t) \le \frac{(\bar{v}_1)^{(6)}+(\bar{C})^{(6)}(\bar{v}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{v}_1)^{(6)}-(\bar{v}_2)^{(6)}\right)t\right]}} \le (v_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(6)}(t)$:-

$$(m_2)^{(6)} \le v^{(6)}(t) \le (m_1)^{(6)}, \quad \boxed{v^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \le u^{(6)}(t) \le (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(v_1)^{(6)} = (\bar{v}_1)^{(6)}$ if in addition $(v_0)^{(6)} = (v_1)^{(6)}$ then $v^{(6)}(t) = (v_0)^{(6)}$ and as a consequence $G_{32}(t) = (v_0)^{(6)}G_{33}(t)$ **this also defines** $(v_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(v_1)^{(6)}$ and $(\bar{v}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Proof : From global equations we obtain

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left((a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37},t)\right) - (a''_{37})^{(7)}(T_{37},t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

Definition of $v^{(7)}$:- $\boxed{v^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2+(\sigma_2)^{(7)}v^{(7)}-(a_{36})^{(7)}\right)\leq\frac{dv^{(7)}}{dt}\leq-\left((a_{37})^{(7)}\left(v^{(7)}\right)^2+(\sigma_1)^{(7)}v^{(7)}-(a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)},(\nu_0)^{(7)}$:-

$$\text{For } 0<\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}<(\nu_1)^{(7)}<(\bar{\nu}_1)^{(7)}$$

$$v^{(7)}(t)\geq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}\quad,\quad\boxed{(C)^{(7)}=\frac{(\nu_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)}\leq v^{(7)}(t)\leq(\nu_1)^{(7)}$

In the same manner , we get 417

$$v^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\quad,\quad\boxed{(\bar{C})^{(7)}=\frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)}\leq v^{(7)}(t)\leq(\bar{\nu}_1)^{(7)}$

If $0<(\nu_1)^{(7)}<(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}<(\bar{\nu}_1)^{(7)}$ we find like in the previous case, 418

$$(\nu_1)^{(7)}\leq\frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}\leq v^{(7)}(t)\leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\bar{\nu}_1)^{(7)}$$

If $0<(\nu_1)^{(7)}\leq(\bar{\nu}_1)^{(7)}\leq\boxed{(\nu_0)^{(7)}=\frac{G_{36}^0}{G_{37}^0}}$, we obtain 419

$$(\nu_1)^{(7)}\leq v^{(7)}(t)\leq\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}\leq(\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(7)}(t)$:-

$$(m_2)^{(7)}\leq v^{(7)}(t)\leq(m_1)^{(7)},\quad\boxed{v^{(7)}(t)=\frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:- 420

$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}$, $\boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

Proof : From global equations we obtain 421

$$\frac{d\nu^{(8)}}{dt} = (a_{40})^{(8)} - \left((a'_{40})^{(8)} - (a'_{41})^{(8)} + (a''_{40})^{(8)}(T_{41}, t)\right) - (a''_{41})^{(8)}(T_{41}, t)\nu^{(8)} - (a_{41})^{(8)}\nu^{(8)}$$

Definition of $\nu^{(8)}$:- $\boxed{\nu^{(8)} = \frac{G_{40}}{G_{41}}}$

It follows

$$-\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_2)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right) \leq \frac{d\nu^{(8)}}{dt} \leq -\left((a_{41})^{(8)}\left(\nu^{(8)}\right)^2 + (\sigma_1)^{(8)}\nu^{(8)} - (a_{40})^{(8)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(8)}, (\nu_0)^{(8)}$:-

For $0 < \boxed{(\nu_0)^{(8)} = \frac{G^0_{40}}{G^0_{41}}} < (\nu_1)^{(8)} < (\bar{\nu}_1)^{(8)}$

$$\nu^{(8)}(t) \geq \frac{(\nu_1)^{(8)} + (C)^{(8)}(\nu_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}}{1 + (C)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\nu_1)^{(8)} - (\nu_0)^{(8)}\right)t\right]}} \quad , \quad \boxed{(C)^{(8)} = \frac{(\nu_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\nu_2)^{(8)}}}$$

it follows $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\nu_1)^{(8)}$

In the same manner , we get 422

$$\nu^{(8)}(t) \leq \frac{(\bar{\nu}_1)^{(8)} + (\bar{C})^{(8)}(\bar{\nu}_2)^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}}{1 + (\bar{C})^{(8)} e^{\left[-(a_{41})^{(8)}\left((\bar{\nu}_1)^{(8)} - (\bar{\nu}_2)^{(8)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(8)} = \frac{(\bar{\nu}_1)^{(8)} - (\nu_0)^{(8)}}{(\nu_0)^{(8)} - (\bar{\nu}_2)^{(8)}}}$$

From which we deduce $(\nu_0)^{(8)} \leq \nu^{(8)}(t) \leq (\bar{\nu}_8)^{(8)}$

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If $0 < (v_1)^{(8)} < (v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0} < (\bar{v}_1)^{(8)}$ we find like in the previous case,

$$(v_1)^{(8)} \leq \frac{(v_1)^{(8)}+(C)^{(8)}(v_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((v_1)^{(8)}-(v_2)^{(8)}\right)t\right]}}{1+(C)^{(8)}e^{\left[-(a_{41})^{(8)}\left((v_1)^{(8)}-(v_2)^{(8)}\right)t\right]}} \leq v^{(8)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(8)}+(\bar{C})^{(8)}(\bar{v}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}} \leq (\bar{v}_1)^{(8)}$$

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If $0 < (v_1)^{(8)} \leq (\bar{v}_1)^{(8)} \leq \boxed{(v_0)^{(8)} = \frac{G_{40}^0}{G_{41}^0}}$, we obtain

$$(v_1)^{(8)} \leq v^{(8)}(t) \leq \frac{(\bar{v}_1)^{(8)}+(\bar{C})^{(8)}(\bar{v}_2)^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}}{1+(\bar{C})^{(8)}e^{\left[-(a_{41})^{(8)}\left((\bar{v}_1)^{(8)}-(\bar{v}_2)^{(8)}\right)t\right]}} \leq (v_0)^{(8)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(8)}(t)$:-

$$(m_2)^{(8)} \leq v^{(8)}(t) \leq (m_1)^{(8)}, \quad \boxed{v^{(8)}(t) = \frac{G_{40}(t)}{G_{41}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(8)}(t)$:-

$$(\mu_2)^{(8)} \leq u^{(8)}(t) \leq (\mu_1)^{(8)}, \quad \boxed{u^{(8)}(t) = \frac{T_{40}(t)}{T_{41}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{40})^{(8)} = (a''_{41})^{(8)}, then\ (\sigma_1)^{(8)} = (\sigma_2)^{(8)}$ and in this case $(v_1)^{(8)} = (\bar{v}_1)^{(8)}$ if in addition $(v_0)^{(8)} = (v_1)^{(8)}$ then $v^{(8)}(t) = (v_0)^{(8)}$ and as a consequence $G_{40}(t) = (v_0)^{(8)}G_{41}(t)$ **this also defines** $(v_0)^{(8)}$ **for the special case** .

Analogously if $(b''_{40})^{(8)} = (b''_{41})^{(8)}, then\ (\tau_1)^{(8)} = (\tau_2)^{(8)}$ and then
$(u_1)^{(8)} = (\bar{u}_1)^{(8)}$if in addition $(u_0)^{(8)} = (u_1)^{(8)}$ then $T_{40}(t) = (u_0)^{(8)}T_{41}(t)$ This is an important consequence of the relation between $(v_1)^{(8)}$ and $(\bar{v}_1)^{(8)}$, **and definition of** $(u_0)^{(8)}$.

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Proof : From 99,20,44,22,23,44 we obtain

$$\frac{dv^{(9)}}{dt} = (a_{44})^{(9)} - \left((a'_{44})^{(9)} - (a'_{45})^{(9)} + (a''_{44})^{(9)}(T_{45}, t)\right) - (a''_{45})^{(9)}(T_{45}, t)v^{(9)} - (a_{45})^{(9)}v^{(9)}$$

Definition of $v^{(9)}$:- $\quad \boxed{v^{(9)} = \frac{G_{44}}{G_{45}}}$

It follows

$$-\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_2)^{(9)}v^{(9)} - (a_{44})^{(9)}\right) \leq \frac{dv^{(9)}}{dt} \leq -\left((a_{45})^{(9)}\left(v^{(9)}\right)^2 + (\sigma_1)^{(9)}v^{(9)} - (a_{44})^{(9)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(9)}, (\nu_0)^{(9)}$:-

For $0 < \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}} < (\nu_1)^{(9)} < (\bar{\nu}_1)^{(9)}$

$$\nu^{(9)}(t) \geq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_0)^{(9)}\right)t\right]}} \quad , \quad \boxed{(C)^{(9)} = \frac{(\nu_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\nu_2)^{(9)}}}$$

it follows $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\nu_9)^{(9)}$

In the same manner , we get

$$\nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(9)} = \frac{(\bar{\nu}_1)^{(9)} - (\nu_0)^{(9)}}{(\nu_0)^{(9)} - (\bar{\nu}_2)^{(9)}}}$$

From which we deduce $(\nu_0)^{(9)} \leq \nu^{(9)}(t) \leq (\bar{\nu}_1)^{(9)}$

If $0 < (\nu_1)^{(9)} < (\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0} < (\bar{\nu}_1)^{(9)}$ we find like in the previous case,

$$(\nu_1)^{(9)} \leq \frac{(\nu_1)^{(9)} + (C)^{(9)}(\nu_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}}{1 + (C)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\nu_1)^{(9)} - (\nu_2)^{(9)}\right)t\right]}} \leq \nu^{(9)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\bar{\nu}_1)^{(9)}$$

If $0 < (\nu_1)^{(9)} \leq (\bar{\nu}_1)^{(9)} \leq \boxed{(\nu_0)^{(9)} = \frac{G_{44}^0}{G_{45}^0}}$, we obtain

$$(\nu_1)^{(9)} \leq \nu^{(9)}(t) \leq \frac{(\bar{\nu}_1)^{(9)} + (\bar{C})^{(9)}(\bar{\nu}_2)^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}}{1 + (\bar{C})^{(9)} e^{\left[-(a_{45})^{(9)}\left((\bar{\nu}_1)^{(9)} - (\bar{\nu}_2)^{(9)}\right)t\right]}} \leq (\nu_0)^{(9)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(9)}(t)$:-

$$(m_2)^{(9)} \leq \nu^{(9)}(t) \leq (m_1)^{(9)}, \quad \boxed{\nu^{(9)}(t) = \frac{G_{44}(t)}{G_{45}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(9)}(t)$:-

$$(\mu_2)^{(9)} \leq u^{(9)}(t) \leq (\mu_1)^{(9)}, \quad \boxed{u^{(9)}(t) = \frac{T_{44}(t)}{T_{45}(t)}}$$

Now, using this result and replacing it in 99, 20,44,22,23, and 44 we get easily the result stated in the theorem.

Particular case :

If $(a''_{44})^{(9)} = (a''_{45})^{(9)}, then\ (\sigma_1)^{(9)} = (\sigma_2)^{(9)}$ and in this case $(\nu_1)^{(9)} = (\bar{\nu}_1)^{(9)}$ if in addition $(\nu_0)^{(9)} = (\nu_1)^{(9)}$ then $\nu^{(9)}(t) = (\nu_0)^{(9)}$ and as a consequence $G_{44}(t) = (\nu_0)^{(9)} G_{45}(t)$ **this also defines** $(\nu_0)^{(9)}$ **for**

the special case .

Analogously if $(b''_{44})^{(9)} = (b''_{45})^{(9)}, then\ (\tau_1)^{(9)} = (\tau_2)^{(9)}$ and then
$(u_1)^{(9)} = (\bar{u}_1)^{(9)}$if in addition $(u_0)^{(9)} = (u_1)^{(9)}$ then $T_{44}(t) = (u_0)^{(9)}T_{45}(t)$ This is an important consequence of the relation between $(v_1)^{(9)}$ and $(\bar{v}_1)^{(9)}$, **and definition of** $(u_0)^{(9)}$.
We can prove the following 425

Theorem : If $(a_i'')^{(1)} and\ (b_i'')^{(1)}$ are independent on t , and the conditions with the notations

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ \ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(2)} and\ (b_i'')^{(2)}$ are independent on t , and the conditions with the notations 426

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$ 427

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$ 428

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$, 429

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$ 430

$with\ \ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(3)} and\ (b_i'')^{(3)}$ are independent on t , and the conditions with the notations 431

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ \ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation are satisfied , then the system

We can prove the following 432

Theorem : If $(a_i'')^{(4)} and\ (b_i'')^{(4)}$ are independent on t , and the conditions with the notations

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ \ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(5)} and\ (b_i'')^{(5)}$ are independent on t , and the conditions with the notations 433

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ \ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation are satisfied , then the system

Theorem If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions with the notations 434

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ \ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(7)} and\ (b_i'')^{(7)}$ are independent on t , and the conditions with the notations 435

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$,

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$

$with\ \ (p_{36})^{(7)}, (r_{37})^{(7)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(8)} and\ (b_i'')^{(8)}$ are independent on t , and the conditions with the notations 436

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} < 0$

$(a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a_{40})^{(8)}(p_{40})^{(8)} + (a'_{41})^{(8)}(p_{41})^{(8)} + (p_{40})^{(8)}(p_{41})^{(8)} > 0$

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} > 0$,

$(b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} - (b'_{40})^{(8)}(r_{41})^{(8)} - (b'_{41})^{(8)}(r_{41})^{(8)} + (r_{40})^{(8)}(r_{41})^{(8)} < 0$

$with\ \ (p_{40})^{(8)}, (r_{41})^{(8)}$ as defined by equation are satisfied , then the system

Theorem : If $(a_i'')^{(9)} and\ (b_i'')^{(9)}$ are independent on t , and the conditions (with the notations 45,46,27,28) 436 A

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} < 0$

$(a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a_{44})^{(9)}(p_{44})^{(9)} + (a'_{45})^{(9)}(p_{45})^{(9)} + (p_{44})^{(9)}(p_{45})^{(9)} > 0$

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} > 0$,

$(b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} - (b'_{44})^{(9)}(r_{45})^{(9)} - (b'_{45})^{(9)}(r_{45})^{(9)} + (r_{44})^{(9)}(r_{45})^{(9)} < 0$

$with\ \ (p_{44})^{(9)}, (r_{45})^{(9)}$ as defined by equation 45 are satisfied , then the system

$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0$ 437

$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0$ 438

$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0$ 439

$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0$ 440

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0$ 441

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0$ 442

has a unique positive solution , which is an equilibrium solution for the system

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 443

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 444

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 445

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})\ ]T_{16} = 0$ 446

$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})\ ]T_{17} = 0$ 447

$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})\ ]T_{18} = 0$ 448

has a unique positive solution , which is an equilibrium solution

$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0$ 449

$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0$ 450

$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0$ 451

$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})\ ]T_{20} = 0$ 452

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})\,]T_{21} = 0 \quad 453$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})\,]T_{22} = 0 \quad 454$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0 \quad 455$$

$$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0 \quad 456$$

$$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0 \quad 457$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\big((G_{27})\big)\,]T_{24} = 0 \quad 458$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\big((G_{27})\big)\,]T_{25} = 0 \quad 459$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\big((G_{27})\big)\,]T_{26} = 0 \quad 460$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0 \quad 461$$

$$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0 \quad 462$$

$$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0 \quad 463$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})\,]T_{28} = 0 \quad 464$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})\,]T_{29} = 0 \quad 465$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0 \quad 466$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0 \quad 467$$

$$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0 \quad 468$$

$$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0 \quad 469$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0 \quad 470$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0 \quad 471$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0 \quad 472$$

has a unique positive solution , which is an equilibrium solution

$$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0 \quad 473$$

$$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0 \quad 474$$

$$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0 \quad 475$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad 476$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0 \quad 477$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0 \quad 478$$

$$(a_{40})^{(8)}G_{41} - \left[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41})\right]G_{40} = 0 \quad 479$$

$$(a_{41})^{(8)}G_{40} - \left[(a'_{41})^{(8)} + (a''_{41})^{(8)}(T_{41})\right]G_{41} = 0 \quad 480$$

$$(a_{42})^{(8)}G_{41} - \left[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41})\right]G_{42} = 0 \quad 481$$

$$(b_{40})^{(8)}T_{41} - [(b'_{40})^{(8)} - (b''_{40})^{(8)}(G_{43})\,]T_{40} = 0 \quad 482$$

$$(b_{41})^{(8)}T_{40} - [(b'_{41})^{(8)} - (b''_{41})^{(8)}(G_{43})\,]T_{41} = 0 \quad 483$$

$$(b_{42})^{(8)}T_{41} - [(b'_{42})^{(8)} - (b''_{42})^{(8)}(G_{43})\,]T_{42} = 0 \quad 484$$

$$(a_{44})^{(9)}G_{45} - \left[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45})\right]G_{44} = 0 \quad 484\text{A}$$

$$(a_{45})^{(9)}G_{44} - \left[(a'_{45})^{(9)} + (a''_{45})^{(9)}(T_{45})\right]G_{45} = 0$$

$$(a_{46})^{(9)}G_{45} - \left[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45})\right]G_{46} = 0$$

$$(b_{44})^{(9)}T_{45} - [(b'_{44})^{(9)} - (b''_{44})^{(9)}(G_{47})\,]T_{44} = 0$$

$$(b_{45})^{(9)}T_{44} - [(b'_{45})^{(9)} - (b''_{45})^{(9)}(G_{47})\,]T_{45} = 0$$

$$(b_{46})^{(9)}T_{45} - [(b'_{46})^{(9)} - (b''_{46})^{(9)}(G_{47})\,]T_{46} = 0$$

Proof: 485

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$

Proof: 486

(jj) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if

$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$

Proof: 488

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

Proof: 489

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

Proof: 490

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Proof: 491

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$

Proof: 492

(a) Indeed the first two equations have a nontrivial solution G_{40}, G_{41} if

$F(T_{43}) = (a'_{40})^{(8)}(a'_{41})^{(8)} - (a_{40})^{(8)}(a_{41})^{(8)} + (a'_{40})^{(8)}(a''_{41})^{(8)}(T_{41}) + (a'_{41})^{(8)}(a''_{40})^{(8)}(T_{41}) + (a''_{40})^{(8)}(T_{41})(a''_{41})^{(8)}(T_{41}) = 0$

Proof: 492 A

(a) Indeed the first two equations have a nontrivial solution G_{44}, G_{45} if

$$F(T_{47}) = (a'_{44})^{(9)}(a'_{45})^{(9)} - (a_{44})^{(9)}(a_{45})^{(9)} + (a'_{44})^{(9)}(a''_{45})^{(9)}(T_{45}) + (a'_{45})^{(9)}(a''_{44})^{(9)}(T_{45}) + (a''_{44})^{(9)}(T_{45})(a''_{45})^{(9)}(T_{45}) = 0$$

Definition and uniqueness of T^*_{14} :- 493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T^*_{14} for which $f(T^*_{14}) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T^*_{14})]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T^*_{14})]}$$

Definition and uniqueness of T^*_{17} :- 494

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T^*_{17} for which $f(T^*_{17}) = 0$. With this value , we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T^*_{17})]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T^*_{17})]}$$ 495

Definition and uniqueness of T^*_{21} :- 496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T^*_{21} for which $f(T^*_{21}) = 0$. With this value , we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T^*_{21})]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T^*_{21})]}$$

Definition and uniqueness of T^*_{25} :- 497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T^*_{25} for which $f(T^*_{25}) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T^*_{25})]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T^*_{25})]}$$

Definition and uniqueness of T^*_{29} :- 498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T^*_{29} for which $f(T^*_{29}) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T^*_{29})]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T^*_{29})]}$$

Definition and uniqueness of T_{33}^{*} :- 499

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^{*} for which $f(T_{33}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^{*})]}$$

Definition and uniqueness of T_{37}^{*} :- 500

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T_{37}^{*} for which $f(T_{37}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)} G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^{*})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)} G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^{*})]}$$

Definition and uniqueness of T_{41}^{*} :- 501

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(8)}(T_{41})$ being increasing, it follows that there exists a unique T_{41}^{*} for which $f(T_{41}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{40} = \frac{(a_{40})^{(8)} G_{41}}{[(a'_{40})^{(8)} + (a''_{40})^{(8)}(T_{41}^{*})]} \quad , \quad G_{42} = \frac{(a_{42})^{(8)} G_{41}}{[(a'_{42})^{(8)} + (a''_{42})^{(8)}(T_{41}^{*})]}$$

Definition and uniqueness of T_{45}^{*} :- 501 A

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(9)}(T_{45})$ being increasing, it follows that there exists a unique T_{45}^{*} for which $f(T_{45}^{*}) = 0$. With this value , we obtain from the three first equations

$$G_{44} = \frac{(a_{44})^{(9)} G_{45}}{[(a'_{44})^{(9)} + (a''_{44})^{(9)}(T_{45}^{*})]} \quad , \quad G_{46} = \frac{(a_{46})^{(9)} G_{45}}{[(a'_{46})^{(9)} + (a''_{46})^{(9)}(T_{45}^{*})]}$$

By the same argument, the equations admit solutions G_{13}, G_{14} if 502

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi(G^{*}) = 0$

By the same argument, the equations admit solutions G_{16}, G_{17} if 503

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right]+(b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 504

By the same argument, the equations admit solutions G_{20}, G_{21} if 505

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right]+(b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

By the same argument, the equations admit solutions G_{24}, G_{25} if 506

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right]+(b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

By the same argument, the equations admit solutions G_{28}, G_{29} if 507

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$\left[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})\right]+(b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

By the same argument, the equations admit solutions G_{32}, G_{33} if 508

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$\left[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})\right]+(b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi({G_{35}}^{*}) = 0$

By the same argument, the equations admit solutions G_{36}, G_{37} if 509

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right]+(b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that

there exists a unique G_{37}^{*} such that $\varphi(G_{39}{}^{*}) = 0$

By the same argument, the equations admit solutions G_{40}, G_{41} if 510

$\varphi(G_{43}) = (b'_{40})^{(8)}(b'_{41})^{(8)} - (b_{40})^{(8)}(b_{41})^{(8)} -$
$\left[(b'_{40})^{(8)}(b''_{41})^{(8)}(G_{43}) + (b'_{41})^{(8)}(b''_{40})^{(8)}(G_{43})\right] + (b''_{40})^{(8)}(G_{43})(b''_{41})^{(8)}(G_{43}) = 0$

Where in $(G_{43})(G_{40}, G_{41}, G_{42}), G_{40}, G_{42}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{41} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{41}^{*} such that $\varphi(G_{43}{}^{*}) = 0$

By the same argument, the equations 92,93 admit solutions G_{44}, G_{45} if
$\varphi(G_{47}) = (b'_{44})^{(9)}(b'_{45})^{(9)} - (b_{44})^{(9)}(b_{45})^{(9)} -$

$\left[(b'_{44})^{(9)}(b''_{45})^{(9)}(G_{47}) + (b'_{45})^{(9)}(b''_{44})^{(9)}(G_{47})\right] + (b''_{44})^{(9)}(G_{47})(b''_{45})^{(9)}(G_{47}) = 0$

Where in $(G_{47})(G_{44}, G_{45}, G_{46}), G_{44}, G_{46}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{45} taking into account the hypothesis $\varphi(0) > 0$, $\varphi(\infty) < 0$ it follows that there exists a unique G_{45}^{*} such that $\varphi((G_{47})^{*}) = 0$

Finally we obtain the unique solution 511

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T_{14}^{*})]} \quad , \quad G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T_{14}^{*})]}$$

$$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^{*})]} \quad , \quad T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^{*})]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

G_{17}^{*} given by $\varphi((G_{19})^{*}) = 0$, T_{17}^{*} given by $f(T_{17}^{*}) = 0$ and 512

$$G_{16}^{*} = \frac{(a_{16})^{(2)}G_{17}^{*}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T_{17}^{*})]} \quad , \quad G_{18}^{*} = \frac{(a_{18})^{(2)}G_{17}^{*}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T_{17}^{*})]} \quad 513$$

$$T_{16}^{*} = \frac{(b_{16})^{(2)}T_{17}^{*}}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^{*})]} \quad , \quad T_{18}^{*} = \frac{(b_{18})^{(2)}T_{17}^{*}}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^{*})]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{21}^{*} given by $\varphi((G_{23})^{*}) = 0$, T_{21}^{*} given by $f(T_{21}^{*}) = 0$ and

$$G_{20}^{*} = \frac{(a_{20})^{(3)}G_{21}^{*}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^{*})]} \quad , \quad G_{22}^{*} = \frac{(a_{22})^{(3)}G_{21}^{*}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^{*})]}$$

$$T_{20}^{*} = \frac{(b_{20})^{(3)}T_{21}^{*}}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}{}^{*})]} \quad , \quad T_{22}^{*} = \frac{(b_{22})^{(3)}T_{21}^{*}}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}{}^{*})]}$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 516

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27})^*)]} \qquad 517$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 518

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}^*)]}$$

$$T_{28}^* = \frac{(b_{28})^{(5)} T_{29}^*}{[(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31})^*)]} \quad , \quad T_{30}^* = \frac{(b_{30})^{(5)} T_{29}^*}{[(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31})^*)]} \qquad 519$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 520

G_{33}^* given by $\varphi((G_{35})^*) = 0$, T_{33}^* given by $f(T_{33}^*) = 0$ and

$$G_{32}^* = \frac{(a_{32})^{(6)} G_{33}^*}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34}^* = \frac{(a_{34})^{(6)} G_{33}^*}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}^*)]}$$

$$T_{32}^* = \frac{(b_{32})^{(6)} T_{33}^*}{[(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35})^*)]} \quad , \quad T_{34}^* = \frac{(b_{34})^{(6)} T_{33}^*}{[(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35})^*)]} \qquad 521$$

Obviously, these values represent an equilibrium solution of global equations

Finally we obtain the unique solution 522

G_{37}^* given by $\varphi((G_{39})^*) = 0$, T_{37}^* given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39})^*)]}$$

Finally we obtain the unique solution 523

G_{41}^* given by $\varphi((G_{43})^*) = 0$, T_{41}^* given by $f(T_{41}^*) = 0$ and

$$G_{40}^{*}=\frac{(a_{40})^{(8)}G_{41}^{*}}{[(a'_{40})^{(8)}+(a''_{40})^{(8)}(T_{41}^{*})]}\quad,\quad G_{42}^{*}=\frac{(a_{42})^{(8)}G_{41}^{*}}{[(a'_{42})^{(8)}+(a''_{42})^{(8)}(T_{41}^{*})]}$$

$$T_{40}^{*}=\frac{(b_{40})^{(8)}T_{41}^{*}}{[(b'_{40})^{(8)}-(b''_{40})^{(8)}((G_{43})^{*})]}\quad,\quad T_{42}^{*}=\frac{(b_{42})^{(8)}T_{41}^{*}}{[(b'_{42})^{(8)}-(b''_{42})^{(8)}((G_{43})^{*})]}$$

Finally we obtain the unique solution of 89 to 99 523
A

G_{45}^{*} given by $\varphi((G_{47})^{*})=0$, T_{45}^{*} given by $f(T_{45}^{*})=0$ and

$$G_{44}^{*}=\frac{(a_{44})^{(9)}G_{45}^{*}}{[(a'_{44})^{(9)}+(a''_{44})^{(9)}(T_{45}^{*})]}\quad,\quad G_{46}^{*}=\frac{(a_{46})^{(9)}G_{45}^{*}}{[(a'_{46})^{(9)}+(a''_{46})^{(9)}(T_{45}^{*})]}$$

$$T_{44}^{*}=\frac{(b_{44})^{(9)}T_{45}^{*}}{[(b'_{44})^{(9)}-(b''_{44})^{(9)}((G_{47})^{*})]}\quad,\quad T_{46}^{*}=\frac{(b_{46})^{(9)}T_{45}^{*}}{[(b'_{46})^{(9)}-(b''_{46})^{(9)}((G_{47})^{*})]}$$

ASYMPTOTIC STABILITY ANALYSIS 524

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}\ and\ (b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i=G_i^{*}+\mathbb{G}_i\qquad,T_i=T_i^{*}+\mathbb{T}_i$$

$$\frac{\partial(a''_{14})^{(1)}}{\partial T_{14}}(T_{14}^{*})=(q_{14})^{(1)}\quad,\ \frac{\partial(b_i'')^{(1)}}{\partial G_j}(\,G^{*}\,)=s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt}=-\big((a'_{13})^{(1)}+(p_{13})^{(1)}\big)\mathbb{G}_{13}+(a_{13})^{(1)}\mathbb{G}_{14}-(q_{13})^{(1)}G_{13}^{*}\mathbb{T}_{14} \quad 525$$

$$\frac{d\mathbb{G}_{14}}{dt}=-\big((a'_{14})^{(1)}+(p_{14})^{(1)}\big)\mathbb{G}_{14}+(a_{14})^{(1)}\mathbb{G}_{13}-(q_{14})^{(1)}G_{14}^{*}\mathbb{T}_{14} \quad 526$$

$$\frac{d\mathbb{G}_{15}}{dt}=-\big((a'_{15})^{(1)}+(p_{15})^{(1)}\big)\mathbb{G}_{15}+(a_{15})^{(1)}\mathbb{G}_{14}-(q_{15})^{(1)}G_{15}^{*}\mathbb{T}_{14} \quad 527$$

$$\frac{d\mathbb{T}_{13}}{dt}=-\big((b'_{13})^{(1)}-(r_{13})^{(1)}\big)\mathbb{T}_{13}+(b_{13})^{(1)}\mathbb{T}_{14}+\textstyle\sum_{j=13}^{15}\big(s_{(13)(j)}T_{13}^{*}\mathbb{G}_j\big) \quad 528$$

$$\frac{d\mathbb{T}_{14}}{dt}=-\big((b'_{14})^{(1)}-(r_{14})^{(1)}\big)\mathbb{T}_{14}+(b_{14})^{(1)}\mathbb{T}_{13}+\textstyle\sum_{j=13}^{15}\big(s_{(14)(j)}T_{14}^{*}\mathbb{G}_j\big) \quad 529$$

$$\frac{d\mathbb{T}_{15}}{dt}=-\big((b'_{15})^{(1)}-(r_{15})^{(1)}\big)\mathbb{T}_{15}+(b_{15})^{(1)}\mathbb{T}_{14}+\textstyle\sum_{j=13}^{15}\big(s_{(15)(j)}T_{15}^{*}\mathbb{G}_j\big) \quad 530$$

ASYMPTOTIC STABILITY ANALYSIS 531

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions

$(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i \tag{532}$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \ , \ \frac{\partial(b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij} \tag{533}$$

taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17} \tag{534}$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17} \tag{535}$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17} \tag{536}$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right) \tag{537}$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}')^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T_{17}^*\mathbb{G}_j\right) \tag{538}$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}')^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T_{18}^*\mathbb{G}_j\right) \tag{539}$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \ , \ \frac{\partial(b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}')^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21} \tag{541}$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}')^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21} \tag{542}$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}')^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21} \tag{543}$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}')^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T_{20}^*\mathbb{G}_j\right) \tag{544}$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b'_{21})^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^{*}\mathbb{G}_j\right) \quad 545$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b'_{22})^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T_{22}^{*}\mathbb{G}_j\right) \quad 546$$

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Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 548

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial(b_i'')^{(4)}}{\partial G_j}((G_{27})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a'_{24})^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^{*}\mathbb{T}_{25} \quad 549$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a'_{25})^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^{*}\mathbb{T}_{25} \quad 550$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a'_{26})^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^{*}\mathbb{T}_{25} \quad 551$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b'_{24})^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^{*}\mathbb{G}_j\right) \quad 552$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b'_{25})^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^{*}\mathbb{G}_j\right) \quad 553$$

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b'_{26})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^{*}\mathbb{G}_j\right) \quad 554$$

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Theorem 5: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 556

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial(b_i'')^{(5)}}{\partial G_j}(\,(G_{31})^* \,) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -\left((a'_{28})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^{*}\mathbb{T}_{29} \quad 557$$

$$\frac{d\mathbb{G}_{29}}{dt} = -\left((a'_{29})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G^*_{29}\mathbb{T}_{29} \quad 558$$

$$\frac{d\mathbb{G}_{30}}{dt} = -\left((a'_{30})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G^*_{30}\mathbb{T}_{29} \quad 559$$

$$\frac{d\mathbb{T}_{28}}{dt} = -\left((b'_{28})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T^*_{28}\mathbb{G}_j\right) \quad 560$$

$$\frac{d\mathbb{T}_{29}}{dt} = -\left((b'_{29})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T^*_{29}\mathbb{G}_j\right) \quad 561$$

$$\frac{d\mathbb{T}_{30}}{dt} = -\left((b'_{30})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T^*_{30}\mathbb{G}_j\right) \quad 562$$

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Theorem 6: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 564

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{33})^{(6)}}{\partial T_{33}}(T^*_{33}) = (q_{33})^{(6)} \quad , \frac{\partial(b_i'')^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -\left((a'_{32})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G^*_{32}\mathbb{T}_{33} \quad 565$$

$$\frac{d\mathbb{G}_{33}}{dt} = -\left((a'_{33})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G^*_{33}\mathbb{T}_{33} \quad 566$$

$$\frac{d\mathbb{G}_{34}}{dt} = -\left((a'_{34})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G^*_{34}\mathbb{T}_{33} \quad 567$$

$$\frac{d\mathbb{T}_{32}}{dt} = -\left((b'_{32})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T^*_{32}\mathbb{G}_j\right) \quad 568$$

$$\frac{d\mathbb{T}_{33}}{dt} = -\left((b'_{33})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T^*_{33}\mathbb{G}_j\right) \quad 569$$

$$\frac{d\mathbb{T}_{34}}{dt} = -\left((b'_{34})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T^*_{34}\mathbb{G}_j\right) \quad 570$$

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Theorem 7: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 572

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^{*}) = (q_{37})^{(7)} \quad , \frac{\partial(b_i'')^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain from

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}')^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^{*}\mathbb{T}_{37} \quad 573$$

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}')^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^{*}\mathbb{T}_{37} \quad 574$$

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}')^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^{*}\mathbb{T}_{37} \quad 575$$

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}')^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^{*}\mathbb{G}_j\right) \quad 576$$

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}')^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^{*}\mathbb{G}_j\right) \quad 578$$

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}')^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^{*}\mathbb{G}_j\right) \quad 579$$

Obviously, these values represent an equilibrium solution

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Theorem 8: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(8)}$ *and* $(b_i'')^{(8)}$ Belong to $C^{(8)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 580

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$

$$\frac{\partial(a_{41}'')^{(8)}}{\partial T_{41}}(T_{41}^{*}) = (q_{41})^{(8)} \quad , \frac{\partial(b_i'')^{(8)}}{\partial G_j}\left((G_{43})^{*} \right) = s_{ij}$$

Then taking into account equations and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{40}}{dt} = -\left((a_{40}')^{(8)} + (p_{40})^{(8)}\right)\mathbb{G}_{40} + (a_{40})^{(8)}\mathbb{G}_{41} - (q_{40})^{(8)}G_{40}^{*}\mathbb{T}_{41} \quad 581$$

$$\frac{d\mathbb{G}_{41}}{dt} = -\left((a_{41}')^{(8)} + (p_{41})^{(8)}\right)\mathbb{G}_{41} + (a_{41})^{(8)}\mathbb{G}_{40} - (q_{41})^{(8)}G_{41}^{*}\mathbb{T}_{41} \quad 582$$

$$\frac{d\mathbb{G}_{42}}{dt} = -\left((a_{42}')^{(8)} + (p_{42})^{(8)}\right)\mathbb{G}_{42} + (a_{42})^{(8)}\mathbb{G}_{41} - (q_{42})^{(8)}G_{42}^{*}\mathbb{T}_{41} \quad 583$$

$$\frac{d\mathbb{T}_{40}}{dt} = -\left((b_{40}')^{(8)} - (r_{40})^{(8)}\right)\mathbb{T}_{40} + (b_{40})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(40)(j)}T_{40}^{*}\mathbb{G}_j\right) \quad 584$$

$$\frac{d\mathbb{T}_{41}}{dt} = -\left((b'_{41})^{(8)} - (r_{41})^{(8)}\right)\mathbb{T}_{41} + (b_{41})^{(8)}\mathbb{T}_{40} + \sum_{j=40}^{42}\left(s_{(41)(j)}T^*_{41}\mathbb{G}_j\right) \quad 585$$

$$\frac{d\mathbb{T}_{42}}{dt} = -\left((b'_{42})^{(8)} - (r_{42})^{(8)}\right)\mathbb{T}_{42} + (b_{42})^{(8)}\mathbb{T}_{41} + \sum_{j=40}^{42}\left(s_{(42)(j)}T^*_{42}\mathbb{G}_j\right) \quad 586$$

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Theorem 9: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(9)}\ and\ (b_i'')^{(9)}$ Belong to $C^{(9)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a''_{45})^{(9)}}{\partial T_{45}}(T^*_{45}) = (q_{45})^{(9)} \ , \ \frac{\partial(b''_i)^{(9)}}{\partial G_j}(\ (G_{47})^*\ ) = s_{ij}$$

Then taking into account equations 89 to 99 and neglecting the terms of power 2, we obtain from 99 to 44

$$\frac{d\mathbb{G}_{44}}{dt} = -\left((a'_{44})^{(9)} + (p_{44})^{(9)}\right)\mathbb{G}_{44} + (a_{44})^{(9)}\mathbb{G}_{45} - (q_{44})^{(9)}G^*_{44}\mathbb{T}_{45} \quad 586\ B$$

$$\frac{d\mathbb{G}_{45}}{dt} = -\left((a'_{45})^{(9)} + (p_{45})^{(9)}\right)\mathbb{G}_{45} + (a_{45})^{(9)}\mathbb{G}_{44} - (q_{45})^{(9)}G^*_{45}\mathbb{T}_{45} \quad 586\ C$$

$$\frac{d\mathbb{G}_{46}}{dt} = -\left((a'_{46})^{(9)} + (p_{46})^{(9)}\right)\mathbb{G}_{46} + (a_{46})^{(9)}\mathbb{G}_{45} - (q_{46})^{(9)}G^*_{46}\mathbb{T}_{45} \quad 586\ D$$

$$\frac{d\mathbb{T}_{44}}{dt} = -\left((b'_{44})^{(9)} - (r_{44})^{(9)}\right)\mathbb{T}_{44} + (b_{44})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(44)(j)}T^*_{44}\mathbb{G}_j\right) \quad 586\ E$$

$$\frac{d\mathbb{T}_{45}}{dt} = -\left((b'_{45})^{(9)} - (r_{45})^{(9)}\right)\mathbb{T}_{45} + (b_{45})^{(9)}\mathbb{T}_{44} + \sum_{j=44}^{46}\left(s_{(45)(j)}T^*_{45}\mathbb{G}_j\right) \quad 586\ F$$

$$\frac{d\mathbb{T}_{46}}{dt} = -\left((b'_{46})^{(9)} - (r_{46})^{(9)}\right)\mathbb{T}_{46} + (b_{46})^{(9)}\mathbb{T}_{45} + \sum_{j=44}^{46}\left(s_{(46)(j)}T^*_{46}\mathbb{G}_j\right) \quad 586\ G$$

The characteristic equation of this system is 587

$$\left((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right)$$

$$\left[\left(\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right)\right]$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T^*_{14} + (b_{14})^{(1)}s_{(13),(14)}T^*_{14}\right)$$

$$+\left(\left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G^*_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G^*_{14}\right)$$

$$\left(\left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T^*_{14} + (b_{14})^{(1)}s_{(13),(13)}T^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left(\ (a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)\ (\lambda)^{(1)}\right)$$

$$\left(\left((\lambda)^{(1)}\right)^2 + \left(\ (b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)\ (\lambda)^{(1)}\right)$$

$$+\left(\left((\lambda)^{(1)}\right)^2+\left((a'_{13})^{(1)}+(a'_{14})^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a'_{13})^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G^*_{14}+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right)$$

$$\left(\left((\lambda)^{(1)}+(b'_{13})^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T^*_{14}+(b_{14})^{(1)}s_{(13),(15)}T^*_{13}\right)\}=0$$

$$+$$

$$(\lambda)^{(2)}+(b'_{18})^{(2)}-(r_{18})^{(2)}\}\{\left((\lambda)^{(2)}+(a'_{18})^{(2)}+(p_{18})^{(2)}\right)$$

$$\left[\left(\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)\right]$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(17)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(17)}\mathrm{T}^*_{17}\right)$$

$$+\left(\left((\lambda)^{(2)}+(a'_{17})^{(2)}+(p_{17})^{(2)}\right)(q_{16})^{(2)}\mathrm{G}^*_{16}+(a_{16})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(16)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(16)}\mathrm{T}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2+\left((b'_{16})^{(2)}+(b'_{17})^{(2)}-(r_{16})^{(2)}+(r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$+\left(\left((\lambda)^{(2)}\right)^2+\left((a'_{16})^{(2)}+(a'_{17})^{(2)}+(p_{16})^{(2)}+(p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}\mathrm{G}_{18}$$

$$+\left((\lambda)^{(2)}+(a'_{16})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$

$$\left(\left((\lambda)^{(2)}+(b'_{16})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}^*_{17}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}^*_{16}\right)\}=0$$

$$+$$

$$(\lambda)^{(3)}+(b'_{22})^{(3)}-(r_{22})^{(3)}\}\{\left((\lambda)^{(3)}+(a'_{22})^{(3)}+(p_{22})^{(3)}\right)$$

$$\left[\left(\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(q_{20})^{(3)}G^*_{20}\right)\right]$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T^*_{21}+(b_{21})^{(3)}s_{(20),(21)}T^*_{21}\right)$$

$$+\left(\left((\lambda)^{(3)}+(a'_{21})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20}+(a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T^*_{21}+(b_{21})^{(3)}s_{(20),(20)}T^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$\left(\left((\lambda)^{(3)}\right)^2+\left((b'_{20})^{(3)}+(b'_{21})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$

$$+\left(\left((\lambda)^{(3)}\right)^2+\left((a'_{20})^{(3)}+(a'_{21})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$

$$+\left((\lambda)^{(3)}+(a'_{20})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$

$$\left(\left((\lambda)^{(3)}+(b'_{20})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T^*_{21}+(b_{21})^{(3)}s_{(20),(22)}T^*_{20}\right)\}=0$$

$$+$$

$$(\lambda)^{(4)}+(b'_{26})^{(4)}-(r_{26})^{(4)}\{\left((\lambda)^{(4)}+(a'_{26})^{(4)}+(p_{26})^{(4)}\right)$$

$$\left[\left(\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(q_{24})^{(4)}G^*_{24}\right)\right]$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T^*_{25}+(b_{25})^{(4)}s_{(24),(25)}T^*_{25}\right)$$

$$+\left(\left((\lambda)^{(4)}+(a'_{25})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24}+(a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T^*_{25}+(b_{25})^{(4)}s_{(24),(24)}T^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$\left(\left((\lambda)^{(4)}\right)^2+\left((b'_{24})^{(4)}+(b'_{25})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$

$$+\left(\left((\lambda)^{(4)}\right)^2+\left((a'_{24})^{(4)}+(a'_{25})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$

$$+\left((\lambda)^{(4)}+(a'_{24})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$

$$\left(\left((\lambda)^{(4)}+(b'_{24})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T^*_{25}+(b_{25})^{(4)}s_{(24),(26)}T^*_{24}\right)\}=0$$

$$+$$

$$(\lambda)^{(5)}+(b'_{30})^{(5)}-(r_{30})^{(5)}\{\left((\lambda)^{(5)}+(a'_{30})^{(5)}+(p_{30})^{(5)}\right)$$

$$\left[\left(\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(q_{28})^{(5)}G^*_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T^*_{29}+(b_{29})^{(5)}s_{(28),(29)}T^*_{29}\right)$$

$$+\left(\left((\lambda)^{(5)}+(a'_{29})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G^*_{28}+(a_{28})^{(5)}(q_{29})^{(5)}G^*_{29}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T^*_{29}+(b_{29})^{(5)}s_{(28),(28)}T^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$\left(\left((\lambda)^{(5)}\right)^2+\left((b'_{28})^{(5)}+(b'_{29})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$

$$+\left(\left((\lambda)^{(5)}\right)^2+\left((a'_{28})^{(5)}+(a'_{29})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$

$$+\left((\lambda)^{(5)}+(a'_{28})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G^*_{29}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G^*_{28}\right)$$

$$\left(\left((\lambda)^{(5)}+(b'_{28})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T^*_{29}+(b_{29})^{(5)}s_{(28),(30)}T^*_{28}\right)\}=0$$

$$+$$

$$(\lambda)^{(6)}+(b'_{34})^{(6)}-(r_{34})^{(6)}\{\left((\lambda)^{(6)}+(a'_{34})^{(6)}+(p_{34})^{(6)}\right)$$

$$\left[\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(q_{32})^{(6)}G^*_{32}\right)\Big]$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T^*_{33}+(b_{33})^{(6)}s_{(32),(33)}T^*_{33}\right)$$

$$+\left(\left((\lambda)^{(6)}+(a'_{33})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G^*_{32}+(a_{32})^{(6)}(q_{33})^{(6)}G^*_{33}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T^*_{33}+(b_{33})^{(6)}s_{(32),(32)}T^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$\left(\left((\lambda)^{(6)}\right)^2+\left((b'_{32})^{(6)}+(b'_{33})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$

$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^*_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^*_{32}\right)$$

$$\left(\left((\lambda)^{(6)}+(b'_{32})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T^*_{33}+(b_{33})^{(6)}s_{(32),(34)}T^*_{32}\right)\}=0$$

$$+$$

$$(\lambda)^{(7)}+(b'_{38})^{(7)}-(r_{38})^{(7)}\{\left((\lambda)^{(7)}+(a'_{38})^{(7)}+(p_{38})^{(7)}\right)$$

$$\left[\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(q_{36})^{(7)}G^*_{36}\right)\Big]$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T^*_{37}+(b_{37})^{(7)}s_{(36),(37)}T^*_{37}\right)$$

$$+\left(\left((\lambda)^{(7)}+(a'_{37})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G^*_{36}+(a_{36})^{(7)}(q_{37})^{(7)}G^*_{37}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T^*_{37}+(b_{37})^{(7)}s_{(36),(36)}T^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$\left(\left((\lambda)^{(7)}\right)^2+\left((b'_{36})^{(7)}+(b'_{37})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$

$$+\left(\left((\lambda)^{(7)}\right)^2+\left((a'_{36})^{(7)}+(a'_{37})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$

$$+\left((\lambda)^{(7)}+(a'_{36})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G^*_{37}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G^*_{36}\right)$$

$$\left(\left((\lambda)^{(7)}+(b'_{36})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T^*_{37}+(b_{37})^{(7)}s_{(36),(38)}T^*_{36}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(8)}+(b'_{42})^{(8)}-(r_{42})^{(8)}\right)\{\left((\lambda)^{(8)}+(a'_{42})^{(8)}+(p_{42})^{(8)}\right)$$

$$\left[\left(\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(q_{40})^{(8)}G^*_{40}\right)\right]$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(41)}T^*_{41}+(b_{41})^{(8)}s_{(40),(41)}T^*_{41}\right)$$

$$+\left(\left((\lambda)^{(8)}+(a'_{41})^{(8)}+(p_{41})^{(8)}\right)(q_{40})^{(8)}G^*_{40}+(a_{40})^{(8)}(q_{41})^{(8)}G^*_{41}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(40)}T^*_{41}+(b_{41})^{(8)}s_{(40),(40)}T^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$\left(\left((\lambda)^{(8)}\right)^2+\left((b'_{40})^{(8)}+(b'_{41})^{(8)}-(r_{40})^{(8)}+(r_{41})^{(8)}\right)(\lambda)^{(8)}\right)$$

$$+\left(\left((\lambda)^{(8)}\right)^2+\left((a'_{40})^{(8)}+(a'_{41})^{(8)}+(p_{40})^{(8)}+(p_{41})^{(8)}\right)(\lambda)^{(8)}\right)(q_{42})^{(8)}G_{42}$$

$$+\left((\lambda)^{(8)}+(a'_{40})^{(8)}+(p_{40})^{(8)}\right)\left((a_{42})^{(8)}(q_{41})^{(8)}G^*_{41}+(a_{41})^{(8)}(a_{42})^{(8)}(q_{40})^{(8)}G^*_{40}\right)$$

$$\left(\left((\lambda)^{(8)}+(b'_{40})^{(8)}-(r_{40})^{(8)}\right)s_{(41),(42)}T^*_{41}+(b_{41})^{(8)}s_{(40),(42)}T^*_{40}\right)\}=0$$

$$+$$

$$\left((\lambda)^{(9)}+(b'_{46})^{(9)}-(r_{46})^{(9)}\right)\{\left((\lambda)^{(9)}+(a'_{46})^{(9)}+(p_{46})^{(9)}\right)$$

$$\left[\left(\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(q_{44})^{(9)}G^*_{44}\right)\right]$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(45)}T^*_{45}+(b_{45})^{(9)}s_{(44),(45)}T^*_{45}\right)$$

$$+\left(\left((\lambda)^{(9)}+(a'_{45})^{(9)}+(p_{45})^{(9)}\right)(q_{44})^{(9)}G^*_{44}+(a_{44})^{(9)}(q_{45})^{(9)}G^*_{45}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(44)}T^*_{45}+(b_{45})^{(9)}s_{(44),(44)}T^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$\left(\left((\lambda)^{(9)}\right)^2+\left((b'_{44})^{(9)}+(b'_{45})^{(9)}-(r_{44})^{(9)}+(r_{45})^{(9)}\right)(\lambda)^{(9)}\right)$$

$$+\left(\left((\lambda)^{(9)}\right)^2+\left((a'_{44})^{(9)}+(a'_{45})^{(9)}+(p_{44})^{(9)}+(p_{45})^{(9)}\right)(\lambda)^{(9)}\right)(q_{46})^{(9)}G_{46}$$

$$+\left((\lambda)^{(9)}+(a'_{44})^{(9)}+(p_{44})^{(9)}\right)\left((a_{46})^{(9)}(q_{45})^{(9)}G^*_{45}+(a_{45})^{(9)}(a_{46})^{(9)}(q_{44})^{(9)}G^*_{44}\right)$$

$$\left(\left((\lambda)^{(9)}+(b'_{44})^{(9)}-(r_{44})^{(9)}\right)s_{(45),(46)}T^*_{45}+(b_{45})^{(9)}s_{(44),(46)}T^*_{44}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

References:

(1) Communications in Mathematical Physics 104 (2): 207–226. Bibcode:1986CMaPh.104..207B. doi:10.1007/BF01211590.
(2) Carlip, Steven (2003). Quantum Gravity in 2+1 Dimensions. Cambridge Monographs on Mathematical Physics. ISBN 978-0-521-54588-4.
(3) Castro, Alejandra; Maloney, Alexander; Strominger, Andrew (2010). "Hidden conformal symmetry of the Kerr black hole". Physical Review D 82 (2). arXiv:1004.0996. Bibcode:2010PhRvD..82b4008C. doi:10.1103/PhysRevD.82.024008.
(4) de Sitter, W. (1917), "On the relativity of inertia: Remarks concerning Einstein's latest hypothesis", Proc. Kon. Ned. Acad. Wet. 19: 1217–1225
(5) de Sitter, W. (1917), "On the curvature of space", Proc. Kon. Ned. Acad. Wet. 20: 229–243
(6) Nomizu, Katsumi (1982), "The Lorentz–Poincaré metric on the upper half-space and its extension", Hokkaido Mathematical Journal 11 (3): 253–261
(7) Processing in the synaptodendritic web. New Ideas in Psychology, 17(3), 205–214. doi:http://dx.doi.org/10.1016/S0732-118X(99)00024-0
(8) ^ Jump up to: a b c d e f g h i j Pribram, K. H. (1999). Quantum holography: Is it relevant to brain function? Information Sciences, 115(1–4), 97–102. doi:http://dx.doi.org/10.1016/S0020-0255(98)10082-8
(9) ^ Jump up to: a b c Vandervert, L. R. (1995). Chaos theory and the evolution of consciousness and mind: A thermodynamic-halographic resolution to the mind-body problem. New Ideas in Psychology, 13(2), 107–127. doi:http://dx.doi.org/10.1016/0732-118X(94)00047-7
(10) Jump up ^ Berger, D.H., & Pribram, K.H. (1992). The Relationship between the Gabor elementary function and a stochastic model of the inter-spike interval distribution in the responses of the visual cortex neurons. Biological Cybernetics, 67, 191–194.
(11) Jump up ^ Pribram, K.H. (2004). Consciousness Reassessed. Mind and Matter, 2, 7–35.
(12) ^ Jump up to: a b c Gabor, D. (1972). Holography, 1948–1971. Science, 177(4046), 299–313.
(13) ^ Jump up to: a b c Borsellino, A., & Poggio, T. (1972). Holographic aspects of temporal memory and optomotor responses. Kybernetik, 10(1), 58–60.
(14) ^ Jump up to: a b c d e István Bókkon. (2005). Dreams and neurohолography: An interdisciplinary interpretation of development of homeotherm state in evolution. Sleep and Hypnosis, 7(2), 47–62.
(15) Jump up ^ Gabor, D. (1968). Holographic Model of Temporal Recall. Nature, 217, 584.
(16) ^ Jump up to: a b Kelly, M. A., Blostein, D., & Mewhort, D. J. K. (2013). Encoding structure in holographic reduced representations. Canadian Journal of Experimental Psychology, 67(2), 79–93.

(17) Jump up ^ P. J. van Heerden (1963). A New Optical Method of Storing and Retrieving Information. Applied Optics, Vol. 2, Issue 4, pp. 387-392 (1963). DOI 10.1364/AO.2.000387
(18) Jump up ^ P. J. van Heerden (1963). Theory of Optical Information Storage in Solids. Applied Optics, Vol. 2, Issue 4, pp. 393-400. DOI 10.1364/AO.2.000393
(19) ^ Jump up to: a b Van Heerden, P. J. (1970). Models for the brain. Nature, 225(5228), 177–178.
(20) ^ Jump up to: a b c Pribram, H.H. (2011). Recollections. Neuroquantology, 9(3), 370–374.
(21) Jump up ^ Emmett N. Leith and Juris Upatnieks (1965). hotography by Laser. Scientific American Volume 212, Issue 6, June 1, 1965
(22) Jump up ^ K. Pribram (1969). The Neurophysiology of Remembering. American Volume 220, Issue 1, January 1, 1969
(23) ^ Jump up to: a b c Globus, G. G., & O'Carroll, C. P. (2010). Nonlocal neurology: Beyond localization to holonomy. Medical Hypotheses, 75, 425+
(24) Jump up ^ The implicate brain by Karl H. Pribram, karlhpribram.com
(25) Jump up ^ DeValois and DeValois, 1980
(26) Jump up ^ "Pribram, 1987"
(27) ^ Jump up to: a b c Srivastava, V., & Edwards, S. F. (2004). A mathematical model of capacious and efficient memory that survives trauma. Physica A: Statistical Mechanics and its Applications, 333(1–4), 465–477.
(28) Jump up ^ Longuet-Higgins, H. C. (1968). Holographic model of temporal recall [50]. Nature, 217(5123), 104.
(29) ^ Jump up to: a b Baev, K.V. (2012). Solution of the Problem of Central Pattern Generators and a New Concept of Brain Functions. Neurophysiology, 4, 414–432.
(30) ^ Jump up to: a b c Persinger, M.A., & Lavallee, C. (2012). The Σn=n Concept and the Quantitative Support for the Cerebral-Holographic and Electromagnetic Configuration of Consciousness
(31) Coussaert, Oliver; Henneaux, Marc; van Driel, Peter (1995). "The asymptotic dynamics of three-dimensional Einstein gravity with a negative cosmological constant". Classical and Quantum Gravity 12 (12): 2961. arXiv:gr-qc/9506019. Bibcode:1995CQGra..12.2961C. doi:10.1088/0264-9381/12/12/012.
(32) Cowen, Ron (2013). "Simulations back up theory that Universe is a hologram". Nature News & Comment. doi:10.1038/nature.2013.14328. Retrieved 21 December 2013.
(33) de Haro, Sebastian; Dieks, Dennis; 't Hooft, Gerard; Verlinde, Erik (2013). "Forty Years of String Theory Reflecting on the Foundations". Foundations of Physics 43 (1): 1–7. Bibcode:2013FoPh...43....1D. doi:10.1007/s10701-012-9691-3.
(34) Deligne, Pierre; Etingof, Pavel; Freed, Daniel; Jeffery, Lisa; Kazhdan, David; Morgan, John; Morrison, David; Witten, Edward, eds. (1999). Quantum Fields and Strings: A Course for Mathematicians. American Mathematical Society. ISBN 978-0-8218-2014-8.
(35) Fefferman, Charles; Graham, Robin (1985). "Conformal invariants". Asterisque: 95–116.
(36) Fefferman, Charles; Graham, Robin (2011). The Ambient Metric. Princeton University Press. ISBN 978-1-4008-4058-8.
(37) Giombi, Simone; Yin, Xi (2010). "Higher spin gauge theory and holography: the three-point functions". Journal of High Energy Physics 2010 (9): 1–80. arXiv:0912.3462. Bibcode:2010JHEP...09..115G. doi:10.1007/JHEP09(2010)115.
(38) Greene, Brian (2000). The Elegant Universe: Superstrings, Hidden Dimensions, and the Quest for the Ultimate Theory. Random House. ISBN 978-0-9650888-0-0.
(39) Griffiths, David (2004). Introduction to Quantum Mechanics. Pearson Prentice Hall. ISBN 978-0-13-111892-8.
(40) Gubser, Steven; Klebanov, Igor; Polyakov, Alexander (1998). "Gauge theory correlators from non-critical string theory". Physics Letters B 428: 105–114. arXiv:hep-th/9802109. Bibcode:1998PhLB..428..105G. doi:10.1016/S0370-2693(98)00377-3.

(41) Guica, Monica; Hartman, Thomas; Song, Wei; Strominger, Andrew (2009). "The Kerr/CFT Correspondence". Physical Review D 80 (12). arXiv:0809.4266. Bibcode:2009PhRvD..80l4008G. doi:10.1103/PhysRevD.80.124008.

(42) Hawking, Stephen (1975). "Particle creation by black holes". Communications in mathematical physics 43 (3): 199–220. Bibcode:1975CMaPh..43..199H. doi:10.1007/BF02345020.

(43) Hawking, Stephen (2005). "Information loss in black holes". Physical Review D 72 (8). arXiv:hep-th/0507171. Bibcode:2005PhRvD..72h4013H. doi:10.1103/PhysRevD.72.084013.

(44) Klebanov, Igor; Maldacena, Juan (2009). "Solving Quantum Field Theories via Curved Spacetimes" (PDF). Physics Today 62: 28. Bibcode:2009PhT....62a..28K. doi:10.1063/1.3074260. Retrieved May 2013.

(45) Klebanov, Igor; Polyakov, Alexander (2002). "The AdS dual of the critical O(N) vector model". Physics Letters B 550 (3–4): 213–219. arXiv:hep-th/0210114. Bibcode:2002PhLB..550..213K. doi:10.1016/S0370-2693(02)02980-5.

(46) Kovtun, P. K.; Son, Dam T.; Starinets, A. O. (2001). "Viscosity in strongly interacting quantum field theories from black hole physics". Physical review letters 94 (11): 111601. arXiv:hep-th/0405231. Bibcode:2005PhRvL..94k1601K. doi:10.1103/PhysRevLett.94.111601. PMID 15903845.

(47) Luzum, Matthew; Romatschke, Paul (2008). "Conformal relativistic viscous hydrodynamics: Applications to RHIC results at \sqrt{s_{NN}}=200 GeV". Physical Review C 78 (3). arXiv:0804.4015. doi:10.1103/PhysRevC.78.034915.

(48) Maldacena, Juan (1998). "The Large N limit of superconformal field theories and supergravity". Advances in Theoretical and Mathematical Physics 2: 231–252. arXiv:hep-th/9711200. Bibcode:1998AdTMP...2..231M. doi:10.1063/1.59653.

(49) Maldacena, Juan (2005). "The Illusion of Gravity". Scientific American 293 (5): 56–63. Bibcode:2005SciAm.293e..56M. doi:10.1038/scientificamerican1105-56. PMID 16318027. Retrieved July 2013.

(50) McLerran, Larry (2007). "Theory Summary : Quark Matter 2006". Journal of Physics G: Nuclear and Particle Physics 34 (8): S583. arXiv:hep-ph/0702004. Bibcode:2007JPhG...34..583M. doi:10.1088/0954-3899/34/8/S50.

(51) Merali, Zeeya (2011). "Collaborative physics: string theory finds a bench mate". Nature 478 (7369): 302–304. Bibcode:2011Natur.478..302M. doi:10.1038/478302a. PMID 22012369.

(52) Moore, Gregory (2012). "Applications of the six-dimensional (2,0) theories to Physical Mathematics". Retrieved 14 August 2013.

(53) Daniel, M., Viallet, C., The geometric setting of gauge symmetries of the Yang–Mills type, Rev. Mod. Phys. 52 (1980) 175.

(54) Eguchi, T., Gilkey, P., Hanson, A., Gravitation, gauge theories and differential geometry, Phys. Rep. 66 (1980) 213.

(55) Gotay, M., Marsden, J., Stress-energy-momentum tensors and the Belinfante–Rosenfeld formula, Contemp. Math. 132 (1992) 367.

(56) Marathe, K., Martucci, G., The Mathematical Foundation of Gauge Theories (North Holland, 1992) ISBN 0-444-89708-9.

(57) Fatibene, L., Ferraris, M., Francaviglia, M., Noether formalism for conserved quantities in classical gauge field theories, J. Math. Phys. 35 (1994) 1644.

(58) Gomis, J., Paris, J., Samuel, S., Antibracket, antifields and gauge theory quantization, Phys. Rep. 295 (1995) 1; arXiv: hep-th/9412228.

(59) Giachetta, G. (2008), Mangiarotti, L., Sardanashvily, G., On the notion of gauge symmetries of generic Lagrangian field theory, J. Math. Phys. 50 (2009) 012903; arXiv: 0807.3003.

(60) Sachdev, Subir (2013). "Strange and stringy". Scientific American 308 (44): 44. Bibcode:2012SciAm.308a..44S. doi:10.1038/scientificamerican0113-44.

(61) Scherk, Joel; Schwarz, John (1974). "Dual models for non-hadrons". Nuclear Physics B 81 (1): 118–144. Bibcode:1974NuPhB..81..118S. doi:10.1016/0550-3213(74)90010-8.

(62) Strominger, Andrew (2001). "The dS/CFT correspondence". Journal of High Energy Physics 2001 (10): 034. doi:10.1088/1126-6708/2001/10/034.

(63) Susskind, Leonard (1995). "The World as a Hologram". Journal of Mathematical Physics 36 (11): 6377–6396. arXiv:hep-th/9409089. Bibcode:1995JMP....36.6377S

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